

```
In[*]:= $k = 1;
SetAlgebra[gl4,ε];
$PBWRule
MatrixForm@Table[{b1, b2} → B[b1, b2], {b1, $Basis}, {b2, $Basis}]
MatrixForm@
Table[{b1, b2} → (DefRep[b1].DefRep[b2] - DefRep[b2].DefRep[b1] == DefRep[B[b1, b2]]),
{b1, $Basis}, {b2, $Basis}]
```

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Out[*]=
{y1,2 → 1, y1,3 → 2, y2,3 → 3, y1,4 → 4, y2,4 → 5, y3,4 → 6, x1,1 → 7, x1,2 → 8,
x2,2 → 9, x1,3 → 10, x2,3 → 11, x3,3 → 12, x1,4 → 13, x2,4 → 14, x3,4 → 15, x4,4 → 16}
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```
Out[*]//MatrixForm=
{
  {y1,2, y1,2} → 0, {y1,2, y1,3} → 0, {y1,2, y2,3} → -y1,3, {y1,2, y1,4}
  {y1,3, y1,2} → 0, {y1,3, y1,3} → 0, {y1,3, y2,3} → 0, {y1,3, y1,4}
  {y2,3, y1,2} → y1,3, {y2,3, y1,3} → 0, {y2,3, y2,3} → 0, {y2,3, y1,4}
  {y1,4, y1,2} → 0, {y1,4, y1,3} → 0, {y1,4, y2,3} → 0, {y1,4, y1,4}
  {y2,4, y1,2} → y1,4, {y2,4, y1,3} → 0, {y2,4, y2,3} → 0, {y2,4, y1,4}
  {y3,4, y1,2} → 0, {y3,4, y1,3} → y1,4, {y3,4, y2,3} → y2,4, {y3,4, y1,4}
  {x1,1, y1,2} → -y1,2, {x1,1, y1,3} → -y1,3, {x1,1, y2,3} → 0, {x1,1, y1,4}
  {x1,2, y1,2} → y1,2, {x1,2, y1,3} → -y2,3, {x1,2, y2,3} → 0, {x1,2, y1,4}
  {x2,2, y1,2} → y1,2, {x2,2, y1,3} → 0, {x2,2, y2,3} → -y2,3, {x2,2, y1,4}
  {x1,3, y1,2} → -x2,3, {x1,3, y1,3} → x1,1 - x3,3, {x1,3, y2,3} → x1,2, {x1,3, y1,4}
  {x2,3, y1,2} → 0, {x2,3, y1,3} → y1,2, {x2,3, y2,3} → x2,2 - x3,3, {x2,3, y1,4}
  {x3,3, y1,2} → 0, {x3,3, y1,3} → y1,3, {x3,3, y2,3} → y2,3, {x3,3, y1,4}
  {x1,4, y1,2} → -x2,4, {x1,4, y1,3} → -x3,4, {x1,4, y2,3} → 0, {x1,4, y1,4} → y1,4
  {x2,4, y1,2} → 0, {x2,4, y1,3} → 0, {x2,4, y2,3} → -x3,4, {x2,4, y1,4}
  {x3,4, y1,2} → 0, {x3,4, y1,3} → 0, {x3,4, y2,3} → 0, {x3,4, y1,4}
  {x4,4, y1,2} → 0, {x4,4, y1,3} → 0, {x4,4, y2,3} → 0, {x4,4, y1,4}
```

```
Out[*]//MatrixForm=
{
  {y1,2, y1,2} → True, {y1,2, y1,3} → True, {y1,2, y2,3} → True, {y1,2, y1,4} → True, {y1,2, y2,4} → True, {
  {y1,3, y1,2} → True, {y1,3, y1,3} → True, {y1,3, y2,3} → True, {y1,3, y1,4} → True, {y1,3, y2,4} → True, {
  {y2,3, y1,2} → True, {y2,3, y1,3} → True, {y2,3, y2,3} → True, {y2,3, y1,4} → True, {y2,3, y2,4} → True, {
  {y1,4, y1,2} → True, {y1,4, y1,3} → True, {y1,4, y2,3} → True, {y1,4, y1,4} → True, {y1,4, y2,4} → True, {
  {y2,4, y1,2} → True, {y2,4, y1,3} → True, {y2,4, y2,3} → True, {y2,4, y1,4} → True, {y2,4, y2,4} → True, {
  {y3,4, y1,2} → True, {y3,4, y1,3} → True, {y3,4, y2,3} → True, {y3,4, y1,4} → True, {y3,4, y2,4} → True, {
  {x1,1, y1,2} → True, {x1,1, y1,3} → True, {x1,1, y2,3} → True, {x1,1, y1,4} → True, {x1,1, y2,4} → True, {
  {x1,2, y1,2} → True, {x1,2, y1,3} → True, {x1,2, y2,3} → True, {x1,2, y1,4} → True, {x1,2, y2,4} → True, {
  {x2,2, y1,2} → True, {x2,2, y1,3} → True, {x2,2, y2,3} → True, {x2,2, y1,4} → True, {x2,2, y2,4} → True, {
  {x1,3, y1,2} → True, {x1,3, y1,3} → True, {x1,3, y2,3} → True, {x1,3, y1,4} → True, {x1,3, y2,4} → True, {
  {x2,3, y1,2} → True, {x2,3, y1,3} → True, {x2,3, y2,3} → True, {x2,3, y1,4} → True, {x2,3, y2,4} → True, {
  {x3,3, y1,2} → True, {x3,3, y1,3} → True, {x3,3, y2,3} → True, {x3,3, y1,4} → True, {x3,3, y2,4} → True, {
  {x1,4, y1,2} → True, {x1,4, y1,3} → True, {x1,4, y2,3} → True, {x1,4, y1,4} → True, {x1,4, y2,4} → True, {
  {x2,4, y1,2} → True, {x2,4, y1,3} → True, {x2,4, y2,3} → True, {x2,4, y1,4} → True, {x2,4, y2,4} → True, {
  {x3,4, y1,2} → True, {x3,4, y1,3} → True, {x3,4, y2,3} → True, {x3,4, y1,4} → True, {x3,4, y2,4} → True, {
  {x4,4, y1,2} → True, {x4,4, y1,3} → True, {x4,4, y2,3} → True, {x4,4, y1,4} → True, {x4,4, y2,4} → True, {
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In[*]:= SBasis = SortBy[$Basis, If[#[[1]] === x, {0, #[[3]] - #[[2]], {1, #[[2]] - #[[3]]}] &];
sol[-1] = Table[If[SSQ[bb], ((bb /. x -> ξ1) + (bb /. x -> ξ2)), 0], {bb, $Basis}];
Do[
  Block[{$k = k},
    s0 = Thread[$Basis -> sol[k - 1]];
    lhs = λTangent[
      Join[Table[If[NilQ[bb], λ, 1] bb (bb /. {x -> ξ1, y -> η1}), {bb, $Basis}],
        Table[If[NilQ[bb], λ, 1] bb (bb /. {x -> ξ2, y -> η2}), {bb, $Basis}]]];
    rhs = λTangent[
      $Basis ($Basis /. {xαβ -> (xαβ /. s0) + εk fαβ[λ], yαβ -> (yαβ /. s0) + εk gαβ[λ]}]);
    eqns = Transpose@{
      (Coefficient[CF[lhs - rhs], #] == 0) & /@ SBasis,
      SBasis /. {xαβ -> fαβ[0] == 0, yαβ -> gαβ[0] == 0}];
    unknowns = SBasis /. {xαβ -> fαβ[λ], yαβ -> gαβ[λ]};
    s = TriangularDSolve[eqns, unknowns, λ];
    sol[k] = CF[sol[k - 1] + εk ($Basis /. {xαβ -> fαβ[λ], yαβ -> gαβ[λ]} /. s)]],
  {k, 0, $k}];
cm[i_, j_ -> k_] := Δ2E{i,j} -> {k} [sol[$k].$Basis /. {xαβ -> xαβ[k], yαβ -> yαβ[k],
  ξ1αβ -> ξαβ[i], η1αβ -> ηαβ[i], ξ2αβ -> ξαβ[j], η2αβ -> ηαβ[j], λ -> 1}];
cm[i, j -> k]
» {f1,1[λ] -> 0}
» {f2,2[λ] -> 0}
» {f3,3[λ] -> 0}
» {f4,4[λ] -> 0}
» {f1,2[λ] -> e-ε21,1 λ ξ11,2 + e-ε12,2 λ ξ21,2}
» {f2,3[λ] -> e-ε22,2 λ ξ12,3 + e-ε13,3 λ ξ22,3}
» {f3,4[λ] -> e-ε23,3 λ ξ13,4 + e-ε14,4 λ ξ23,4}
» {f1,3[λ] -> e-ε21,1 λ ξ11,3 - λ2 ξ12,3 ξ21,2 + e-ε13,3 λ ξ21,3}
» {f2,4[λ] -> e-ε22,2 λ ξ12,4 - λ2 ξ13,4 ξ22,3 + e-ε14,4 λ ξ22,4}
» {f1,4[λ] -> e-ε21,1 λ ξ11,4 - λ2 ξ12,4 ξ21,2 - λ2 ξ13,4 ξ21,3 + e-ε14,4 λ ξ21,4}
» {g1,4[λ] -> λ η11,4 + e-ε11,1+ε14,4 λ η21,4}
» {g1,3[λ] -> λ η11,3 + e-ε11,1+ε13,3 λ η21,3 + e-ε11,1+ε13,3+ε14,4 λ2 η21,4 ξ13,4}
» {g2,4[λ] -> λ η12,4 + e-ε12,2+ε14,4 λ η22,4 - eε14,4 λ2 η21,4 ξ11,2}
» {g1,2[λ] -> λ η11,2 + e-ε11,1+ε12,2 λ η21,2 + e-ε11,1+ε12,2+ε13,3 λ2 η21,3 ξ12,3 +
  e-ε11,1+ε12,2+ε14,4 λ2 η21,4 ξ12,4 + e-ε11,1+ε12,2+ε13,3+ε14,4 λ3 η21,4 ξ12,3 ξ13,4}

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- $$\gg \{g_{2,4}[\lambda] \rightarrow e^{-\epsilon_{1,2} + \epsilon_{1,3}} \lambda^2 \eta_{1,3,4} \eta_{2,2,3} - e^{\epsilon_{1,3}} \lambda^3 \eta_{1,3,4} \eta_{2,1,3} \epsilon_{1,1,2} + e^{\epsilon_{1,4}} \lambda^3 \eta_{2,1,2} \eta_{2,2,4} \epsilon_{1,1,2} - e^{-\epsilon_{1,2} + \epsilon_{1,3} + \epsilon_{1,4}} \lambda^3 \eta_{2,1,4} \eta_{2,2,3} \epsilon_{1,1,3} + e^{\epsilon_{1,3} + \epsilon_{1,4}} \lambda^4 \eta_{2,1,3} \eta_{2,1,4} \epsilon_{1,1,2} \epsilon_{1,1,3} - e^{-\epsilon_{1,2} + 2\epsilon_{1,4}} \lambda^3 \eta_{2,1,4} \eta_{2,2,4} \epsilon_{1,1,4} + e^{2\epsilon_{1,4}} \lambda^4 \eta_{2,1,4}^2 \epsilon_{1,1,2} \epsilon_{1,1,4} - e^{-\epsilon_{1,2} + \epsilon_{1,3} + \epsilon_{1,4}} \lambda^3 \eta_{2,2,3} \eta_{2,2,4} \epsilon_{1,2,3} + e^{\epsilon_{1,3} + \epsilon_{1,4}} \lambda^4 \eta_{2,1,3} \eta_{2,2,4} \epsilon_{1,1,2} \epsilon_{1,2,3} - e^{-\epsilon_{1,2} + 2\epsilon_{1,4}} \lambda^3 \eta_{2,2,4}^2 \epsilon_{1,2,4} + e^{2\epsilon_{1,4}} \lambda^4 \eta_{2,1,4} \eta_{2,2,4} \epsilon_{1,1,2} \epsilon_{1,2,4} + e^{-\epsilon_{1,2} + \epsilon_{1,3} + \epsilon_{1,4}} \lambda^3 \eta_{1,3,4} \eta_{2,2,4} \epsilon_{1,3,4} - e^{\epsilon_{1,3} + \epsilon_{1,4}} \lambda^4 \eta_{1,3,4} \eta_{2,1,4} \epsilon_{1,1,2} \epsilon_{1,3,4} - e^{-\epsilon_{1,2} + \epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^4 \eta_{2,1,4} \eta_{2,2,4} \epsilon_{1,1,3} \epsilon_{1,3,4} + e^{\epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^5 \eta_{2,1,4}^2 \epsilon_{1,1,2} \epsilon_{1,1,3} \epsilon_{1,3,4} - e^{-\epsilon_{1,2} + \epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^4 \eta_{2,2,4}^2 \epsilon_{1,2,3} \epsilon_{1,3,4} + e^{\epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^5 \eta_{2,1,4} \eta_{2,2,4} \epsilon_{1,1,2} \epsilon_{1,2,3} \epsilon_{1,3,4}\}$$
- $$\gg \{g_{1,2}[\lambda] \rightarrow -e^{-\epsilon_{1,1} + 2\epsilon_{1,2}} \lambda^3 \eta_{2,1,2}^2 \epsilon_{1,1,2} - e^{-\epsilon_{1,1} + \epsilon_{1,2} + \epsilon_{1,3}} \lambda^3 \eta_{2,1,2} \eta_{2,1,3} \epsilon_{1,1,3} - e^{-\epsilon_{1,1} + \epsilon_{1,2} + \epsilon_{1,4}} \lambda^3 \eta_{2,1,2} \eta_{2,1,4} \epsilon_{1,1,4} - 2e^{-\epsilon_{1,1} + 2\epsilon_{1,2} + \epsilon_{1,3}} \lambda^4 \eta_{2,1,2} \eta_{2,1,3} \epsilon_{1,1,2} \epsilon_{1,2,3} - e^{-\epsilon_{1,1} + \epsilon_{1,2} + 2\epsilon_{1,3}} \lambda^4 \eta_{2,1,3}^2 \epsilon_{1,1,3} \epsilon_{1,2,3} - e^{-\epsilon_{1,1} + \epsilon_{1,2} + \epsilon_{1,3} + \epsilon_{1,4}} \lambda^4 \eta_{2,1,3} \eta_{2,1,4} \epsilon_{1,1,4} \epsilon_{1,2,3} - e^{-\epsilon_{1,1} + 2\epsilon_{1,2} + 2\epsilon_{1,3}} \lambda^5 \eta_{2,1,3}^2 \epsilon_{1,1,2} \epsilon_{1,2,3} - 2e^{-\epsilon_{1,1} + 2\epsilon_{1,2} + \epsilon_{1,4}} \lambda^4 \eta_{2,1,2} \eta_{2,1,4} \epsilon_{1,1,2} \epsilon_{1,2,4} - e^{-\epsilon_{1,1} + \epsilon_{1,2} + \epsilon_{1,3} + \epsilon_{1,4}} \lambda^4 \eta_{2,1,3} \eta_{2,1,4} \epsilon_{1,1,3} \epsilon_{1,2,4} - e^{-\epsilon_{1,1} + \epsilon_{1,2} + 2\epsilon_{1,4}} \lambda^4 \eta_{2,1,4}^2 \epsilon_{1,1,4} \epsilon_{1,2,4} - 2e^{-\epsilon_{1,1} + 2\epsilon_{1,2} + \epsilon_{1,3} + \epsilon_{1,4}} \lambda^5 \eta_{2,1,3} \eta_{2,1,4} \epsilon_{1,1,2} \epsilon_{1,2,3} \epsilon_{1,2,4} - e^{-\epsilon_{1,1} + 2\epsilon_{1,2} + 2\epsilon_{1,4}} \lambda^5 \eta_{2,1,4}^2 \epsilon_{1,1,2} \epsilon_{1,2,4} - e^{-\epsilon_{1,1} + \epsilon_{1,2} + \epsilon_{1,3} + \epsilon_{1,4}} \lambda^4 \eta_{2,1,2} \eta_{2,1,4} \epsilon_{1,1,3} \epsilon_{1,3,4} - 2e^{-\epsilon_{1,1} + 2\epsilon_{1,2} + 2\epsilon_{1,3} + \epsilon_{1,4}} \lambda^5 \eta_{2,1,3} \eta_{2,1,4} \epsilon_{1,1,3} \epsilon_{1,2,3} \epsilon_{1,3,4} - e^{-\epsilon_{1,1} + \epsilon_{1,2} + \epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^5 \eta_{2,1,4}^2 \epsilon_{1,1,4} \epsilon_{1,2,3} \epsilon_{1,3,4} - 2e^{-\epsilon_{1,1} + 2\epsilon_{1,2} + 2\epsilon_{1,3} + \epsilon_{1,4}} \lambda^6 \eta_{2,1,3} \eta_{2,1,4} \epsilon_{1,1,2} \epsilon_{1,2,3}^2 \epsilon_{1,3,4} - e^{-\epsilon_{1,1} + \epsilon_{1,2} + \epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^5 \eta_{2,1,4}^2 \epsilon_{1,1,3} \epsilon_{1,2,4} \epsilon_{1,3,4} - 2e^{-\epsilon_{1,1} + 2\epsilon_{1,2} + \epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^6 \eta_{2,1,4}^2 \epsilon_{1,1,2} \epsilon_{1,2,3} \epsilon_{1,2,4} \epsilon_{1,3,4} - e^{-\epsilon_{1,1} + \epsilon_{1,2} + 2\epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^6 \eta_{2,1,4}^2 \epsilon_{1,1,3} \epsilon_{1,2,3} \epsilon_{1,3,4}^2 - e^{-\epsilon_{1,1} + 2\epsilon_{1,2} + 2\epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^7 \eta_{2,1,4}^2 \epsilon_{1,1,2} \epsilon_{1,2,3}^2 \epsilon_{1,3,4}^2\}$$
- $$\gg \{g_{2,3}[\lambda] \rightarrow e^{\epsilon_{1,3}} \lambda^3 \eta_{2,1,2} \eta_{2,2,3} \epsilon_{1,1,2} - e^{-\epsilon_{1,2} + 2\epsilon_{1,3}} \lambda^3 \eta_{2,1,3} \eta_{2,2,3} \epsilon_{1,1,3} + e^{2\epsilon_{1,3}} \lambda^4 \eta_{2,1,3}^2 \epsilon_{1,1,2} \epsilon_{1,1,3} - e^{-\epsilon_{1,2} + \epsilon_{1,3} + \epsilon_{1,4}} \lambda^3 \eta_{2,1,3} \eta_{2,2,4} \epsilon_{1,1,4} + e^{\epsilon_{1,3} + \epsilon_{1,4}} \lambda^4 \eta_{2,1,3} \eta_{2,1,4} \epsilon_{1,1,2} \epsilon_{1,1,4} - e^{-\epsilon_{1,2} + 2\epsilon_{1,3}} \lambda^3 \eta_{2,2,3}^2 \epsilon_{1,2,3} + e^{2\epsilon_{1,3}} \lambda^4 \eta_{2,1,3} \eta_{2,2,3} \epsilon_{1,1,2} \epsilon_{1,2,3} - e^{-\epsilon_{1,2} + \epsilon_{1,3} + \epsilon_{1,4}} \lambda^3 \eta_{2,2,3} \eta_{2,2,4} \epsilon_{1,2,4} + e^{\epsilon_{1,3} + \epsilon_{1,4}} \lambda^4 \eta_{2,1,4} \eta_{2,2,3} \epsilon_{1,1,2} \epsilon_{1,2,4} + e^{\epsilon_{1,3} + \epsilon_{1,4}} \lambda^4 \eta_{2,1,2} \eta_{2,2,4} \epsilon_{1,1,2} \epsilon_{1,3,4} - e^{-\epsilon_{1,2} + 2\epsilon_{1,3} + \epsilon_{1,4}} \lambda^4 \eta_{2,1,3} \eta_{2,2,4} \epsilon_{1,1,3} \epsilon_{1,3,4} + 2e^{2\epsilon_{1,3} + \epsilon_{1,4}} \lambda^5 \eta_{2,1,3} \eta_{2,1,4} \epsilon_{1,1,2} \epsilon_{1,1,3} \epsilon_{1,3,4} - e^{-\epsilon_{1,2} + \epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^4 \eta_{2,1,4} \eta_{2,2,4} \epsilon_{1,1,4} \epsilon_{1,3,4} + e^{\epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^5 \eta_{2,1,4}^2 \epsilon_{1,1,2} \epsilon_{1,1,4} \epsilon_{1,3,4} - 2e^{-\epsilon_{1,2} + 2\epsilon_{1,3} + \epsilon_{1,4}} \lambda^4 \eta_{2,2,3} \eta_{2,2,4} \epsilon_{1,2,3} \epsilon_{1,3,4} + e^{2\epsilon_{1,3} + \epsilon_{1,4}} \lambda^5 \eta_{2,1,3} \eta_{2,2,4} \epsilon_{1,1,2} \epsilon_{1,2,3} \epsilon_{1,3,4} - e^{-\epsilon_{1,2} + \epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^4 \eta_{2,2,4}^2 \epsilon_{1,2,4} \epsilon_{1,3,4} + e^{\epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^5 \eta_{2,1,4} \eta_{2,2,4} \epsilon_{1,1,2} \epsilon_{1,2,4} \epsilon_{1,3,4} - e^{-\epsilon_{1,2} + 2\epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^5 \eta_{2,1,4} \eta_{2,2,4} \epsilon_{1,1,3} \epsilon_{1,3,4}^2 + e^{2\epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^6 \eta_{2,1,4}^2 \epsilon_{1,1,2} \epsilon_{1,1,3} \epsilon_{1,3,4}^2 - e^{-\epsilon_{1,2} + 2\epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^5 \eta_{2,2,4}^2 \epsilon_{1,2,3} \epsilon_{1,3,4}^2 + e^{2\epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^6 \eta_{2,1,4} \eta_{2,2,4} \epsilon_{1,1,2} \epsilon_{1,2,3} \epsilon_{1,3,4}^2\}$$
- $$\gg \{g_{3,4}[\lambda] \rightarrow e^{\epsilon_{1,4}} \lambda^3 \eta_{2,1,2} \eta_{2,2,4} \epsilon_{1,1,3} + e^{\epsilon_{1,4}} \lambda^3 \eta_{2,1,3} \eta_{2,3,4} \epsilon_{1,1,3} - e^{-\epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^3 \eta_{2,1,4} \eta_{2,3,4} \epsilon_{1,1,4} + e^{2\epsilon_{1,4}} \lambda^4 \eta_{2,1,4}^2 \epsilon_{1,1,3} \epsilon_{1,1,4} + e^{\epsilon_{1,4}} \lambda^3 \eta_{2,2,3} \eta_{2,3,4} \epsilon_{1,2,3} + e^{2\epsilon_{1,4}} \lambda^4 \eta_{2,1,4} \eta_{2,2,4} \epsilon_{1,1,4} \epsilon_{1,2,3} - e^{-\epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^3 \eta_{2,2,4} \eta_{2,3,4} \epsilon_{1,2,4} + e^{2\epsilon_{1,4}} \lambda^4 \eta_{2,1,4} \eta_{2,2,4} \epsilon_{1,1,3} \epsilon_{1,2,4} + e^{2\epsilon_{1,4}} \lambda^4 \eta_{2,2,4}^2 \epsilon_{1,2,3} \epsilon_{1,2,4} - e^{-\epsilon_{1,3} + 2\epsilon_{1,4}} \lambda^3 \eta_{2,3,4}^2 \epsilon_{1,3,4} + e^{2\epsilon_{1,4}} \lambda^4 \eta_{2,1,4} \eta_{2,3,4} \epsilon_{1,1,3} \epsilon_{1,3,4} + e^{2\epsilon_{1,4}} \lambda^4 \eta_{2,2,4} \eta_{2,3,4} \epsilon_{1,2,3} \epsilon_{1,3,4}\}$$

Out[*]=

$$\begin{aligned} & E_{\{i,j\} \rightarrow \{k\}} [y_{1,4}[k] (\eta_{1,4}[i] + e^{-\epsilon_{1,1}[i] + \epsilon_{4,4}[i]} \eta_{1,4}[j]) + x_{1,1}[k] (\epsilon_{1,1}[i] + \epsilon_{1,1}[j]) + \\ & y_{2,4}[k] (\eta_{2,4}[i] + e^{-\epsilon_{2,2}[i] + \epsilon_{4,4}[i]} \eta_{2,4}[j] - e^{\epsilon_{4,4}[i]} \eta_{1,4}[j] \epsilon_{1,2}[i]) + \\ & x_{1,2}[k] (e^{-\epsilon_{1,1}[j]} \epsilon_{1,2}[i] + e^{-\epsilon_{2,2}[i]} \epsilon_{1,2}[j]) + x_{2,2}[k] (\epsilon_{2,2}[i] + \epsilon_{2,2}[j]) + \\ & y_{3,4}[k] (\eta_{3,4}[i] + e^{-\epsilon_{3,3}[i] + \epsilon_{4,4}[i]} \eta_{3,4}[j] - e^{\epsilon_{4,4}[i]} \eta_{1,4}[j] \epsilon_{1,3}[i] - e^{\epsilon_{4,4}[i]} \eta_{2,4}[j] \epsilon_{2,3}[i]) + \\ & x_{1,3}[k] (e^{-\epsilon_{1,1}[j]} \epsilon_{1,3}[i] + e^{-\epsilon_{3,3}[i]} \epsilon_{1,3}[j] - \epsilon_{1,2}[j] \epsilon_{2,3}[i]) + \\ & x_{2,3}[k] (e^{-\epsilon_{2,2}[j]} \epsilon_{2,3}[i] + e^{-\epsilon_{3,3}[i]} \epsilon_{2,3}[j]) + x_{3,3}[k] (\epsilon_{3,3}[i] + \epsilon_{3,3}[j]) + \\ & y_{1,3}[k] (\eta_{1,3}[i] + e^{-\epsilon_{1,1}[i] + \epsilon_{3,3}[i]} \eta_{1,3}[j] + e^{-\epsilon_{1,1}[i] + \epsilon_{3,3}[i] + \epsilon_{4,4}[i]} \eta_{1,4}[j] \epsilon_{3,4}[i]) + \\ & y_{2,3}[k] (\eta_{2,3}[i] + e^{-\epsilon_{2,2}[i] + \epsilon_{3,3}[i]} \eta_{2,3}[j] - e^{\epsilon_{3,3}[i]} \eta_{1,3}[j] \epsilon_{1,2}[i] + \\ & e^{-\epsilon_{2,2}[i] + \epsilon_{3,3}[i] + \epsilon_{4,4}[i]} \eta_{2,4}[j] \epsilon_{3,4}[i] - e^{\epsilon_{3,3}[i] + \epsilon_{4,4}[i]} \eta_{1,4}[j] \epsilon_{1,2}[i] \epsilon_{3,4}[i]) + \\ & x_{1,4}[k] (e^{-\epsilon_{1,1}[j]} \epsilon_{1,4}[i] + e^{-\epsilon_{4,4}[i]} \epsilon_{1,4}[j] - \epsilon_{1,2}[j] \epsilon_{2,4}[i] - \epsilon_{1,3}[j] \epsilon_{3,4}[i]) + \\ & e^{-\epsilon_{1,1}[i]} y_{1,2}[k] (e^{\epsilon_{1,1}[i]} \eta_{1,2}[i] + e^{\epsilon_{2,2}[i]} \eta_{1,2}[j] + e^{\epsilon_{2,2}[i] + \epsilon_{3,3}[i]} \eta_{1,3}[j] \epsilon_{2,3}[i] + \end{aligned}$$

$$\begin{aligned}
 & e^{-\epsilon_{1,1}[j]+\epsilon_{4,4}[i]} \eta_{1,4}[j] \xi_{1,2}[i] \xi_{1,4}[i] - e^{-\epsilon_{2,2}[i]+\epsilon_{3,3}[i]} \eta_{2,3}[j] \xi_{1,2}[j] \xi_{2,3}[i] + \\
 & e^{\epsilon_{3,3}[i]} \eta_{1,3}[j] \xi_{1,2}[i] \xi_{1,2}[j] \xi_{2,3}[i] - e^{-\epsilon_{2,2}[i]+\epsilon_{4,4}[i]} \eta_{2,4}[j] \xi_{1,2}[j] \xi_{2,4}[i] + \\
 & e^{\epsilon_{4,4}[i]} \eta_{1,4}[j] \xi_{1,2}[i] \xi_{1,2}[j] \xi_{2,4}[i] + e^{-\epsilon_{1,1}[j]-\epsilon_{2,2}[i]+\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{2,4}[j] \xi_{1,3}[i] \xi_{3,4}[i] - \\
 & e^{-\epsilon_{1,1}[j]+\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{1,4}[j] \xi_{1,2}[i] \xi_{1,3}[i] \xi_{3,4}[i] - e^{-\epsilon_{2,2}[i]+\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{2,4}[j] \\
 & \xi_{1,2}[j] \xi_{2,3}[i] \xi_{3,4}[i] + e^{\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{1,4}[j] \xi_{1,2}[i] \xi_{1,2}[j] \xi_{2,3}[i] \xi_{3,4}[i]) + \\
 x_{1,3}[k] & (\eta_{1,2}[j] \xi_{1,2}[j] \xi_{1,3}[i] + \eta_{1,3}[j] \xi_{1,3}[i] \xi_{1,3}[j] + e^{-\epsilon_{1,1}[j]-\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{3,4}[j] \xi_{1,4}[i] - \\
 & e^{-\epsilon_{1,1}[j]+\epsilon_{4,4}[i]} \eta_{1,4}[j] \xi_{1,3}[i] \xi_{1,4}[i] + \eta_{2,3}[j] \xi_{1,3}[j] \xi_{2,3}[i] - e^{-\epsilon_{1,1}[j]+\epsilon_{4,4}[i]} \eta_{2,4}[j] \xi_{1,4}[i] \\
 & \xi_{2,3}[i] - e^{-\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{3,4}[j] \xi_{1,2}[j] \xi_{2,4}[i] + e^{\epsilon_{4,4}[i]} \eta_{1,4}[j] \xi_{1,2}[j] \xi_{1,3}[i] \xi_{2,4}[i] + \\
 & e^{\epsilon_{4,4}[i]} \eta_{2,4}[j] \xi_{1,2}[j] \xi_{2,3}[i] \xi_{2,4}[i] - e^{-\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{3,4}[j] \xi_{1,3}[j] \xi_{3,4}[i] + \\
 & e^{\epsilon_{4,4}[i]} \eta_{1,4}[j] \xi_{1,3}[i] \xi_{1,3}[j] \xi_{3,4}[i] + e^{\epsilon_{4,4}[i]} \eta_{2,4}[j] \xi_{1,3}[j] \xi_{2,3}[i] \xi_{3,4}[i]) + \\
 x_{2,3}[k] & (-e^{-\epsilon_{2,2}[j]} \eta_{1,2}[j] \xi_{1,3}[i] + \eta_{1,3}[j] \xi_{1,3}[i] \xi_{2,3}[j] + \eta_{2,3}[j] \xi_{2,3}[i] \xi_{2,3}[j] + \\
 & e^{-\epsilon_{2,2}[j]-\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{3,4}[j] \xi_{2,4}[i] - e^{-\epsilon_{2,2}[j]+\epsilon_{4,4}[i]} \eta_{1,4}[j] \xi_{1,3}[i] \xi_{2,4}[i] - \\
 & e^{-\epsilon_{2,2}[j]+\epsilon_{4,4}[i]} \eta_{2,4}[j] \xi_{2,3}[i] \xi_{2,4}[i] - e^{-\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{3,4}[j] \xi_{2,3}[j] \xi_{3,4}[i] + \\
 & e^{\epsilon_{4,4}[i]} \eta_{1,4}[j] \xi_{1,3}[i] \xi_{2,3}[j] \xi_{3,4}[i] + e^{\epsilon_{4,4}[i]} \eta_{2,4}[j] \xi_{2,3}[i] \xi_{2,3}[j] \xi_{3,4}[i]) + \\
 x_{2,4}[k] & (-e^{-\epsilon_{2,2}[j]} \eta_{1,2}[j] \xi_{1,4}[i] + \eta_{1,3}[j] \xi_{1,4}[i] \xi_{2,3}[j] + \eta_{2,3}[j] \xi_{2,3}[j] \xi_{2,4}[i] + \\
 & \eta_{1,4}[j] \xi_{1,4}[i] \xi_{2,4}[j] + \eta_{2,4}[j] \xi_{2,4}[i] \xi_{2,4}[j] + \eta_{3,4}[j] \xi_{2,4}[j] \xi_{3,4}[i]) - \\
 e^{-\epsilon_{1,1}[i]} y_{1,3}[k] & (-e^{\epsilon_{2,2}[i]} \eta_{1,2}[j] \eta_{2,3}[i] + e^{\epsilon_{2,2}[i]+\epsilon_{3,3}[i]} \eta_{1,2}[j] \eta_{1,3}[j] \xi_{1,2}[i] + \\
 & e^{2\epsilon_{3,3}[i]} \eta_{1,3}[j]^2 \xi_{1,3}[i] + e^{\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{1,3}[j] \eta_{1,4}[j] \xi_{1,4}[i] - \\
 & e^{\epsilon_{2,2}[i]+\epsilon_{3,3}[i]} \eta_{1,3}[j] \eta_{2,3}[i] \xi_{2,3}[i] + e^{\epsilon_{2,2}[i]+2\epsilon_{3,3}[i]} \eta_{1,3}[j]^2 \xi_{1,2}[i] \xi_{2,3}[i] - \\
 & e^{\epsilon_{2,2}[i]+\epsilon_{4,4}[i]} \eta_{1,4}[j] \eta_{2,3}[i] \xi_{2,4}[i] + e^{\epsilon_{2,2}[i]+\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{1,3}[j] \eta_{1,4}[j] \xi_{1,2}[i] \xi_{2,4}[i] + \\
 & e^{\epsilon_{2,2}[i]+\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{1,2}[j] \eta_{1,4}[j] \xi_{1,2}[i] \xi_{3,4}[i] + \\
 & 2e^{2\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{1,3}[j] \eta_{1,4}[j] \xi_{1,3}[i] \xi_{3,4}[i] + e^{\epsilon_{3,3}[i]+2\epsilon_{4,4}[i]} \eta_{1,4}[j]^2 \xi_{1,4}[i] \xi_{3,4}[i] - \\
 & e^{\epsilon_{2,2}[i]+\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{1,4}[j] \eta_{2,3}[i] \xi_{2,3}[i] \xi_{3,4}[i] + 2e^{\epsilon_{2,2}[i]+2\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{1,3}[j] \eta_{1,4}[j] \\
 & \xi_{1,2}[i] \xi_{2,3}[i] \xi_{3,4}[i] + e^{\epsilon_{2,2}[i]+\epsilon_{3,3}[i]+2\epsilon_{4,4}[i]} \eta_{1,4}[j]^2 \xi_{1,2}[i] \xi_{2,4}[i] \xi_{3,4}[i] + e^{2\epsilon_{3,3}[i]+2\epsilon_{4,4}[i]} \\
 & \eta_{1,4}[j]^2 \xi_{1,3}[i] \xi_{3,4}[i]^2 + e^{\epsilon_{2,2}[i]+2\epsilon_{3,3}[i]+2\epsilon_{4,4}[i]} \eta_{1,4}[j]^2 \xi_{1,2}[i] \xi_{2,3}[i] \xi_{3,4}[i]^2) + \\
 e^{-\epsilon_{2,2}[i]+\epsilon_{3,3}[i]} y_{2,3}[k] & (e^{\epsilon_{2,2}[i]} \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,2}[i] - e^{\epsilon_{3,3}[i]} \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i] + \\
 & e^{\epsilon_{2,2}[i]+\epsilon_{3,3}[i]} \eta_{1,3}[j]^2 \xi_{1,2}[i] \xi_{1,3}[i] - e^{\epsilon_{4,4}[i]} \eta_{1,3}[j] \eta_{2,4}[j] \xi_{1,4}[i] + \\
 & e^{\epsilon_{2,2}[i]+\epsilon_{4,4}[i]} \eta_{1,3}[j] \eta_{1,4}[j] \xi_{1,2}[i] \xi_{1,4}[i] - e^{\epsilon_{3,3}[i]} \eta_{2,3}[j]^2 \xi_{2,3}[i] + \\
 & e^{\epsilon_{2,2}[i]+\epsilon_{3,3}[i]} \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,2}[i] \xi_{2,3}[i] - e^{\epsilon_{4,4}[i]} \eta_{2,3}[j] \eta_{2,4}[j] \xi_{2,4}[i] + \\
 & e^{\epsilon_{2,2}[i]+\epsilon_{4,4}[i]} \eta_{1,4}[j] \eta_{2,3}[j] \xi_{1,2}[i] \xi_{2,4}[i] + e^{\epsilon_{2,2}[i]+\epsilon_{4,4}[i]} \eta_{1,2}[j] \eta_{2,4}[j] \xi_{1,2}[i] \xi_{3,4}[i] - \\
 & e^{\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{1,4}[j] \eta_{2,3}[j] \xi_{1,3}[i] \xi_{3,4}[i] - e^{\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{1,3}[j] \eta_{2,4}[j] \xi_{1,3}[i] \xi_{3,4}[i] + \\
 & 2e^{\epsilon_{2,2}[i]+\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{1,3}[j] \eta_{1,4}[j] \xi_{1,2}[i] \xi_{1,3}[i] \xi_{3,4}[i] - \\
 & e^{2\epsilon_{4,4}[i]} \eta_{1,4}[j] \eta_{2,4}[j] \xi_{1,4}[i] \xi_{3,4}[i] + e^{\epsilon_{2,2}[i]+2\epsilon_{4,4}[i]} \eta_{1,4}[j]^2 \xi_{1,2}[i] \xi_{1,4}[i] \xi_{3,4}[i] - \\
 & 2e^{\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{2,3}[j] \eta_{2,4}[j] \xi_{2,3}[i] \xi_{3,4}[i] + e^{\epsilon_{2,2}[i]+\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{1,4}[j] \eta_{2,3}[j] \\
 & \xi_{1,2}[i] \xi_{2,3}[i] \xi_{3,4}[i] + e^{\epsilon_{2,2}[i]+\epsilon_{3,3}[i]+\epsilon_{4,4}[i]} \eta_{1,3}[j] \eta_{2,4}[j] \xi_{1,2}[i] \xi_{2,3}[i] \xi_{3,4}[i] - \\
 & e^{2\epsilon_{4,4}[i]} \eta_{2,4}[j]^2 \xi_{2,4}[i] \xi_{3,4}[i] + e^{\epsilon_{2,2}[i]+2\epsilon_{4,4}[i]} \eta_{1,4}[j] \eta_{2,4}[j] \xi_{1,2}[i] \xi_{2,4}[i] \xi_{3,4}[i] - \\
 & e^{\epsilon_{3,3}[i]+2\epsilon_{4,4}[i]} \eta_{1,4}[j] \eta_{2,4}[j] \xi_{1,3}[i] \xi_{3,4}[i]^2 + \\
 & e^{\epsilon_{2,2}[i]+\epsilon_{3,3}[i]+2\epsilon_{4,4}[i]} \eta_{1,4}[j]^2 \xi_{1,2}[i] \xi_{1,3}[i] \xi_{3,4}[i]^2 - e^{\epsilon_{3,3}[i]+2\epsilon_{4,4}[i]} \eta_{2,4}[j]^2 \xi_{2,3}[i] \xi_{3,4}[i]^2 + \\
 & e^{\epsilon_{2,2}[i]+\epsilon_{3,3}[i]+2\epsilon_{4,4}[i]} \eta_{1,4}[j] \eta_{2,4}[j] \xi_{1,2}[i] \xi_{2,3}[i] \xi_{3,4}[i]^2) + \\
 x_{3,4}[k] & (-e^{-\epsilon_{3,3}[j]} \eta_{1,3}[j] \xi_{1,4}[i] - e^{-\epsilon_{3,3}[j]} \eta_{2,3}[j] \xi_{2,4}[i] + \eta_{1,4}[j] \xi_{1,4}[i] \xi_{3,4}[j] + \\
 & \eta_{2,4}[j] \xi_{2,4}[i] \xi_{3,4}[j] + \eta_{3,4}[j] \xi_{3,4}[i] \xi_{3,4}[j])]
 \end{aligned}$$

$$\ln[*]:= (\mathbf{cm}[1, 2 \rightarrow 2] // \mathbf{cm}[2, 3 \rightarrow 1]) \equiv (\mathbf{cm}[2, 3 \rightarrow 2] // \mathbf{cm}[1, 2 \rightarrow 1])$$

Out[*n*]=

True