

```
In[*]:= $k = 2;
SetAlgebra[gl3,ε];
$PBWRule
MatrixForm@Table[{b1, b2} → B[b1, b2], {b1, $Basis}, {b2, $Basis}]
MatrixForm@
Table[{b1, b2} → (DefRep[b1].DefRep[b2] - DefRep[b2].DefRep[b1] == DefRep[B[b1, b2]]),
{b1, $Basis}, {b2, $Basis}]
```

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Out[*]=
{y1,2 → 1, y1,3 → 2, y2,3 → 3, x1,1 → 4, x1,2 → 5, x2,2 → 6, x1,3 → 7, x2,3 → 8, x3,3 → 9}
```

```
Out[*]//MatrixForm=
{
  {y1,2, y1,2} → 0, {y1,2, y1,3} → 0, {y1,2, y2,3} → -ε y1,3, {y1,2, x1,1} → y1,2,
  {y1,3, y1,2} → 0, {y1,3, y1,3} → 0, {y1,3, y2,3} → 0, {y1,3, x1,1} → y1,3,
  {y2,3, y1,2} → ε y1,3, {y2,3, y1,3} → 0, {y2,3, y2,3} → 0, {y2,3, x1,1} → y2,3,
  {x1,1, y1,2} → -y1,2, {x1,1, y1,3} → -y1,3, {x1,1, y2,3} → 0, {x1,1, x1,1} → 0,
  {x1,2, y1,2} → ε x1,1 - ε x2,2, {x1,2, y1,3} → -y2,3, {x1,2, y2,3} → 0, {x1,2, x1,1} → -x1,2,
  {x2,2, y1,2} → y1,2, {x2,2, y1,3} → 0, {x2,2, y2,3} → -y2,3, {x2,2, x1,1} → -x2,2,
  {x1,3, y1,2} → -ε x2,3, {x1,3, y1,3} → ε x1,1 - ε x3,3, {x1,3, y2,3} → ε x1,2, {x1,3, x1,1} → -x1,3,
  {x2,3, y1,2} → 0, {x2,3, y1,3} → y1,2, {x2,3, y2,3} → ε x2,2 - ε x3,3, {x2,3, x1,1} → -x2,3,
  {x3,3, y1,2} → 0, {x3,3, y1,3} → y1,3, {x3,3, y2,3} → y2,3, {x3,3, x1,1} → -x3,3
}
```

```
Out[*]//MatrixForm=
{
  {y1,2, y1,2} → True, {y1,2, y1,3} → True, {y1,2, y2,3} → True, {y1,2, x1,1} → True, {y1,2, x1,2} → True,
  {y1,3, y1,2} → True, {y1,3, y1,3} → True, {y1,3, y2,3} → True, {y1,3, x1,1} → True, {y1,3, x1,2} → True,
  {y2,3, y1,2} → True, {y2,3, y1,3} → True, {y2,3, y2,3} → True, {y2,3, x1,1} → True, {y2,3, x1,2} → True,
  {x1,1, y1,2} → True, {x1,1, y1,3} → True, {x1,1, y2,3} → True, {x1,1, x1,1} → True, {x1,1, x1,2} → True,
  {x1,2, y1,2} → True, {x1,2, y1,3} → True, {x1,2, y2,3} → True, {x1,2, x1,1} → True, {x1,2, x1,2} → True,
  {x2,2, y1,2} → True, {x2,2, y1,3} → True, {x2,2, y2,3} → True, {x2,2, x1,1} → True, {x2,2, x1,2} → True,
  {x1,3, y1,2} → True, {x1,3, y1,3} → True, {x1,3, y2,3} → True, {x1,3, x1,1} → True, {x1,3, x1,2} → True,
  {x2,3, y1,2} → True, {x2,3, y1,3} → True, {x2,3, y2,3} → True, {x2,3, x1,1} → True, {x2,3, x1,2} → True,
  {x3,3, y1,2} → True, {x3,3, y1,3} → True, {x3,3, y2,3} → True, {x3,3, x1,1} → True, {x3,3, x1,2} → True
}
```

```

In[*]:= SBasis = SortBy[$Basis, If[#[[1]] === x, {0, #[[3]] - #[[2]]}, {1, #[[2]] - #[[3]]}] &];
sol[-1] = 0 $Basis;
Do[
  Block[{$k = k},
    s0 = Thread[$Basis -> sol[k - 1]];
    lhs = λTangent[
      Join[λ $Basis ($Basis /. {x -> ξ1, y -> η1}), λ $Basis ($Basis /. {x -> ξ2, y -> η2})]
    ];
    rhs = λTangent[
      $Basis ($Basis /. {xαβ -> (xαβ /. s0) + εk fαβ[λ], yαβ -> (yαβ /. s0) + εk gαβ[λ]})
    ];
    eqns = Transpose@{
      (Coefficient[CF[lhs - rhs], #] == 0) & /@ SBasis,
      SBasis /. {xαβ -> fαβ[0] == 0, yαβ -> gαβ[0] == 0}
    };
    unknowns = SBasis /. {xαβ -> fαβ[λ], yαβ -> gαβ[λ]};
    s = TriangularDSolve[eqns, unknowns, λ];
    sol[k] = CF[sol[k - 1] + εk ($Basis /. {xαβ -> fαβ[λ], yαβ -> gαβ[λ]} /. s)]
  ],
  {k, 0, $k}
];
cm[i_, j_ -> k_] := Δ2E{i,j} -> {k}[sol[$k].$Basis /. {xαβ -> xαβ[k], yαβ -> yαβ[k],
  ξ1αβ -> ξαβ[i], η1αβ -> ηαβ[i], ξ2αβ -> ξαβ[j], η2αβ -> ηαβ[j], λ -> 1}];
cm[i, j -> k]
» {f1,1[λ] -> λ ξ1,1 + λ ξ2,1}
» {f2,2[λ] -> λ ξ1,2 + λ ξ2,2}
» {f3,3[λ] -> λ ξ1,3 + λ ξ2,3}
» {f1,2[λ] -> e-λ ξ2,1 λ ξ1,2 + e-λ ξ1,2 λ ξ2,2}
» {f2,3[λ] -> e-λ ξ2,2 λ ξ1,3 + e-λ ξ1,3 λ ξ2,3}
» {f1,3[λ] -> e-λ ξ2,1 λ ξ1,3 - λ2 ξ1,3 ξ2,2 + e-λ ξ1,3 λ ξ2,3}
» {g1,3[λ] -> λ η1,3 + e-λ ξ1,1 + λ ξ1,3 λ η2,3}
» {g1,2[λ] -> λ η1,2 + e-λ ξ1,1 + λ ξ1,2 λ η2,2 + e-λ ξ1,1 + λ ξ1,2 + λ ξ1,3 λ2 η2,3 ξ1,3}
» {g2,3[λ] -> λ η1,3 + e-λ ξ1,2 + λ ξ1,3 λ η2,3 - eλ ξ1,3 λ2 η2,3 ξ1,2}
» {f1,1[λ] -> eλ ξ1,2 λ2 η2,2 ξ1,2 + eλ ξ1,3 λ2 η2,3 ξ1,3 + eλ ξ1,2 + λ ξ1,3 λ3 η2,3 ξ1,2 ξ1,3}
» {f2,2[λ] -> -eλ ξ1,2 λ2 η2,2 ξ1,2 + eλ ξ1,3 λ2 η2,3 ξ1,3 - eλ ξ1,2 + λ ξ1,3 λ3 η2,3 ξ1,2 ξ1,3}
» {f3,3[λ] -> -eλ ξ1,3 λ2 η2,3 ξ1,3 - eλ ξ1,3 λ2 η2,3 ξ1,3}
» {f1,2[λ] -> -eλ ξ1,2 + λ ξ1,3 λ2 η2,3 ξ1,3 - eλ ξ1,3 λ2 η2,3 ξ1,3 - eλ ξ1,3 λ3 η2,3 ξ1,2 ξ1,3 +
  λ3 η2,2 ξ1,2 ξ2,2 - e-λ ξ1,2 + λ ξ1,3 λ3 η2,3 ξ1,2 ξ2,2 + eλ ξ1,3 λ4 η2,3 ξ1,2 ξ1,3 ξ2,2}
» {f2,3[λ] -> -e-λ ξ2,2 λ2 η2,2 ξ1,3 + λ3 η2,3 ξ1,3 ξ2,3 + λ3 η2,3 ξ1,2 ξ2,3}
» {f1,3[λ] -> λ3 η2,2 ξ1,3 ξ2,2 + λ3 η2,3 ξ1,3 ξ2,3 + λ3 η2,3 ξ1,2 ξ2,3}

```

- $$\gg \{g_{1,3}[\lambda] \rightarrow e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2,2}} \lambda^2 \eta_{1,2,3} \eta_{2,1,2} - e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^3 \eta_{2,1,2} \eta_{2,1,3} \xi_{1,1,2} - e^{-\lambda \xi_{1,1} + 2\lambda \xi_{1,3,3}} \lambda^3 \eta_{2,1,3}^2 \xi_{1,1,3} + e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,2,3} - e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^4 \eta_{2,1,3}^2 \xi_{1,1,2} \xi_{1,2,3}\}$$
- $$\gg \{g_{1,2}[\lambda] \rightarrow -e^{-\lambda \xi_{1,1} + 2\lambda \xi_{1,2,2}} \lambda^3 \eta_{2,1,2}^2 \xi_{1,1,2} - e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^3 \eta_{2,1,2} \eta_{2,1,3} \xi_{1,1,3} - 2 e^{-\lambda \xi_{1,1} + 2\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^4 \eta_{2,1,2} \eta_{2,1,3} \xi_{1,1,2} \xi_{1,2,3} - e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^4 \eta_{2,1,3}^2 \xi_{1,1,3} \xi_{1,2,3} - e^{-\lambda \xi_{1,1} + 2\lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^5 \eta_{2,1,3}^2 \xi_{1,1,2} \xi_{1,2,3}\}$$
- $$\gg \{g_{2,3}[\lambda] \rightarrow e^{\lambda \xi_{1,3,3}} \lambda^3 \eta_{2,1,2} \eta_{2,2,3} \xi_{1,1,2} - e^{-\lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^3 \eta_{2,1,3} \eta_{2,2,3} \xi_{1,1,3} + e^{2\lambda \xi_{1,3,3}} \lambda^4 \eta_{2,1,3}^2 \xi_{1,1,2} \xi_{1,1,3} - e^{-\lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^3 \eta_{2,2,3}^2 \xi_{1,2,3} + e^{2\lambda \xi_{1,3,3}} \lambda^4 \eta_{2,1,3} \eta_{2,2,3} \xi_{1,1,2} \xi_{1,2,3}\}$$
- $$\gg \left\{ f_{1,1}[\lambda] \rightarrow -\frac{1}{2} e^{2\lambda \xi_{1,2,2}} \lambda^4 \eta_{2,1,2}^2 \xi_{1,1,2}^2 - e^{\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^4 \eta_{2,1,2} \eta_{2,1,3} \xi_{1,1,2} \xi_{1,1,3} - \frac{1}{2} e^{2\lambda \xi_{1,3,3}} \lambda^4 \eta_{2,1,3}^2 \xi_{1,1,3}^2 - e^{2\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^5 \eta_{2,1,2} \eta_{2,1,3} \xi_{1,1,2}^2 \xi_{1,2,3} - e^{\lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^5 \eta_{2,1,3}^2 \xi_{1,1,2} \xi_{1,1,3} \xi_{1,2,3} - \frac{1}{2} e^{2\lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^6 \eta_{2,1,3}^2 \xi_{1,1,2}^2 \xi_{1,2,3}^2 \right\}$$
- $$\gg \left\{ f_{2,2}[\lambda] \rightarrow \frac{1}{2} e^{2\lambda \xi_{1,2,2}} \lambda^4 \eta_{2,1,2}^2 \xi_{1,1,2}^2 - e^{\lambda \xi_{1,3,3}} \lambda^3 \eta_{2,1,2} \eta_{2,2,3} \xi_{1,1,3} + e^{\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^4 \eta_{2,1,2} \eta_{2,1,3} \xi_{1,1,2} \xi_{1,1,3} + e^{2\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^5 \eta_{2,1,2} \eta_{2,1,3} \xi_{1,1,2}^2 \xi_{1,2,3} - e^{2\lambda \xi_{1,3,3}} \lambda^4 \eta_{2,1,3} \eta_{2,2,3} \xi_{1,1,3} \xi_{1,2,3} + e^{\lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^5 \eta_{2,1,3}^2 \xi_{1,1,2} \xi_{1,1,3} \xi_{1,2,3} - \frac{1}{2} e^{2\lambda \xi_{1,3,3}} \lambda^4 \eta_{2,2,3}^2 \xi_{1,2,3}^2 + \frac{1}{2} e^{2\lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^6 \eta_{2,1,3}^2 \xi_{1,1,2}^2 \xi_{1,2,3}^2 \right\}$$
- $$\gg \left\{ f_{3,3}[\lambda] \rightarrow e^{\lambda \xi_{1,3,3}} \lambda^3 \eta_{2,1,2} \eta_{2,2,3} \xi_{1,1,3} + \frac{1}{2} e^{2\lambda \xi_{1,3,3}} \lambda^4 \eta_{2,1,3}^2 \xi_{1,1,3}^2 + e^{2\lambda \xi_{1,3,3}} \lambda^4 \eta_{2,1,3} \eta_{2,2,3} \xi_{1,1,3} \xi_{1,2,3} + \frac{1}{2} e^{2\lambda \xi_{1,3,3}} \lambda^4 \eta_{2,2,3}^2 \xi_{1,2,3}^2 \right\}$$
- $$\gg \left\{ f_{1,2}[\lambda] \rightarrow e^{\lambda \xi_{1,3,3} - \lambda \xi_{2,1,1}} \lambda^4 \eta_{2,1,2} \eta_{2,2,3} \xi_{1,1,2} \xi_{1,1,3} - e^{-\lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3} - \lambda \xi_{2,1,1}} \lambda^4 \eta_{2,1,3} \eta_{2,2,3} \xi_{1,1,3} + e^{2\lambda \xi_{1,3,3} - \lambda \xi_{2,1,1}} \lambda^5 \eta_{2,1,3}^2 \xi_{1,1,2} \xi_{1,1,3}^2 - e^{-\lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3} - \lambda \xi_{2,1,1}} \lambda^4 \eta_{2,2,3}^2 \xi_{1,1,3} \xi_{1,2,3} + e^{2\lambda \xi_{1,3,3} - \lambda \xi_{2,1,1}} \lambda^5 \eta_{2,1,3} \eta_{2,2,3} \xi_{1,1,2} \xi_{1,1,3} \xi_{1,2,3} + e^{-\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^4 \eta_{2,1,2} \eta_{2,2,3} \xi_{1,1,3} \xi_{2,1,2} - e^{\lambda \xi_{1,3,3}} \lambda^5 \eta_{2,1,2} \eta_{2,1,3} \xi_{1,1,2} \xi_{1,1,3} \xi_{2,1,2} - e^{\lambda \xi_{1,3,3}} \lambda^5 \eta_{2,1,2} \eta_{2,2,3} \xi_{1,1,2} \xi_{1,2,3} \xi_{2,1,2} + e^{-\lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^5 \eta_{2,1,3} \eta_{2,2,3} \xi_{1,1,3} \xi_{1,2,3} \xi_{2,1,2} - e^{2\lambda \xi_{1,3,3}} \lambda^6 \eta_{2,1,3}^2 \xi_{1,1,2} \xi_{1,1,3} \xi_{1,2,3} \xi_{2,1,2} + e^{-\lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^5 \eta_{2,2,3}^2 \xi_{1,2,3} \xi_{2,1,2} - e^{2\lambda \xi_{1,3,3}} \lambda^6 \eta_{2,1,3} \eta_{2,2,3} \xi_{1,1,2} \xi_{1,2,3} \xi_{2,1,2} \right\}$$
- $$\gg \{f_{2,3}[\lambda] \rightarrow -\lambda^4 \eta_{2,1,2} \eta_{2,2,3} \xi_{1,1,3} \xi_{2,2,3}\}$$
- $$\gg \{f_{1,3}[\lambda] \rightarrow -\lambda^4 \eta_{2,1,2} \eta_{2,2,3} \xi_{1,1,3} \xi_{2,1,3}\}$$
- $$\gg \left\{ g_{1,3}[\lambda] \rightarrow -e^{-\lambda \xi_{1,1} + 2\lambda \xi_{1,2,2}} \lambda^4 \eta_{1,2,3} \eta_{2,1,2}^2 \xi_{1,1,2} + e^{-\lambda \xi_{1,1} + 2\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^5 \eta_{2,1,2}^2 \eta_{2,1,3} \xi_{1,1,2}^2 - e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^4 \eta_{1,2,3} \eta_{2,1,2} \eta_{2,1,3} \xi_{1,1,3} + 2 e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^5 \eta_{2,1,2} \eta_{2,1,3}^2 \xi_{1,1,2} \xi_{1,1,3} + e^{-\lambda \xi_{1,1} + 3\lambda \xi_{1,3,3}} \lambda^5 \eta_{2,1,3}^3 \xi_{1,1,3}^2 - 2 e^{-\lambda \xi_{1,1} + 2\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^5 \eta_{1,2,3} \eta_{2,1,2} \eta_{2,1,3} \xi_{1,1,2} \xi_{1,2,3} + 2 e^{-\lambda \xi_{1,1} + 2\lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^6 \eta_{2,1,2} \eta_{2,1,3}^2 \xi_{1,1,2}^2 \xi_{1,2,3} - e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^5 \eta_{1,2,3} \eta_{2,1,3}^2 \xi_{1,1,3} \xi_{1,2,3} + 2 e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2,2} + 3\lambda \xi_{1,3,3}} \lambda^6 \eta_{2,1,3}^3 \xi_{1,1,2} \xi_{1,1,3} \xi_{1,2,3} - e^{-\lambda \xi_{1,1} + 2\lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^6 \eta_{1,2,3} \eta_{2,1,3}^2 \xi_{1,1,2} \xi_{1,2,3} + e^{-\lambda \xi_{1,1} + 2\lambda \xi_{1,2,2} + 3\lambda \xi_{1,3,3}} \lambda^7 \eta_{2,1,3}^3 \xi_{1,1,2}^2 \xi_{1,2,3}^2 \right\}$$
- $$\gg \left\{ g_{1,2}[\lambda] \rightarrow e^{-\lambda \xi_{1,1} + 3\lambda \xi_{1,2,2}} \lambda^5 \eta_{2,1,2}^3 \xi_{1,1,2}^2 + 2 e^{-\lambda \xi_{1,1} + 2\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^5 \eta_{2,1,2}^2 \eta_{2,1,3} \xi_{1,1,2} \xi_{1,1,3} + e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^5 \eta_{2,1,2} \eta_{2,1,3}^2 \xi_{1,1,3}^2 + 3 e^{-\lambda \xi_{1,1} + 3\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^6 \eta_{2,1,2}^2 \eta_{2,1,3} \xi_{1,1,2}^2 \xi_{1,2,3} + 4 e^{-\lambda \xi_{1,1} + 2\lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^6 \eta_{2,1,2} \eta_{2,1,3}^2 \xi_{1,1,2} \xi_{1,1,3} \xi_{1,2,3} + e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2,2} + 3\lambda \xi_{1,3,3}} \lambda^6 \eta_{2,1,3}^3 \xi_{1,1,3}^2 \xi_{1,2,3} + 3 e^{-\lambda \xi_{1,1} + 3\lambda \xi_{1,2,2} + 2\lambda \xi_{1,3,3}} \lambda^7 \eta_{2,1,2} \eta_{2,1,3}^2 \xi_{1,1,2}^2 \xi_{1,2,3}^2 + 2 e^{-\lambda \xi_{1,1} + 2\lambda \xi_{1,2,2} + 3\lambda \xi_{1,3,3}} \lambda^7 \eta_{2,1,3}^3 \xi_{1,1,2} \xi_{1,1,3} \xi_{1,2,3} + e^{-\lambda \xi_{1,1} + 3\lambda \xi_{1,2,2} + 3\lambda \xi_{1,3,3}} \lambda^8 \eta_{2,1,3}^3 \xi_{1,1,2}^2 \xi_{1,2,3}^3 \right\}$$

$$\gg \{g_{2,3}[\lambda] \rightarrow e^{-\lambda \xi_{1,2} + 2\lambda \xi_{1,3}} \lambda^4 \eta_{2,1,2} \eta_{2,3}^2 \xi_{1,3} - 2 e^{2\lambda \xi_{1,3}} \lambda^5 \eta_{2,1,2} \eta_{2,1,3} \eta_{2,3} \xi_{1,2} \xi_{1,3} + e^{-\lambda \xi_{1,2} + 3\lambda \xi_{1,3}} \lambda^5 \eta_{2,1,3}^2 \eta_{2,3} \xi_{1,3}^2 - e^{3\lambda \xi_{1,3}} \lambda^6 \eta_{2,1,3}^3 \xi_{1,2} \xi_{1,3}^2 - e^{2\lambda \xi_{1,3}} \lambda^5 \eta_{2,1,2} \eta_{2,3}^2 \xi_{1,2} \xi_{1,3} + 2 e^{-\lambda \xi_{1,2} + 3\lambda \xi_{1,3}} \lambda^5 \eta_{2,1,3} \eta_{2,3}^2 \xi_{1,3} \xi_{1,2,3} - 2 e^{3\lambda \xi_{1,3}} \lambda^6 \eta_{2,1,3}^2 \eta_{2,3} \xi_{1,2} \xi_{1,3} \xi_{1,2,3} + e^{-\lambda \xi_{1,2} + 3\lambda \xi_{1,3}} \lambda^5 \eta_{2,3}^3 \xi_{1,2,3} - e^{3\lambda \xi_{1,3}} \lambda^6 \eta_{2,1,3} \eta_{2,3}^2 \xi_{1,2} \xi_{1,2,3}\}$$

Out[*]=

$$\begin{aligned} & \mathbb{E}_{(i,j) \rightarrow (k)} \left[y_{1,3}[k] (\eta_{1,3}[i] + e^{-\xi_{1,1}[i] + \xi_{3,3}[i]} \eta_{1,3}[j]) + x_{1,1}[k] (\xi_{1,1}[i] + \xi_{1,1}[j]) + \right. \\ & y_{2,3}[k] (\eta_{2,3}[i] + e^{-\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{2,3}[j] - e^{\xi_{3,3}[i]} \eta_{1,3}[j] \xi_{1,2}[i]) + \\ & x_{1,2}[k] (e^{-\xi_{1,1}[j]} \xi_{1,2}[i] + e^{-\xi_{2,2}[i]} \xi_{1,2}[j]) + x_{2,2}[k] (\xi_{2,2}[i] + \xi_{2,2}[j]) + \\ & y_{1,2}[k] (\eta_{1,2}[i] + e^{-\xi_{1,1}[i] + \xi_{2,2}[i]} \eta_{1,2}[j] + e^{-\xi_{1,1}[i] + \xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,3}[j] \xi_{2,3}[i]) + \\ & x_{1,3}[k] (e^{-\xi_{1,1}[j]} \xi_{1,3}[i] + e^{-\xi_{3,3}[i]} \xi_{1,3}[j] - \xi_{1,2}[j] \xi_{2,3}[i]) + \\ & x_{2,3}[k] (e^{-\xi_{2,2}[j]} \xi_{2,3}[i] + e^{-\xi_{3,3}[i]} \xi_{2,3}[j]) + x_{3,3}[k] (\xi_{3,3}[i] + \xi_{3,3}[j]), \\ & - e^{\xi_{3,3}[i]} x_{3,3}[k] (\eta_{1,3}[j] \xi_{1,3}[i] + \eta_{2,3}[j] \xi_{2,3}[i]) + \\ & x_{1,1}[k] (e^{\xi_{2,2}[i]} \eta_{1,2}[j] \xi_{1,2}[i] + e^{\xi_{3,3}[i]} \eta_{1,3}[j] \xi_{1,3}[i] + e^{\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,3}[j] \xi_{1,2}[i] \xi_{2,3}[i]) - \\ & e^{-\xi_{1,1}[i] + \xi_{2,2}[i]} y_{1,2}[k] (\eta_{1,2}[j] + e^{\xi_{3,3}[i]} \eta_{1,3}[j] \xi_{2,3}[i]) \\ & (e^{\xi_{2,2}[i]} \eta_{1,2}[j] \xi_{1,2}[i] + e^{\xi_{3,3}[i]} \eta_{1,3}[j] \xi_{1,3}[i] + e^{\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,3}[j] \xi_{1,2}[i] \xi_{2,3}[i]) - \\ & x_{2,2}[k] (e^{\xi_{2,2}[i]} \eta_{1,2}[j] \xi_{1,2}[i] - e^{\xi_{3,3}[i]} \eta_{2,3}[j] \xi_{2,3}[i] + e^{\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,3}[j] \xi_{1,2}[i] \xi_{2,3}[i]) + \\ & e^{-\xi_{1,1}[i]} y_{1,3}[k] (e^{\xi_{2,2}[i]} \eta_{1,2}[j] \eta_{2,3}[i] - e^{\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,2}[j] \eta_{1,3}[j] \xi_{1,2}[i] - e^{2\xi_{3,3}[i]} \eta_{1,3}[j]^2 \\ & \xi_{1,3}[i] + e^{\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,3}[j] \eta_{2,3}[i] \xi_{2,3}[i] - e^{\xi_{2,2}[i] + 2\xi_{3,3}[i]} \eta_{1,3}[j]^2 \xi_{1,2}[i] \xi_{2,3}[i]) + \\ & y_{2,3}[k] (e^{\xi_{3,3}[i]} \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,2}[i] - e^{-\xi_{2,2}[i] + 2\xi_{3,3}[i]} \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i] + e^{2\xi_{3,3}[i]} \eta_{1,3}[j]^2 \\ & \xi_{1,2}[i] \xi_{1,3}[i] - e^{-\xi_{2,2}[i] + 2\xi_{3,3}[i]} \eta_{2,3}[j]^2 \xi_{2,3}[i] + e^{2\xi_{3,3}[i]} \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,2}[i] \xi_{2,3}[i]) + \\ & x_{1,2}[k] (\eta_{1,2}[j] \xi_{1,2}[i] \xi_{1,2}[j] + e^{-\xi_{1,1}[j] - \xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{2,3}[j] \xi_{1,3}[i] - e^{-\xi_{1,1}[j] + \xi_{3,3}[i]} \eta_{1,3}[j] \xi_{1,2}[i] \\ & i \xi_{1,3}[i] - e^{-\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{2,3}[j] \xi_{1,2}[j] \xi_{2,3}[i] + e^{\xi_{3,3}[i]} \eta_{1,3}[j] \xi_{1,2}[i] \xi_{1,2}[j] \xi_{2,3}[i]) + \\ & x_{1,3}[k] (\eta_{1,2}[j] \xi_{1,2}[j] \xi_{1,3}[i] + \eta_{1,3}[j] \xi_{1,3}[i] \xi_{1,3}[j] + \eta_{2,3}[j] \xi_{1,3}[j] \xi_{2,3}[i]) + \\ & x_{2,3}[k] (-e^{-\xi_{2,2}[j]} \eta_{1,2}[j] \xi_{1,3}[i] + \eta_{1,3}[j] \xi_{1,3}[i] \xi_{2,3}[j] + \eta_{2,3}[j] \xi_{2,3}[i] \xi_{2,3}[j]), \\ & - x_{1,3}[k] \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,3}[i] \xi_{1,3}[j] - \frac{1}{2} x_{1,1}[k] \\ & (e^{\xi_{2,2}[i]} \eta_{1,2}[j] \xi_{1,2}[i] + e^{\xi_{3,3}[i]} \eta_{1,3}[j] \xi_{1,3}[i] + e^{\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,3}[j] \xi_{1,2}[i] \xi_{2,3}[i])^2 + \\ & e^{-\xi_{1,1}[i] + \xi_{2,2}[i]} y_{1,2}[k] (\eta_{1,2}[j] + e^{\xi_{3,3}[i]} \eta_{1,3}[j] \xi_{2,3}[i]) \\ & (e^{\xi_{2,2}[i]} \eta_{1,2}[j] \xi_{1,2}[i] + e^{\xi_{3,3}[i]} \eta_{1,3}[j] \xi_{1,3}[i] + e^{\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,3}[j] \xi_{1,2}[i] \xi_{2,3}[i])^2 + e^{-\xi_{1,1}[i]} \\ & y_{1,3}[k] (e^{\xi_{2,2}[i]} \eta_{1,2}[j] \xi_{1,2}[i] + e^{\xi_{3,3}[i]} \eta_{1,3}[j] \xi_{1,3}[i] + e^{\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,3}[j] \xi_{1,2}[i] \xi_{2,3}[i]) \\ & (-e^{\xi_{2,2}[i]} \eta_{1,2}[j] \eta_{2,3}[i] + e^{\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,2}[j] \eta_{1,3}[j] \xi_{1,2}[i] + e^{2\xi_{3,3}[i]} \eta_{1,3}[j]^2 \xi_{1,3}[i] - \\ & e^{\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,3}[j] \eta_{2,3}[i] \xi_{2,3}[i] + e^{\xi_{2,2}[i] + 2\xi_{3,3}[i]} \eta_{1,3}[j]^2 \xi_{1,2}[i] \xi_{2,3}[i]) + \\ & \frac{1}{2} e^{\xi_{3,3}[i]} x_{3,3}[k] (2 \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,3}[i] + e^{\xi_{3,3}[i]} \eta_{1,3}[j]^2 \xi_{1,3}[i]^2 + \\ & 2 e^{\xi_{3,3}[i]} \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i] \xi_{2,3}[i] + e^{\xi_{3,3}[i]} \eta_{2,3}[j]^2 \xi_{2,3}[i]^2) - e^{-\xi_{2,2}[i] + 2\xi_{3,3}[i]} \\ & y_{2,3}[k] (-\eta_{1,2}[j] \eta_{2,3}[j]^2 \xi_{1,3}[i] + 2 e^{\xi_{2,2}[i]} \eta_{1,2}[j] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,2}[i] \xi_{1,3}[i] - \\ & e^{\xi_{3,3}[i]} \eta_{1,3}[j]^2 \eta_{2,3}[j] \xi_{1,3}[i]^2 + e^{\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,3}[j]^3 \xi_{1,2}[i] \xi_{1,3}[i]^2 + \\ & e^{\xi_{2,2}[i]} \eta_{1,2}[j] \eta_{2,3}[j]^2 \xi_{1,2}[i] \xi_{2,3}[i] - 2 e^{\xi_{3,3}[i]} \eta_{1,3}[j] \eta_{2,3}[j]^2 \xi_{1,3}[i] \xi_{2,3}[i] + \\ & 2 e^{\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,3}[j]^2 \eta_{2,3}[j] \xi_{1,2}[i] \xi_{1,3}[i] \xi_{2,3}[i] - \\ & e^{\xi_{3,3}[i]} \eta_{2,3}[j]^3 \xi_{2,3}[i]^2 + e^{\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,3}[j] \eta_{2,3}[j]^2 \xi_{1,2}[i] \xi_{2,3}[i]^2) + \\ & \frac{1}{2} x_{2,2}[k] (e^{2\xi_{2,2}[i]} \eta_{1,2}[j]^2 \xi_{1,2}[i]^2 - 2 e^{\xi_{3,3}[i]} \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,3}[i] + 2 e^{\xi_{2,2}[i] + \xi_{3,3}[i]} \end{aligned}$$

$$\begin{aligned}
 & \eta_{1,2}[j] \eta_{1,3}[j] \xi_{1,2}[i] \xi_{1,3}[i] + 2 e^{2 \xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,2}[j] \eta_{1,3}[j] \xi_{1,2}[i]^2 \xi_{2,3}[i] - \\
 & 2 e^{2 \xi_{3,3}[i]} \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i] \xi_{2,3}[i] + 2 e^{\xi_{2,2}[i] + 2 \xi_{3,3}[i]} \eta_{1,3}[j]^2 \xi_{1,2}[i] \xi_{1,3}[i] \xi_{2,3}[i] - \\
 & e^{2 \xi_{3,3}[i]} \eta_{2,3}[j]^2 \xi_{2,3}[i]^2 + e^{2 \xi_{2,2}[i] + 2 \xi_{3,3}[i]} \eta_{1,3}[j]^2 \xi_{1,2}[i]^2 \xi_{2,3}[i]^2) - e^{-\xi_{1,1}[j] - \xi_{2,2}[i] + \xi_{3,3}[i]} \\
 & x_{1,2}[k] \left(-e^{\xi_{2,2}[i]} \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,2}[i] \xi_{1,3}[i] - e^{\xi_{1,1}[j]} \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,2}[j] \xi_{1,3}[i] + \right. \\
 & e^{\xi_{1,1}[j] + \xi_{2,2}[i]} \eta_{1,2}[j] \eta_{1,3}[j] \xi_{1,2}[i] \xi_{1,2}[j] \xi_{1,3}[i] + e^{\xi_{3,3}[i]} \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i]^2 - \\
 & e^{\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,3}[j]^2 \xi_{1,2}[i] \xi_{1,3}[i]^2 + e^{\xi_{1,1}[j] + \xi_{2,2}[i]} \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,2}[i] \xi_{1,2}[j] \xi_{2,3}[i] + \\
 & e^{\xi_{3,3}[i]} \eta_{2,3}[j]^2 \xi_{1,3}[i] \xi_{2,3}[i] - e^{\xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,2}[i] \xi_{1,3}[i] \xi_{2,3}[i] - \\
 & e^{\xi_{1,1}[j] + \xi_{3,3}[i]} \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,2}[j] \xi_{1,3}[i] \xi_{2,3}[i] + e^{\xi_{1,1}[j] + \xi_{2,2}[i] + \xi_{3,3}[i]} \eta_{1,3}[j]^2 \xi_{1,2}[i] \\
 & \left. \xi_{1,2}[j] \xi_{1,3}[i] \xi_{2,3}[i] - e^{\xi_{1,1}[j] + \xi_{3,3}[i]} \eta_{2,3}[j]^2 \xi_{1,2}[j] \xi_{2,3}[i]^2 + e^{\xi_{1,1}[j] + \xi_{2,2}[i] + \xi_{3,3}[i]} \right. \\
 & \left. \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,2}[i] \xi_{1,2}[j] \xi_{2,3}[i]^2) - x_{2,3}[k] \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,3}[i] \xi_{2,3}[j] \right]
 \end{aligned}$$

In[*]:= (cm[1, 2 → 2] // cm[2, 3 → 1]) ≡ (cm[2, 3 → 2] // cm[1, 2 → 1])

Out[*]=

True