

```

In[ ]:= PrintProfile[]
Out[ ]:=
ProfileRoot is root. Profiled time: 7.859
( 1) 0.063/ 7.859 above sol[1, 3]
TTR: called 1 times, time in 2.329/4.672
( 1) 2.329/ 4.672 under sol[1, 3]
( 15) 0.062/ 0.062 above CF
( 63) 1.299/ 2.234 above UB
( 88) 0.016/ 0.047 above UCF
UB: called 126 times, time in 2.207/4.109
( 63) 1.299/ 2.234 under TTR
( 24) 0.110/ 0.235 under SG
( 39) 0.798/ 1.640 under TT
( 3733) 0.981/ 1.902 above UCF
CF: called 7228 times, time in 1.562/1.562
( 55) 0.032/ 0.032 under DSolve
( 15) 0.062/ 0.062 under TTR
( 10) 0/ 0 under SG
( 7148) 1.468/ 1.468 under UCF
UCF: called 3852 times, time in 1.012/2.48
( 1) 0.015/ 0.015 under diff
( 88) 0.016/ 0.047 under TTR
( 14) 0/ 0 under SG
( 16) 0/ 0.516 under TT
( 3733) 0.981/ 1.902 under UB
( 7148) 1.468/ 1.468 above CF
SG: called 1 times, time in 0.343/0.578
( 1) 0.343/ 0.578 under sol[1, 3]
( 10) 0/ 0 above CF
( 24) 0.110/ 0.235 above UB
( 14) 0/ 0 above UCF
DSolve: called 1 times, time in 0.343/0.375
( 1) 0.343/ 0.375 under sol[1, 3]
( 55) 0.032/ 0.032 above CF
sol[1, 3]: called 1 times, time in 0.063/7.859
( 1) 0.063/ 7.859 under ProfileRoot
( 1) 0/ 0.015 above diff
( 1) 0.343/ 0.375 above DSolve
( 1) 0/ 0 above eqns
( 1) 0/ 2.156 above TTL
( 1) 2.329/ 4.672 above TTR
( 1) 0.343/ 0.578 above SG
TT: called 10 times, time in 0./6.187
( 1) 0/ 2.156 under TTL
( 9) 0/ 4.031 under TT
( 9) 0/ 4.031 above TT
( 39) 0.798/ 1.640 above UB

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( 16)      0/ 0.516 above UCF
TTL: called 1 times, time in 0./2.156
( 1)      0/ 2.156 under sol[1, 3]
( 1)      0/ 2.156 above TT
eqns: called 1 times, time in 0./0.
( 1)      0/ 0 under sol[1, 3]
diff: called 1 times, time in 0./0.015
( 1)      0/ 0.015 under sol[1, 3]
( 1)      0.015/ 0.015 above UCF

```

```

In[*]:= PotQ[0] = True;
PotQ[c_. U_i_[aj_] (EUs : U_[] ...)] /; (τ[aj] == "a") ∧ FreeQ[c, U] = True;
PotQ[x_Plus] := PotQ /@ (And @@ x);

```

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In[*]:= Ad[x_][y_] /; PotQ[x] ∧ (Head[y] != Plus) := eUB[y,x]/y y;
Ad[x_][y_Plus] /; PotQ[x] := Ad[x] /@ y;

```

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In[*]:= Ad[x_][y_] /; ¬ TrueQ[PotQ[x]] := Module[{k = 1, s = y, t = y},
  While[(t = UB[t, x]) != 0, s += t / k!; ++k];
  UCF[s]
]

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In[*]:= ¬ 3

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Out[*]=
! 3

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In[*]:= $n = 3

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Out[*]=
3

```

```

In[*]:= Cases[$PBWGens, g_ -> Which[
  τ[g] === "x", λ (g*)i Δi→j,k[Ui[g]],
  τ[g] === "a", (g*)i Δi→j,k[Ui[g]]
]]

```

```

Out[*]=
{
  λ ξ12i (Uj[A1[-1]] Uk[x12] + Uj[x12] Uk[A1[1]]),
  λ ξ13i (Uj[A1[-1], A2[-1]] Uk[x13] + (-1/q2 + q2) Uj[x12, A2[-1]] Uk[x23, A1[1]] +
    Uj[x13] Uk[A1[1], A2[1]]), λ ξ23i (Uj[A2[-1]] Uk[x23] + Uj[x23] Uk[A2[1]]),
  α1i (Uj[a1] Uk[] + Uj[] Uk[a1]), α2i (Uj[a2] Uk[] + Uj[] Uk[a2])
}

```

In[*]:= \$k = 1; \$n = 3;

TTL = TT_λ@Cases[\$PBWGens, g_ -> Which[
 τ[g] === "x", UCF[λ (g*)_i Δ_{i→j,k}[U_i[g]]],
 τ[g] === "a", UCF[(g*)_i Δ_{i→j,k}[U_i[g]]]
]]

Out[*]=

$$\begin{aligned}
 & e^{-\frac{1}{2}\gamma(2\alpha_1-\alpha_2)} \xi_{12_i} U_j[x_{12}] U_k[] + \\
 & \left(e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \xi_{13_i} + e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \lambda \xi_{12_i} \xi_{23_i} + e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \gamma \in \lambda \hbar \xi_{12_i} \xi_{23_i} \right) U_j[x_{13}] U_k[] + \\
 & e^{\frac{1}{2}\gamma(\alpha_1-2\alpha_2)} \xi_{23_i} U_j[x_{23}] U_k[] + 2 e^{-\frac{3\gamma\alpha_1}{2}} \gamma \in \lambda \hbar \xi_{12_i} \xi_{13_i} U_j[x_{12}, x_{13}] U_k[] - \\
 & 2 e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \gamma \in \lambda \hbar \xi_{12_i} \xi_{23_i} U_j[x_{12}, x_{23}] U_k[] + \\
 & 2 e^{-\gamma(\alpha_1+\alpha_2)} \gamma \in \lambda^2 \hbar \xi_{12_i} \xi_{13_i} \xi_{23_i} U_j[x_{13}, x_{13}] U_k[] + \\
 & 2 e^{-\frac{3\gamma\alpha_1}{2}} \gamma \in \lambda \hbar \xi_{13_i} \xi_{23_i} U_j[x_{13}, x_{23}] U_k[] - 2 e^{-\frac{1}{2}\gamma(2\alpha_1-\alpha_2)} \in \hbar \xi_{12_i} U_j[x_{12}] U_k[a_1] + \\
 & \left(-2 e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \in \hbar \xi_{13_i} - 2 e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \in \lambda \hbar \xi_{12_i} \xi_{23_i} \right) U_j[x_{13}] U_k[a_1] + \\
 & \left(-2 e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \in \hbar \xi_{13_i} - 2 e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \in \lambda \hbar \xi_{12_i} \xi_{23_i} \right) U_j[x_{13}] U_k[a_2] - \\
 & 2 e^{\frac{1}{2}\gamma(\alpha_1-2\alpha_2)} \in \hbar \xi_{23_i} U_j[x_{23}] U_k[a_2] + e^{-\frac{1}{2}\gamma(2\alpha_1-\alpha_2)} \xi_{12_i} U_j[] U_k[x_{12}] + \\
 & 2 e^{-\frac{1}{2}\gamma(2\alpha_1-\alpha_2)} \in \hbar \xi_{12_i} U_j[a_1] U_k[x_{12}] + 2 e^{-\frac{3\gamma\alpha_1}{2}} \gamma \in \lambda \hbar \xi_{12_i} \xi_{13_i} U_j[x_{13}] U_k[x_{12}] - \\
 & 2 e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \gamma \in \lambda \hbar \xi_{12_i} \xi_{23_i} U_j[x_{23}] U_k[x_{12}] + \\
 & \left(e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \xi_{13_i} + e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \lambda \xi_{12_i} \xi_{23_i} + e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \gamma \in \lambda \hbar \xi_{12_i} \xi_{23_i} \right) U_j[] U_k[x_{13}] + \\
 & \left(2 e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \in \hbar \xi_{13_i} + 2 e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \in \lambda \hbar \xi_{12_i} \xi_{23_i} \right) U_j[a_1] U_k[x_{13}] + \\
 & \left(2 e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \in \hbar \xi_{13_i} + 2 e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \in \lambda \hbar \xi_{12_i} \xi_{23_i} \right) U_j[a_2] U_k[x_{13}] + \\
 & 2 e^{-\frac{3\gamma\alpha_1}{2}} \gamma \in \lambda \hbar \xi_{12_i} \xi_{13_i} U_j[x_{12}] U_k[x_{13}] + \\
 & 4 e^{-\gamma(\alpha_1+\alpha_2)} \gamma \in \lambda^2 \hbar \xi_{12_i} \xi_{13_i} \xi_{23_i} U_j[x_{13}] U_k[x_{13}] + 2 e^{-\frac{3\gamma\alpha_1}{2}} \gamma \in \lambda \hbar \xi_{13_i} \xi_{23_i} U_j[x_{23}] U_k[x_{13}] + \\
 & e^{\frac{1}{2}\gamma(\alpha_1-2\alpha_2)} \xi_{23_i} U_j[] U_k[x_{23}] + 2 e^{\frac{1}{2}\gamma(\alpha_1-2\alpha_2)} \in \hbar \xi_{23_i} U_j[a_2] U_k[x_{23}] + \\
 & \left(4 e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \gamma \in \hbar \xi_{13_i} + 2 e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \gamma \in \lambda \hbar \xi_{12_i} \xi_{23_i} \right) U_j[x_{12}] U_k[x_{23}] + \\
 & 2 e^{-\frac{3\gamma\alpha_1}{2}} \gamma \in \lambda \hbar \xi_{13_i} \xi_{23_i} U_j[x_{13}] U_k[x_{23}] + 2 e^{-\frac{3\gamma\alpha_1}{2}} \gamma \in \lambda \hbar \xi_{12_i} \xi_{13_i} U_j[] U_k[x_{12}, x_{13}] - \\
 & 2 e^{-\frac{1}{2}\gamma(\alpha_1+\alpha_2)} \gamma \in \lambda \hbar \xi_{12_i} \xi_{23_i} U_j[] U_k[x_{12}, x_{23}] + \\
 & 2 e^{-\gamma(\alpha_1+\alpha_2)} \gamma \in \lambda^2 \hbar \xi_{12_i} \xi_{13_i} \xi_{23_i} U_j[] U_k[x_{13}, x_{13}] + 2 e^{-\frac{3\gamma\alpha_1}{2}} \gamma \in \lambda \hbar \xi_{13_i} \xi_{23_i} U_j[] U_k[x_{13}, x_{23}]
 \end{aligned}$$

```
In[*]:= mons[-1] = {};
mons[p_] :=
  mons[p-1] ∪ Union@Cases[Coefficient[TTL, ε, p], c_. Uj[x___] Uk[y___] :> Uj[x] Uk[y], ∞];
mons /@ {0, 1}
```

```
Out[*]=
{ {Uj[x12] Uk[], Uj[x13] Uk[], Uj[x23] Uk[], Uj[] Uk[x12], Uj[] Uk[x13], Uj[] Uk[x23]},
  {Uj[x12] Uk[], Uj[x13] Uk[], Uj[x23] Uk[], Uj[x12, x13] Uk[], Uj[x12, x23] Uk[],
   Uj[x13, x13] Uk[], Uj[x13, x23] Uk[], Uj[x12] Uk[a1], Uj[x13] Uk[a1], Uj[x13] Uk[a2],
   Uj[x23] Uk[a2], Uj[] Uk[x12], Uj[a1] Uk[x12], Uj[x13] Uk[x12], Uj[x23] Uk[x12],
   Uj[] Uk[x13], Uj[a1] Uk[x13], Uj[a2] Uk[x13], Uj[x12] Uk[x13], Uj[x13] Uk[x13],
   Uj[x23] Uk[x13], Uj[] Uk[x23], Uj[a2] Uk[x23], Uj[x12] Uk[x23], Uj[x13] Uk[x23],
   Uj[] Uk[x12, x13], Uj[] Uk[x12, x23], Uj[] Uk[x13, x13], Uj[] Uk[x13, x23]} }
```

```
In[*]:= $k = 2
```

```
Out[*]=
2
```

```
In[*]:= Δx14 =
```

$$\text{UCF}\left[U_j[A1[-1], A2[-1], A3[-1]] U_k[x14] + \left(-\frac{1}{q^2} + q^2\right) U_j[x12, A2[-1], A3[-1]] U_k[x24, A1[1]] + \right. \\ \left. \left(-\frac{1}{q^2} + q^2\right) U_j[x13, A3[-1]] U_k[x34, A1[1], A2[1]] + U_j[x14] U_k[A1[1], A2[1], A3[1]]\right]$$

```
Out[*]=
Uj[x14] Uk[] - 2 ∈ ħ Uj[x14] Uk[a1] - 2 ∈ ħ Uj[x14] Uk[a2] -
  2 ∈ ħ Uj[x14] Uk[a3] + Uj[] Uk[x14] + 2 ∈ ħ Uj[a1] Uk[x14] + 2 ∈ ħ Uj[a2] Uk[x14] +
  2 ∈ ħ Uj[a3] Uk[x14] + 2 ∈^2 ħ^2 Uj[a1, a1] Uk[x14] + 4 ∈^2 ħ^2 Uj[a1, a2] Uk[x14] +
  4 ∈^2 ħ^2 Uj[a1, a3] Uk[x14] + 2 ∈^2 ħ^2 Uj[a2, a2] Uk[x14] + 4 ∈^2 ħ^2 Uj[a2, a3] Uk[x14] +
  2 ∈^2 ħ^2 Uj[a3, a3] Uk[x14] + 4 γ ∈ ħ Uj[x12] Uk[x24] + 8 γ ∈^2 ħ^2 Uj[x12, a2] Uk[x24] +
  8 γ ∈^2 ħ^2 Uj[x12, a3] Uk[x24] + 4 γ ∈ ħ Uj[x13] Uk[x34] + 8 γ ∈^2 ħ^2 Uj[x13, a3] Uk[x34] +
  2 ∈^2 ħ^2 Uj[x14] Uk[a1, a1] + 4 ∈^2 ħ^2 Uj[x14] Uk[a1, a2] + 4 ∈^2 ħ^2 Uj[x14] Uk[a1, a3] +
  2 ∈^2 ħ^2 Uj[x14] Uk[a2, a2] + 4 ∈^2 ħ^2 Uj[x14] Uk[a2, a3] + 2 ∈^2 ħ^2 Uj[x14] Uk[a3, a3] -
  8 γ ∈^2 ħ^2 Uj[x12] Uk[x24, a1] - 8 γ ∈^2 ħ^2 Uj[x13] Uk[x34, a1] - 8 γ ∈^2 ħ^2 Uj[x13] Uk[x34, a2]
```

```
In[*]:= Δx14 /. ε -> 0
```

```
Out[*]=
Uj[x14] Uk[] + Uj[] Uk[x14]
```