

Pensieve header: The PBW multiplication tensor for $\mathfrak{sl}_n$ using the λ -tangent formalism. Some material from pensieve://Projects/UEA.

```
In[ ]:= SetDirectory@"C:\\drorbn\\AcademicPensieve\\Projects\\SolvablePBW";
<< KnotTheory`
```

Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at <http://katlas.org/wiki/KnotTheory>.

Prolog

```
In[ ]:= BeginPackage["UEA`"];
Print["UEA` does computations in general universal enveloping
algebras and PBW algebras. It is in the public domain, available
at http://drorbn.net/AcademicPensieve/Projects/SolvablePBW/.
Dror Bar-Natan is committed to support it within
reason until June 1, 2027. This is version 260601."];
Print["UEA` implements / extends ",
Sort@{"**",  $\epsilon$ , $k, CF, SSQ, NilQ, B, m, SetAlgebra, U,
UB, UProducts, USimp, UU, $Basis, $PBWRule,  $\delta$ , adPower, adExp},
"."];
Begin["`Private`"];
```

UEA` does computations in general universal enveloping algebras and PBW algebras. It is in the public domain, available at <http://drorbn.net/AcademicPensieve/Projects/SolvablePBW/>. Dror Bar-Natan is committed to support it within reason until June 1, 2027. This is version 260601.

UEA` implements / extends {**, adExp, adPower, B, CF, m, NilQ, SetAlgebra, SSQ, U, UB, UProducts, USimp, UU, δ , ϵ , \$Basis, \$k, \$PBWRule}.

Utilities

```
In[ ]:=  $\delta_{i,j}$  := If[i == j, 1, 0];
```

```
In[ ]:= SSQ[c_.*x_] /; MemberQ[$Basis, x] :=  $\neg$  NilQ[x]; (* Semi-Simple Q *)
NilQ[c_.*x_] /; MemberQ[$Basis, x] := NilQ[x];
SSQ[x_Plus] := And @@ (SSQ /@ (List @@ x));
NilQ[x_Plus] := And @@ (NilQ /@ (List @@ x));
```

```
In[ ]:= $k = 1;
```

```
In[ ]:= CF[ $\mathcal{E}$ _] := Expand[ $\mathcal{E}$  /.  $e^L \rightarrow e^{\text{Expand}[L]}$ ] /. { $e^{k\cdot}$  /;  $k > \$k \rightarrow 0$ };
```

Implementing general universal enveloping algebras

```
In[*]:= B[0, _] = 0; B[_ , 0] = 0;
B[c_ * x_, y_] /; MemberQ[$Basis, x] := CF[c B[x, y]];
B[y_, c_ * x_] /; MemberQ[$Basis, x] := CF[c B[y, x]];
B[x_Plus, y_] := B[#, y] & /@ x;
B[x_, y_Plus] := B[x, #] & /@ y;
B[x_, x_] = 0;
B[y_, x_] := CF[-B[x, y]];
```

```
In[*]:= x_ ≤ y_ := OrderedQ[{x, y} /. $PBWRule]; x_ < y_ := ! OrderedQ[{y, x} /. $PBWRule];
UU_i_[1] := U_i[];
UU_i_[x_P_] := UU_i_@@Table[x, {p}];
UU_i_[ε_] := ε /. {
  U[xs_] => U_i[xs],
  x_ /; MemberQ[$Basis, x] => U_i[x]
};
UU_i_[x_, xs_] := UU_t1[x] UU_t2[xs] // Expand // m_{t1,t2→i};
USimp[ε_] := Collect[ε, Times[U_[] ..], Expand];
USimp[ε_] := Expand[ε];
```

```
In[*]:= m_s_[0] = 0;
m_s_[x_Plus] := m_s_ /@ x;
m_s_[sd_SeriesData] := MapAt[m_s, sd, {3, All}];
m_{i→j}[ε_] := ε /. U_i → U_j;
```

```
In[*]:= m_{i,j→k}[c_. U_i[x_] U_j[]] := c U_k[x];
m_{i,j→k}[c_. U_i[] U_j[y_]] := c U_k[y];
m_{i,j→k}[c_. U_i[xx_, x_] U_j[y_, yy_]] := If[x ≤ y,
  c U_k[xx, x, y, yy],
  ((U_i[xx] (U_j[y, x] + UU_j[B[x, y]])) // Expand // m_{i,j→i} U_j[yy] // Expand // m_{i,j→k}
  c // USimp
];
```

```
In[*]:= UProducts[{}, 0] = {1}; UProducts[{}, d_Integer] /; d > 0 = {};
UProducts[{i_, is_}, d_Integer] := Sort@
  Flatten@Table[(U_i@@@Subsets[$Basis, {j}]) u, {j, 0, d}, {u, UProducts[{is, d - j]}}];
```

```
In[*]:= Supp[ε_] := Union@Cases[{ε}, U_i[___] => i, ∞];
```

```
In[*]:= Unprotect[NonCommutativeMultiply];
NonCommutativeMultiply[x_] := x;
x_ ⋈ y_ := Module[{is = Supp[x] ∩ Supp[y], σ, z},
  z = x; Do[z = mi→σ@i[z], {i, is}];
  z = Expand[y z]; Do[z = mσ@i, i→i[z], {i, is}]; z];
UB[x_, y_] := USimp[x ⋈ y - y ⋈ x];
```

Epilog

```
In[*]:= End[]; EndPackage[];
```

Predefined Algebras

sl(2)

```
In[*]:= Print["UEA`SetAlgebra knows \"sl2\"."];
```

UEA`SetAlgebra knows "sl2".

```
In[*]:= SetAlgebra["sl2"] := (
  Print["In sl2: ⟨e,h,f⟩ / ([h,e]=2e, [h,f]=-2f, [e,f]=h)."];
  B[h, e] = 2 e; B[h, f] = -2 f; B[e, f] = h;
  $Algebra = "sl2";
  $Basis = {e, h, f};
  $PBWRule = {e → 1, h → 2, f → 3};
  NilQ[e] = NilQ[f] = True; NilQ[h] = False;
);
```

$gl_{n,\epsilon}$

```
In[*]:= Print["UEA`SetAlgebra knows the ε-nilpotent algebra gln,ε."];
```

UEA`SetAlgebra knows the ϵ -nilpotent algebra $gl_{n,\epsilon}$.

```

In[ ]:= SetAlgebra[gln,ε] := (
  $Algebra = gln,ε;
  $Basis = Flatten@{
    Table[yα,β, {β, 2, n}, {α, 1, β - 1}],
    Table[xα,β, {β, 1, n}, {α, 1, β}]
  };
  NilQ[y_] = True; NilQ[xα,β] := (α != β); (* Nilpotent Q *)
  $PBWRule = Thread[$Basis → Range@Length@$Basis];
  B[xi,j, xk,l] := δj,k xi,l - δl,i xk,j;
  B[yi,j, yk,l] := CF[ε δj,k yi,l - ε δl,i yk,j];
  B[xi,j, yk,l] := CF[δj,k xi,l - δl,i xk,j /. xα,β → If[α ≤ β, ε xα,β, yβ,α]];
);

```

```

In[ ]:= SetAlgebra[gl2,ε];
$PBWRule
MatrixForm@Table[{b1, b2} → B[b1, b2], {b1, $Basis}, {b2, $Basis}]

```

```

Out[ ]:=
{y1,2 → 1, x1,1 → 2, x1,2 → 3, x2,2 → 4}

```

```

Out[ ]//MatrixForm=

$$\left( \begin{array}{cccc} \{y_{1,2}, y_{1,2}\} \rightarrow 0 & \{y_{1,2}, x_{1,1}\} \rightarrow y_{1,2} & \{y_{1,2}, x_{1,2}\} \rightarrow -\epsilon x_{1,1} + \epsilon x_{2,2} & \{y_{1,2}, x_{2,2}\} \rightarrow -y_{1,2} \\ \{x_{1,1}, y_{1,2}\} \rightarrow -y_{1,2} & \{x_{1,1}, x_{1,1}\} \rightarrow 0 & \{x_{1,1}, x_{1,2}\} \rightarrow x_{1,2} & \{x_{1,1}, x_{2,2}\} \rightarrow 0 \\ \{x_{1,2}, y_{1,2}\} \rightarrow \epsilon x_{1,1} - \epsilon x_{2,2} & \{x_{1,2}, x_{1,1}\} \rightarrow -x_{1,2} & \{x_{1,2}, x_{1,2}\} \rightarrow 0 & \{x_{1,2}, x_{2,2}\} \rightarrow x_{1,2} \\ \{x_{2,2}, y_{1,2}\} \rightarrow y_{1,2} & \{x_{2,2}, x_{1,1}\} \rightarrow 0 & \{x_{2,2}, x_{1,2}\} \rightarrow -x_{1,2} & \{x_{2,2}, x_{2,2}\} \rightarrow 0 \end{array} \right)$$


```

```

In[ ]:= adPower[x_, k_, Env_] [c_ * y_] /; MemberQ[$Basis, y] := CF[c adPower[x, k, Env] [y]];
adPower[x_, k_, Env_] [y_Plus] := adPower[x, k, Env] /@ y;
adPower[_ , 0, Env_] [y_] := y;
adPower[x_, k_, Env_] [0] = 0;
adPower[x_, k_, Env_] [y_] /; MemberQ[$Basis, y] :=
  adPower[x, k, Env] [y] = B[adPower[x, k - 1, Env] [y], x];
adPower[x_, k_] [y_] := adPower[x, k, {$Algebra, $k}] [y];

```

```

In[ ]:= x0 = ($Basis /. {x → ξ0, y → η0}).$Basis
x1 = ($Basis /. {x → ξ1, y → η1}).$Basis

```

```

Out[ ]:=
y1,2 η0,1,2 + x1,1 ξ0,1,1 + x1,2 ξ0,1,2 + x2,2 ξ0,2,2

```

```

Out[ ]:=
y1,2 η1,1,2 + x1,1 ξ1,1,1 + x1,2 ξ1,1,2 + x2,2 ξ1,2,2

```

```

In[ ]:= adExp[x_] [y_] /; SSQ[x] := Expand@Module[{A, b, c},
  A = Table[Coefficient[B[b, x], c], {c, $Basis}, {b, $Basis}];
  CF[$Basis.Normal[Series[MatrixExp[A], {ε, 0, $k}]]].
  Table[Coefficient[y, b], {b, $Basis}]
]

```

In[*]:= **SSQ**[$\xi_{0,1}$ $x_{1,1}$]

Out[*]=

True

In[*]:= **adExp**[$\xi_{0,1}$ $x_{1,1}$] [**x1**]

Out[*]=

$e^{\xi_{0,1}} y_{1,2} \eta_{1,2} + x_{1,1} \xi_{1,1} + e^{-\xi_{0,1}} x_{1,2} \xi_{1,2} + x_{2,2} \xi_{1,2}$

```
adExp[x_][y_] /; NilQ[x] := Expand@Module[{k = 0, s = 0},
  While[adPower[x, k][y] != 0,
    s += adPower[x, k][y] / k!;
    ++k
  ];
  s
]
```

In[*]:= **adExp**[$\xi_{0,2}$ $x_{1,2}$] [**x1**]

Out[*]=

$y_{1,2} \eta_{1,2} - \epsilon x_{1,1} \eta_{1,2} \xi_{0,2} + \epsilon x_{2,2} \eta_{1,2} \xi_{0,2} - \epsilon x_{1,2} \eta_{1,2} \xi_{0,2}^2 +$
 $x_{1,1} \xi_{1,1} + x_{1,2} \xi_{0,2} \xi_{1,1} + x_{1,2} \xi_{1,2} + x_{2,2} \xi_{1,2} - x_{1,2} \xi_{0,2} \xi_{1,2}$

```
λTangent[ ] = 0;
λTangent[xs___, x_] := adExp[x][λTangent[xs]] +  $\partial_{\lambda}$  x;
λTangent[xs_List] := λTangent@@xs
```

In[*]:= **\$Basis** (**\$Basis** /. {**x** → ξ , **y** → η })

Out[*]=

{ $y_{1,2} \eta_{1,2}$, $x_{1,1} \xi_{1,1}$, $x_{1,2} \xi_{1,2}$, $x_{2,2} \xi_{2,2}$ }

In[*]:= **λTangent**@(**λ** **\$Basis** (**\$Basis** /. {**x** → ξ , **y** → η }))

Out[*]=

$e^{\lambda \xi_{1,1} - \lambda \xi_{2,2}} y_{1,2} \eta_{1,2} + x_{1,1} \xi_{1,1} + e^{\lambda \xi_{2,2}} x_{1,2} \xi_{1,2} - e^{\lambda \xi_{1,1}} \epsilon \lambda x_{1,1} \eta_{1,2} \xi_{1,2} +$
 $e^{\lambda \xi_{1,1}} \epsilon \lambda x_{2,2} \eta_{1,2} \xi_{1,2} + e^{\lambda \xi_{2,2}} \lambda x_{1,2} \xi_{1,1} \xi_{1,2} - e^{\lambda \xi_{1,1} + \lambda \xi_{2,2}} \epsilon \lambda^2 x_{1,2} \eta_{1,2} \xi_{1,2}^2 + x_{2,2} \xi_{2,2}$

In[*]:= **\$Basis**

Out[*]=

{ $y_{1,2}$, $x_{1,1}$, $x_{1,2}$, $x_{2,2}$ }

```

In[*]:= lhs =  $\lambda$ Tangent[
  Join[ $\lambda$ $Basis ($Basis /. {x  $\rightarrow$   $\xi_1$ , y  $\rightarrow$   $\eta_1$ }),  $\lambda$ $Basis ($Basis /. {x  $\rightarrow$   $\xi_2$ , y  $\rightarrow$   $\eta_2$ })]
]

Out[*]=

$$\begin{aligned}
& e^{\lambda \xi_{1,1} - \lambda \xi_{1,2} + \lambda \xi_{2,1} - \lambda \xi_{2,2}} y_{1,2} \eta_{1,2} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} y_{1,2} \eta_{2,2} + x_{1,1} \xi_{1,1} - \\
& e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda y_{1,2} \eta_{2,2} \xi_{1,1} + e^{\lambda \xi_{1,2} - \lambda \xi_{2,1} + \lambda \xi_{2,2}} x_{1,2} \xi_{1,2} - e^{\lambda \xi_{1,1}} \in \lambda x_{1,1} \eta_{1,2} \xi_{1,2} + \\
& e^{\lambda \xi_{1,1}} \in \lambda x_{2,2} \eta_{1,2} \xi_{1,2} + e^{\lambda \xi_{1,2}} \in \lambda x_{1,1} \eta_{2,2} \xi_{1,2} - e^{\lambda \xi_{1,2}} \in \lambda x_{2,2} \eta_{2,2} \xi_{1,2} + \\
& 2 e^{\lambda \xi_{1,1} + \lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^2 y_{1,2} \eta_{1,2} \eta_{2,2} \xi_{1,2} - e^{\lambda \xi_{1,2} + \lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^2 y_{1,2} \eta_{2,2}^2 \xi_{1,2} + \\
& e^{\lambda \xi_{1,2} - \lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda x_{1,2} \xi_{1,1} \xi_{1,2} + e^{\lambda \xi_{1,2}} \in \lambda^2 x_{1,1} \eta_{2,2} \xi_{1,1} \xi_{1,2} - e^{\lambda \xi_{1,2}} \in \lambda^2 x_{2,2} \eta_{2,2} \xi_{1,1} \xi_{1,2} - \\
& e^{\lambda \xi_{1,2} + \lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^3 y_{1,2} \eta_{2,2}^2 \xi_{1,1} \xi_{1,2} - e^{\lambda \xi_{1,1} + \lambda \xi_{1,2} - \lambda \xi_{2,1} + \lambda \xi_{2,2}} \in \lambda^2 x_{1,2} \eta_{1,2} \xi_{1,2}^2 + x_{2,2} \xi_{1,2} + \\
& e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda y_{1,2} \eta_{2,2} \xi_{1,2} + x_{1,1} \xi_{2,1} + e^{\lambda \xi_{2,2}} x_{1,2} \xi_{2,2} - e^{\lambda \xi_{1,1} - \lambda \xi_{1,2} + \lambda \xi_{2,1}} \in \lambda x_{1,1} \eta_{1,2} \xi_{2,2} + \\
& e^{\lambda \xi_{1,1} - \lambda \xi_{1,2} + \lambda \xi_{2,1}} \in \lambda x_{2,2} \eta_{1,2} \xi_{2,2} - e^{\lambda \xi_{2,1}} \in \lambda x_{1,1} \eta_{2,2} \xi_{2,2} + e^{\lambda \xi_{2,1}} \in \lambda x_{2,2} \eta_{2,2} \xi_{2,2} + \\
& e^{\lambda \xi_{2,2}} \lambda x_{1,2} \xi_{1,1} \xi_{2,2} + e^{\lambda \xi_{2,1}} \in \lambda^2 x_{1,1} \eta_{2,2} \xi_{1,1} \xi_{2,2} - e^{\lambda \xi_{2,1}} \in \lambda^2 x_{2,2} \eta_{2,2} \xi_{1,1} \xi_{2,2} - \\
& 2 e^{\lambda \xi_{1,1} + \lambda \xi_{2,2}} \in \lambda^2 x_{1,2} \eta_{1,2} \xi_{1,2} \xi_{2,2} + 2 e^{\lambda \xi_{1,2} + \lambda \xi_{2,2}} \in \lambda^2 x_{1,2} \eta_{2,2} \xi_{1,2} \xi_{2,2} + \\
& 2 e^{\lambda \xi_{1,2} + \lambda \xi_{2,2}} \in \lambda^3 x_{1,2} \eta_{2,2} \xi_{1,1} \xi_{1,2} \xi_{2,2} - e^{\lambda \xi_{2,2}} \lambda x_{1,2} \xi_{1,2} \xi_{2,2} - \\
& e^{\lambda \xi_{2,1}} \in \lambda^2 x_{1,1} \eta_{2,2} \xi_{1,2} \xi_{2,2} + e^{\lambda \xi_{2,1}} \in \lambda^2 x_{2,2} \eta_{2,2} \xi_{1,2} \xi_{2,2} + e^{\lambda \xi_{2,2}} \lambda x_{1,2} \xi_{2,1} \xi_{2,2} - \\
& e^{\lambda \xi_{1,1} - \lambda \xi_{1,2} + \lambda \xi_{2,1} + \lambda \xi_{2,2}} \in \lambda^2 x_{1,2} \eta_{1,2} \xi_{2,2}^2 - e^{\lambda \xi_{2,1} + \lambda \xi_{2,2}} \in \lambda^2 x_{1,2} \eta_{2,2} \xi_{2,2}^2 + \\
& e^{\lambda \xi_{2,1} + \lambda \xi_{2,2}} \in \lambda^3 x_{1,2} \eta_{2,2} \xi_{1,1} \xi_{2,2}^2 - e^{\lambda \xi_{2,1} + \lambda \xi_{2,2}} \in \lambda^3 x_{1,2} \eta_{2,2} \xi_{1,2} \xi_{2,2}^2 + x_{2,2} \xi_{2,2}
\end{aligned}$$


In[*]:= rhs =  $\lambda$ Tangent@($Basis ($Basis /. {x $\alpha$ ,  $\alpha$   $\rightarrow$  CF[f $\alpha$ ,  $\alpha$ ][ $\lambda$ ], x $\alpha\beta$   $\rightarrow$  f $\alpha\beta$ [ $\lambda$ ], y $\alpha\beta$   $\rightarrow$  g $\alpha\beta$ [ $\lambda$ ]}))

Out[*]=

$$\begin{aligned}
& x_{1,1} f_{1,1}'[\lambda] + e^{f_{2,2}[\lambda]} x_{1,2} f_{1,2}[\lambda] f_{1,1}'[\lambda] + e^{f_{2,2}[\lambda]} x_{1,2} f_{1,2}'[\lambda] + \\
& x_{2,2} f_{2,2}'[\lambda] + e^{f_{1,1}[\lambda] - f_{2,2}[\lambda]} y_{1,2} g_{1,2}'[\lambda] - e^{f_{1,1}[\lambda]} \in x_{1,1} f_{1,2}[\lambda] g_{1,2}'[\lambda] + \\
& e^{f_{1,1}[\lambda]} \in x_{2,2} f_{1,2}[\lambda] g_{1,2}'[\lambda] - e^{f_{1,1}[\lambda] + f_{2,2}[\lambda]} \in x_{1,2} f_{1,2}[\lambda]^2 g_{1,2}'[\lambda]
\end{aligned}$$


In[*]:= SSBasis = Select[$Basis, SSQ]

Out[*]=
{ $x_{1,1}$ ,  $x_{2,2}$ }

In[*]:= eqns = Simplify[(Coefficient[lhs - rhs, #] == 0) & /@ $Basis]

Out[*]=

$$\begin{aligned}
& \{ e^{\lambda (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})} \eta_{1,2} (1 + 2 e^{\lambda \xi_{1,2}} \in \lambda^2 \eta_{2,2} \xi_{1,2}) = \\
& e^{\lambda (\xi_{1,2} + \xi_{2,1} - \xi_{2,2})} \in \lambda^2 \eta_{2,2}^2 (1 + \lambda \xi_{1,1}) \xi_{1,2} + \\
& e^{\lambda (\xi_{2,1} - \xi_{2,2})} \eta_{2,2} (-1 + \lambda \xi_{1,1} - \lambda \xi_{1,2}) + e^{f_{1,1}[\lambda] - f_{2,2}[\lambda]} g_{1,2}'[\lambda], \\
& \xi_{2,1} + \xi_{1,1} (1 + \in \lambda^2 \eta_{2,2} (e^{\lambda \xi_{1,2}} \xi_{1,2} + e^{\lambda \xi_{2,1}} \xi_{2,2})) + e^{f_{1,1}[\lambda]} \in f_{1,2}[\lambda] g_{1,2}'[\lambda] = \\
& e^{\lambda (\xi_{1,1} - \xi_{1,2})} \in \lambda \eta_{1,2} (e^{\lambda \xi_{1,2}} \xi_{1,2} + e^{\lambda \xi_{2,1}} \xi_{2,2}) + \\
& \in \lambda \eta_{2,2} (-e^{\lambda \xi_{1,2}} \xi_{1,2} + e^{\lambda \xi_{2,1}} (1 + \lambda \xi_{1,2}) \xi_{2,2}) + f_{1,1}'[\lambda], \\
& e^{-\lambda (\xi_{1,2} + \xi_{2,1})} (-e^{\lambda (\xi_{1,1} + 2 \xi_{1,2} + \xi_{2,2})} \in \lambda^2 \eta_{1,2} \xi_{1,2}^2 + e^{\lambda (\xi_{1,2} + \xi_{2,2})} \xi_{1,2} (-2 e^{\lambda (\xi_{1,1} + \xi_{2,1})} \in \lambda^2 \eta_{1,2} \\
& \xi_{2,2} + e^{\lambda \xi_{2,2}} (1 + 2 e^{\lambda \xi_{2,1}} \in \lambda^2 \eta_{2,2} \xi_{2,2}) + e^{\lambda \xi_{1,2}} \lambda \xi_{1,1} (1 + 2 e^{\lambda \xi_{2,1}} \in \lambda^2 \eta_{2,2} \xi_{2,2})) - \\
& e^{\lambda \xi_{2,1}} (e^{\lambda (\xi_{1,2} + \xi_{2,2})} (-1 - \lambda \xi_{1,1} + \lambda \xi_{1,2} - \lambda \xi_{2,1}) \xi_{2,2} + \\
& e^{\lambda (\xi_{2,1} + \xi_{2,2})} \in \lambda^2 (e^{\lambda \xi_{1,1}} \eta_{1,2} + e^{\lambda \xi_{1,2}} \eta_{2,2} (1 - \lambda \xi_{1,1} + \lambda \xi_{1,2})) \xi_{2,2}^2 + \\
& e^{\lambda \xi_{1,2} + f_{2,2}[\lambda]} (f_{1,2}[\lambda] f_{1,1}'[\lambda] + f_{1,2}'[\lambda] - e^{f_{1,1}[\lambda]} \in f_{1,2}[\lambda]^2 g_{1,2}'[\lambda])) = 0, \\
& \xi_{1,2} + \in \lambda \eta_{1,2} (e^{\lambda \xi_{1,1}} \xi_{1,2} + e^{\lambda (\xi_{1,1} - \xi_{1,2} + \xi_{2,1})} \xi_{2,2}) + \xi_{2,2} = \\
& \in \lambda \eta_{2,2} (e^{\lambda \xi_{1,2}} (1 + \lambda \xi_{1,1}) \xi_{1,2} + e^{\lambda \xi_{2,1}} (-1 + \lambda \xi_{1,1} - \lambda \xi_{1,2}) \xi_{2,2}) + \\
& f_{2,2}'[\lambda] + e^{f_{1,1}[\lambda]} \in f_{1,2}[\lambda] g_{1,2}'[\lambda] \}
\end{aligned}$$


```

```

In[*]:= Length@eqns
Out[*]=
4

In[*]:= unknowns = $Basis /. {xαβ__ → fαβ[λ], yαβ__ → gαβ[λ]}
Out[*]=
{g1,2[λ], f1,1[λ], f1,2[λ], f2,2[λ]}

In[*]:= $Basis /. {xαβ__ ⇒ fαβ[0] == 0, yαβ__ ⇒ gαβ[0] == 0}
Out[*]=
{g1,2[0] == 0, f1,1[0] == 0, f1,2[0] == 0, f2,2[0] == 0}

In[*]:= {sol0} = Block[{ $k = 0}, Simplify@DSolve[
  lhs = λTangent[
    Join[λ $Basis ($Basis /. {x → ξ1, y → η1}), λ $Basis ($Basis /. {x → ξ2, y → η2})]
  ];
  rhs = λTangent@($Basis ($Basis /. {xα, α ⇒ CF[fα,α[λ]], xαβ__ → fαβ[λ], yαβ__ → gαβ[λ]}));
  Join[
    (Coefficient[CF[lhs - rhs], #] == 0) & /@ $Basis,
    $Basis /. {xαβ__ ⇒ fαβ[0] == 0, yαβ__ ⇒ gαβ[0] == 0}
  ],
  $Basis /. {xαβ__ → fαβ[λ], yαβ__ → gαβ[λ]},
  λ
]]
Out[*]=
{ {f1,1[λ] → λ (ξ1,1 + ξ2,1), f2,2[λ] → λ (ξ1,2 + ξ2,2),
  f1,2[λ] → e-λ ξ2,1 λ ξ1,2 + e-λ ξ1,2 λ ξ2,2, g1,2[λ] → λ (η1,2 + eλ (-ξ1,1 + ξ1,2) η2,2) } }

In[*]:= {sol1} = Block[{ $k = 1}, Simplify@DSolve[
  lhs = λTangent[
    Join[λ $Basis ($Basis /. {x → ξ1, y → η1}), λ $Basis ($Basis /. {x → ξ2, y → η2})]
  ];
  rhs = λTangent[
    $Basis ($Basis /. {xα, α ⇒ CF[fα,α[λ]], xαβ__ → fαβ[λ], yαβ__ → gαβ[λ]})) /.
    (sol0 /. ((a → b) ⇒ (a → e a + b)))
  ];
  Join[
    (Coefficient[CF[lhs - rhs], #] == 0) & /@ $Basis,
    $Basis /. {xαβ__ ⇒ fαβ[0] == 0, yαβ__ ⇒ gαβ[0] == 0}
  ],
  $Basis /. {xαβ__ → fαβ[λ], yαβ__ → gαβ[λ]},
  λ
]]
Out[*]=
{ {f1,1[λ] → eλ ξ1,2 λ2 η2,2 ξ1,2, f2,2[λ] → -eλ ξ1,2 λ2 η2,2 ξ1,2,
  f1,2[λ] → λ3 η2,2 ξ1,2 ξ2,2, g1,2[λ] → -e-λ ξ1,1 + 2 λ ξ1,2 λ3 η2,22 ξ1,2} }

```

```

In[ ]:= TriangularDSolve[eqns_, funs_, iv_] := Module[{sol = {}, e, fun, der},
  MapThread[
    {e, fun} \[Mapsto] sol = Echo@Union[sol,
      MapAt[CF, First@DSolve[Echo@CF[e /. Derivative[1][f_][v_] \[Rrightarrow] der[f[v], v] /. sol /.
        der[f_, v_] \[Rrightarrow] \partial_v f], fun, iv], {1, 2}]
    ],
  {eqns, funs}
];
sol
]

```

```

In[ ]:= SetAlgebra[gl3,ε];
$k = 1
$PBWRule
MatrixForm@Table[{b1, b2} \[Mapsto] B[b1, b2], {b1, $Basis}, {b2, $Basis}]

```

Out[]=

1

Out[]=

{y_{1,2} \[Mapsto] 1, y_{1,3} \[Mapsto] 2, y_{2,3} \[Mapsto] 3, x_{1,1} \[Mapsto] 4, x_{1,2} \[Mapsto] 5, x_{2,2} \[Mapsto] 6, x_{1,3} \[Mapsto] 7, x_{2,3} \[Mapsto] 8, x_{3,3} \[Mapsto] 9}

Out[]//MatrixForm=

{y _{1,2} , y _{1,2} } \[Mapsto] 0	{y _{1,2} , y _{1,3} } \[Mapsto] 0	{y _{1,2} , y _{2,3} } \[Mapsto] \in y _{1,3}	{y _{1,2} , x _{1,1} } \[Mapsto] y _{1,2}
{y _{1,3} , y _{1,2} } \[Mapsto] 0	{y _{1,3} , y _{1,3} } \[Mapsto] 0	{y _{1,3} , y _{2,3} } \[Mapsto] 0	{y _{1,3} , x _{1,1} } \[Mapsto] y _{1,3}
{y _{2,3} , y _{1,2} } \[Mapsto] -\in y _{1,3}	{y _{2,3} , y _{1,3} } \[Mapsto] 0	{y _{2,3} , y _{2,3} } \[Mapsto] 0	{y _{2,3} , x _{1,1} } \[Mapsto] 0
{x _{1,1} , y _{1,2} } \[Mapsto] -y _{1,2}	{x _{1,1} , y _{1,3} } \[Mapsto] -y _{1,3}	{x _{1,1} , y _{2,3} } \[Mapsto] 0	{x _{1,1} , x _{1,1} } \[Mapsto] 0
{x _{1,2} , y _{1,2} } \[Mapsto] \in x _{1,1} - \in x _{2,2}	{x _{1,2} , y _{1,3} } \[Mapsto] -y _{2,3}	{x _{1,2} , y _{2,3} } \[Mapsto] 0	{x _{1,2} , x _{1,1} } \[Mapsto] -y _{2,3}
{x _{2,2} , y _{1,2} } \[Mapsto] y _{1,2}	{x _{2,2} , y _{1,3} } \[Mapsto] 0	{x _{2,2} , y _{2,3} } \[Mapsto] -y _{2,3}	{x _{2,2} , x _{1,1} } \[Mapsto] 0
{x _{1,3} , y _{1,2} } \[Mapsto] -\in x _{2,3}	{x _{1,3} , y _{1,3} } \[Mapsto] \in x _{1,1} - \in x _{3,3}	{x _{1,3} , y _{2,3} } \[Mapsto] \in x _{1,2}	{x _{1,3} , x _{1,1} } \[Mapsto] -y _{2,3}
{x _{2,3} , y _{1,2} } \[Mapsto] 0	{x _{2,3} , y _{1,3} } \[Mapsto] y _{1,2}	{x _{2,3} , y _{2,3} } \[Mapsto] \in x _{2,2} - \in x _{3,3}	{x _{2,3} , x _{1,1} } \[Mapsto] 0
{x _{3,3} , y _{1,2} } \[Mapsto] 0	{x _{3,3} , y _{1,3} } \[Mapsto] y _{1,3}	{x _{3,3} , y _{2,3} } \[Mapsto] y _{2,3}	{x _{3,3} , x _{1,1} } \[Mapsto] 0

In[]:= \$Basis

Out[]=

{y_{1,2}, x_{1,1}, x_{1,2}, x_{2,2}}

In[]:= SortBy[\$Basis, If[#[[1]] === x, {0, #[[3]] - #[[2]]}, {1, #[[2]] - #[[3]]}] &]

Out[]=

{x_{1,1}, x_{2,2}, x_{1,2}, y_{1,2}}

```

In[*]:= SBasis = SortBy[$Basis, If[#[[1]] === x, {0, #[[3]] - #[[2]]}, {1, #[[2]] - #[[3]]}] &];
sol[-1] = 0 SBasis;
Do[
  Block[{ $k = k },
    s0 = Thread[SBasis → sol[k - 1]];
    lhs = λTangent[
      Join[λ SBasis (SBasis /. {x → ξ1, y → η1}), λ SBasis (SBasis /. {x → ξ2, y → η2})]
    ];
    rhs = λTangent[
      SBasis (SBasis /. {xαβ__ ⇒ (xαβ /. s0) + εk fαβ[λ], yαβ__ ⇒ (yαβ /. s0) + εk gαβ[λ]})
    ];
    Echo@"computed lhs and rhs";
    eqns = EchoLabel["eqns"]@Transpose@{
      (Coefficient[CF[lhs - rhs], #] == 0) & /@ SBasis,
      SBasis /. {xαβ__ ⇒ fαβ[0] == 0, yαβ__ ⇒ gαβ[0] == 0}
    };
    unknowns = SBasis /. {xαβ__ → fαβ[λ], yαβ__ → gαβ[λ]};
    s = TriangularDSolve[eqns, unknowns, λ];
    sol[k] = CF[sol[k - 1] + εk (SBasis /. {xαβ__ → fαβ[λ], yαβ__ → gαβ[λ]} /. s)]
  ],
  {k, 0, $k}
];
sol[$k].SBasis
» computed lhs and rhs

```

» eqns $\{ \{ \xi_{1,1} + \xi_{2,1} - f_{1,1}'[\lambda] = 0, f_{1,1}[0] = 0 \},$
 $\{ \xi_{1,2} + \xi_{2,2} - f_{2,2}'[\lambda] = 0, f_{2,2}[0] = 0 \}, \{ \xi_{1,3} + \xi_{2,3} - f_{3,3}'[\lambda] = 0, f_{3,3}[0] = 0 \},$
 $\{ e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \xi_{1,2} + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda \xi_{1,1} \xi_{1,2} - e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda \xi_{1,2} \xi_{1,2} + \xi_{2,2} + \lambda \xi_{1,1} \xi_{2,2} - \lambda \xi_{1,2}$
 $\xi_{2,2} + \lambda \xi_{2,1} \xi_{2,2} - \lambda \xi_{2,2} \xi_{2,2} - f_{1,2}[\lambda] f_{1,1}'[\lambda] - f_{1,2}'[\lambda] + f_{1,2}[\lambda] f_{2,2}'[\lambda] = 0, f_{1,2}[0] = 0 \},$
 $\{ e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \xi_{1,2,3} + e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda \xi_{1,2} \xi_{1,2,3} - e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda \xi_{1,2,3} \xi_{1,3,3} + \xi_{2,2,3} + \lambda \xi_{1,2} \xi_{2,2,3} -$
 $\lambda \xi_{1,3,3} \xi_{2,2,3} + \lambda \xi_{2,2,2} \xi_{2,2,3} - \lambda \xi_{2,2,3} \xi_{2,3,3} - f_{2,3}[\lambda] f_{2,2}'[\lambda] - f_{2,3}'[\lambda] + f_{2,3}[\lambda] f_{3,3}'[\lambda] = 0,$
 $f_{2,3}[0] = 0 \}, \{ e^{-\lambda \xi_{2,1} + \lambda \xi_{2,3}} \xi_{1,1,3} + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,3}} \lambda \xi_{1,1,1} \xi_{1,1,3} + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,3}} \lambda \xi_{1,1,2} \xi_{1,2,3} +$
 $e^{-\lambda \xi_{2,1} + \lambda \xi_{2,3}} \lambda^2 \xi_{1,1,1} \xi_{1,1,2} \xi_{1,2,3} - e^{-\lambda \xi_{2,1} + \lambda \xi_{2,3}} \lambda^2 \xi_{1,1,2} \xi_{1,2,2} \xi_{1,2,3} - e^{-\lambda \xi_{2,1} + \lambda \xi_{2,3}} \lambda \xi_{1,1,3} \xi_{1,3,3} -$
 $e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda \xi_{1,2,3} \xi_{2,1,2} - e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda^2 \xi_{1,2,2} \xi_{1,2,3} \xi_{2,1,2} + e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda^2 \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2} + \xi_{2,1,3} +$
 $\lambda \xi_{1,1,1} \xi_{2,1,3} - \lambda \xi_{1,3,3} \xi_{2,1,3} + \lambda \xi_{2,1,1} \xi_{2,1,3} + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda \xi_{1,1,2} \xi_{2,2,3} + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda^2 \xi_{1,1,1} \xi_{1,1,2} \xi_{2,2,3} -$
 $e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda^2 \xi_{1,1,2} \xi_{1,2,2} \xi_{2,2,3} + \lambda \xi_{1,2} \xi_{2,2,3} + \lambda^2 \xi_{1,1,1} \xi_{2,1,2} \xi_{2,2,3} - \lambda^2 \xi_{1,2,2} \xi_{2,1,2} \xi_{2,2,3} +$
 $\lambda^2 \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3} - \lambda^2 \xi_{2,1,2} \xi_{2,2,2} \xi_{2,2,3} - \lambda \xi_{2,1,3} \xi_{2,3,3} - f_{1,3}[\lambda] f_{1,1}'[\lambda] - f_{1,2}[\lambda] f_{2,3}[\lambda] f_{1,1}'[\lambda] -$
 $f_{2,3}[\lambda] f_{1,2}'[\lambda] - f_{1,3}'[\lambda] + f_{1,2}[\lambda] f_{2,3}[\lambda] f_{2,2}'[\lambda] + f_{1,3}[\lambda] f_{3,3}'[\lambda] = 0, f_{1,3}[0] = 0 \},$
 $\{ e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \eta_{1,1,3} + \eta_{2,1,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda \eta_{1,1,3} \xi_{1,1,1} - \lambda \eta_{2,1,3} \xi_{1,1,1} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda \eta_{1,1,3} \xi_{1,3,3} + \lambda \eta_{2,1,3} \xi_{1,3,3} -$
 $\lambda \eta_{2,1,3} \xi_{2,1,1} + \lambda \eta_{2,1,3} \xi_{2,3,3} + g_{1,3}[\lambda] f_{1,1}'[\lambda] - g_{1,3}[\lambda] f_{3,3}'[\lambda] - g_{1,3}'[\lambda] = 0, g_{1,3}[0] = 0 \},$
 $\{ e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \eta_{1,1,2} + \eta_{2,1,2} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda \eta_{1,1,2} \xi_{1,1,1} - \lambda \eta_{2,1,2} \xi_{1,1,1} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda \eta_{1,1,2} \xi_{1,2,2} +$
 $\lambda \eta_{2,1,2} \xi_{1,2,2} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda \eta_{1,1,3} \xi_{1,2,3} + e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda \eta_{2,1,3} \xi_{1,2,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^2 \eta_{1,1,3} \xi_{1,2,2} \xi_{1,2,3} +$
 $e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda^2 \eta_{2,1,3} \xi_{1,2,2} \xi_{1,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^2 \eta_{1,1,3} \xi_{1,2,3} \xi_{1,3,3} - e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda^2 \eta_{2,1,3} \xi_{1,2,3} \xi_{1,3,3} -$
 $\lambda \eta_{2,1,2} \xi_{2,1,1} + \lambda \eta_{2,1,2} \xi_{2,2,2} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda \eta_{1,1,3} \xi_{2,2,3} + \lambda \eta_{2,1,3} \xi_{2,2,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,1,3} \xi_{1,1,1} \xi_{2,2,3} +$
 $\lambda^2 \eta_{2,1,3} \xi_{1,2,2} \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,1,3} \xi_{1,3,3} \xi_{2,2,3} - \lambda^2 \eta_{2,1,3} \xi_{1,3,3} \xi_{2,2,3} + \lambda^2 \eta_{2,1,3} \xi_{2,2,2} \xi_{2,2,3} -$
 $\lambda^2 \eta_{2,1,3} \xi_{2,2,3} \xi_{2,3,3} + g_{1,2}[\lambda] f_{1,1}'[\lambda] - g_{1,2}[\lambda] f_{2,2}'[\lambda] - f_{2,3}[\lambda] g_{1,3}[\lambda] f_{2,2}'[\lambda] -$
 $g_{1,3}[\lambda] f_{2,3}'[\lambda] + f_{2,3}[\lambda] g_{1,3}[\lambda] f_{3,3}'[\lambda] - g_{1,2}'[\lambda] = 0, g_{1,2}[0] = 0 \},$
 $\{ e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \eta_{1,2,3} + \eta_{2,2,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda \eta_{1,1,3} \xi_{1,1,2} - e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda \eta_{2,1,3} \xi_{1,1,2} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}}$
 $\lambda^2 \eta_{1,1,3} \xi_{1,1,1} \xi_{1,1,2} - e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda^2 \eta_{2,1,3} \xi_{1,1,1} \xi_{1,1,2} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda \eta_{1,2,3} \xi_{1,2,2} - \lambda \eta_{2,2,3} \xi_{1,2,2} +$
 $e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,1,3} \xi_{1,1,2} \xi_{1,2,2} + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda^2 \eta_{2,1,3} \xi_{1,1,2} \xi_{1,2,2} + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda \eta_{1,2,3} \xi_{1,3,3} +$
 $\lambda \eta_{2,2,3} \xi_{1,3,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda \eta_{1,1,3} \xi_{2,1,2} - \lambda \eta_{2,1,3} \xi_{2,1,2} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,1,3} \xi_{1,1,1} \xi_{2,1,2} -$
 $\lambda^2 \eta_{2,1,3} \xi_{1,1,1} \xi_{2,1,2} + \lambda^2 \eta_{2,1,3} \xi_{1,2,2} \xi_{2,1,2} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,1,3} \xi_{1,3,3} \xi_{2,1,2} - \lambda^2 \eta_{2,1,3} \xi_{2,1,1} \xi_{2,1,2} -$
 $\lambda \eta_{2,2,3} \xi_{2,2,2} + \lambda^2 \eta_{2,1,3} \xi_{2,1,2} \xi_{2,2,2} + \lambda \eta_{2,2,3} \xi_{2,3,3} + f_{1,2}[\lambda] g_{1,3}[\lambda] f_{1,1}'[\lambda] + g_{1,3}[\lambda] f_{1,2}'[\lambda] -$
 $f_{1,2}[\lambda] g_{1,3}[\lambda] f_{2,2}'[\lambda] + g_{2,3}[\lambda] f_{2,2}'[\lambda] - g_{2,3}[\lambda] f_{3,3}'[\lambda] - g_{2,3}'[\lambda] = 0, g_{2,3}[0] = 0 \} \}$

» $\{ \xi_{1,1} + \xi_{2,1} - f_{1,1}'[\lambda] = 0, f_{1,1}[0] = 0 \}$

» $\{ f_{1,1}[\lambda] \rightarrow \lambda \xi_{1,1} + \lambda \xi_{2,1} \}$

» $\{ \xi_{1,2} + \xi_{2,2} - f_{2,2}'[\lambda] = 0, f_{2,2}[0] = 0 \}$

» $\{ f_{1,1}[\lambda] \rightarrow \lambda \xi_{1,1} + \lambda \xi_{2,1}, f_{2,2}[\lambda] \rightarrow \lambda \xi_{1,2} + \lambda \xi_{2,2} \}$

» $\{ \xi_{1,3} + \xi_{2,3} - f_{3,3}'[\lambda] = 0, f_{3,3}[0] = 0 \}$

» $\{ f_{1,1}[\lambda] \rightarrow \lambda \xi_{1,1} + \lambda \xi_{2,1}, f_{2,2}[\lambda] \rightarrow \lambda \xi_{1,2} + \lambda \xi_{2,2}, f_{3,3}[\lambda] \rightarrow \lambda \xi_{1,3} + \lambda \xi_{2,3} \}$

» $\{ e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \xi_{1,1,2} + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda \xi_{1,1,1} \xi_{1,1,2} - e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda \xi_{1,1,2} \xi_{1,2,2} +$
 $\xi_{2,1,2} + \lambda \xi_{1,1,1} \xi_{2,1,2} - \lambda \xi_{1,2,2} \xi_{2,1,2} + \lambda \xi_{2,1,1} \xi_{2,1,2} - \lambda \xi_{2,1,2} \xi_{2,2,2} - \xi_{1,1,1} f_{1,2}[\lambda] +$
 $\xi_{1,2,2} f_{1,2}[\lambda] - \xi_{2,1,1} f_{1,2}[\lambda] + \xi_{2,2,2} f_{1,2}[\lambda] - f_{1,2}'[\lambda] = 0, f_{1,2}[0] = 0 \}$

» $\{ f_{1,1}[\lambda] \rightarrow \lambda \xi_{1,1} + \lambda \xi_{2,1}, f_{1,2}[\lambda] \rightarrow e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda \xi_{1,1,2} + \lambda \xi_{2,1,2},$
 $f_{2,2}[\lambda] \rightarrow \lambda \xi_{1,2} + \lambda \xi_{2,2}, f_{3,3}[\lambda] \rightarrow \lambda \xi_{1,3} + \lambda \xi_{2,3} \}$

» $\{ e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \xi_{1,2,3} + e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda \xi_{1,2,2} \xi_{1,2,3} - e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda \xi_{1,2,3} \xi_{1,3,3} +$
 $\xi_{2,2,3} + \lambda \xi_{1,2} \xi_{2,2,3} - \lambda \xi_{1,3,3} \xi_{2,2,3} + \lambda \xi_{2,2,2} \xi_{2,2,3} - \lambda \xi_{2,2,3} \xi_{2,3,3} - \xi_{1,2,2} f_{2,3}[\lambda] +$
 $\xi_{1,3,3} f_{2,3}[\lambda] - \xi_{2,2,2} f_{2,3}[\lambda] + \xi_{2,3,3} f_{2,3}[\lambda] - f_{2,3}'[\lambda] = 0, f_{2,3}[0] = 0 \}$

[illegible]

$$\begin{aligned}
& e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} + e^{\lambda \eta_{1,2,3} \xi_{1,1,3} \xi_{1,3,3} \xi_{2,2,3} + 2e^{\lambda^3 \eta_{1,1,2} \xi_{1,1,2} \xi_{2,1,2} \xi_{2,2,3} + 2e^{\lambda^4 \eta_{1,1,2} \xi_{1,1,1} \xi_{1,2,2} \xi_{2,1,2} \xi_{2,2,3} +} \\
& e^{\lambda^3 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,2} \xi_{2,2,3} + e^{\lambda^4 \eta_{1,1,3} \xi_{1,1,1} \xi_{1,1,3} \xi_{2,1,2} \xi_{2,2,3} - 2e^{\lambda^4 \eta_{1,1,2} \xi_{1,1,2} \xi_{2,1,2} \xi_{2,2,3} -} \\
& e^{\lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} + e^{\lambda^4 \eta_{1,1,3} \xi_{1,1,2} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} + e^{\lambda^5 \eta_{1,1,3} \xi_{1,1,1} \xi_{1,1,2} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} -} \\
& e^{\lambda^4 \eta_{1,2,3} \xi_{1,2,2} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda^5 \eta_{1,1,3} \xi_{1,1,2} \xi_{1,2,2} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} -} \\
& e^{\lambda^4 \eta_{1,1,3} \xi_{1,1,3} \xi_{1,3,3} \xi_{2,1,2} \xi_{2,2,3} + e^{\lambda^4 \eta_{1,2,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} e^{\lambda^3 \eta_{1,1,2} \xi_{2,1,2}^2 \xi_{2,2,3} +} \\
& e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} e^{\lambda^4 \eta_{1,1,2} \xi_{1,1,1} \xi_{2,1,2}^2 \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} e^{\lambda^4 \eta_{1,1,2} \xi_{1,2,2} \xi_{2,1,2}^2 \xi_{2,2,3} -} \\
& e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} e^{\lambda^4 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2}^2 \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} e^{\lambda^5 \eta_{1,1,3} \xi_{1,2,2} \xi_{1,2,3} \xi_{2,1,2}^2 \xi_{2,2,3} +} \\
& e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} e^{\lambda^5 \eta_{1,1,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2}^2 \xi_{2,2,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} e^{\lambda^2 \eta_{1,2,3} \xi_{2,1,3} \xi_{2,2,3} +} \\
& e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} e^{\lambda^3 \eta_{1,1,3} \xi_{1,1,2} \xi_{2,1,3} \xi_{2,2,3} + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} e^{\lambda^4 \eta_{1,1,3} \xi_{1,1,1} \xi_{1,1,2} \xi_{2,1,3} \xi_{2,2,3} +} \\
& e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} e^{\lambda^3 \eta_{1,2,3} \xi_{1,2,2} \xi_{2,1,3} \xi_{2,2,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} e^{\lambda^4 \eta_{1,1,3} \xi_{1,1,2} \xi_{1,2,2} \xi_{2,1,3} \xi_{2,2,3} -} \\
& e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} e^{\lambda^3 \eta_{1,2,3} \xi_{1,3,3} \xi_{2,1,3} \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} e^{\lambda^3 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,3} +} \\
& e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} e^{\lambda^4 \eta_{1,1,3} \xi_{1,1,1} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} e^{\lambda^4 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,3} -} \\
& e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} e^{\lambda \xi_{1,1,1} \xi_{1,2,3} f_{1,2}[\lambda] + e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} e^{\lambda \xi_{1,2,2} \xi_{1,2,3} f_{1,2}[\lambda] -} \\
& e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} e^{\lambda \xi_{1,2,3} \xi_{2,1,1} f_{1,2}[\lambda] + e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} e^{\lambda \xi_{1,2,3} \xi_{2,2,2} f_{1,2}[\lambda] - e^{\lambda \xi_{1,1,1} \xi_{2,2,3} f_{1,2}[\lambda] +} \\
& e^{\lambda \xi_{1,2,2} \xi_{2,2,3} f_{1,2}[\lambda] - e^{\lambda \xi_{2,1,1} \xi_{2,2,3} f_{1,2}[\lambda] + e^{\lambda \xi_{2,2,2} \xi_{2,2,3} f_{1,2}[\lambda] - e^{\lambda \xi_{1,1,1} f_{1,3}[\lambda] + e^{\lambda \xi_{1,3,3} f_{1,3}[\lambda] -} \\
& e^{\lambda \xi_{2,1,1} f_{1,3}[\lambda] + e^{\lambda \xi_{2,3,3} f_{1,3}[\lambda] - e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,2}} e^{\lambda \xi_{1,1,2} f_{2,3}[\lambda] - e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,2}} e^{\lambda \xi_{1,1,1} \xi_{1,1,2} f_{2,3}[\lambda] +} \\
& e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,2}} e^{\lambda \xi_{1,1,2} \xi_{1,2,2} f_{2,3}[\lambda] - e^{\lambda \xi_{2,1,2} f_{2,3}[\lambda] - e^{\lambda \xi_{1,1,1} \xi_{2,1,2} f_{2,3}[\lambda] + e^{\lambda \xi_{1,2,2} \xi_{2,1,2} f_{2,3}[\lambda] -} \\
& e^{\lambda \xi_{2,1,1} \xi_{2,1,2} f_{2,3}[\lambda] + e^{\lambda \xi_{2,1,2} \xi_{2,2,2} f_{2,3}[\lambda] - e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,3}} e^{\lambda \xi_{1,1,3} f_{1,1}'[\lambda] -} \\
& e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,3}} e^{\lambda^2 \xi_{1,1,2} \xi_{1,2,3} f_{1,1}'[\lambda] - e^{\lambda \xi_{2,1,3} f_{1,1}'[\lambda] - e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,2}} e^{\lambda^2 \xi_{1,1,2} \xi_{2,2,3} f_{1,1}'[\lambda] -} \\
& e^{\lambda^2 \xi_{2,1,2} \xi_{2,2,3} f_{1,1}'[\lambda] - e^{-\lambda \xi_{2,2,2} + \lambda \xi_{2,3}} e^{\lambda \xi_{1,2,3} f_{1,2}'[\lambda] - e^{\lambda \xi_{2,2,3} f_{1,2}'[\lambda] - e^{\lambda \xi_{1,3,3} f_{1,2}'[\lambda] +} \\
& e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,3}} e^{\lambda^2 \xi_{1,1,2} \xi_{1,2,3} f_{2,2}'[\lambda] + e^{-\lambda \xi_{2,2,2} + \lambda \xi_{2,3}} e^{\lambda^2 \xi_{1,2,3} \xi_{2,1,2} f_{2,2}'[\lambda] +} \\
& e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,2}} e^{\lambda^2 \xi_{1,1,2} \xi_{2,2,3} f_{2,2}'[\lambda] + e^{\lambda^2 \xi_{2,1,2} \xi_{2,2,3} f_{2,2}'[\lambda] + e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,3}} e^{\lambda \xi_{1,1,3} f_{3,3}'[\lambda] -} \\
& e^{-\lambda \xi_{2,2,2} + \lambda \xi_{2,3}} e^{\lambda^2 \xi_{1,2,3} \xi_{2,1,2} f_{3,3}'[\lambda] + e^{\lambda \xi_{2,1,3} f_{3,3}'[\lambda] = 0, f_{1,3}[0] = 0\}, \\
& -2e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} e^{\lambda \eta_{1,2,3} \eta_{2,1,2} + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} e^{\lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{1,1,1} - 2e^{\lambda^2 \eta_{1,1,2} \eta_{2,1,3} \xi_{1,1,2} -} \\
& 2e^{\lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{1,1,1} \xi_{1,1,2} + 2e^{\lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{1,1,2} \xi_{1,2,2} - 2e^{\lambda^2 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,2,3} -} \\
& 2e^{\lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,2,2} \xi_{1,2,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} e^{\lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{1,3,3} + 2e^{\lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,2,3} \xi_{1,3,3} +} \\
& e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} e^{\lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{2,1,1} - 2e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} e^{\lambda^2 \eta_{1,1,2} \eta_{1,1,3} \xi_{2,1,2} -} \\
& 3e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} e^{\lambda^2 \eta_{1,1,3} \eta_{2,1,2} \xi_{2,1,2} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} e^{\lambda^3 \eta_{1,1,3} \eta_{2,1,2} \xi_{1,1,1} \xi_{2,1,2} -} \\
& 2e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} e^{\lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{1,1,1} \xi_{2,1,2} + 2e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} e^{\lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{1,2,2} \xi_{2,1,2} -} \\
& 2e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} e^{\lambda^3 \eta_{1,1,3}^2 \xi_{1,2,3} \xi$$

[illegible]

$$\begin{aligned}
& \gg \left\{ 2 e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \in \lambda \eta_{1,3} \xi_{2,1,3} + e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \in \lambda^2 \eta_{1,3} \xi_{2,1,1} \xi_{2,1,3} + 2 e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda \eta_{1,2,3} \xi_{2,2,3} + \right. \\
& \quad 3 e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \in \lambda^2 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,2,3} + e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \in \lambda^3 \eta_{1,1,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3} + \\
& \quad e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^2 \eta_{1,2,3} \xi_{2,2,2} \xi_{2,2,3} - e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \in \lambda^2 \eta_{1,1,3} \xi_{2,1,3} \xi_{2,3,3} - \\
& \quad \left. e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^2 \eta_{1,2,3} \xi_{2,2,3} \xi_{2,3,3} - e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \in \lambda^3 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,2,3} \xi_{2,3,3} - \in f_{3,3}'[\lambda] = 0, f_{3,3}[0] = 0 \right\} \\
& \gg \left\{ f_{1,1}[\lambda] \rightarrow -e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,2,2}} \lambda^2 \eta_{1,1,2} \xi_{2,1,2} - e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \lambda^2 \eta_{1,1,3} \xi_{2,1,3}, \right. \\
& \quad f_{2,2}[\lambda] \rightarrow e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,2,2}} \lambda^2 \eta_{1,1,2} \xi_{2,1,2} - e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \lambda^2 \eta_{1,2,3} \xi_{2,2,3} - e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \lambda^3 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,2,3}, \\
& \quad \left. f_{3,3}[\lambda] \rightarrow e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \lambda^2 \eta_{1,1,3} \xi_{2,1,3} + e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \lambda^2 \eta_{1,2,3} \xi_{2,2,3} + e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \lambda^3 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,2,3} \right\} \\
& \gg \left\{ 6 \in \lambda^2 \eta_{1,1,2} \xi_{1,1,2} \xi_{2,1,2} + 2 \in \lambda^3 \eta_{1,1,2} \xi_{1,1,1} \xi_{1,1,2} \xi_{2,1,2} - 2 \in \lambda^3 \eta_{1,1,2} \xi_{1,1,2} \xi_{1,2,2} \xi_{2,1,2} + \right. \\
& \quad 2 \in \lambda^3 \eta_{1,1,2} \xi_{1,1,2} \xi_{2,1,1} \xi_{2,1,2} + 3 e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,2,2}} \in \lambda^2 \eta_{1,1,2} \xi_{2,1,2}^2 + e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,2,2}} \in \lambda^3 \eta_{1,1,2} \xi_{1,1,1} \xi_{2,1,2}^2 - \\
& \quad e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,2,2}} \in \lambda^3 \eta_{1,1,2} \xi_{1,1,2} \xi_{2,1,2}^2 + 2 e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,2,2}} \in \lambda^3 \eta_{1,1,2} \xi_{2,1,1} \xi_{2,1,2}^2 - \\
& \quad 2 e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda \eta_{1,2,3} \xi_{2,1,3} - e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^2 \eta_{1,2,3} \xi_{1,1,1} \xi_{2,1,3} + 3 e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^2 \eta_{1,1,3} \xi_{1,1,2} \xi_{2,1,3} + \\
& \quad e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^3 \eta_{1,1,3} \xi_{1,1,1} \xi_{1,1,2} \xi_{2,1,3} + e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^2 \eta_{1,2,3} \xi_{1,2,2} \xi_{2,1,3} - \\
& \quad e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^3 \eta_{1,1,3} \xi_{1,1,2} \xi_{1,2,2} \xi_{2,1,3} - e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^2 \eta_{1,2,3} \xi_{2,1,1} \xi_{2,1,3} + \\
& \quad e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^3 \eta_{1,1,3} \xi_{1,1,2} \xi_{2,1,1} \xi_{2,1,3} - 2 \in \lambda^3 \eta_{1,1,2} \xi_{1,1,2} \xi_{2,1,2} \xi_{2,2,2} - \\
& \quad 2 e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,2,2}} \in \lambda^3 \eta_{1,1,2} \xi_{2,1,2}^2 \xi_{2,2,2} - 3 e^{-\lambda \xi_{2,1,1} + 2 \lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^2 \eta_{1,2,3} \xi_{1,1,2} \xi_{2,2,3} - \\
& \quad e^{-\lambda \xi_{2,1,1} + 2 \lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,1} \xi_{1,1,2} \xi_{2,2,3} + e^{-\lambda \xi_{2,1,1} + 2 \lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,2} \xi_{1,2,2} \xi_{2,2,3} - \\
& \quad 3 e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^2 \eta_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,1} \xi_{2,1,2} \xi_{2,2,3} - \\
& \quad 4 e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^3 \eta_{1,1,3} \xi_{1,1,2} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^4 \eta_{1,1,3} \xi_{1,1,1} \xi_{1,1,2} \xi_{2,1,2} \xi_{2,2,3} + \\
& \quad e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,2,2} \xi_{2,1,2} \xi_{2,2,3} + e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^4 \eta_{1,1,3} \xi_{1,1,2} \xi_{1,2,2} \xi_{2,1,2} \xi_{2,2,3} - \\
& \quad e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^3 \eta_{1,2,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^4 \eta_{1,1,3} \xi_{1,1,2} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3} - \\
& \quad 4 e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \in \lambda^3 \eta_{1,1,3} \xi_{2,1,2}^2 \xi_{2,2,3} - e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \in \lambda^4 \eta_{1,1,3} \xi_{1,1,1} \xi_{2,1,2}^2 \xi_{2,2,3} + \\
& \quad e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \in \lambda^4 \eta_{1,1,3} \xi_{1,1,2} \xi_{2,1,2}^2 \xi_{2,2,3} - 2 e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \in \lambda^4 \eta_{1,1,3} \xi_{2,1,1} \xi_{2,1,2}^2 \xi_{2,2,3} - \\
& \quad e^{-\lambda \xi_{2,1,1} + 2 \lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,2} \xi_{2,2,2} \xi_{2,2,3} + e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \in \lambda^4 \eta_{1,1,3} \xi_{2,1,2}^2 \xi_{2,2,2} \xi_{2,2,3} + \\
& \quad e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^2 \eta_{1,2,3} \xi_{2,1,3} \xi_{2,3,3} - e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^3 \eta_{1,1,3} \xi_{1,1,2} \xi_{2,1,3} \xi_{2,3,3} + \\
& \quad e^{-\lambda \xi_{2,1,1} + 2 \lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,2} \xi_{2,2,3} \xi_{2,3,3} + e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^3 \eta_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} \xi_{2,3,3} + \\
& \quad e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \in \lambda^4 \eta_{1,1,3} \xi_{1,1,2} \xi_{2,1,2} \xi_{2,2,3} \xi_{2,3,3} + e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \in \lambda^4 \eta_{1,1,3} \xi_{2,1,2}^2 \xi_{2,2,3} \xi_{2,3,3} - \\
& \quad \in \xi_{1,1,1} f_{1,2}[\lambda] + \in \xi_{1,2,2} f_{1,2}[\lambda] - \in \xi_{2,1,1} f_{1,2}[\lambda] + \in \xi_{2,2,2} f_{1,2}[\lambda] - \in f_{1,2}'[\lambda] = 0, f_{1,2}[0] = 0 \right\} \\
& \gg \left\{ f_{1,1}[\lambda] \rightarrow -e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,2,2}} \lambda^2 \eta_{1,1,2} \xi_{2,1,2} - e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \lambda^2 \eta_{1,1,3} \xi_{2,1,3}, \right. \\
& \quad f_{1,2}[\lambda] \rightarrow 2 \lambda^3 \eta_{1,1,2} \xi_{1,1,2} \xi_{2,1,2} + e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,2,2}} \lambda^3 \eta_{1,1,2} \xi_{2,1,2}^2 - e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \lambda^2 \eta_{1,2,3} \xi_{2,1,3} + \\
& \quad e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \lambda^3 \eta_{1,1,3} \xi_{1,1,2} \xi_{2,1,3} - e^{-\lambda \xi_{2,1,1} + 2 \lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \lambda^3 \eta_{1,2,3} \xi_{1,1,2} \xi_{2,2,3} - \\
& \quad e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \lambda^3 \eta_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \lambda^4 \eta_{1,1,3} \xi_{1,1,2} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \lambda^4 \eta_{1,1,3} \xi_{2,1,2}^2 \xi_{2,2,3}, \\
& \quad f_{2,2}[\lambda] \rightarrow e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,2,2}} \lambda^2 \eta_{1,1,2} \xi_{2,1,2} - e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \lambda^2 \eta_{1,2,3} \xi_{2,2,3} - e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \lambda^3 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,2,3}, \\
& \quad \left. f_{3,3}[\lambda] \rightarrow e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \lambda^2 \eta_{1,1,3} \xi_{2,1,3} + e^{\lambda \xi_{2,2,2-\lambda} \xi_{2,3,3}} \lambda^2 \eta_{1,2,3} \xi_{2,2,3} + e^{\lambda \xi_{2,1,1-\lambda} \xi_{2,3,3}} \lambda^3 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,2,3} \right\}
\end{aligned}$$

$$\begin{aligned}
& \gg \left\{ -3 e^{\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2} \xi_{1,2,3} \xi_{2,1,2} - e^{\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2} \xi_{1,2,2} \xi_{1,2,3} \xi_{2,1,2} + \right. \\
& \quad e^{\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2} - e^{\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2} \xi_{1,2,3} \xi_{2,1,1} \xi_{2,1,2} + \\
& \quad 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda \eta_{1,2} \xi_{2,1,3} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,2} \xi_{1,2,2} \xi_{2,1,3} + 3 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,3} \xi_{1,2,3} \xi_{2,1,3} + \\
& \quad e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,3} \xi_{1,2,2} \xi_{1,2,3} \xi_{2,1,3} - e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,2} \xi_{1,3,3} \xi_{2,1,3} - \\
& \quad e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,3} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,2} \xi_{2,1,1} \xi_{2,1,3} + \\
& \quad e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,3} \xi_{1,2,3} \xi_{2,1,1} \xi_{2,1,3} + e^{\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,2} + \\
& \quad 6 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,2,3} + 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,2,3} \xi_{1,2,2} \xi_{1,2,3} \xi_{2,2,3} - 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,2,3} + \\
& \quad 8 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} + 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^4 \eta_{1,3} \xi_{1,2,2} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - \\
& \quad 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^4 \eta_{1,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2} \xi_{2,2,3} + 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^4 \eta_{1,3} \xi_{1,2,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3} + \\
& \quad 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,2,2} \xi_{2,2,3} + 3 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2,3} \xi_{2,2,3}^2 + \\
& \quad e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,2,2} \xi_{2,2,3}^2 - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,3,3} \xi_{2,2,3}^2 + \\
& \quad 4 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,3}^2 + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{1,2,2} \xi_{2,1,2} \xi_{2,2,3}^2 - \\
& \quad e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{1,3,3} \xi_{2,1,2} \xi_{2,2,3}^2 + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3}^2 + \\
& \quad 2 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{2,2,2} \xi_{2,2,3}^2 + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,2} \xi_{2,2,3}^2 - \\
& \quad e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,2} \xi_{2,1,3} \xi_{2,3,3} - e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,3} \xi_{1,2,3} \xi_{2,1,3} \xi_{2,3,3} - \\
& \quad 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,2,3} \xi_{2,3,3} - 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^4 \eta_{1,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} \xi_{2,3,3} - \\
& \quad 2 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{2,2,3}^2 \xi_{2,3,3} - 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,3}^2 \xi_{2,3,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \xi_{1,2,2} f_{2,3}[\lambda] + \\
& \quad e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \xi_{1,3,3} f_{2,3}[\lambda] - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \xi_{2,2,2} f_{2,3}[\lambda] + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \xi_{2,3,3} f_{2,3}[\lambda] - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in f_{2,3}[\lambda] = 0, f_{2,3}[0] = 0 \} \\
& \gg \{ f_{1,1}[\lambda] \rightarrow -e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \lambda^2 \eta_{1,2} \xi_{2,1,2} - e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,3} \xi_{2,1,3}, \\
& \quad f_{1,2}[\lambda] \rightarrow 2 \lambda^3 \eta_{1,2} \xi_{1,2,2} \xi_{2,1,2} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \lambda^3 \eta_{1,2} \xi_{2,1,2}^2 - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,2,3} \xi_{2,1,3} + \\
& \quad e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,3} \xi_{1,2,2} \xi_{2,1,3} - e^{\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda^3 \eta_{1,2,3} \xi_{1,2,2} \xi_{2,2,3} - \\
& \quad e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^4 \eta_{1,3} \xi_{1,2,2} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \lambda^4 \eta_{1,3} \xi_{2,1,2}^2 \xi_{2,2,3}, \\
& \quad f_{2,2}[\lambda] \rightarrow e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \lambda^2 \eta_{1,2} \xi_{2,1,2} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,2,3} \xi_{2,2,3} - e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,3}, \\
& \quad f_{2,3}[\lambda] \rightarrow -e^{\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda^3 \eta_{1,2} \xi_{1,2,3} \xi_{2,1,2} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \lambda^2 \eta_{1,2} \xi_{2,1,3} + \\
& \quad e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \lambda^3 \eta_{1,3} \xi_{1,2,3} \xi_{2,1,3} + 2 \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,2,3} + 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \lambda^4 \eta_{1,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} + \\
& \quad e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,2,3} \xi_{2,2,3}^2 + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \lambda^4 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,3}^2, \\
& \quad f_{3,3}[\lambda] \rightarrow e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,3} \xi_{2,1,3} + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,2,3} \xi_{2,2,3} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,3} \}
\end{aligned}$$

$$\begin{aligned}
& \gg \{ 3 e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2} \xi_{1,3} \xi_{2,1,2} + e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2} \xi_{1,1,1} \xi_{1,3} \xi_{2,1,2} - \\
& \quad e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2} \xi_{1,1,3} \xi_{1,3,3} \xi_{2,1,2} + e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2} \xi_{1,1,3} \xi_{2,1,1} \xi_{2,1,2} + \\
& \quad 6 \in \lambda^2 \eta_{1,3} \xi_{1,1,3} \xi_{2,1,3} + 2 \in \lambda^3 \eta_{1,1,3} \xi_{1,1,1} \xi_{1,3} \xi_{2,1,3} + 3 \in \lambda^2 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,3} + \\
& \quad \in \lambda^3 \eta_{1,2,3} \xi_{1,1,1} \xi_{1,2,3} \xi_{2,1,3} - 2 \in \lambda^3 \eta_{1,1,3} \xi_{1,1,3} \xi_{1,3,3} \xi_{2,1,3} - \in \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,3} + \\
& \quad 2 \in \lambda^3 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,1} \xi_{2,1,3} + \in \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,1} \xi_{2,1,3} - 4 e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,1,3} - \\
& \quad e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^4 \eta_{1,1,3} \xi_{1,1,1} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,1,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^4 \eta_{1,1,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2} \xi_{2,1,3} - \\
& \quad 2 e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^4 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,1,3} + 3 e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,1,3} \xi_{2,1,3}^2 + \\
& \quad e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3} \xi_{1,1,1} \xi_{2,1,3}^2 - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3} \xi_{1,3,3} \xi_{2,1,3}^2 + 2 e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3} \xi_{2,1,1} \xi_{2,1,3}^2 - \\
& \quad e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,2} \xi_{1,1,3} \xi_{2,1,2} \xi_{2,2,2} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^4 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,2} + \\
& \quad 3 e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,2,3} \xi_{1,1,3} \xi_{2,2,3} + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,1} \xi_{1,3} \xi_{2,2,3} - \\
& \quad e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,3} \xi_{1,3,3} \xi_{2,2,3} + 4 \in \lambda^3 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,2} \xi_{2,2,3} + \in \lambda^4 \eta_{1,1,3} \xi_{1,1,1} \xi_{1,3} \xi_{2,1,2} \xi_{2,2,3} - \\
& \quad 4 \in \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - \in \lambda^4 \eta_{1,2,3} \xi_{1,1,1} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - \in \lambda^4 \eta_{1,1,3} \xi_{1,1,3} \xi_{1,3,3} \xi_{2,1,2} \xi_{2,2,3} + \\
& \quad \in \lambda^4 \eta_{1,2,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2} \xi_{2,2,3} + \in \lambda^4 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3} - \in \lambda^4 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3} - \\
& \quad 5 e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^4 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2}^2 \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^5 \eta_{1,1,3} \xi_{1,1,1} \xi_{1,2,3} \xi_{2,1,2}^2 \xi_{2,2,3} + \\
& \quad e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^5 \eta_{1,1,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2}^2 \xi_{2,2,3} - 2 e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^5 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,1} \xi_{2,1,2}^2 \xi_{2,2,3} + \\
& \quad 3 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2,3} \xi_{2,1,3} \xi_{2,2,3} + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,1} \xi_{2,1,3} \xi_{2,2,3} - \\
& \quad e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,3,3} \xi_{2,1,3} \xi_{2,2,3} + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{2,1,1} \xi_{2,1,3} \xi_{2,2,3} + \\
& \quad 4 e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3} \xi_{1,1,1} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,3} - \\
& \quad e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3} \xi_{1,3,3} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,3} + 2 e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,3} + \\
& \quad e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,3} \xi_{2,2,2} \xi_{2,2,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^5 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2}^2 \xi_{2,2,2} \xi_{2,2,3} + \\
& \quad e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{2,1,3} \xi_{2,2,2} \xi_{2,2,3} - 2 \in \lambda^3 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,3} \xi_{2,3,3} - \in \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,3} \xi_{2,3,3} + \\
& \quad e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^4 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,3,3} - 2 e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3} \xi_{2,1,3}^2 \xi_{2,3,3} - \\
& \quad e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,3} \xi_{2,2,3} \xi_{2,3,3} - \in \lambda^4 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,2} \xi_{2,2,3} \xi_{2,3,3} + \\
& \quad \in \lambda^4 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} \xi_{2,3,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^5 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2}^2 \xi_{2,2,3} \xi_{2,3,3} - \\
& \quad 2 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{2,1,3} \xi_{2,2,3} \xi_{2,3,3} - 2 e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,3} \xi_{2,3,3} - \\
& \quad \in \xi_{1,1,1} f_{1,3}[\lambda] + \in \xi_{1,3,3} f_{1,3}[\lambda] - \in \xi_{2,1,1} f_{1,3}[\lambda] + \in \xi_{2,3,3} f_{1,3}[\lambda] - \in f_{1,3}'[\lambda] = 0, f_{1,3}[0] = 0 \} \\
& \gg \{ f_{1,1}[\lambda] \rightarrow -e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^2 \eta_{1,1,2} \xi_{2,1,2} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,1,3} \xi_{2,1,3}, \\
& \quad f_{1,2}[\lambda] \rightarrow 2 \lambda^3 \eta_{1,1,2} \xi_{1,1,2} \xi_{2,1,2} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^3 \eta_{1,1,2} \xi_{2,1,2}^2 - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,2,3} \xi_{2,1,3} + \\
& \quad e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,1,3} \xi_{1,1,2} \xi_{2,1,3} - e^{-\lambda \xi_{2,1} + 2 \lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,2,3} \xi_{1,1,2} \xi_{2,2,3} - \\
& \quad e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^4 \eta_{1,1,3} \xi_{1,1,2} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^4 \eta_{1,1,3} \xi_{2,1,2}^2 \xi_{2,2,3}, \\
& \quad f_{1,3}[\lambda] \rightarrow e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda^3 \eta_{1,1,2} \xi_{1,1,3} \xi_{2,1,2} + 2 \lambda^3 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,3} + \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,3} - \\
& \quad e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^4 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,1,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,1,3} \xi_{2,1,3}^2 + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda^3 \eta_{1,2,3} \xi_{1,1,3} \xi_{2,2,3} + \\
& \quad \lambda^4 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,2} \xi_{2,2,3} - \lambda^4 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^5 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2}^2 \xi_{2,2,3} + \\
& \quad e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,2,3} \xi_{2,1,3} \xi_{2,2,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^4 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,3}, \\
& \quad f_{2,2}[\lambda] \rightarrow e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^2 \eta_{1,1,2} \xi_{2,1,2} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,2,3} \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,2,3}, \\
& \quad f_{2,3}[\lambda] \rightarrow -e^{\lambda \xi_{2,1} - 2 \lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda^3 \eta_{1,1,2} \xi_{1,2,3} \xi_{2,1,2} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^2 \eta_{1,1,2} \xi_{2,1,3} + \\
& \quad e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^3 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,3} + 2 \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,2,3} + 2 e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^4 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} + \\
& \quad e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,2,3} \xi_{2,2,3}^2 + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^4 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,2,3}^2, \\
& \quad f_{3,3}[\lambda] \rightarrow e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,1,3} \xi_{2,1,3} + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,2,3} \xi_{2,2,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,2,3} \}
\end{aligned}$$

[illegible]

Out[]=

\$Aborted

Dot: Tensors

$$\{\lambda \xi_{1,1} + \lambda \xi_{2,1} - e^{\lambda \xi_{1,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,2} \xi_{2,2}, \lambda \xi_{1,2} + e^{\lambda \xi_{1,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,2} \xi_{2,1} + \lambda \xi_{2,2}, e^{-\lambda \xi_{1,1} + \lambda \xi_{2,2}} \lambda \xi_{1,2} + \lambda \xi_{2,1} + 2 \in \lambda^3 \eta_{1,2} \xi_{1,2} \xi_{2,2} + e^{\lambda \xi_{1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,2} \xi_{2,1}^2, e^{\lambda \xi_{1,1} - \lambda \xi_{2,2}} \lambda \eta_{1,2} + \lambda \eta_{2,1} - e^{-\lambda \xi_{1,1} - 2 \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,2}^2 \xi_{2,1}\} \text{ and } \{x_{1,1}, x_{2,2}, x_{3,3}, x_{1,2}, x_{2,3}, x_{1,3}, y_{1,3}, y_{1,2}, y_{2,3}\} \text{ have incompatible shapes.}$$

Out[•]=

$$\begin{aligned} & \{ \lambda \xi_{1,1} + \lambda \xi_{2,1} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,2} \xi_{2,2}, \lambda \xi_{1,2} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,2} \xi_{2,2} + \lambda \xi_{2,2}, \\ & e^{-\lambda \xi_{1,1} + \lambda \xi_{2,2}} \lambda \xi_{1,2} + \lambda \xi_{2,2} + 2 \in \lambda^3 \eta_{1,2} \xi_{1,2} \xi_{2,2} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,2} \xi_{2,2}^2, \\ & e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda \eta_{1,2} + \lambda \eta_{2,2} - e^{2 \lambda \xi_{2,1} - 2 \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,2}^2 \xi_{2,2} \}. \\ & \{ x_{1,1}, x_{2,2}, x_{3,3}, x_{1,2}, x_{2,3}, x_{1,3}, y_{1,3}, y_{1,2}, y_{2,3} \} \end{aligned}$$

$$\begin{aligned} \text{osol} = & \mathbb{E}_{\{i,j\} \rightarrow \{k\}} \left[\mathbf{y}_{1,2}[k] \, \eta_{1,2}[i] + e^{-\xi_{1,1}[i] + \xi_{2,2}[i]} \, \mathbf{y}_{1,2}[k] \, \eta_{1,2}[j] + \mathbf{y}_{1,3}[k] \, \eta_{1,3}[i] + \right. \\ & e^{-\xi_{1,1}[i] + \xi_{3,3}[i]} \, \mathbf{y}_{1,3}[k] \, \eta_{1,3}[j] + \mathbf{y}_{2,3}[k] \, \eta_{2,3}[i] + e^{-\xi_{2,2}[i] + \xi_{3,3}[i]} \, \mathbf{y}_{2,3}[k] \, \eta_{2,3}[j] + \\ & \mathbf{x}_{1,1}[k] \, \xi_{1,1}[i] + \mathbf{x}_{1,1}[k] \, \xi_{1,1}[j] + e^{-\xi_{1,1}[j]} \, \mathbf{x}_{1,2}[k] \, \xi_{1,2}[i] - e^{\xi_{3,3}[i]} \, \mathbf{y}_{2,3}[k] \, \eta_{1,3}[j] \, \xi_{1,2}[i] + \\ & \left. e^{-\xi_{2,2}[i]} \, \mathbf{x}_{1,2}[k] \, \xi_{1,2}[j] + e^{-\xi_{1,1}[j]} \, \mathbf{x}_{1,3}[k] \, \xi_{1,3}[i] + e^{-\xi_{3,3}[i]} \, \mathbf{x}_{1,3}[k] \, \xi_{1,3}[j] + \mathbf{x}_{2,2}[k] \, \xi_{2,2}[i] + \right. \end{aligned}$$

$$\begin{aligned}
& x_{2,2}[k] \xi_{2,2}[j] + e^{-\xi_{2,2}[j]} x_{2,3}[k] \xi_{2,3}[i] + e^{-\xi_{1,1}[i] + \xi_{2,2}[i] + \xi_{3,3}[i]} y_{1,2}[k] \eta_{1,3}[j] \xi_{2,3}[i] - \\
& x_{1,3}[k] \xi_{1,2}[j] \xi_{2,3}[i] + e^{-\xi_{3,3}[i]} x_{2,3}[k] \xi_{2,3}[j] + x_{3,3}[k] \xi_{3,3}[i] + x_{3,3}[k] \xi_{3,3}[j], \\
& - e^{-\xi_{2,2}[i] + \xi_{3,3}[i]} y_{1,3}[k] \eta_{1,2}[i] \eta_{2,3}[j] + e^{\xi_{2,2}[i]} x_{1,1}[k] \eta_{1,2}[j] \xi_{1,2}[i] - \\
& e^{\xi_{2,2}[i]} x_{2,2}[k] \eta_{1,2}[j] \xi_{1,2}[i] - e^{-\xi_{1,1}[i] + 2 \xi_{2,2}[i]} y_{1,2}[k] \eta_{1,2}[j]^2 \xi_{1,2}[i] + \\
& e^{\xi_{3,3}[i]} y_{1,3}[k] \eta_{1,2}[i] \eta_{1,3}[j] \xi_{1,2}[i] + x_{1,2}[k] \eta_{1,2}[j] \xi_{1,2}[i] \xi_{1,2}[j] - \\
& e^{-\xi_{2,2}[j]} x_{2,3}[k] \eta_{1,2}[j] \xi_{1,3}[i] + e^{\xi_{3,3}[i]} x_{1,1}[k] \eta_{1,3}[j] \xi_{1,3}[i] - e^{\xi_{3,3}[i]} x_{3,3}[k] \eta_{1,3}[j] \xi_{1,3}[i] - \\
& e^{-\xi_{1,1}[i] + \xi_{2,2}[i] + \xi_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j] \eta_{1,3}[j] \xi_{1,3}[i] - e^{-\xi_{1,1}[i] + 2 \xi_{3,3}[i]} y_{1,3}[k] \eta_{1,3}[j]^2 \xi_{1,3}[i] + \\
& e^{-\xi_{1,1}[j] - \xi_{2,2}[i] + \xi_{3,3}[i]} x_{1,2}[k] \eta_{2,3}[j] \xi_{1,3}[i] - e^{-\xi_{2,2}[i] + 2 \xi_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i] - \\
& e^{-\xi_{1,1}[j] + \xi_{3,3}[i]} x_{1,2}[k] \eta_{1,3}[j] \xi_{1,2}[i] \xi_{1,3}[i] + e^{2 \xi_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j]^2 \xi_{1,2}[i] \xi_{1,3}[i] + \\
& x_{1,3}[k] \eta_{1,2}[j] \xi_{1,2}[j] \xi_{1,3}[i] + x_{1,3}[k] \eta_{1,3}[j] \xi_{1,3}[i] \xi_{1,3}[j] + e^{\xi_{3,3}[i]} x_{2,2}[k] \eta_{2,3}[j] \xi_{2,3}[i] - \\
& e^{\xi_{3,3}[i]} x_{3,3}[k] \eta_{2,3}[j] \xi_{2,3}[i] + e^{-\xi_{1,1}[i] + \xi_{2,2}[i] + \xi_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j] \eta_{2,3}[j] \xi_{2,3}[i] - \\
& e^{-\xi_{1,1}[i] + 2 \xi_{3,3}[i]} y_{1,3}[k] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{2,3}[i] - e^{-\xi_{2,2}[i] + 2 \xi_{3,3}[i]} y_{2,3}[k] \eta_{2,3}[j]^2 \xi_{2,3}[i] + \\
& e^{\xi_{2,2}[i] + \xi_{3,3}[i]} x_{1,1}[k] \eta_{1,3}[j] \xi_{1,2}[i] \xi_{2,3}[i] - e^{\xi_{2,2}[i] + \xi_{3,3}[i]} x_{2,2}[k] \eta_{1,3}[j] \xi_{1,2}[i] \xi_{2,3}[i] - \\
& 2 e^{-\xi_{1,1}[i] + 2 \xi_{2,2}[i] + \xi_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j] \eta_{1,3}[j] \xi_{1,2}[i] \xi_{2,3}[i] + \\
& e^{2 \xi_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,2}[i] \xi_{2,3}[i] - \\
& e^{-\xi_{2,2}[i] + \xi_{3,3}[i]} x_{1,2}[k] \eta_{2,3}[j] \xi_{1,2}[j] \xi_{2,3}[i] + e^{\xi_{3,3}[i]} x_{1,2}[k] \eta_{1,3}[j] \xi_{1,2}[i] \xi_{1,2}[j] \xi_{2,3}[i] - \\
& e^{-\xi_{1,1}[i] + \xi_{2,2}[i] + 2 \xi_{3,3}[i]} y_{1,2}[k] \eta_{1,3}[j]^2 \xi_{1,3}[i] \xi_{2,3}[i] + x_{1,3}[k] \eta_{2,3}[j] \xi_{1,3}[j] \xi_{2,3}[i] - \\
& e^{-\xi_{1,1}[i] + 2 \xi_{2,2}[i] + 2 \xi_{3,3}[i]} y_{1,2}[k] \eta_{1,3}[j]^2 \xi_{1,2}[i] \xi_{2,3}[i]^2 + \\
& x_{2,3}[k] \eta_{1,3}[j] \xi_{1,3}[i] \xi_{2,3}[j] + x_{2,3}[k] \eta_{2,3}[j] \xi_{2,3}[i] \xi_{2,3}[j], \\
& - \frac{1}{2} e^{2 \xi_{2,2}[i]} x_{1,1}[k] \eta_{1,2}[j]^2 \xi_{1,2}[i]^2 + \frac{1}{2} e^{2 \xi_{2,2}[i]} x_{2,2}[k] \eta_{1,2}[j]^2 \xi_{1,2}[i]^2 + \\
& e^{-\xi_{1,1}[i] + 3 \xi_{2,2}[i]} y_{1,2}[k] \eta_{1,2}[j]^3 \xi_{1,2}[i]^2 + e^{\xi_{3,3}[i]} x_{1,1}[k] \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,3}[i] - \\
& e^{\xi_{3,3}[i]} x_{2,2}[k] \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,3}[i] - e^{-\xi_{1,1}[i] + \xi_{2,2}[i] + \xi_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j]^2 \eta_{2,3}[j] \xi_{1,3}[i] + \\
& e^{-\xi_{2,2}[i] + 2 \xi_{3,3}[i]} y_{1,3}[k] \eta_{1,2}[i] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i] - e^{\xi_{2,2}[i] + \xi_{3,3}[i]} x_{1,1}[k] \eta_{1,2}[j] \\
& \eta_{1,3}[j] \xi_{1,2}[i] \xi_{1,3}[i] + e^{\xi_{2,2}[i] + \xi_{3,3}[i]} x_{2,2}[k] \eta_{1,2}[j] \eta_{1,3}[j] \xi_{1,2}[i] \xi_{1,3}[i] + \\
& 2 e^{-\xi_{1,1}[i] + 2 \xi_{2,2}[i] + \xi_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j]^2 \eta_{1,3}[j] \xi_{1,2}[i] \xi_{1,3}[i] - e^{2 \xi_{3,3}[i]} y_{1,3}[k] \\
& \eta_{1,2}[i] \eta_{1,3}[j]^2 \xi_{1,2}[i] \xi_{1,3}[i] + e^{-\xi_{2,2}[i] + \xi_{3,3}[i]} x_{1,2}[k] \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,2}[j] \xi_{1,3}[i] - \\
& e^{\xi_{3,3}[i]} x_{1,2}[k] \eta_{1,2}[j] \eta_{1,3}[j] \xi_{1,2}[i] \xi_{1,2}[j] \xi_{1,3}[i] - \frac{1}{2} e^{2 \xi_{3,3}[i]} x_{1,1}[k] \eta_{1,3}[j]^2 \xi_{1,3}[i]^2 + \\
& \frac{1}{2} e^{2 \xi_{3,3}[i]} x_{3,3}[k] \eta_{1,3}[j]^2 \xi_{1,3}[i]^2 + e^{-\xi_{1,1}[i] + \xi_{2,2}[i] + 2 \xi_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j] \eta_{1,3}[j]^2 \xi_{1,3}[i]^2 + \\
& e^{-\xi_{1,1}[i] + 3 \xi_{3,3}[i]} y_{1,3}[k] \eta_{1,3}[j]^3 \xi_{1,3}[i]^2 - e^{-\xi_{1,1}[j] - \xi_{2,2}[i] + 2 \xi_{3,3}[i]} x_{1,2}[k] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i]^2 + \\
& e^{-\xi_{2,2}[i] + 3 \xi_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j]^2 \eta_{2,3}[j] \xi_{1,3}[i]^2 + \\
& e^{-\xi_{1,1}[j] + 2 \xi_{3,3}[i]} x_{1,2}[k] \eta_{1,3}[j]^2 \xi_{1,2}[i] \xi_{1,3}[i]^2 - e^{3 \xi_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j]^3 \xi_{1,2}[i] \xi_{1,3}[i]^2 + \\
& e^{-\xi_{2,2}[i] + 2 \xi_{3,3}[i]} y_{1,3}[k] \eta_{1,2}[i] \eta_{2,3}[j]^2 \xi_{2,3}[i] + e^{\xi_{2,2}[i] + \xi_{3,3}[i]} x_{1,1}[k] \eta_{1,2}[j] \\
& \eta_{2,3}[j] \xi_{1,2}[i] \xi_{2,3}[i] - e^{\xi_{2,2}[i] + \xi_{3,3}[i]} x_{2,2}[k] \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,2}[i] \xi_{2,3}[i] - \\
& 2 e^{-\xi_{1,1}[i] + 2 \xi_{2,2}[i] + \xi_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j]^2 \eta_{2,3}[j] \xi_{1,2}[i] \xi_{2,3}[i] - \\
& e^{2 \xi_{3,3}[i]} y_{1,3}[k] \eta_{1,2}[i] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,2}[i] \xi_{2,3}[i] - e^{2 \xi_{2,2}[i] + \xi_{3,3}[i]} x_{1,1}[k] \eta_{1,2}[j] \\
& \eta_{1,3}[j] \xi_{1,2}[i]^2 \xi_{2,3}[i] + e^{2 \xi_{2,2}[i] + \xi_{3,3}[i]} x_{2,2}[k] \eta_{1,2}[j] \eta_{1,3}[j] \xi_{1,2}[i]^2 \xi_{2,3}[i] + \\
& 3 e^{-\xi_{1,1}[i] + 3 \xi_{2,2}[i] + \xi_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j]^2 \eta_{1,3}[j] \xi_{1,2}[i]^2 \xi_{2,3}[i] - \\
& e^{2 \xi_{3,3}[i]} x_{2,2}[k] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i] \xi_{2,3}[i] + e^{2 \xi_{3,3}[i]} x_{3,3}[k] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i] \xi_{2,3}[i] - \\
& 2 e^{-\xi_{1,1}[i] + \xi_{2,2}[i] + 2 \xi_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i] \xi_{2,3}[i] + \\
& 2 e^{-\xi_{1,1}[i] + 3 \xi_{3,3}[i]} y_{1,3}[k] \eta_{1,3}[j]^2 \eta_{2,3}[j] \xi_{1,3}[i] \xi_{2,3}[i] - \\
& e^{-\xi_{1,1}[j] - \xi_{2,2}[i] + 2 \xi_{3,3}[i]} x_{1,2}[k] \eta_{2,3}[j]^2 \xi_{1,3}[i] \xi_{2,3}[i] +
\end{aligned}$$

$$\begin{aligned}
& 2 e^{-\epsilon_{2,2}[i]+3 \epsilon_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j] \eta_{2,3}[j]^2 \epsilon_{1,3}[i] \epsilon_{2,3}[i] - e^{\epsilon_{2,2}[i]+2 \epsilon_{3,3}[i]} x_{1,1}[k] \eta_{1,3}[j]^2 \\
& \epsilon_{1,2}[i] \epsilon_{1,3}[i] \epsilon_{2,3}[i] + e^{\epsilon_{2,2}[i]+2 \epsilon_{3,3}[i]} x_{2,2}[k] \eta_{1,3}[j]^2 \epsilon_{1,2}[i] \epsilon_{1,3}[i] \epsilon_{2,3}[i] + \\
& 4 e^{-\epsilon_{1,1}[i]+2 \epsilon_{2,2}[i]+2 \epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j] \eta_{1,3}[j]^2 \epsilon_{1,2}[i] \epsilon_{1,3}[i] \epsilon_{2,3}[i] + \\
& e^{-\epsilon_{1,1}[j]+2 \epsilon_{3,3}[i]} x_{1,2}[k] \eta_{1,3}[j] \eta_{2,3}[j] \epsilon_{1,2}[i] \epsilon_{1,3}[i] \epsilon_{2,3}[i] - \\
& 2 e^{3 \epsilon_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j]^2 \eta_{2,3}[j] \epsilon_{1,2}[i] \epsilon_{1,3}[i] \epsilon_{2,3}[i] + \\
& e^{-\epsilon_{2,2}[i]+2 \epsilon_{3,3}[i]} x_{1,2}[k] \eta_{1,3}[j] \eta_{2,3}[j] \epsilon_{1,2}[j] \epsilon_{1,3}[i] \epsilon_{2,3}[i] - \\
& e^{2 \epsilon_{3,3}[i]} x_{1,2}[k] \eta_{1,3}[j]^2 \epsilon_{1,2}[i] \epsilon_{1,2}[j] \epsilon_{1,3}[i] \epsilon_{2,3}[i] + \\
& e^{-\epsilon_{1,1}[i]+\epsilon_{2,2}[i]+3 \epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,3}[j]^3 \epsilon_{1,3}[i]^2 \epsilon_{2,3}[i] - \\
& \frac{1}{2} e^{2 \epsilon_{3,3}[i]} x_{2,2}[k] \eta_{2,3}[j]^2 \epsilon_{2,3}[i]^2 + \frac{1}{2} e^{2 \epsilon_{3,3}[i]} x_{3,3}[k] \eta_{2,3}[j]^2 \epsilon_{2,3}[i]^2 + \\
& e^{-\epsilon_{1,1}[i]+3 \epsilon_{3,3}[i]} y_{1,3}[k] \eta_{1,3}[j] \eta_{2,3}[j]^2 \epsilon_{2,3}[i]^2 + e^{-\epsilon_{2,2}[i]+3 \epsilon_{3,3}[i]} y_{2,3}[k] \eta_{2,3}[j]^3 \epsilon_{2,3}[i]^2 - \\
& 2 e^{-\epsilon_{1,1}[i]+2 \epsilon_{2,2}[i]+2 \epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j] \eta_{1,3}[j] \eta_{2,3}[j] \epsilon_{1,2}[i] \epsilon_{2,3}[i]^2 - \\
& e^{3 \epsilon_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j] \eta_{2,3}[j]^2 \epsilon_{1,2}[i] \epsilon_{2,3}[i]^2 - \frac{1}{2} e^{2 \epsilon_{2,2}[i]+2 \epsilon_{3,3}[i]} x_{1,1}[k] \\
& \eta_{1,3}[j]^2 \epsilon_{1,2}[i]^2 \epsilon_{2,3}[i]^2 + \frac{1}{2} e^{2 \epsilon_{2,2}[i]+2 \epsilon_{3,3}[i]} x_{2,2}[k] \eta_{1,3}[j]^2 \epsilon_{1,2}[i]^2 \epsilon_{2,3}[i]^2 + \\
& 3 e^{-\epsilon_{1,1}[i]+3 \epsilon_{2,2}[i]+2 \epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j] \eta_{1,3}[j]^2 \epsilon_{1,2}[i]^2 \epsilon_{2,3}[i]^2 + \\
& e^{-\epsilon_{2,2}[i]+2 \epsilon_{3,3}[i]} x_{1,2}[k] \eta_{2,3}[j]^2 \epsilon_{1,2}[j] \epsilon_{2,3}[i]^2 - e^{2 \epsilon_{3,3}[i]} x_{1,2}[k] \eta_{1,3}[j] \eta_{2,3}[j] \epsilon_{1,2}[i] \\
& \epsilon_{1,2}[j] \epsilon_{2,3}[i]^2 + 2 e^{-\epsilon_{1,1}[i]+2 \epsilon_{2,2}[i]+3 \epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,3}[j]^3 \epsilon_{1,2}[i] \epsilon_{1,3}[i] \epsilon_{2,3}[i]^2 + \\
& e^{-\epsilon_{1,1}[i]+3 \epsilon_{2,2}[i]+3 \epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,3}[j]^3 \epsilon_{1,2}[i]^2 \epsilon_{2,3}[i]^3];
\end{aligned}$$

osol = (List@@osol).Table[e^k , {k, 0, Length[osol] - 1}]

Out[*]=

$$\begin{aligned}
& y_{1,2}[k] \eta_{1,2}[i] + e^{-\epsilon_{1,1}[i]+\epsilon_{2,2}[i]} y_{1,2}[k] \eta_{1,2}[j] + \\
& x_{1,1}[k] \epsilon_{1,1}[i] + x_{1,1}[k] \epsilon_{1,1}[j] + e^{-\epsilon_{1,1}[j]} x_{1,2}[k] \epsilon_{1,2}[i] + \\
& e^2 \left(-\frac{1}{2} e^{2 \epsilon_{2,2}[i]} x_{1,1}[k] \eta_{1,2}[j]^2 \epsilon_{1,2}[i]^2 + \frac{1}{2} e^{2 \epsilon_{2,2}[i]} x_{2,2}[k] \eta_{1,2}[j]^2 \epsilon_{1,2}[i]^2 + \right. \\
& \left. e^{-\epsilon_{1,1}[i]+3 \epsilon_{2,2}[i]} y_{1,2}[k] \eta_{1,2}[j]^3 \epsilon_{1,2}[i]^2 \right) + e^{-\epsilon_{2,2}[i]} x_{1,2}[k] \epsilon_{1,2}[j] + \\
& e \left(e^{\epsilon_{2,2}[i]} x_{1,1}[k] \eta_{1,2}[j] \epsilon_{1,2}[i] - e^{\epsilon_{2,2}[i]} x_{2,2}[k] \eta_{1,2}[j] \epsilon_{1,2}[i] - e^{-\epsilon_{1,1}[i]+2 \epsilon_{2,2}[i]} y_{1,2}[k] \right. \\
& \left. \eta_{1,2}[j]^2 \epsilon_{1,2}[i] + x_{1,2}[k] \eta_{1,2}[j] \epsilon_{1,2}[i] \epsilon_{1,2}[j] \right) + x_{2,2}[k] \epsilon_{2,2}[i] + x_{2,2}[k] \epsilon_{2,2}[j]
\end{aligned}$$

In[*]:= CF[(Expand@sol[\$k].\$Basis /. { $\epsilon_{1\alpha\beta} \Rightarrow \epsilon_{\alpha\beta}[i]$, $\eta_{1\alpha\beta} \Rightarrow \eta_{\alpha\beta}[i]$,
 $\epsilon_{2\alpha\beta} \Rightarrow \epsilon_{\alpha\beta}[j]$, $\eta_{2\alpha\beta} \Rightarrow \eta_{\alpha\beta}[j]$, $\lambda \rightarrow 1$ }) - (osol /. $v_{-}[k] \Rightarrow v$)]

Out[*]=

0