```
In[•]:= $k = 2;
        SetAlgebra[gl_{3,ϵ}];
        $PBWRule
        MatrixForm@Table[{b1, b2} → B[b1, b2], {b1, $Basis}, {b2, $Basis}]
        MatrixForm@
         Table[{b1, b2} → (DefRep[b1].DefRep[b2] - DefRep[b2].DefRep[b1] == DefRep[B[b1, b2]]),
          {b1, $Basis}, {b2, $Basis}]
```

Out[•]=

$$\{y_{1,2} \to 1,\ y_{1,3} \to 2,\ y_{2,3} \to 3,\ x_{1,1} \to 4,\ x_{1,2} \to 5,\ x_{2,2} \to 6,\ x_{1,3} \to 7,\ x_{2,3} \to 8,\ x_{3,3} \to 9\}$$

Out[•]//MatrixForm=

$$\begin{pmatrix}
\{y_{1,2}, y_{1,2}\} \to 0 & \{y_{1,2}, y_{1,3}\} \to 0 & \{y_{1,2}, y_{2,3}\} \to -\epsilon\, y_{1,3} & \{y_{1,2}, x_{1,1}\} \to y\\
\{y_{1,3}, y_{1,2}\} \to 0 & \{y_{1,3}, y_{1,3}\} \to 0 & \{y_{1,3}, y_{2,3}\} \to 0 & \{y_{1,3}, x_{1,1}\} \to y\\
\{y_{2,3}, y_{1,2}\} \to \epsilon\, y_{1,3} & \{y_{2,3}, y_{1,3}\} \to 0 & \{y_{2,3}, y_{2,3}\} \to 0 & \{y_{2,3}, x_{1,1}\} \to \\
\{x_{1,1}, y_{1,2}\} \to -y_{1,2} & \{x_{1,1}, y_{1,3}\} \to -y_{1,3} & \{x_{1,1}, y_{2,3}\} \to 0 & \{x_{1,1}, x_{1,1}\} \to \\
\{x_{1,2}, y_{1,2}\} \to \epsilon\, x_{1,1} - \epsilon\, x_{2,2} & \{x_{1,2}, y_{1,3}\} \to -y_{2,3} & \{x_{1,2}, y_{2,3}\} \to 0 & \{x_{1,2}, x_{1,1}\} \to -x\\
\{x_{2,2}, y_{1,2}\} \to y_{1,2} & \{x_{2,2}, y_{1,3}\} \to 0 & \{x_{2,2}, y_{2,3}\} \to -y_{2,3} & \{x_{2,2}, x_{1,1}\} \to \\
\{x_{1,3}, y_{1,2}\} \to -\epsilon\, x_{2,3} & \{x_{1,3}, y_{1,3}\} \to \epsilon\, x_{1,1} - \epsilon\, x_{3,3} & \{x_{1,3}, y_{2,3}\} \to \epsilon\, x_{1,2} & \{x_{1,3}, x_{1,1}\} \to -x\\
\{x_{2,3}, y_{1,2}\} \to 0 & \{x_{2,3}, y_{1,3}\} \to y_{1,2} & \{x_{2,3}, y_{2,3}\} \to \epsilon\, x_{2,2} - \epsilon\, x_{3,3} & \{x_{2,3}, x_{1,1}\} \to \\
\{x_{3,3}, y_{1,2}\} \to 0 & \{x_{3,3}, y_{1,3}\} \to y_{1,3} & \{x_{3,3}, y_{2,3}\} \to y_{2,3} & \{x_{3,3}, x_{1,1}\} \to
\end{pmatrix}$$

Out[•]//MatrixForm=

$$\begin{pmatrix}
\{y_{1,2}, y_{1,2}\} \to \text{True} & \{y_{1,2}, y_{1,3}\} \to \text{True} & \{y_{1,2}, y_{2,3}\} \to \text{True} & \{y_{1,2}, x_{1,1}\} \to \text{True} & \{y_{1,2}, x_{1,2}\} \to \text{True} & \{\\
\{y_{1,3}, y_{1,2}\} \to \text{True} & \{y_{1,3}, y_{1,3}\} \to \text{True} & \{y_{1,3}, y_{2,3}\} \to \text{True} & \{y_{1,3}, x_{1,1}\} \to \text{True} & \{y_{1,3}, x_{1,2}\} \to \text{True} & \{\\
\{y_{2,3}, y_{1,2}\} \to \text{True} & \{y_{2,3}, y_{1,3}\} \to \text{True} & \{y_{2,3}, y_{2,3}\} \to \text{True} & \{y_{2,3}, x_{1,1}\} \to \text{True} & \{y_{2,3}, x_{1,2}\} \to \text{True} & \{\\
\{x_{1,1}, y_{1,2}\} \to \text{True} & \{x_{1,1}, y_{1,3}\} \to \text{True} & \{x_{1,1}, y_{2,3}\} \to \text{True} & \{x_{1,1}, x_{1,1}\} \to \text{True} & \{x_{1,1}, x_{1,2}\} \to \text{True} & \{\\
\{x_{1,2}, y_{1,2}\} \to \text{True} & \{x_{1,2}, y_{1,3}\} \to \text{True} & \{x_{1,2}, y_{2,3}\} \to \text{True} & \{x_{1,2}, x_{1,1}\} \to \text{True} & \{x_{1,2}, x_{1,2}\} \to \text{True} & \{\\
\{x_{2,2}, y_{1,2}\} \to \text{True} & \{x_{2,2}, y_{1,3}\} \to \text{True} & \{x_{2,2}, y_{2,3}\} \to \text{True} & \{x_{2,2}, x_{1,1}\} \to \text{True} & \{x_{2,2}, x_{1,2}\} \to \text{True} & \{\\
\{x_{1,3}, y_{1,2}\} \to \text{True} & \{x_{1,3}, y_{1,3}\} \to \text{True} & \{x_{1,3}, y_{2,3}\} \to \text{True} & \{x_{1,3}, x_{1,1}\} \to \text{True} & \{x_{1,3}, x_{1,2}\} \to \text{True} & \{\\
\{x_{2,3}, y_{1,2}\} \to \text{True} & \{x_{2,3}, y_{1,3}\} \to \text{True} & \{x_{2,3}, y_{2,3}\} \to \text{True} & \{x_{2,3}, x_{1,1}\} \to \text{True} & \{x_{2,3}, x_{1,2}\} \to \text{True} & \{\\
\{x_{3,3}, y_{1,2}\} \to \text{True} & \{x_{3,3}, y_{1,3}\} \to \text{True} & \{x_{3,3}, y_{2,3}\} \to \text{True} & \{x_{3,3}, x_{1,1}\} \to \text{True} & \{x_{3,3}, x_{1,2}\} \to \text{True} & \{
\end{pmatrix}$$

```
In[•]:= SBasis = SortBy[$Basis, If[#[[1]] === x, {0, #[[3]] - #[[2]]}, {1, #[[2]] - #[[3]]}] &];
sol[-1] = 0 $Basis;
Do[
  Block[{$k = k},
   s0 = Thread[$Basis → sol[k - 1]];
   lhs = λTangent[
     Join[λ $Basis ($Basis /. {x → ξ1, y → η1}), λ $Basis ($Basis /. {x → ξ2, y → η2})]
    ];
   rhs = λTangent[
     $Basis ($Basis /. {x_αβ__ :> (x_αβ /. s0) + ϵ^k f_αβ[λ], y_αβ__ :> (y_αβ /. s0) + ϵ^k g_αβ[λ]})
    ];
   eqns = Transpose@{
       (Coefficient[CF[lhs - rhs], #] == 0) & /@ SBasis,
       SBasis /. {x_αβ__ :> f_αβ[0] == 0, y_αβ__ :> g_αβ[0] == 0}
     };
   unknowns = SBasis /. {x_αβ__ → f_αβ[λ], y_αβ__ → g_αβ[λ]};
   s = TriangularDSolve[eqns, unknowns, λ];
   sol[k] = CF[sol[k - 1] + ϵ^k ($Basis /. {x_αβ__ → f_αβ[λ], y_αβ__ → g_αβ[λ]} /. s)]
  ],
  {k, 0, $k}
 ];
cm[i_, j_ → k_] := Λ2𝔼_{i,j}→{k}[sol[$k].$Basis /. {x_αβ__ :> x_αβ[k], y_αβ__ :> y_αβ[k],
     ξ1_αβ__ :> ξ_αβ[i], η1_αβ__ :> η_αβ[i], ξ2_αβ__ :> ξ_αβ[j], η2_αβ__ :> η_αβ[j], λ → 1}];
cm[i, j → k]
```

» $\{f_{1,1}[\lambda] \to \lambda\, \xi1_{1,1} + \lambda\, \xi2_{1,1}\}$

» $\{f_{2,2}[\lambda] \to \lambda\, \xi1_{2,2} + \lambda\, \xi2_{2,2}\}$

» $\{f_{3,3}[\lambda] \to \lambda\, \xi1_{3,3} + \lambda\, \xi2_{3,3}\}$

» $\{f_{1,2}[\lambda] \to e^{-\lambda\, \xi2_{1,1}}\, \lambda\, \xi1_{1,2} + e^{-\lambda\, \xi1_{2,2}}\, \lambda\, \xi2_{1,2}\}$

» $\{f_{2,3}[\lambda] \to e^{-\lambda\, \xi2_{2,2}}\, \lambda\, \xi1_{2,3} + e^{-\lambda\, \xi1_{3,3}}\, \lambda\, \xi2_{2,3}\}$

» $\{f_{1,3}[\lambda] \to e^{-\lambda\, \xi2_{1,1}}\, \lambda\, \xi1_{1,3} - \lambda^2\, \xi1_{2,3}\, \xi2_{1,2} + e^{-\lambda\, \xi1_{3,3}}\, \lambda\, \xi2_{1,3}\}$

» $\{g_{1,3}[\lambda] \to \lambda\, \eta1_{1,3} + e^{-\lambda\, \xi1_{1,1} + \lambda\, \xi1_{3,3}}\, \lambda\, \eta2_{1,3}\}$

» $\{g_{1,2}[\lambda] \to \lambda\, \eta1_{1,2} + e^{-\lambda\, \xi1_{1,1} + \lambda\, \xi1_{2,2}}\, \lambda\, \eta2_{1,2} + e^{-\lambda\, \xi1_{1,1} + \lambda\, \xi1_{2,2} + \lambda\, \xi1_{3,3}}\, \lambda^2\, \eta2_{1,3}\, \xi1_{2,3}\}$

» $\{g_{2,3}[\lambda] \to \lambda\, \eta1_{2,3} + e^{-\lambda\, \xi1_{2,2} + \lambda\, \xi1_{3,3}}\, \lambda\, \eta2_{2,3} - e^{\lambda\, \xi1_{3,3}}\, \lambda^2\, \eta2_{1,3}\, \xi1_{1,2}\}$

» $\{f_{1,1}[\lambda] \to e^{\lambda\, \xi1_{2,2}}\, \lambda^2\, \eta2_{1,2}\, \xi1_{1,2} + e^{\lambda\, \xi1_{3,3}}\, \lambda^2\, \eta2_{1,3}\, \xi1_{1,3} + e^{\lambda\, \xi1_{2,2} + \lambda\, \xi1_{3,3}}\, \lambda^3\, \eta2_{1,3}\, \xi1_{1,2}\, \xi1_{2,3}\}$

» $\{f_{2,2}[\lambda] \to -e^{\lambda\, \xi1_{2,2}}\, \lambda^2\, \eta2_{1,2}\, \xi1_{1,2} + e^{\lambda\, \xi1_{3,3}}\, \lambda^2\, \eta2_{2,3}\, \xi1_{2,3} - e^{\lambda\, \xi1_{2,2} + \lambda\, \xi1_{3,3}}\, \lambda^3\, \eta2_{1,3}\, \xi1_{1,2}\, \xi1_{2,3}\}$

» $\{f_{3,3}[\lambda] \to -e^{\lambda\, \xi1_{3,3}}\, \lambda^2\, \eta2_{1,3}\, \xi1_{1,3} - e^{\lambda\, \xi1_{3,3}}\, \lambda^2\, \eta2_{2,3}\, \xi1_{2,3}\}$

» $\{f_{1,2}[\lambda] \to e^{-\lambda\, \xi1_{2,2} + \lambda\, \xi1_{3,3} - \lambda\, \xi2_{1,1}}\, \lambda^2\, \eta2_{2,3}\, \xi1_{1,3} - e^{\lambda\, \xi1_{3,3} - \lambda\, \xi2_{1,1}}\, \lambda^3\, \eta2_{1,3}\, \xi1_{1,2}\, \xi1_{1,3} + \lambda^3\, \eta2_{1,2}\, \xi1_{1,2}\, \xi2_{1,2} - e^{-\lambda\, \xi1_{2,2} + \lambda\, \xi1_{3,3}}\, \lambda^3\, \eta2_{2,3}\, \xi1_{2,3}\, \xi2_{1,2} + e^{\lambda\, \xi1_{3,3}}\, \lambda^4\, \eta2_{1,3}\, \xi1_{1,2}\, \xi1_{2,3}\, \xi2_{1,2}\}$

» $\{f_{2,3}[\lambda] \to -e^{-\lambda\, \xi2_{2,2}}\, \lambda^2\, \eta2_{1,2}\, \xi1_{1,3} + \lambda^3\, \eta2_{1,3}\, \xi1_{1,3}\, \xi2_{2,3} + \lambda^3\, \eta2_{2,3}\, \xi1_{2,3}\, \xi2_{2,3}\}$

» $\{f_{1,3}[\lambda] \to \lambda^3\, \eta2_{1,2}\, \xi1_{1,3}\, \xi2_{1,2} + \lambda^3\, \eta2_{1,3}\, \xi1_{1,3}\, \xi2_{1,3} + \lambda^3\, \eta2_{2,3}\, \xi1_{2,3}\, \xi2_{1,3}\}$

» $\left\{g_{1,3}[\lambda] \to e^{-\lambda \xi 1_{1,1}+\lambda \xi 1_{2,2}} \lambda^2 \eta 1_{2,3} \eta 2_{1,2} - e^{-\lambda \xi 1_{1,1}+\lambda \xi 1_{2,2}+\lambda \xi 1_{3,3}} \lambda^3 \eta 2_{1,2} \eta 2_{1,3} \xi 1_{1,2} - e^{-\lambda \xi 1_{1,1}+2 \lambda \xi 1_{3,3}} \lambda^3 \eta 2_{1,3}^2 \xi 1_{1,3} + e^{-\lambda \xi 1_{1,1}+\lambda \xi 1_{2,2}+\lambda \xi 1_{3,3}} \lambda^3 \eta 1_{2,3} \eta 2_{1,3} \xi 1_{2,3} - e^{-\lambda \xi 1_{1,1}+\lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^4 \eta 2_{1,3}^2 \xi 1_{1,2} \xi 1_{2,3}\right\}$

» $\left\{g_{1,2}[\lambda] \to -e^{-\lambda \xi 1_{1,1}+2 \lambda \xi 1_{2,2}} \lambda^3 \eta 2_{1,2}^2 \xi 1_{1,2} - e^{-\lambda \xi 1_{1,1}+\lambda \xi 1_{2,2}+\lambda \xi 1_{3,3}} \lambda^3 \eta 2_{1,2} \eta 2_{1,3} \xi 1_{1,3} - 2 e^{-\lambda \xi 1_{1,1}+2 \lambda \xi 1_{2,2}+\lambda \xi 1_{3,3}} \lambda^4 \eta 2_{1,2} \eta 2_{1,3} \xi 1_{1,2} \xi 1_{2,3} - e^{-\lambda \xi 1_{1,1}+\lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^4 \eta 2_{1,3}^2 \xi 1_{1,3} \xi 1_{2,3} - e^{-\lambda \xi 1_{1,1}+2 \lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,3}^2 \xi 1_{1,2} \xi 1_{2,3}^2\right\}$

» $\left\{g_{2,3}[\lambda] \to e^{\lambda \xi 1_{3,3}} \lambda^3 \eta 2_{1,2} \eta 2_{2,3} \xi 1_{1,2} - e^{-\lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^3 \eta 2_{1,3} \eta 2_{2,3} \xi 1_{1,3} + e^{2 \lambda \xi 1_{3,3}} \lambda^4 \eta 2_{1,3}^2 \xi 1_{1,2} \xi 1_{1,3} - e^{-\lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^3 \eta 2_{2,3}^2 \xi 1_{2,3} + e^{2 \lambda \xi 1_{3,3}} \lambda^4 \eta 2_{1,3} \eta 2_{2,3} \xi 1_{1,2} \xi 1_{2,3}\right\}$

» $\left\{f_{1,1}[\lambda] \to -\frac{1}{2} e^{2 \lambda \xi 1_{2,2}} \lambda^4 \eta 2_{1,2}^2 \xi 1_{1,2}^2 - e^{\lambda \xi 1_{2,2}+\lambda \xi 1_{3,3}} \lambda^4 \eta 2_{1,2} \eta 2_{1,3} \xi 1_{1,2} \xi 1_{1,3} - \frac{1}{2} e^{2 \lambda \xi 1_{3,3}} \lambda^4 \eta 2_{1,3}^2 \xi 1_{1,3}^2 - e^{2 \lambda \xi 1_{2,2}+\lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,2} \eta 2_{1,3} \xi 1_{1,2}^2 \xi 1_{2,3} - e^{\lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,3}^2 \xi 1_{1,2} \xi 1_{1,3} \xi 1_{2,3} - \frac{1}{2} e^{2 \lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^6 \eta 2_{1,3}^2 \xi 1_{1,2}^2 \xi 1_{2,3}^2\right\}$

» $\left\{f_{2,2}[\lambda] \to \frac{1}{2} e^{2 \lambda \xi 1_{2,2}} \lambda^4 \eta 2_{1,2}^2 \xi 1_{1,2}^2 - e^{\lambda \xi 1_{3,3}} \lambda^3 \eta 2_{1,2} \eta 2_{2,3} \xi 1_{1,3} + e^{\lambda \xi 1_{2,2}+\lambda \xi 1_{3,3}} \lambda^4 \eta 2_{1,2} \eta 2_{1,3} \xi 1_{1,2} \xi 1_{1,3} + e^{2 \lambda \xi 1_{2,2}+\lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,2} \eta 2_{1,3} \xi 1_{1,2}^2 \xi 1_{2,3} - e^{2 \lambda \xi 1_{3,3}} \lambda^4 \eta 2_{1,3} \eta 2_{2,3} \xi 1_{1,3} \xi 1_{2,3} + e^{\lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,3}^2 \xi 1_{1,2} \xi 1_{1,3} \xi 1_{2,3} - \frac{1}{2} e^{2 \lambda \xi 1_{3,3}} \lambda^4 \eta 2_{2,3}^2 \xi 1_{2,3}^2 + \frac{1}{2} e^{2 \lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^6 \eta 2_{1,3}^2 \xi 1_{1,2}^2 \xi 1_{2,3}^2\right\}$

» $\left\{f_{3,3}[\lambda] \to e^{\lambda \xi 1_{3,3}} \lambda^3 \eta 2_{1,2} \eta 2_{2,3} \xi 1_{1,3} + \frac{1}{2} e^{2 \lambda \xi 1_{3,3}} \lambda^4 \eta 2_{1,3}^2 \xi 1_{1,3}^2 + e^{2 \lambda \xi 1_{3,3}} \lambda^4 \eta 2_{1,3} \eta 2_{2,3} \xi 1_{1,3} \xi 1_{2,3} + \frac{1}{2} e^{2 \lambda \xi 1_{3,3}} \lambda^4 \eta 2_{2,3}^2 \xi 1_{2,3}^2\right\}$

» $\left\{f_{1,2}[\lambda] \to e^{\lambda \xi 1_{3,3}-\lambda \xi 2_{1,1}} \lambda^4 \eta 2_{1,2} \eta 2_{2,3} \xi 1_{1,2} \xi 1_{1,3} - e^{-\lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}-\lambda \xi 2_{1,1}} \lambda^4 \eta 2_{1,3} \eta 2_{2,3} \xi 1_{1,3}^2 + e^{2 \lambda \xi 1_{3,3}-\lambda \xi 2_{1,1}} \lambda^5 \eta 2_{1,3}^2 \xi 1_{1,2} \xi 1_{1,3}^2 - e^{-\lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}-\lambda \xi 2_{1,1}} \lambda^4 \eta 2_{2,3}^2 \xi 1_{1,3} \xi 1_{2,3} + e^{2 \lambda \xi 1_{3,3}-\lambda \xi 2_{1,1}} \lambda^5 \eta 2_{1,3} \eta 2_{2,3} \xi 1_{1,2} \xi 1_{1,3} \xi 1_{2,3} + e^{-\lambda \xi 1_{1,1}+\lambda \xi 1_{3,3}} \lambda^4 \eta 2_{1,2} \eta 2_{2,3} \xi 1_{1,3} \xi 2_{1,2} - e^{\lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,2} \eta 2_{1,3} \xi 1_{1,2} \xi 1_{1,3} \xi 2_{1,2} - e^{\lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,2} \eta 2_{2,3} \xi 1_{1,2} \xi 1_{2,3} \xi 2_{1,2} + e^{-\lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,3} \eta 2_{2,3} \xi 1_{1,3} \xi 1_{2,3} \xi 2_{1,2} - e^{2 \lambda \xi 1_{3,3}} \lambda^6 \eta 2_{1,3}^2 \xi 1_{1,2} \xi 1_{1,3} \xi 1_{2,3} \xi 2_{1,2} + e^{-\lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^5 \eta 2_{2,3}^2 \xi 1_{2,3}^2 \xi 2_{1,2} - e^{2 \lambda \xi 1_{3,3}} \lambda^6 \eta 2_{1,3} \eta 2_{2,3} \xi 1_{1,2} \xi 1_{2,3}^2 \xi 2_{1,2}\right\}$

» $\left\{f_{2,3}[\lambda] \to -\lambda^4 \eta 2_{1,2} \eta 2_{2,3} \xi 1_{1,3} \xi 2_{2,3}\right\}$

» $\left\{f_{1,3}[\lambda] \to -\lambda^4 \eta 2_{1,2} \eta 2_{2,3} \xi 1_{1,3} \xi 2_{1,3}\right\}$

» $\left\{g_{1,3}[\lambda] \to -e^{-\lambda \xi 1_{1,1}+2 \lambda \xi 1_{2,2}} \lambda^4 \eta 1_{2,3} \eta 2_{1,2}^2 \xi 1_{1,2} + e^{-\lambda \xi 1_{1,1}+2 \lambda \xi 1_{2,2}+\lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,2}^2 \eta 2_{1,3} \xi 1_{1,2}^2 - e^{-\lambda \xi 1_{1,1}+\lambda \xi 1_{2,2}+\lambda \xi 1_{3,3}} \lambda^4 \eta 1_{2,3} \eta 2_{1,2} \eta 2_{1,3} \xi 1_{1,3} + 2 e^{-\lambda \xi 1_{1,1}+\lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,2} \eta 2_{1,3}^2 \xi 1_{1,2} \xi 1_{1,3} + e^{-\lambda \xi 1_{1,1}+3 \lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,3}^3 \xi 1_{1,2}^2 - 2 e^{-\lambda \xi 1_{1,1}+2 \lambda \xi 1_{2,2}+\lambda \xi 1_{3,3}} \lambda^5 \eta 1_{2,3} \eta 2_{1,2} \eta 2_{1,3}^2 \xi 1_{1,2} \xi 1_{2,3} + 2 e^{-\lambda \xi 1_{1,1}+2 \lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^6 \eta 2_{1,2} \eta 2_{1,3}^2 \xi 1_{1,2}^2 \xi 1_{2,3} - e^{-\lambda \xi 1_{1,1}+\lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^5 \eta 1_{2,3} \eta 2_{1,3}^2 \xi 1_{1,3} \xi 1_{2,3} + 2 e^{-\lambda \xi 1_{1,1}+\lambda \xi 1_{2,2}+3 \lambda \xi 1_{3,3}} \lambda^6 \eta 2_{1,3}^3 \xi 1_{1,2} \xi 1_{1,3} \xi 1_{2,3} - e^{-\lambda \xi 1_{1,1}+2 \lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^6 \eta 1_{2,3} \eta 2_{1,3}^2 \xi 1_{1,2} \xi 1_{2,3}^2 + e^{-\lambda \xi 1_{1,1}+2 \lambda \xi 1_{2,2}+3 \lambda \xi 1_{3,3}} \lambda^7 \eta 2_{1,3}^3 \xi 1_{1,2}^2 \xi 1_{2,3}^2\right\}$

» $\left\{g_{1,2}[\lambda] \to e^{-\lambda \xi 1_{1,1}+3 \lambda \xi 1_{2,2}} \lambda^5 \eta 2_{1,2}^3 \xi 1_{1,2}^2 + 2 e^{-\lambda \xi 1_{1,1}+2 \lambda \xi 1_{2,2}+\lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,2}^2 \eta 2_{1,3} \xi 1_{1,2} \xi 1_{1,3} + e^{-\lambda \xi 1_{1,1}+\lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,2} \eta 2_{1,3}^2 \xi 1_{1,3}^2 + 3 e^{-\lambda \xi 1_{1,1}+3 \lambda \xi 1_{2,2}+\lambda \xi 1_{3,3}} \lambda^6 \eta 2_{1,2}^2 \eta 2_{1,3} \xi 1_{1,2}^2 \xi 1_{2,3} + 4 e^{-\lambda \xi 1_{1,1}+2 \lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^6 \eta 2_{1,2} \eta 2_{1,3}^2 \xi 1_{1,2} \xi 1_{1,3} \xi 1_{2,3} + e^{-\lambda \xi 1_{1,1}+\lambda \xi 1_{2,2}+3 \lambda \xi 1_{3,3}} \lambda^6 \eta 2_{1,3}^3 \xi 1_{1,2}^2 \xi 1_{2,3} + 3 e^{-\lambda \xi 1_{1,1}+3 \lambda \xi 1_{2,2}+2 \lambda \xi 1_{3,3}} \lambda^7 \eta 2_{1,2} \eta 2_{1,3}^2 \xi 1_{1,2}^2 \xi 1_{2,3}^2 + 2 e^{-\lambda \xi 1_{1,1}+2 \lambda \xi 1_{2,2}+3 \lambda \xi 1_{3,3}} \lambda^7 \eta 2_{1,3}^3 \xi 1_{1,2}^2 \xi 1_{2,3}^2 + e^{-\lambda \xi 1_{1,1}+3 \lambda \xi 1_{2,2}+3 \lambda \xi 1_{3,3}} \lambda^8 \eta 2_{1,3}^3 \xi 1_{1,2}^2 \xi 1_{2,3}^3\right\}$

» $$\left\{g_{2,3}[\lambda] \to e^{-\lambda \xi 1_{2,2}+2\lambda \xi 1_{3,3}} \lambda^4 \eta 2_{1,2} \eta 2_{2,3}^2 \xi 1_{1,3} - 2 e^{2\lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,2} \eta 2_{1,3} \eta 2_{2,3} \xi 1_{1,2} \xi 1_{1,3} + e^{-\lambda \xi 1_{2,2}+3\lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,3}^2 \eta 2_{2,3} \xi 1_{1,3}^2 - e^{3\lambda \xi 1_{3,3}} \lambda^6 \eta 2_{1,3}^3 \xi 1_{1,2} \xi 1_{1,3}^2 - e^{2\lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,2} \eta 2_{2,3}^2 \xi 1_{1,2} \xi 1_{2,3} + 2 e^{-\lambda \xi 1_{2,2}+3\lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,3} \eta 2_{2,3}^2 \xi 1_{1,3} \xi 1_{2,3} - 2 e^{3\lambda \xi 1_{3,3}} \lambda^6 \eta 2_{1,3}^2 \eta 2_{2,3} \xi 1_{1,2} \xi 1_{1,3} \xi 1_{2,3} + e^{-\lambda \xi 1_{2,2}+3\lambda \xi 1_{3,3}} \lambda^5 \eta 2_{1,3}^3 \xi 1_{2,3}^2 - e^{3\lambda \xi 1_{3,3}} \lambda^6 \eta 2_{1,3} \eta 2_{2,3}^2 \xi 1_{1,2} \xi 1_{2,3}^2\right\}$$

Out[•]=

$$\begin{aligned}
&\mathbb{E}_{\{i,j\}\to\{k\}}\Big[y_{1,3}[k]\left(\eta_{1,3}[i] + e^{-\xi_{1,1}[i]+\xi_{3,3}[i]}\eta_{1,3}[j]\right) + x_{1,1}[k]\left(\xi_{1,1}[i] + \xi_{1,1}[j]\right) + \\
&\quad y_{2,3}[k]\left(\eta_{2,3}[i] + e^{-\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{2,3}[j] - e^{\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{1,2}[i]\right) + \\
&\quad x_{1,2}[k]\left(e^{-\xi_{1,1}[j]}\xi_{1,2}[i] + e^{-\xi_{2,2}[i]}\xi_{1,2}[j]\right) + x_{2,2}[k]\left(\xi_{2,2}[i] + \xi_{2,2}[j]\right) + \\
&\quad y_{1,2}[k]\left(\eta_{1,2}[i] + e^{-\xi_{1,1}[i]+\xi_{2,2}[i]}\eta_{1,2}[j] + e^{-\xi_{1,1}[i]+\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{2,3}[i]\right) + \\
&\quad x_{1,3}[k]\left(e^{-\xi_{1,1}[j]}\xi_{1,3}[i] + e^{-\xi_{3,3}[i]}\xi_{1,3}[j] - \xi_{1,2}[j]\,\xi_{2,3}[i]\right) + \\
&\quad x_{2,3}[k]\left(e^{-\xi_{2,2}[j]}\xi_{2,3}[i] + e^{-\xi_{3,3}[i]}\xi_{2,3}[j]\right) + x_{3,3}[k]\left(\xi_{3,3}[i] + \xi_{3,3}[j]\right),\\
&-e^{\xi_{3,3}[i]}x_{3,3}[k]\left(\eta_{1,3}[j]\,\xi_{1,3}[i] + \eta_{2,3}[j]\,\xi_{2,3}[i]\right) + \\
&\quad x_{1,1}[k]\left(e^{\xi_{2,2}[i]}\eta_{1,2}[j]\,\xi_{1,2}[i] + e^{\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{1,3}[i] + e^{\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{1,2}[i]\,\xi_{2,3}[i]\right) - \\
&\quad e^{-\xi_{1,1}[i]+\xi_{2,2}[i]}y_{1,2}[k]\left(\eta_{1,2}[j] + e^{\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{2,3}[i]\right) \\
&\qquad \left(e^{\xi_{2,2}[i]}\eta_{1,2}[j]\,\xi_{1,2}[i] + e^{\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{1,3}[i] + e^{\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{1,2}[i]\,\xi_{2,3}[i]\right) - \\
&\quad x_{2,2}[k]\left(e^{\xi_{2,2}[i]}\eta_{1,2}[j]\,\xi_{1,2}[i] - e^{\xi_{3,3}[i]}\eta_{2,3}[j]\,\xi_{2,3}[i] + e^{\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{1,2}[i]\,\xi_{2,3}[i]\right) + \\
&\quad e^{-\xi_{1,1}[i]}y_{1,3}[k]\left(e^{\xi_{2,2}[i]}\eta_{1,2}[j]\,\eta_{2,3}[i] - e^{\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{1,2}[j]\,\eta_{1,3}[j]\,\xi_{1,2}[i] - e^{2\xi_{3,3}[i]}\eta_{1,3}[j]^2\right. \\
&\qquad \left.\xi_{1,3}[i] + e^{\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{1,3}[j]\,\eta_{2,3}[i]\,\xi_{2,3}[i] - e^{\xi_{2,2}[i]+2\xi_{3,3}[i]}\eta_{1,3}[j]^2\,\xi_{1,2}[i]\,\xi_{2,3}[i]\right) + \\
&\quad y_{2,3}[k]\left(e^{\xi_{3,3}[i]}\eta_{1,2}[j]\,\eta_{2,3}[j]\,\xi_{1,2}[i] - e^{-\xi_{2,2}[i]+2\xi_{3,3}[i]}\eta_{1,3}[j]\,\eta_{2,3}[j]\,\xi_{1,3}[i] + e^{2\xi_{3,3}[i]}\eta_{1,3}[j]^2\right. \\
&\qquad \left.\xi_{1,2}[i]\,\xi_{1,3}[i] - e^{-\xi_{2,2}[i]+2\xi_{3,3}[i]}\eta_{2,3}[j]^2\,\xi_{2,3}[i] + e^{2\xi_{3,3}[i]}\eta_{1,3}[j]\,\eta_{2,3}[j]\,\xi_{1,2}[i]\,\xi_{2,3}[i]\right) + \\
&\quad x_{1,2}[k]\left(\eta_{1,2}[j]\,\xi_{1,2}[i]\,\xi_{1,2}[j] + e^{-\xi_{1,1}[j]-\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{2,3}[j]\,\xi_{1,3}[i] - e^{-\xi_{1,1}[j]+\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{1,2}[\right. \\
&\qquad \left.i]\,\xi_{1,3}[i] - e^{-\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{2,3}[j]\,\xi_{1,2}[j]\,\xi_{2,3}[i] + e^{\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{1,2}[i]\,\xi_{1,2}[j]\,\xi_{2,3}[i]\right) + \\
&\quad x_{1,3}[k]\left(\eta_{1,2}[j]\,\xi_{1,2}[j]\,\xi_{1,3}[i] + \eta_{1,3}[j]\,\xi_{1,3}[i]\,\xi_{1,3}[j] + \eta_{2,3}[j]\,\xi_{1,3}[j]\,\xi_{2,3}[i]\right) + \\
&\quad x_{2,3}[k]\left(-e^{-\xi_{2,2}[j]}\eta_{1,2}[j]\,\xi_{1,3}[i] + \eta_{1,3}[j]\,\xi_{1,3}[i]\,\xi_{2,3}[j] + \eta_{2,3}[j]\,\xi_{2,3}[i]\,\xi_{2,3}[j]\right),\\
&-x_{1,3}[k]\,\eta_{1,2}[j]\,\eta_{2,3}[j]\,\xi_{1,3}[i]\,\xi_{1,3}[j] - \frac{1}{2}x_{1,1}[k] \\
&\qquad \left(e^{\xi_{2,2}[i]}\eta_{1,2}[j]\,\xi_{1,2}[i] + e^{\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{1,3}[i] + e^{\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{1,2}[i]\,\xi_{2,3}[i]\right)^2 + \\
&\quad e^{-\xi_{1,1}[i]+\xi_{2,2}[i]}y_{1,2}[k]\left(\eta_{1,2}[j] + e^{\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{2,3}[i]\right) \\
&\qquad \left(e^{\xi_{2,2}[i]}\eta_{1,2}[j]\,\xi_{1,2}[i] + e^{\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{1,3}[i] + e^{\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{1,2}[i]\,\xi_{2,3}[i]\right)^2 + e^{-\xi_{1,1}[i]} \\
&\quad y_{1,3}[k]\left(e^{\xi_{2,2}[i]}\eta_{1,2}[j]\,\xi_{1,2}[i] + e^{\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{1,3}[i] + e^{\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{1,3}[j]\,\xi_{1,2}[i]\,\xi_{2,3}[i]\right) \\
&\qquad \left(-e^{\xi_{2,2}[i]}\eta_{1,2}[j]\,\eta_{2,3}[i] + e^{\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{1,2}[j]\,\eta_{1,3}[j]\,\xi_{1,2}[i] + e^{2\xi_{3,3}[i]}\eta_{1,3}[j]^2\,\xi_{1,3}[i] - \right. \\
&\qquad \left. e^{\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{1,3}[j]\,\eta_{2,3}[i]\,\xi_{2,3}[i] + e^{\xi_{2,2}[i]+2\xi_{3,3}[i]}\eta_{1,3}[j]^2\,\xi_{1,2}[i]\,\xi_{2,3}[i]\right) + \\
&\quad \frac{1}{2}e^{\xi_{3,3}[i]}x_{3,3}[k]\left(2\,\eta_{1,2}[j]\,\eta_{2,3}[j]\,\xi_{1,3}[i] + e^{\xi_{3,3}[i]}\eta_{1,3}[j]^2\,\xi_{1,3}[i]^2 + \right. \\
&\qquad \left. 2\,e^{\xi_{3,3}[i]}\eta_{1,3}[j]\,\eta_{2,3}[j]\,\xi_{1,3}[i]\,\xi_{2,3}[i] + e^{\xi_{3,3}[i]}\eta_{2,3}[j]^2\,\xi_{2,3}[i]^2\right) - e^{-\xi_{2,2}[i]+2\xi_{3,3}[i]} \\
&\quad y_{2,3}[k]\left(-\eta_{1,2}[j]\,\eta_{2,3}[j]^2\,\xi_{1,3}[i] + 2\,e^{\xi_{2,2}[i]}\eta_{1,2}[j]\,\eta_{1,3}[j]\,\eta_{2,3}[j]\,\xi_{1,2}[i]\,\xi_{1,3}[i] - \right. \\
&\qquad e^{\xi_{3,3}[i]}\eta_{1,3}[j]^2\,\eta_{2,3}[j]\,\xi_{1,3}[i]^2 + e^{\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{1,3}[j]^3\,\xi_{1,2}[i]\,\xi_{1,3}[i]^2 + \\
&\qquad e^{\xi_{2,2}[i]}\eta_{1,2}[j]\,\eta_{2,3}[j]^2\,\xi_{1,2}[i]\,\xi_{2,3}[i] - 2\,e^{\xi_{3,3}[i]}\eta_{1,3}[j]\,\eta_{2,3}[j]^2\,\xi_{1,3}[i]\,\xi_{2,3}[i] + \\
&\qquad 2\,e^{\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{1,3}[j]^2\,\eta_{2,3}[j]\,\xi_{1,2}[i]\,\xi_{1,3}[i]\,\xi_{2,3}[i] - \\
&\qquad \left. e^{\xi_{3,3}[i]}\eta_{2,3}[j]^3\,\xi_{2,3}[i]^2 + e^{\xi_{2,2}[i]+\xi_{3,3}[i]}\eta_{1,3}[j]\,\eta_{2,3}[j]^2\,\xi_{1,2}[i]\,\xi_{2,3}[i]^2\right) + \\
&\quad \frac{1}{2}x_{2,2}[k]\left(e^{2\xi_{2,2}[i]}\eta_{1,2}[j]^2\,\xi_{1,2}[i]^2 - 2\,e^{\xi_{3,3}[i]}\eta_{1,2}[j]\,\eta_{2,3}[j]\,\xi_{1,3}[i] + 2\,e^{\xi_{2,2}[i]+\xi_{3,3}[i]}\right.
\end{aligned}$$

$$\eta_{1,2}[\mathbf{j}]\,\eta_{1,3}[\mathbf{j}]\,\xi_{1,2}[\mathbf{i}]\,\xi_{1,3}[\mathbf{i}] + 2\,e^{2\,\xi_{2,2}[\mathbf{i}]+\xi_{3,3}[\mathbf{i}]}\,\eta_{1,2}[\mathbf{j}]\,\eta_{1,3}[\mathbf{j}]\,\xi_{1,2}[\mathbf{i}]^2\,\xi_{2,3}[\mathbf{i}] - 2\,e^{2\,\xi_{3,3}[\mathbf{i}]}\,\eta_{1,3}[\mathbf{j}]\,\eta_{2,3}[\mathbf{j}]\,\xi_{1,3}[\mathbf{i}]\,\xi_{2,3}[\mathbf{i}] + 2\,e^{\xi_{2,2}[\mathbf{i}]+2\,\xi_{3,3}[\mathbf{i}]}\,\eta_{1,3}[\mathbf{j}]^2\,\xi_{1,2}[\mathbf{i}]\,\xi_{1,3}[\mathbf{i}]\,\xi_{2,3}[\mathbf{i}] - e^{2\,\xi_{3,3}[\mathbf{i}]}\,\eta_{2,3}[\mathbf{j}]^2\,\xi_{2,3}[\mathbf{i}]^2 + e^{2\,\xi_{2,2}[\mathbf{i}]+2\,\xi_{3,3}[\mathbf{i}]}\,\eta_{1,3}[\mathbf{j}]^2\,\xi_{1,2}[\mathbf{i}]^2\,\xi_{2,3}[\mathbf{i}]^2\Big) - e^{-\xi_{1,1}[\mathbf{j}]-\xi_{2,2}[\mathbf{i}]+\xi_{3,3}[\mathbf{i}]}\,x_{1,2}[\mathbf{k}]\,\Big(-e^{\xi_{2,2}[\mathbf{i}]}\,\eta_{1,2}[\mathbf{j}]\,\eta_{2,3}[\mathbf{j}]\,\xi_{1,2}[\mathbf{i}]\,\xi_{1,3}[\mathbf{i}] - e^{\xi_{1,1}[\mathbf{j}]}\,\eta_{1,2}[\mathbf{j}]\,\eta_{2,3}[\mathbf{j}]\,\xi_{1,2}[\mathbf{j}]\,\xi_{1,3}[\mathbf{i}] + e^{\xi_{1,1}[\mathbf{j}]+\xi_{2,2}[\mathbf{i}]}\,\eta_{1,2}[\mathbf{j}]\,\eta_{1,3}[\mathbf{j}]\,\xi_{1,2}[\mathbf{i}]\,\xi_{1,2}[\mathbf{j}]\,\xi_{1,3}[\mathbf{i}] + e^{\xi_{3,3}[\mathbf{i}]}\,\eta_{1,3}[\mathbf{j}]\,\eta_{2,3}[\mathbf{j}]\,\xi_{1,3}[\mathbf{i}]^2 - e^{\xi_{2,2}[\mathbf{i}]+\xi_{3,3}[\mathbf{i}]}\,\eta_{1,3}[\mathbf{j}]^2\,\xi_{1,2}[\mathbf{i}]\,\xi_{1,3}[\mathbf{i}]^2 + e^{\xi_{1,1}[\mathbf{j}]+\xi_{2,2}[\mathbf{i}]}\,\eta_{1,2}[\mathbf{j}]\,\eta_{2,3}[\mathbf{j}]\,\xi_{1,2}[\mathbf{i}]\,\xi_{1,2}[\mathbf{j}]\,\xi_{2,3}[\mathbf{i}] + e^{\xi_{3,3}[\mathbf{i}]}\,\eta_{2,3}[\mathbf{j}]^2\,\xi_{1,3}[\mathbf{i}]\,\xi_{2,3}[\mathbf{i}] - e^{\xi_{2,2}[\mathbf{i}]+\xi_{3,3}[\mathbf{i}]}\,\eta_{1,3}[\mathbf{j}]\,\eta_{2,3}[\mathbf{j}]\,\xi_{1,2}[\mathbf{i}]\,\xi_{1,3}[\mathbf{i}]\,\xi_{2,3}[\mathbf{i}] - e^{\xi_{1,1}[\mathbf{j}]+\xi_{3,3}[\mathbf{i}]}\,\eta_{1,3}[\mathbf{j}]\,\eta_{2,3}[\mathbf{j}]\,\xi_{1,2}[\mathbf{j}]\,\xi_{1,3}[\mathbf{i}]\,\xi_{2,3}[\mathbf{i}] + e^{\xi_{1,1}[\mathbf{j}]+\xi_{2,2}[\mathbf{i}]+\xi_{3,3}[\mathbf{i}]}\,\eta_{1,3}[\mathbf{j}]^2\,\xi_{1,2}[\mathbf{i}]\,\xi_{1,2}[\mathbf{j}]\,\xi_{1,3}[\mathbf{j}]\,\xi_{2,3}[\mathbf{i}]^2 - e^{\xi_{1,1}[\mathbf{j}]+\xi_{3,3}[\mathbf{i}]}\,\eta_{2,3}[\mathbf{j}]^2\,\xi_{1,2}[\mathbf{j}]\,\xi_{2,3}[\mathbf{i}]^2 + e^{\xi_{1,1}[\mathbf{j}]+\xi_{2,2}[\mathbf{i}]+\xi_{3,3}[\mathbf{i}]}\,\eta_{1,3}[\mathbf{j}]\,\eta_{2,3}[\mathbf{j}]\,\xi_{1,2}[\mathbf{i}]\,\xi_{1,2}[\mathbf{j}]\,\xi_{2,3}[\mathbf{j}]\Big]$$

In[•]:=
```
(cm[1, 2 → 2] // cm[2, 3 → 1]) ≡ (cm[2, 3 → 2] // cm[1, 2 → 1])
```

Out[•]=
```
True
```