

```

In[*]:= $k = 1;
SetAlgebra[gl4,ε];
$PBWRule
MatrixForm@Table[{b1, b2} → B[b1, b2], {b1, $Basis}, {b2, $Basis}]
MatrixForm@
Table[{b1, b2} → (DefRep[b1].DefRep[b2] - DefRep[b2].DefRep[b1] == DefRep[B[b1, b2]]),
{b1, $Basis}, {b2, $Basis}]

Out[*]=
{y1,2 → 1, y1,3 → 2, y2,3 → 3, y1,4 → 4, y2,4 → 5, y3,4 → 6, x1,1 → 7, x1,2 → 8,
x2,2 → 9, x1,3 → 10, x2,3 → 11, x3,3 → 12, x1,4 → 13, x2,4 → 14, x3,4 → 15, x4,4 → 16}

Out[*]//MatrixForm=


|                                                                              |                                                                              |                                                                              |                                                           |
|------------------------------------------------------------------------------|------------------------------------------------------------------------------|------------------------------------------------------------------------------|-----------------------------------------------------------|
| {y <sub>1,2</sub> , y <sub>1,2</sub> } → 0                                   | {y <sub>1,2</sub> , y <sub>1,3</sub> } → 0                                   | {y <sub>1,2</sub> , y <sub>2,3</sub> } → -y <sub>1,3</sub>                   | {y <sub>1,2</sub> , y <sub>1,4</sub>                      |
| {y <sub>1,3</sub> , y <sub>1,2</sub> } → 0                                   | {y <sub>1,3</sub> , y <sub>1,3</sub> } → 0                                   | {y <sub>1,3</sub> , y <sub>2,3</sub> } → 0                                   | {y <sub>1,3</sub> , y <sub>1,4</sub>                      |
| {y <sub>2,3</sub> , y <sub>1,2</sub> } → y <sub>1,3</sub>                    | {y <sub>2,3</sub> , y <sub>1,3</sub> } → 0                                   | {y <sub>2,3</sub> , y <sub>2,3</sub> } → 0                                   | {y <sub>2,3</sub> , y <sub>1,4</sub>                      |
| {y <sub>1,4</sub> , y <sub>1,2</sub> } → 0                                   | {y <sub>1,4</sub> , y <sub>1,3</sub> } → 0                                   | {y <sub>1,4</sub> , y <sub>2,3</sub> } → 0                                   | {y <sub>1,4</sub> , y <sub>1,4</sub>                      |
| {y <sub>2,4</sub> , y <sub>1,2</sub> } → y <sub>1,4</sub>                    | {y <sub>2,4</sub> , y <sub>1,3</sub> } → 0                                   | {y <sub>2,4</sub> , y <sub>2,3</sub> } → 0                                   | {y <sub>2,4</sub> , y <sub>1,4</sub>                      |
| {y <sub>3,4</sub> , y <sub>1,2</sub> } → 0                                   | {y <sub>3,4</sub> , y <sub>1,3</sub> } → y <sub>1,4</sub>                    | {y <sub>3,4</sub> , y <sub>2,3</sub> } → y <sub>2,4</sub>                    | {y <sub>3,4</sub> , y <sub>1,4</sub>                      |
| {x <sub>1,1</sub> , y <sub>1,2</sub> } → -y <sub>1,2</sub>                   | {x <sub>1,1</sub> , y <sub>1,3</sub> } → -y <sub>1,3</sub>                   | {x <sub>1,1</sub> , y <sub>2,3</sub> } → 0                                   | {x <sub>1,1</sub> , y <sub>1,4</sub>                      |
| {x <sub>1,2</sub> , y <sub>1,2</sub> } → x <sub>1,1</sub> - x <sub>2,2</sub> | {x <sub>1,2</sub> , y <sub>1,3</sub> } → -y <sub>2,3</sub>                   | {x <sub>1,2</sub> , y <sub>2,3</sub> } → 0                                   | {x <sub>1,2</sub> , y <sub>1,4</sub>                      |
| {x <sub>2,2</sub> , y <sub>1,2</sub> } → y <sub>1,2</sub>                    | {x <sub>2,2</sub> , y <sub>1,3</sub> } → 0                                   | {x <sub>2,2</sub> , y <sub>2,3</sub> } → -y <sub>2,3</sub>                   | {x <sub>2,2</sub> , y <sub>1,4</sub>                      |
| {x <sub>1,3</sub> , y <sub>1,2</sub> } → -x <sub>2,3</sub>                   | {x <sub>1,3</sub> , y <sub>1,3</sub> } → x <sub>1,1</sub> - x <sub>3,3</sub> | {x <sub>1,3</sub> , y <sub>2,3</sub> } → x <sub>1,2</sub>                    | {x <sub>1,3</sub> , y <sub>1,4</sub>                      |
| {x <sub>2,3</sub> , y <sub>1,2</sub> } → 0                                   | {x <sub>2,3</sub> , y <sub>1,3</sub> } → y <sub>1,2</sub>                    | {x <sub>2,3</sub> , y <sub>2,3</sub> } → x <sub>2,2</sub> - x <sub>3,3</sub> | {x <sub>2,3</sub> , y <sub>1,4</sub>                      |
| {x <sub>3,3</sub> , y <sub>1,2</sub> } → 0                                   | {x <sub>3,3</sub> , y <sub>1,3</sub> } → y <sub>1,3</sub>                    | {x <sub>3,3</sub> , y <sub>2,3</sub> } → y <sub>2,3</sub>                    | {x <sub>3,3</sub> , y <sub>1,4</sub>                      |
| {x <sub>1,4</sub> , y <sub>1,2</sub> } → -x <sub>2,4</sub>                   | {x <sub>1,4</sub> , y <sub>1,3</sub> } → -x <sub>3,4</sub>                   | {x <sub>1,4</sub> , y <sub>2,3</sub> } → 0                                   | {x <sub>1,4</sub> , y <sub>1,4</sub> } → x <sub>1,2</sub> |
| {x <sub>2,4</sub> , y <sub>1,2</sub> } → 0                                   | {x <sub>2,4</sub> , y <sub>1,3</sub> } → 0                                   | {x <sub>2,4</sub> , y <sub>2,3</sub> } → -x <sub>3,4</sub>                   | {x <sub>2,4</sub> , y <sub>1,4</sub>                      |
| {x <sub>3,4</sub> , y <sub>1,2</sub> } → 0                                   | {x <sub>3,4</sub> , y <sub>1,3</sub> } → 0                                   | {x <sub>3,4</sub> , y <sub>2,3</sub> } → 0                                   | {x <sub>3,4</sub> , y <sub>1,4</sub>                      |
| {x <sub>4,4</sub> , y <sub>1,2</sub> } → 0                                   | {x <sub>4,4</sub> , y <sub>1,3</sub> } → 0                                   | {x <sub>4,4</sub> , y <sub>2,3</sub> } → 0                                   | {x <sub>4,4</sub> , y <sub>1,4</sub>                      |



Out[*]//MatrixForm=


|                                               |                                               |                                               |                                               |                                               |   |
|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|---|
| {y <sub>1,2</sub> , y <sub>1,2</sub> } → True | {y <sub>1,2</sub> , y <sub>1,3</sub> } → True | {y <sub>1,2</sub> , y <sub>2,3</sub> } → True | {y <sub>1,2</sub> , y <sub>1,4</sub> } → True | {y <sub>1,2</sub> , y <sub>2,4</sub> } → True | { |
| {y <sub>1,3</sub> , y <sub>1,2</sub> } → True | {y <sub>1,3</sub> , y <sub>1,3</sub> } → True | {y <sub>1,3</sub> , y <sub>2,3</sub> } → True | {y <sub>1,3</sub> , y <sub>1,4</sub> } → True | {y <sub>1,3</sub> , y <sub>2,4</sub> } → True | { |
| {y <sub>2,3</sub> , y <sub>1,2</sub> } → True | {y <sub>2,3</sub> , y <sub>1,3</sub> } → True | {y <sub>2,3</sub> , y <sub>2,3</sub> } → True | {y <sub>2,3</sub> , y <sub>1,4</sub> } → True | {y <sub>2,3</sub> , y <sub>2,4</sub> } → True | { |
| {y <sub>1,4</sub> , y <sub>1,2</sub> } → True | {y <sub>1,4</sub> , y <sub>1,3</sub> } → True | {y <sub>1,4</sub> , y <sub>2,3</sub> } → True | {y <sub>1,4</sub> , y <sub>1,4</sub> } → True | {y <sub>1,4</sub> , y <sub>2,4</sub> } → True | { |
| {y <sub>2,4</sub> , y <sub>1,2</sub> } → True | {y <sub>2,4</sub> , y <sub>1,3</sub> } → True | {y <sub>2,4</sub> , y <sub>2,3</sub> } → True | {y <sub>2,4</sub> , y <sub>1,4</sub> } → True | {y <sub>2,4</sub> , y <sub>2,4</sub> } → True | { |
| {y <sub>3,4</sub> , y <sub>1,2</sub> } → True | {y <sub>3,4</sub> , y <sub>1,3</sub> } → True | {y <sub>3,4</sub> , y <sub>2,3</sub> } → True | {y <sub>3,4</sub> , y <sub>1,4</sub> } → True | {y <sub>3,4</sub> , y <sub>2,4</sub> } → True | { |
| {x <sub>1,1</sub> , y <sub>1,2</sub> } → True | {x <sub>1,1</sub> , y <sub>1,3</sub> } → True | {x <sub>1,1</sub> , y <sub>2,3</sub> } → True | {x <sub>1,1</sub> , y <sub>1,4</sub> } → True | {x <sub>1,1</sub> , y <sub>2,4</sub> } → True | { |
| {x <sub>1,2</sub> , y <sub>1,2</sub> } → True | {x <sub>1,2</sub> , y <sub>1,3</sub> } → True | {x <sub>1,2</sub> , y <sub>2,3</sub> } → True | {x <sub>1,2</sub> , y <sub>1,4</sub> } → True | {x <sub>1,2</sub> , y <sub>2,4</sub> } → True | { |
| {x <sub>2,2</sub> , y <sub>1,2</sub> } → True | {x <sub>2,2</sub> , y <sub>1,3</sub> } → True | {x <sub>2,2</sub> , y <sub>2,3</sub> } → True | {x <sub>2,2</sub> , y <sub>1,4</sub> } → True | {x <sub>2,2</sub> , y <sub>2,4</sub> } → True | { |
| {x <sub>1,3</sub> , y <sub>1,2</sub> } → True | {x <sub>1,3</sub> , y <sub>1,3</sub> } → True | {x <sub>1,3</sub> , y <sub>2,3</sub> } → True | {x <sub>1,3</sub> , y <sub>1,4</sub> } → True | {x <sub>1,3</sub> , y <sub>2,4</sub> } → True | { |
| {x <sub>2,3</sub> , y <sub>1,2</sub> } → True | {x <sub>2,3</sub> , y <sub>1,3</sub> } → True | {x <sub>2,3</sub> , y <sub>2,3</sub> } → True | {x <sub>2,3</sub> , y <sub>1,4</sub> } → True | {x <sub>2,3</sub> , y <sub>2,4</sub> } → True | { |
| {x <sub>3,3</sub> , y <sub>1,2</sub> } → True | {x <sub>3,3</sub> , y <sub>1,3</sub> } → True | {x <sub>3,3</sub> , y <sub>2,3</sub> } → True | {x <sub>3,3</sub> , y <sub>1,4</sub> } → True | {x <sub>3,3</sub> , y <sub>2,4</sub> } → True | { |
| {x <sub>1,4</sub> , y <sub>1,2</sub> } → True | {x <sub>1,4</sub> , y <sub>1,3</sub> } → True | {x <sub>1,4</sub> , y <sub>2,3</sub> } → True | {x <sub>1,4</sub> , y <sub>1,4</sub> } → True | {x <sub>1,4</sub> , y <sub>2,4</sub> } → True | { |
| {x <sub>2,4</sub> , y <sub>1,2</sub> } → True | {x <sub>2,4</sub> , y <sub>1,3</sub> } → True | {x <sub>2,4</sub> , y <sub>2,3</sub> } → True | {x <sub>2,4</sub> , y <sub>1,4</sub> } → True | {x <sub>2,4</sub> , y <sub>2,4</sub> } → True | { |
| {x <sub>3,4</sub> , y <sub>1,2</sub> } → True | {x <sub>3,4</sub> , y <sub>1,3</sub> } → True | {x <sub>3,4</sub> , y <sub>2,3</sub> } → True | {x <sub>3,4</sub> , y <sub>1,4</sub> } → True | {x <sub>3,4</sub> , y <sub>2,4</sub> } → True | { |
| {x <sub>4,4</sub> , y <sub>1,2</sub> } → True | {x <sub>4,4</sub> , y <sub>1,3</sub> } → True | {x <sub>4,4</sub> , y <sub>2,3</sub> } → True | {x <sub>4,4</sub> , y <sub>1,4</sub> } → True | {x <sub>4,4</sub> , y <sub>2,4</sub> } → True | { |


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In[*]:= SBasis = SortBy[$Basis, If[#[[1]] === x, {0, #[[3]] - #[[2]]}, {1, #[[2]] - #[[3]]}] &];
sol[-1] = 0 $Basis;
Do[
  Block[{ $k = k },
    s0 = Thread[$Basis -> sol[k - 1]];
    lhs = λTangent[
      Join[λ $Basis ($Basis /. {x -> ξ1, y -> η1}), λ $Basis ($Basis /. {x -> ξ2, y -> η2})]
    ];
    rhs = λTangent[
      $Basis ($Basis /. {xαβ -> (xαβ /. s0) + εk fαβ[λ], yαβ -> (yαβ /. s0) + εk gαβ[λ]})
    ];
    eqns = Transpose@{
      (Coefficient[CF[lhs - rhs], #] == 0) & /@ SBasis,
      SBasis /. {xαβ -> fαβ[0] == 0, yαβ -> gαβ[0] == 0}
    };
    unknowns = SBasis /. {xαβ -> fαβ[λ], yαβ -> gαβ[λ]};
    s = TriangularDSolve[eqns, unknowns, λ];
    sol[k] = CF[sol[k - 1] + εk ($Basis /. {xαβ -> fαβ[λ], yαβ -> gαβ[λ]} /. s)]
  ],
  {k, 0, $k}
];
cm[i_, j_ -> k_] := Δ2E{i,j} -> {k}[sol[$k].$Basis /. {xαβ -> xαβ[k], yαβ -> yαβ[k],
  ξ1αβ -> ξαβ[i], η1αβ -> ηαβ[i], ξ2αβ -> ξαβ[j], η2αβ -> ηαβ[j], λ -> 1}];
cm[i, j -> k]
» {f1,1[λ] -> λ ξ1,1 + λ ξ2,1}
» {f2,2[λ] -> λ ξ1,2 + λ ξ2,2}
» {f3,3[λ] -> λ ξ1,3 + λ ξ2,3}
» {f4,4[λ] -> λ ξ1,4 + λ ξ2,4}
» {f1,2[λ] -> e-λ ξ2,1 λ ξ1,2 + e-λ ξ1,2 λ ξ2,2}
» {f2,3[λ] -> e-λ ξ2,2 λ ξ1,3 + e-λ ξ1,3 λ ξ2,3}
» {f3,4[λ] -> e-λ ξ2,3 λ ξ1,4 + e-λ ξ1,4 λ ξ2,4}
» {f1,3[λ] -> e-λ ξ2,1 λ ξ1,3 - λ2 ξ1,3 ξ2,2 + e-λ ξ1,3 λ ξ2,3}
» {f2,4[λ] -> e-λ ξ2,2 λ ξ1,4 - λ2 ξ1,4 ξ2,3 + e-λ ξ1,4 λ ξ2,4}
» {f1,4[λ] -> e-λ ξ2,1 λ ξ1,4 - λ2 ξ1,4 ξ2,2 - λ2 ξ1,3 ξ2,3 + e-λ ξ1,4 λ ξ2,4}
» {g1,4[λ] -> λ η1,4 + e-λ ξ1,1 λ ξ1,4 λ η2,1,4}
» {g1,3[λ] -> λ η1,3 + e-λ ξ1,1 λ ξ1,3 λ η2,1,3 + e-λ ξ1,1 λ ξ1,3 λ ξ1,4 λ2 η2,1,4 ξ1,3,4}
» {g2,4[λ] -> λ η1,4 + e-λ ξ1,2 λ ξ1,4 λ η2,2,4 - e-λ ξ1,4 λ2 η2,1,4 ξ1,1,2}
» {g1,2[λ] -> λ η1,2 + e-λ ξ1,1 λ ξ1,2 λ η2,1,2 + e-λ ξ1,1 λ ξ1,2 λ ξ1,3 λ2 η2,1,3 ξ1,2,3 +
  e-λ ξ1,1 λ ξ1,2 λ ξ1,4 λ2 η2,1,4 ξ1,2,4 + e-λ ξ1,1 λ ξ1,2 λ ξ1,3 λ ξ1,4 λ3 η2,1,4 ξ1,2,3 ξ1,3,4}
» {g2,3[λ] -> λ η1,3 + e-λ ξ1,2 λ ξ1,3 λ η2,2,3 - e-λ ξ1,3 λ2 η2,1,3 ξ1,1,2 +
  e-λ ξ1,2 λ ξ1,3 λ ξ1,4 λ2 η2,2,4 ξ1,3,4 - e-λ ξ1,3 λ ξ1,3 λ ξ1,4 λ3 η2,1,4 ξ1,1,2 ξ1,3,4}

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- » $\{g_{3,4}[\lambda] \rightarrow \lambda \eta_{1,3,4} + e^{-\lambda \xi_{1,3,3} + \lambda \xi_{1,4,4}} \lambda \eta_{2,3,4} - e^{\lambda \xi_{1,4,4}} \lambda^2 \eta_{2,1,4} \xi_{1,1,3} - e^{\lambda \xi_{1,4,4}} \lambda^2 \eta_{2,2,4} \xi_{1,2,3}\}$
- » $\{f_{1,1}[\lambda] \rightarrow e^{\lambda \xi_{1,2,2}} \lambda^2 \eta_{2,1,2} \xi_{1,1,2} + e^{\lambda \xi_{1,3,3}} \lambda^2 \eta_{2,1,3} \xi_{1,1,3} + e^{\lambda \xi_{1,4,4}} \lambda^2 \eta_{2,1,4} \xi_{1,1,4} + e^{\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^3 \eta_{2,1,3} \xi_{1,1,2} \xi_{1,2,3} + e^{\lambda \xi_{1,2,2} + \lambda \xi_{1,4,4}} \lambda^3 \eta_{2,1,4} \xi_{1,1,2} \xi_{1,2,4} + e^{\lambda \xi_{1,3,3} + \lambda \xi_{1,4,4}} \lambda^3 \eta_{2,1,4} \xi_{1,1,3} \xi_{1,3,4} + e^{\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3} + \lambda \xi_{1,4,4}} \lambda^4 \eta_{2,1,4} \xi_{1,1,2} \xi_{1,2,3} \xi_{1,3,4}\}$
- » $\{f_{2,2}[\lambda] \rightarrow -e^{\lambda \xi_{1,2,2}} \lambda^2 \eta_{2,1,2} \xi_{1,1,2} + e^{\lambda \xi_{1,3,3}} \lambda^2 \eta_{2,2,3} \xi_{1,2,3} - e^{\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^3 \eta_{2,1,3} \xi_{1,1,2} \xi_{1,2,3} + e^{\lambda \xi_{1,4,4}} \lambda^2 \eta_{2,2,4} \xi_{1,2,4} - e^{\lambda \xi_{1,2,2} + \lambda \xi_{1,4,4}} \lambda^3 \eta_{2,1,4} \xi_{1,1,2} \xi_{1,2,4} + e^{\lambda \xi_{1,3,3} + \lambda \xi_{1,4,4}} \lambda^3 \eta_{2,2,4} \xi_{1,2,3} \xi_{1,3,4} - e^{\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3} + \lambda \xi_{1,4,4}} \lambda^4 \eta_{2,1,4} \xi_{1,1,2} \xi_{1,2,3} \xi_{1,3,4}\}$
- » $\{f_{3,3}[\lambda] \rightarrow -e^{\lambda \xi_{1,3,3}} \lambda^2 \eta_{2,1,3} \xi_{1,1,3} - e^{\lambda \xi_{1,3,3}} \lambda^2 \eta_{2,2,3} \xi_{1,2,3} + e^{\lambda \xi_{1,4,4}} \lambda^2 \eta_{2,3,4} \xi_{1,3,4} - e^{\lambda \xi_{1,3,3} + \lambda \xi_{1,4,4}} \lambda^3 \eta_{2,1,4} \xi_{1,1,3} \xi_{1,3,4} - e^{\lambda \xi_{1,3,3} + \lambda \xi_{1,4,4}} \lambda^3 \eta_{2,2,4} \xi_{1,2,3} \xi_{1,3,4}\}$
- » $\{f_{4,4}[\lambda] \rightarrow -e^{\lambda \xi_{1,4,4}} \lambda^2 \eta_{2,1,4} \xi_{1,1,4} - e^{\lambda \xi_{1,4,4}} \lambda^2 \eta_{2,2,4} \xi_{1,2,4} - e^{\lambda \xi_{1,4,4}} \lambda^2 \eta_{2,3,4} \xi_{1,3,4}\}$
- » $\{f_{1,2}[\lambda] \rightarrow e^{-\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3} - \lambda \xi_{2,1,1}} \lambda^2 \eta_{2,2,3} \xi_{1,1,3} - e^{\lambda \xi_{1,3,3} - \lambda \xi_{2,1,1}} \lambda^3 \eta_{2,1,3} \xi_{1,1,2} \xi_{1,1,3} + e^{-\lambda \xi_{1,2,2} + \lambda \xi_{1,4,4} - \lambda \xi_{2,1,1}} \lambda^2 \eta_{2,2,4} \xi_{1,1,4} - e^{\lambda \xi_{1,4,4} - \lambda \xi_{2,1,1}} \lambda^3 \eta_{2,1,4} \xi_{1,1,2} \xi_{1,1,4} + e^{-\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3} + \lambda \xi_{1,4,4} - \lambda \xi_{2,1,1}} \lambda^3 \eta_{2,2,4} \xi_{1,1,3} \xi_{1,3,4} - e^{\lambda \xi_{1,3,3} + \lambda \xi_{1,4,4} - \lambda \xi_{2,1,1}} \lambda^4 \eta_{2,1,4} \xi_{1,1,2} \xi_{1,1,3} \xi_{1,3,4} + \lambda^3 \eta_{2,1,2} \xi_{1,1,2} \xi_{2,1,2} - e^{-\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^3 \eta_{2,2,3} \xi_{1,2,3} \xi_{2,1,2} + e^{\lambda \xi_{1,3,3}} \lambda^4 \eta_{2,1,3} \xi_{1,1,2} \xi_{1,2,3} \xi_{2,1,2} - e^{-\lambda \xi_{1,2,2} + \lambda \xi_{1,4,4}} \lambda^3 \eta_{2,2,4} \xi_{1,2,4} \xi_{2,1,2} + e^{\lambda \xi_{1,4,4}} \lambda^4 \eta_{2,1,4} \xi_{1,1,2} \xi_{1,2,4} \xi_{2,1,2} - e^{-\lambda \xi_{1,2,2} + \lambda \xi_{1,3,3} + \lambda \xi_{1,4,4}} \lambda^4 \eta_{2,2,4} \xi_{1,2,3} \xi_{1,3,4} \xi_{2,1,2} + e^{\lambda \xi_{1,3,3} + \lambda \xi_{1,4,4}} \lambda^5 \eta_{2,1,4} \xi_{1,1,2} \xi_{1,2,3} \xi_{1,3,4} \xi_{2,1,2}\}$
- » $\{f_{2,3}[\lambda] \rightarrow -e^{-\lambda \xi_{2,2,2}} \lambda^2 \eta_{2,1,2} \xi_{1,1,3} + e^{-\lambda \xi_{1,3,3} + \lambda \xi_{1,4,4} - \lambda \xi_{2,2,2}} \lambda^2 \eta_{2,3,4} \xi_{1,2,4} - e^{\lambda \xi_{1,4,4} - \lambda \xi_{2,2,2}} \lambda^3 \eta_{2,1,4} \xi_{1,1,3} \xi_{1,2,4} - e^{\lambda \xi_{1,4,4} - \lambda \xi_{2,2,2}} \lambda^3 \eta_{2,2,4} \xi_{1,2,3} \xi_{1,2,4} + \lambda^3 \eta_{2,1,3} \xi_{1,1,3} \xi_{2,2,3} + \lambda^3 \eta_{2,2,3} \xi_{1,2,3} \xi_{2,2,3} - e^{-\lambda \xi_{1,3,3} + \lambda \xi_{1,4,4}} \lambda^3 \eta_{2,3,4} \xi_{1,3,4} \xi_{2,2,3} + e^{\lambda \xi_{1,4,4}} \lambda^4 \eta_{2,1,4} \xi_{1,1,3} \xi_{1,3,4} \xi_{2,2,3} + e^{\lambda \xi_{1,4,4}} \lambda^4 \eta_{2,2,4} \xi_{1,2,3} \xi_{1,3,4} \xi_{2,2,3}\}$
- » $\{f_{3,4}[\lambda] \rightarrow -e^{-\lambda \xi_{2,3,3}} \lambda^2 \eta_{2,1,3} \xi_{1,1,4} - e^{-\lambda \xi_{2,3,3}} \lambda^2 \eta_{2,2,3} \xi_{1,2,4} + \lambda^3 \eta_{2,1,4} \xi_{1,1,4} \xi_{2,3,4} + \lambda^3 \eta_{2,2,4} \xi_{1,2,4} \xi_{2,3,4} + \lambda^3 \eta_{2,3,4} \xi_{1,3,4} \xi_{2,3,4}\}$
- » $\{f_{1,3}[\lambda] \rightarrow e^{-\lambda \xi_{1,3,3} + \lambda \xi_{1,4,4} - \lambda \xi_{2,1,1}} \lambda^2 \eta_{2,3,4} \xi_{1,1,4} - e^{\lambda \xi_{1,4,4} - \lambda \xi_{2,1,1}} \lambda^3 \eta_{2,1,4} \xi_{1,1,3} \xi_{1,1,4} - e^{\lambda \xi_{1,4,4} - \lambda \xi_{2,1,1}} \lambda^3 \eta_{2,2,4} \xi_{1,1,4} \xi_{1,2,3} + \lambda^3 \eta_{2,1,2} \xi_{1,1,3} \xi_{2,1,2} - e^{-\lambda \xi_{1,3,3} + \lambda \xi_{1,4,4}} \lambda^3 \eta_{2,3,4} \xi_{1,2,4} \xi_{2,1,2} + e^{\lambda \xi_{1,4,4}} \lambda^4 \eta_{2,1,4} \xi_{1,1,3} \xi_{1,2,4} \xi_{2,1,2} + e^{\lambda \xi_{1,4,4}} \lambda^4 \eta_{2,2,4} \xi_{1,2,3} \xi_{1,2,4} \xi_{2,1,2} + \lambda^3 \eta_{2,1,3} \xi_{1,1,3} \xi_{2,1,3} + \lambda^3 \eta_{2,2,3} \xi_{1,2,3} \xi_{2,1,3} - e^{-\lambda \xi_{1,3,3} + \lambda \xi_{1,4,4}} \lambda^3 \eta_{2,3,4} \xi_{1,3,4} \xi_{2,1,3} + e^{\lambda \xi_{1,4,4}} \lambda^4 \eta_{2,1,4} \xi_{1,1,3} \xi_{1,3,4} \xi_{2,1,3} + e^{\lambda \xi_{1,4,4}} \lambda^4 \eta_{2,2,4} \xi_{1,2,3} \xi_{1,3,4} \xi_{2,1,3}\}$
- » $\{f_{2,4}[\lambda] \rightarrow -e^{-\lambda \xi_{2,2,2}} \lambda^2 \eta_{2,1,2} \xi_{1,1,4} + \lambda^3 \eta_{2,1,3} \xi_{1,1,4} \xi_{2,2,3} + \lambda^3 \eta_{2,2,3} \xi_{1,2,4} \xi_{2,2,3} + \lambda^3 \eta_{2,1,4} \xi_{1,1,4} \xi_{2,2,4} + \lambda^3 \eta_{2,2,4} \xi_{1,2,4} \xi_{2,2,4} + \lambda^3 \eta_{2,3,4} \xi_{1,3,4} \xi_{2,2,4}\}$
- » $\{f_{1,4}[\lambda] \rightarrow \lambda^3 \eta_{2,1,2} \xi_{1,1,4} \xi_{2,1,2} + \lambda^3 \eta_{2,1,3} \xi_{1,1,4} \xi_{2,1,3} + \lambda^3 \eta_{2,2,3} \xi_{1,2,4} \xi_{2,1,3} + \lambda^3 \eta_{2,1,4} \xi_{1,1,4} \xi_{2,1,4} + \lambda^3 \eta_{2,2,4} \xi_{1,2,4} \xi_{2,1,4} + \lambda^3 \eta_{2,3,4} \xi_{1,3,4} \xi_{2,1,4}\}$
- » $\{g_{1,4}[\lambda] \rightarrow e^{-\lambda \xi_{1,1,1} + \lambda \xi_{1,2,2}} \lambda^2 \eta_{1,2,4} \eta_{2,1,2} + e^{-\lambda \xi_{1,1,1} + \lambda \xi_{1,3,3}} \lambda^2 \eta_{1,3,4} \eta_{2,1,3} - e^{-\lambda \xi_{1,1,1} + \lambda \xi_{1,2,2} + \lambda \xi_{1,4,4}} \lambda^3 \eta_{2,1,2} \eta_{2,1,4} \xi_{1,1,2} - e^{-\lambda \xi_{1,1,1} + \lambda \xi_{1,3,3} + \lambda \xi_{1,4,4}} \lambda^3 \eta_{2,1,3} \eta_{2,1,4} \xi_{1,1,3} - e^{-\lambda \xi_{1,1,1} + 2 \lambda \xi_{1,4,4}} \lambda^3 \eta_{2,1,4}^2 \xi_{1,1,4} + e^{-\lambda \xi_{1,1,1} + \lambda \xi_{1,2,2} + \lambda \xi_{1,3,3}} \lambda^3 \eta_{1,2,4} \eta_{2,1,3} \xi_{1,1,3} - e^{-\lambda \xi_{1,1,1} + \lambda \xi_{1,2,2} + \lambda \xi_{1,3,3} + \lambda \xi_{1,4,4}} \lambda^4 \eta_{2,1,3} \eta_{2,1,4} \xi_{1,1,2} \xi_{1,2,3} + e^{-\lambda \xi_{1,1,1} + \lambda \xi_{1,2,2} + \lambda \xi_{1,4,4}} \lambda^3 \eta_{1,2,4} \eta_{2,1,4} \xi_{1,2,4} - e^{-\lambda \xi_{1,1,1} + \lambda \xi_{1,2,2} + 2 \lambda \xi_{1,4,4}} \lambda^4 \eta_{2,1,4}^2 \xi_{1,1,2} \xi_{1,2,4} + e^{-\lambda \xi_{1,1,1} + \lambda \xi_{1,3,3} + \lambda \xi_{1,4,4}} \lambda^3 \eta_{1,3,4} \eta_{2,1,4} \xi_{1,3,4} - e^{-\lambda \xi_{1,1,1} + \lambda \xi_{1,3,3} + 2 \lambda \xi_{1,4,4}} \lambda^4 \eta_{2,1,4}^2 \xi_{1,1,3} \xi_{1,3,4} + e^{-\lambda \xi_{1,1,1} + \lambda \xi_{1,2,2} + \lambda \xi_{1,3,3} + \lambda \xi_{1,4,4}} \lambda^4 \eta_{1,2,4} \eta_{2,1,4} \xi_{1,2,3} \xi_{1,3,4} - e^{-\lambda \xi_{1,1,1} + \lambda \xi_{1,2,2} + \lambda \xi_{1,3,3} + 2 \lambda \xi_{1,4,4}} \lambda^5 \eta_{2,1,4}^2 \xi_{1,1,2} \xi_{1,2,3} \xi_{1,3,4}\}$

$$\begin{aligned}
\gg \{ & \mathbf{g}_{1,3}[\lambda] \rightarrow e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2} + 2 \lambda \xi_{1,3}} \lambda^2 \eta_{1,3} \eta_{2,1,2} - e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2} + 2 \lambda \xi_{1,3}} \lambda^3 \eta_{2,1,2} \eta_{2,1,3} \xi_{1,1,2} - \\
& e^{-\lambda \xi_{1,1} + 2 \lambda \xi_{1,3}} \lambda^3 \eta_{2,1,3}^2 \xi_{1,1,3} - e^{-\lambda \xi_{1,1} + \lambda \xi_{1,3} + \lambda \xi_{1,4}} \lambda^3 \eta_{2,1,3} \eta_{2,1,4} \xi_{1,1,4} + \\
& e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2} + 2 \lambda \xi_{1,3}} \lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,2,3} - e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2} + 2 \lambda \xi_{1,3}} \lambda^4 \eta_{2,1,3}^2 \xi_{1,1,2} \xi_{1,2,3} + \\
& e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2} + 2 \lambda \xi_{1,4}} \lambda^3 \eta_{1,2,3} \eta_{2,1,4} \xi_{1,2,4} - e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2} + 2 \lambda \xi_{1,3} + \lambda \xi_{1,4}} \lambda^4 \eta_{2,1,3} \eta_{2,1,4} \xi_{1,1,2} \xi_{1,2,4} - \\
& e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2} + 2 \lambda \xi_{1,3} + \lambda \xi_{1,4}} \lambda^4 \eta_{2,1,2} \eta_{2,1,4} \xi_{1,1,2} \xi_{1,3,4} - 2 e^{-\lambda \xi_{1,1} + 2 \lambda \xi_{1,3} + \lambda \xi_{1,4}} \lambda^4 \eta_{2,1,3} \eta_{2,1,4} \xi_{1,1,3} \xi_{1,3,4} - \\
& e^{-\lambda \xi_{1,1} + \lambda \xi_{1,3} + 2 \lambda \xi_{1,4}} \lambda^4 \eta_{2,1,4}^2 \xi_{1,1,4} \xi_{1,3,4} + e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2} + 2 \lambda \xi_{1,3} + \lambda \xi_{1,4}} \lambda^4 \eta_{1,2,3} \eta_{2,1,4} \xi_{1,2,3} \xi_{1,3,4} - \\
& 2 e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2} + 2 \lambda \xi_{1,3} + \lambda \xi_{1,4}} \lambda^5 \eta_{2,1,3} \eta_{2,1,4} \xi_{1,1,2} \xi_{1,2,3} \xi_{1,3,4} - \\
& e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2} + 2 \lambda \xi_{1,3} + 2 \lambda \xi_{1,4}} \lambda^5 \eta_{2,1,4}^2 \xi_{1,1,2} \xi_{1,2,4} \xi_{1,3,4} - e^{-\lambda \xi_{1,1} + 2 \lambda \xi_{1,3} + 2 \lambda \xi_{1,4}} \lambda^5 \eta_{2,1,4}^2 \xi_{1,1,3} \xi_{1,3,4} - \\
& e^{-\lambda \xi_{1,1} + \lambda \xi_{1,2} + 2 \lambda \xi_{1,3} + 2 \lambda \xi_{1,4}} \lambda^6 \eta_{2,1,4}^2 \xi_{1,1,2} \xi_{1,2,3} \xi_{1,3,4} \} \\
\gg \{ & \mathbf{g}_{2,4}[\lambda] \rightarrow e^{-\lambda \xi_{1,2} + 2 \lambda \xi_{1,3}} \lambda^2 \eta_{1,3,4} \eta_{2,2,3} - e^{\lambda \xi_{1,3}} \lambda^3 \eta_{1,3,4} \eta_{2,1,3} \xi_{1,1,2} + e^{\lambda \xi_{1,4}} \lambda^3 \eta_{2,1,2} \eta_{2,2,4} \xi_{1,1,2} - \\
& e^{-\lambda \xi_{1,2} + 2 \lambda \xi_{1,3} + \lambda \xi_{1,4}} \lambda^3 \eta_{2,1,4} \eta_{2,2,3} \xi_{1,1,3} + e^{\lambda \xi_{1,3} + \lambda \xi_{1,4}} \lambda^4 \eta_{2,1,3} \eta_{2,1,4} \xi_{1,1,2} \xi_{1,1,3} - \\
& e^{-\lambda \xi_{1,2} + 2 \lambda \xi_{1,4}} \lambda^3 \eta_{2,1,4} \eta_{2,2,4} \xi_{1,1,4} + e^{2 \lambda \xi_{1,4}} \lambda^4 \eta_{2,1,4}^2 \xi_{1,1,2} \xi_{1,1,4} - e^{-\lambda \xi_{1,2} + 2 \lambda \xi_{1,3} + \lambda \xi_{1,4}} \lambda^3 \eta_{2,2,3} \eta_{2,2,4} \xi_{1,2,3} + \\
& e^{\lambda \xi_{1,3} + \lambda \xi_{1,4}} \lambda^4 \eta_{2,1,3} \eta_{2,2,4} \xi_{1,1,2} \xi_{1,2,3} - e^{-\lambda \xi_{1,2} + 2 \lambda \xi_{1,4}} \lambda^3 \eta_{2,2,4}^2 \xi_{1,2,4} + e^{2 \lambda \xi_{1,4}} \lambda^4 \eta_{2,1,4} \eta_{2,2,4} \xi_{1,1,2} \xi_{1,2,4} + \\
& e^{-\lambda \xi_{1,2} + 2 \lambda \xi_{1,3} + \lambda \xi_{1,4}} \lambda^3 \eta_{1,3,4} \eta_{2,2,4} \xi_{1,3,4} - e^{\lambda \xi_{1,3} + \lambda \xi_{1,4}} \lambda^4 \eta_{1,3,4} \eta_{2,1,4} \xi_{1,1,2} \xi_{1,3,4} - \\
& e^{-\lambda \xi_{1,2} + 2 \lambda \xi_{1,3} + 2 \lambda \xi_{1,4}} \lambda^4 \eta_{2,1,4} \eta_{2,2,4} \xi_{1,1,3} \xi_{1,3,4} + e^{\lambda \xi_{1,3} + 2 \lambda \xi_{1,4}} \lambda^5 \eta_{2,1,4}^2 \xi_{1,1,2} \xi_{1,1,3} \xi_{1,3,4} - \\
& e^{-\lambda \xi_{1,2} + 2 \lambda \xi_{1,3} + 2 \lambda \xi_{1,4}} \lambda^4 \eta_{2,2,4}^2 \xi_{1,2,3} \xi_{1,3,4} + e^{\lambda \xi_{1,3} + 2 \lambda \xi_{1,4}} \lambda^5 \eta_{2,1,4} \eta_{2,2,4} \xi_{1,1,2} \xi_{1,2,3} \xi_{1,3,4} \}
\end{aligned}$$

In[*]:= (cm[1, 2 → 2] // cm[2, 3 → 1]) ≡ (cm[2, 3 → 2] // cm[1, 2 → 1])

Out[*]=

True