

```
In[*]:= CF[ε_] := ε /. e^L_ -> e^Expand[L]
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In[*]:= msol =
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Echo[# -> Simplify[DSolve[{# - ε ξ1,1 f1,2[λ] + ε ξ1,2 f1,2[λ] - ε ξ2,1 f1,2[λ] + ε ξ2,2 f1,2[λ] -
    ε f1,2'[λ] == 0, f1,2[0] == 0},
    f1,2[λ], λ
  ]]] & /@
(List@@ (6 ε λ2 η1,2 ξ1,2 ξ2,1,2 + 2 ε λ3 η1,2 ξ1,1 ξ1,2 ξ2,1,2 - 2 ε λ3 η1,2 ξ1,2 ξ1,2 ξ2,1,2 +
  2 ε λ3 η1,2 ξ1,2 ξ2,1,1 ξ2,1,2 + 3 eλ ξ2,1,1-λ ξ2,2,2 ε λ2 η1,2 ξ2,1,22 + eλ ξ2,1,1-λ ξ2,2,2 ε λ3 η1,2 ξ1,1 ξ2,1,22 -
  eλ ξ2,1,1-λ ξ2,2,2 ε λ3 η1,2 ξ1,2 ξ2,1,22 + 2 eλ ξ2,1,1-λ ξ2,2,2 ε λ3 η1,2 ξ2,1,1 ξ2,1,22 - 2 eλ ξ2,2,2-λ ξ2,3,3 ε
  λ η1,2,3 ξ2,1,3 - eλ ξ2,2,2-λ ξ2,3,3 ε λ2 η1,2,3 ξ1,1 ξ2,1,3 + 3 eλ ξ2,2,2-λ ξ2,3,3 ε λ2 η1,3 ξ1,1,2 ξ2,1,3 +
  eλ ξ2,2,2-λ ξ2,3,3 ε λ3 η1,3 ξ1,1 ξ1,2 ξ2,1,3 + eλ ξ2,2,2-λ ξ2,3,3 ε λ2 η1,2,3 ξ1,2,2 ξ2,1,3 -
  eλ ξ2,2,2-λ ξ2,3,3 ε λ3 η1,3 ξ1,2 ξ2,1,3 - eλ ξ2,2,2-λ ξ2,3,3 ε λ2 η1,2,3 ξ2,1,1 ξ2,1,3 +
  eλ ξ2,2,2-λ ξ2,3,3 ε λ3 η1,3 ξ1,2 ξ2,1,3 - 2 eλ ξ2,2,2-λ ξ2,3,3 ε λ3 η1,2 ξ1,2 ξ2,1,2 ξ2,2,2 -
  2 eλ ξ2,1,1-λ ξ2,2,2 ε λ3 η1,2 ξ2,1,22 ξ2,2,2 - 3 e-λ ξ2,1,1+2 λ ξ2,2,2-λ ξ2,3,3 ε λ2 η1,2,3 ξ1,1,2 ξ2,2,3 -
  e-λ ξ2,1,1+2 λ ξ2,2,2-λ ξ2,3,3 ε λ3 η1,2,3 ξ1,1,1 ξ1,2 ξ2,2,3 + e-λ ξ2,1,1+2 λ ξ2,2,2-λ ξ2,3,3 ε λ3 η1,2,3 ξ1,1,2 ξ1,2,2 ξ2,2,3 -
  3 eλ ξ2,2,2-λ ξ2,3,3 ε λ2 η1,2,3 ξ2,1,2 ξ2,2,3 - eλ ξ2,2,2-λ ξ2,3,3 ε λ3 η1,2,3 ξ1,1,1 ξ2,1,2 ξ2,2,3 -
  4 eλ ξ2,2,2-λ ξ2,3,3 ε λ3 η1,3 ξ1,1,2 ξ2,1,2 ξ2,2,3 - eλ ξ2,2,2-λ ξ2,3,3 ε λ4 η1,3 ξ1,1,1 ξ1,2 ξ2,1,2 ξ2,2,3 +
  eλ ξ2,2,2-λ ξ2,3,3 ε λ3 η1,2,3 ξ1,2,2 ξ2,1,2 ξ2,2,3 + eλ ξ2,2,2-λ ξ2,3,3 ε λ4 η1,3 ξ1,1,2 ξ1,2 ξ2,1,2 ξ2,2,3 -
  eλ ξ2,2,2-λ ξ2,3,3 ε λ3 η1,2,3 ξ2,1,1 ξ2,1,2 ξ2,2,3 - eλ ξ2,2,2-λ ξ2,3,3 ε λ4 η1,3 ξ1,1,2 ξ2,1,1 ξ2,1,2 ξ2,2,3 -
  4 eλ ξ2,1,1-λ ξ2,3,3 ε λ3 η1,3 ξ2,1,22 ξ2,2,3 - eλ ξ2,1,1-λ ξ2,3,3 ε λ4 η1,3 ξ1,1,1 ξ2,1,22 ξ2,2,3 +
  eλ ξ2,1,1-λ ξ2,3,3 ε λ4 η1,3 ξ1,2,2 ξ2,1,22 ξ2,2,3 - 2 eλ ξ2,1,1-λ ξ2,3,3 ε λ4 η1,3 ξ2,1,1 ξ2,1,22 ξ2,2,3 -
  e-λ ξ2,1,1+2 λ ξ2,2,2-λ ξ2,3,3 ε λ3 η1,2,3 ξ1,1,2 ξ2,2,2 ξ2,2,3 + eλ ξ2,1,1-λ ξ2,3,3 ε λ4 η1,3 ξ2,1,22 ξ2,2,2 ξ2,2,3 +
  eλ ξ2,2,2-λ ξ2,3,3 ε λ2 η1,2,3 ξ2,1,3 ξ2,3,3 - eλ ξ2,2,2-λ ξ2,3,3 ε λ3 η1,3 ξ1,1,2 ξ2,1,3 ξ2,3,3 +
  e-λ ξ2,1,1+2 λ ξ2,2,2-λ ξ2,3,3 ε λ3 η1,2,3 ξ1,1,2 ξ2,2,3 ξ2,3,3 + eλ ξ2,2,2-λ ξ2,3,3 ε λ3 η1,2,3 ξ2,1,2 ξ2,2,3 ξ2,3,3 +
  eλ ξ2,2,2-λ ξ2,3,3 ε λ4 η1,3 ξ1,1,2 ξ2,1,2 ξ2,2,3 ξ2,3,3 + eλ ξ2,1,1-λ ξ2,3,3 ε λ4 η1,3 ξ2,1,22 ξ2,2,3 ξ2,3,3))
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Out[*]=
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In[*]:= Simplify@Total[msol[[All, 2, 1, 1, 2]]]
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Out[*]=
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$$-e^{-(\xi_{2,1,1}+\xi_{2,2,2}+\xi_{2,3,3})} \lambda^2 \left(-e^{(\xi_{2,1,1}+\xi_{2,3,3})} \lambda \eta_{1,2} \xi_{2,1,2} \left(2 e^{\lambda \xi_{2,2,2}} \xi_{1,2} + e^{\lambda \xi_{2,1,1}} \xi_{2,1,2} \right) + \right. \\ \left. e^{2 \lambda \xi_{2,2,2}} \eta_{1,2,3} \left(e^{\lambda \xi_{2,1,1}} \xi_{2,1,3} + \lambda \left(e^{\lambda \xi_{2,2,2}} \xi_{1,2} + e^{\lambda \xi_{2,1,1}} \xi_{2,1,2} \right) \xi_{2,2,3} \right) + \right. \\ \left. e^{\lambda (\xi_{2,1,1}+\xi_{2,2,2})} \lambda \eta_{1,3} \left(e^{\lambda \xi_{2,1,1}} \lambda \xi_{2,1,2}^2 \xi_{2,2,3} - e^{\lambda \xi_{2,2,2}} \xi_{1,2} (\xi_{2,1,3} - \lambda \xi_{2,1,2} \xi_{2,2,3}) \right) \right)$$

```
In[*]:= Expand@CF@Together@Total[msol[[All, 2, 1, 1, 2]]]
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Out[*]=
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$$2 \lambda^3 \eta_{1,2} \xi_{1,2} \xi_{2,1,2} + e^{\lambda \xi_{2,1,1}-\lambda \xi_{2,2,2}} \lambda^3 \eta_{1,2} \xi_{2,1,2}^2 - e^{\lambda \xi_{2,2,2}-\lambda \xi_{2,3,3}} \lambda^2 \eta_{1,2,3} \xi_{2,1,3} + \\ e^{\lambda \xi_{2,2,2}-\lambda \xi_{2,3,3}} \lambda^3 \eta_{1,3} \xi_{1,1,2} \xi_{2,1,3} - e^{-\lambda \xi_{2,1,1}+2 \lambda \xi_{2,2,2}-\lambda \xi_{2,3,3}} \lambda^3 \eta_{1,2,3} \xi_{1,1,2} \xi_{2,2,3} - \\ e^{\lambda \xi_{2,2,2}-\lambda \xi_{2,3,3}} \lambda^3 \eta_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,2,2}-\lambda \xi_{2,3,3}} \lambda^4 \eta_{1,3} \xi_{1,1,2} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,1,1}-\lambda \xi_{2,3,3}} \lambda^4 \eta_{1,3} \xi_{2,1,2}^2 \xi_{2,2,3}$$

In[*]:= **s = 0; k = 0;**

Do[

s = ExpandDenominator@ExpandNumerator@Together@CF[s + t];

Echo[(++k) → $\frac{\text{Short}[\text{Numerator}[s]]}{\text{Factor@Denominator}[s]}$],

{t, msol[[All, 2, 1, 1, 2]]}

]

$$\begin{aligned}
 & \text{1} \rightarrow \frac{e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1}} \left(\ll 22 \gg + 6 \ll 5 \gg \xi_{2,2}^2 \right)}{(\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})^3} \\
 & \text{2} \rightarrow \frac{e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1}} \ll 1 \gg}{(\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})^4} \\
 & \text{3} \rightarrow \frac{e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1}} \left(\ll 84 \gg + 2 \ll 6 \gg \xi_{2,2}^3 \right)}{(\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})^4} \\
 & \text{4} \rightarrow \frac{e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1}} \ll 1 \gg}{(\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})^4} \\
 & \text{5} \rightarrow \frac{e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2}} \left(\ll 685 \gg + 12 \ll 4 \gg \ll 1 \gg \right)}{(\xi_{1,1} - \xi_{1,2} + 2 \xi_{2,1} - 2 \xi_{2,2})^3 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})^4} \\
 & \text{6} \rightarrow \frac{e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2}} \ll 1 \gg}{(\xi_{1,1} - \xi_{1,2} + 2 \xi_{2,1} - 2 \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})^4} \\
 & \text{7} \rightarrow \frac{e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2}} \left(\ll 1227 \gg + 8 \ll 5 \gg \ll 1 \gg \right)}{(\xi_{1,1} - \xi_{1,2} + 2 \xi_{2,1} - 2 \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})^4} \\
 & \text{8} \rightarrow \frac{e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2}} \ll 1 \gg}{(\xi_{1,1} - \xi_{1,2} + 2 \xi_{2,1} - 2 \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})^4} \\
 & \text{9} \rightarrow \left(e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \ll 1 \gg \right) / \\
 & \quad \left((\xi_{1,1} - \xi_{1,2} + 2 \xi_{2,1} - 2 \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,3})^2 \right) \\
 & \text{10} \rightarrow \left(e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \left(\ll 10746 \gg + \ll 1 \gg \right) \right) / \\
 & \quad \left((\xi_{1,1} - \xi_{1,2} + 2 \xi_{2,1} - 2 \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,3})^3 \right) \\
 & \text{11} \rightarrow \left(e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \left(\ll 12809 \gg + \ll 1 \gg \right) \right) / \\
 & \quad \left((\xi_{1,1} - \xi_{1,2} + 2 \xi_{2,1} - 2 \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,3})^3 \right) \\
 & \text{12} \rightarrow \left(e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \ll 1 \gg \right) / \\
 & \quad \left((\xi_{1,1} - \xi_{1,2} + 2 \xi_{2,1} - 2 \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,3})^4 \right) \\
 & \text{13} \rightarrow \left(e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \ll 1 \gg \right) / \\
 & \quad \left((\xi_{1,1} - \xi_{1,2} + 2 \xi_{2,1} - 2 \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,3})^4 \right) \\
 & \text{14} \rightarrow \left(e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \ll 1 \gg \right) / \\
 & \quad \left((\xi_{1,1} - \xi_{1,2} + 2 \xi_{2,1} - 2 \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,3})^4 \right) \\
 & \text{15} \rightarrow \left(e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \ll 1 \gg \right) / \\
 & \quad \left((\xi_{1,1} - \xi_{1,2} + 2 \xi_{2,1} - 2 \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,3})^4 \right) \\
 & \text{16} \rightarrow \left(e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \ll 1 \gg \right) / \\
 & \quad \left((\xi_{1,1} - \xi_{1,2} + 2 \xi_{2,1} - 2 \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})^4 (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,3})^4 \right)
 \end{aligned}$$

<https://drorbn.net/AcademicPensieve/Projects/SolvablePBW/Archive/#MathematicaNotebooks>

$$\begin{aligned}
\gg 37 &\rightarrow \left(e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} (\ll 65 \ 561 \gg + \ll 1 \gg) \right) / \\
&\quad \left((\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,3})^5 (\xi_{1,1} - \xi_{1,2} + 2 \xi_{2,1} - \xi_{2,2} - \xi_{2,3})^5 (\xi_{1,1} - \xi_{1,2} + \xi_{2,2} - \xi_{2,3})^4 \right) \\
&\quad e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \ll 1 \gg \\
\gg 38 &\rightarrow \frac{(\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,3})^5 (\xi_{1,1} - \xi_{1,2} + 2 \xi_{2,1} - \xi_{2,2} - \xi_{2,3})^5}{e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \ll 1 \gg} \\
\gg 39 &\rightarrow \frac{(\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,3})^5 (\xi_{1,1} - \xi_{1,2} + 2 \xi_{2,1} - \xi_{2,2} - \xi_{2,3})^5}{e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \ll 1 \gg} \\
\gg 40 &\rightarrow \frac{(\xi_{1,1} - \xi_{1,2} + 2 \xi_{2,1} - \xi_{2,2} - \xi_{2,3})^5}{e^{-\lambda \xi_{1,1} - \lambda \xi_{2,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \ll 1 \gg} \\
\gg 41 &\rightarrow e^{-\lambda \xi_{2,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \left(2 e^{\lambda \xi_{2,1} + \lambda \ll 1 \gg + \lambda \xi_{2,1}} \lambda^3 \eta_{1,2} \xi_{1,2} \xi_{2,1,2} + \ll 12 \gg \right)
\end{aligned}$$

In[*]:= **Expand@CF@s**

Out[*]=

$$\begin{aligned}
&2 \lambda^3 \eta_{1,2} \xi_{1,2} \xi_{2,1,2} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^3 \eta_{1,2} \xi_{2,1,2}^2 - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,2,3} \xi_{2,1,3} + \\
&e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,3} \xi_{1,2} \xi_{2,1,3} - e^{-\lambda \xi_{2,1} + 2 \lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,2,3} \xi_{1,2} \xi_{2,2,3} - \\
&e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^4 \eta_{1,3} \xi_{1,2} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^4 \eta_{1,3} \xi_{2,1,2}^2 \xi_{2,2,3}
\end{aligned}$$

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In[*]:= msol =
Echo[# -> Simplify[DSolve[{# + e  $\xi_{1,1}$   $g_{1,3}[\lambda]$  - e  $\xi_{1,3,3}$   $g_{1,3}[\lambda]$  + e  $\xi_{2,1,1}$   $g_{1,3}[\lambda]$  - e  $\xi_{2,3,3}$   $g_{1,3}[\lambda]$  -
e  $g_{1,3}'[\lambda]$  == 0,  $g_{1,3}[0]$  == 0},
 $g_{1,3}[\lambda]$ ,  $\lambda$ 
]] & /@
(List @@ (-2 e  $\lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda \eta_{1,2,3} \eta_{2,1,2}$  + e  $\lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{1,1,1}$  - 2 e  $\lambda^2 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,1,2}$  -
2 e  $\lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,1,1} \xi_{1,1,2}$  + 2 e  $\lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,1,2} \xi_{1,2,2}$  - 2 e  $\lambda^2 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,2,3}$  -
2 e  $\lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,2,2} \xi_{1,2,3}$  - e  $\lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{1,3,3}$  + 2 e  $\lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,2,3} \xi_{1,3,3}$  +
e  $\lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{2,1,1}$  - 4 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^2 \eta_{1,2,3} \eta_{2,1,3} \xi_{2,1,2}$  -
3 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}$  e  $\lambda^2 \eta_{1,1,3} \eta_{2,1,2} \xi_{2,1,2}$  - 2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,2}$  e  $\lambda^2 \eta_{1,1,2} \eta_{2,1,3} \xi_{2,1,2}$  +
e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,1,3} \eta_{2,1,2} \xi_{1,1,1} \xi_{2,1,2}$  - 2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,2}$  e  $\lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{1,1,1} \xi_{2,1,2}$  +
2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,2}$  e  $\lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{1,2,2} \xi_{2,1,2}$  - 2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{1,2,3} \xi_{2,1,2}$  -
2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,2}$  e  $\lambda^3 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,2,3} \xi_{2,1,2}$  - 2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3}^2 \xi_{1,2,2} \xi_{2,1,2} \xi_{2,1,3}$  -
2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,2}$  e  $\lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,2,2} \xi_{2,1,2} \xi_{2,1,3}$  - e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,1,3} \eta_{2,1,2} \xi_{1,3,3} \xi_{2,1,2}$  +
2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3}^2 \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2}$  + 2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,2}$  e  $\lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2}$  -
2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{2,1,1} \xi_{2,1,2}$  - 2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,2}$  e  $\lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{2,1,1} \xi_{2,1,2}$  -
3 e  $\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^2 \eta_{1,1,3}^2 \xi_{2,1,3}$  + e  $\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,1,3}^2 \xi_{1,1,1} \xi_{2,1,3}$  -
e  $\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,1,3}^2 \xi_{1,3,3} \xi_{2,1,3}$  - e  $\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,1,3}^2 \xi_{2,1,1} \xi_{2,1,3}$  -
e  $\lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{2,2,2}$  + 2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{2,1,2} \xi_{2,2,2}$  +
2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,2}$  e  $\lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{2,1,2} \xi_{2,2,2}$  - e  $\lambda \xi_{2,1,1} + \lambda \xi_{2,2} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^2 \eta_{1,1,3} \eta_{2,1,3} \xi_{2,2,3}$  -
2 e  $\lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^2 \eta_{1,2,3} \eta_{2,1,3} \xi_{2,2,3}$  - e  $\lambda \xi_{2,1,1} + \lambda \xi_{2,2} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,1,1} \xi_{2,2,3}$  +
2 e  $\lambda \xi_{2,1,1} + \lambda \xi_{2,2} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,1,3}^2 \xi_{1,1,2} \xi_{2,2,3}$  + 2 e  $\lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,1,2} \xi_{2,2,3}$  +
2 e  $\lambda \xi_{2,1,1} + \lambda \xi_{2,2} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3}^2 \xi_{1,1,1} \xi_{1,1,2} \xi_{2,2,3}$  + 2 e  $\lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,1,1} \xi_{1,1,2} \xi_{2,2,3}$  -
2 e  $\lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,2,2} \xi_{2,2,3}$  - 2 e  $\lambda \xi_{2,1,1} + \lambda \xi_{2,2} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3}^2 \xi_{1,1,2} \xi_{1,2,2} \xi_{2,2,3}$  -
2 e  $\lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,1,2} \xi_{1,2,2} \xi_{2,2,3}$  + e  $\lambda \xi_{2,1,1} + \lambda \xi_{2,2} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,3,3} \xi_{2,2,3}$  +
2 e  $\lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,3,3} \xi_{2,2,3}$  + e  $\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3}^2 \xi_{1,1,1} \xi_{2,1,2} \xi_{2,2,3}$  +
2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,1,1} \xi_{2,1,2} \xi_{2,2,3}$  - 2 e  $\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3}^2 \xi_{1,2,2} \xi_{2,1,2} \xi_{2,2,3}$  -
4 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,2,2} \xi_{2,1,2} \xi_{2,2,3}$  + e  $\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3}^2 \xi_{1,3,3} \xi_{2,1,2} \xi_{2,2,3}$  +
2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,3,3} \xi_{2,1,2} \xi_{2,2,3}$  + e  $\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3}^2 \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3}$  +
2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3}$  - e  $\lambda \xi_{2,1,1} + \lambda \xi_{2,2} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,1,3} \eta_{2,1,3} \xi_{2,2,2} \xi_{2,2,3}$  -
2 e  $\lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{2,2,2} \xi_{2,2,3}$  - 2 e  $\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3}^2 \xi_{2,1,2} \xi_{2,2,2} \xi_{2,2,3}$  -
4 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{2,1,2} \xi_{2,2,2} \xi_{2,2,3}$  + e  $\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,1,3}^2 \xi_{2,1,3} \xi_{2,3,3}$  +
e  $\lambda \xi_{2,1,1} + \lambda \xi_{2,2} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,1,3} \eta_{2,1,3} \xi_{2,2,3} \xi_{2,3,3}$  + 2 e  $\lambda \xi_{2,2} - \lambda \xi_{2,3,3}$  e  $\lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{2,2,3} \xi_{2,3,3}$  +
e  $\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3}^2 \xi_{2,1,2} \xi_{2,2,3} \xi_{2,3,3}$  + 2 e  $\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}$  e  $\lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{2,1,2} \xi_{2,2,3} \xi_{2,3,3}$ ))

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Out[*]=

```

In[*]:= Factor[Total@msol[[1 ;; 15, 2, 1, 1, 2]]]

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Out[*]=

\$Aborted

In[*]:= **s = 0; k = 0;**

Do[

s = ExpandDenominator@ExpandNumerator@Together[s + t];

Echo[(++k) → $\frac{\text{Short}[\text{Numerator}[s]]}{\text{Factor@Denominator}[s]}$],

{t, msol[[1 ;; 15, 2, 1, 1, 2]]}

]

$$\begin{aligned}
 & \text{1} \rightarrow -\frac{2 e^{\lambda (\langle\langle 1 \rangle\rangle) + \lambda (\langle\langle 1 \rangle\rangle)} \eta_{1,2,3} \eta_{2,1,2} + \langle\langle 7 \rangle\rangle}{(\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,2,2})^2} \\
 & \text{2} \rightarrow -\frac{e^{\lambda (\xi_{2,2,2} - \xi_{2,3,3})} \lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{1,1,1}^3 + \langle\langle 36 \rangle\rangle}{(\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,2,2})^3} \\
 & \text{3} \rightarrow -\frac{\langle\langle 1 \rangle\rangle}{(\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,2,2})^3 (\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,3,3})^3} \\
 & \text{4} \rightarrow -\frac{e^{\lambda (\xi_{2,2,2} - \xi_{2,3,3})} \lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{1,1,1}^7 + \langle\langle 1614 \rangle\rangle}{(\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,2,2})^3 (\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,3,3})^4} \\
 & \text{5} \rightarrow -\frac{e^{\lambda (\xi_{2,2,2} - \xi_{2,3,3})} \lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{1,1,1}^7 + \langle\langle 2186 \rangle\rangle}{(\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,2,2})^3 (\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,3,3})^4} \\
 & \text{6} \rightarrow -\frac{e^{\lambda (\xi_{2,2,2} - \xi_{2,3,3})} \lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{1,1,1}^7 + \langle\langle 2776 \rangle\rangle}{(\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,2,2})^3 (\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,3,3})^4} \\
 & \text{7} \rightarrow -\frac{e^{\lambda (\xi_{2,2,2} - \xi_{2,3,3})} \lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{1,1,1}^7 + \langle\langle 3344 \rangle\rangle}{(\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,2,2})^3 (\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,3,3})^4} \\
 & \text{8} \rightarrow -\frac{\langle\langle 1 \rangle\rangle}{(\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,2,2})^3 (\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,3,3})^4} \\
 & \text{9} \rightarrow -\frac{e^{\lambda (\xi_{2,2,2} - \xi_{2,3,3})} \lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{1,1,1}^7 + \langle\langle 3627 \rangle\rangle + e^{\lambda (\langle\langle 1 \rangle\rangle)} \lambda^2 \langle\langle 4 \rangle\rangle \xi_{2,1,1}^4}{(\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,2,2})^3 (\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,3,3})^4} \\
 & \text{10} \rightarrow -\frac{\langle\langle 1 \rangle\rangle}{(\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,2,2})^3 (\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,3,3})^4} \\
 & \text{11} \rightarrow -\left(\left(e^{-\lambda (-2 \xi_{2,1,1} + \xi_{2,2,2} + \xi_{2,3,3})} (\langle\langle 12\,240 \rangle\rangle + 4 \langle\langle 5 \rangle\rangle \xi_{2,1,1}^4) \right) / \right. \\
 & \quad \left. \left((\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,2,2})^3 (\xi_{1,1,1} - \xi_{1,3,3} - \xi_{2,1,1} + \xi_{2,2,2})^3 (\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,3,3})^4 \right) \right) \\
 & \text{12} \rightarrow -\left(\left(e^{-\lambda (\xi_{1,3,3} - \xi_{2,1,1} + \xi_{2,3,3}) - \lambda (-2 \xi_{2,1,1} + \xi_{2,2,2} + \xi_{2,3,3})} (\langle\langle 28\,004 \rangle\rangle + 3 \langle\langle 7 \rangle\rangle \langle\langle 1 \rangle\rangle) \right) / \left((\xi_{1,1,1} - \xi_{1,3,3})^3 \right. \right. \\
 & \quad \left. \left. (\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,2,2})^3 (\xi_{1,1,1} - \xi_{1,3,3} - \xi_{2,1,1} + \xi_{2,2,2})^3 (\xi_{1,1,1} - \xi_{1,3,3} + \xi_{2,1,1} - \xi_{2,3,3})^4 \right) \right)
 \end{aligned}$$

Out[*]=

\$Aborted

In[*]:= **Table**[**Simplify**@**DSolve**[**c** λ^k $e^{b\lambda} + a$ **g**[λ] + **g'**[λ] == **0** $\wedge$ **g**[**0**] == **0**, **g**[λ], λ], {**k**, **0**, **5**}]

Out[*]=

$$\left\{ \left\{ \left\{ g[\lambda] \rightarrow \frac{c (e^{-a\lambda} - e^{b\lambda})}{a+b} \right\} \right\}, \left\{ \left\{ g[\lambda] \rightarrow -\frac{c e^{-a\lambda} (1 + e^{(a+b)\lambda} (-1 + a\lambda + b\lambda))}{(a+b)^2} \right\} \right\}, \right. \\ \left. \left\{ \left\{ g[\lambda] \rightarrow -\frac{c e^{-a\lambda} (-2 + e^{(a+b)\lambda} (2 - 2b\lambda + a^2\lambda^2 + b^2\lambda^2 + 2a\lambda (-1 + b\lambda)))}{(a+b)^3} \right\} \right\}, \right. \\ \left. \left\{ \left\{ g[\lambda] \rightarrow -\frac{1}{(a+b)^4} c e^{-a\lambda} \right. \right. \right. \\ \left. \left. \left(6 + e^{(a+b)\lambda} (-6 + 6b\lambda - 3b^2\lambda^2 + a^3\lambda^3 + b^3\lambda^3 + 3a^2\lambda^2 (-1 + b\lambda) + 3a\lambda (2 - 2b\lambda + b^2\lambda^2)) \right) \right\} \right\}, \\ \left\{ \left\{ g[\lambda] \rightarrow -\frac{1}{(a+b)^5} c e^{-a\lambda} (-24 + e^{(a+b)\lambda} (24 - 24b\lambda + 12b^2\lambda^2 - 4b^3\lambda^3 + a^4\lambda^4 + b^4\lambda^4 + \right. \right. \\ \left. \left. 4a^3\lambda^3 (-1 + b\lambda) + 6a^2\lambda^2 (2 - 2b\lambda + b^2\lambda^2) + 4a\lambda (-6 + 6b\lambda - 3b^2\lambda^2 + b^3\lambda^3)) \right) \right\} \right\}, \\ \left\{ \left\{ g[\lambda] \rightarrow -\frac{1}{(a+b)^6} c e^{-a\lambda} (120 + e^{(a+b)\lambda} (-120 + 120b\lambda - 60b^2\lambda^2 + 20b^3\lambda^3 - 5b^4\lambda^4 + \right. \right. \\ \left. \left. a^5\lambda^5 + b^5\lambda^5 + 5a^4\lambda^4 (-1 + b\lambda) + 10a^3\lambda^3 (2 - 2b\lambda + b^2\lambda^2) + \right. \right. \\ \left. \left. 10a^2\lambda^2 (-6 + 6b\lambda - 3b^2\lambda^2 + b^3\lambda^3) + 5a\lambda (24 - 24b\lambda + 12b^2\lambda^2 - 4b^3\lambda^3 + b^4\lambda^4)) \right) \right\} \right\} \right\}$$

In[*]:= **DSolve**[**c** $\lambda^3 + a$ **g**[λ] + **g'**[λ] == **0** $\wedge$ **g**[**0**] == **0**, **g**[λ], λ]

Out[*]=

$$\left\{ \left\{ g[\lambda] \rightarrow -\frac{c e^{-a\lambda} (6 - 6e^{a\lambda} + 6a e^{a\lambda}\lambda - 3a^2 e^{a\lambda}\lambda^2 + a^3 e^{a\lambda}\lambda^3)}{a^4} \right\} \right\}$$