```
Kas[K_, ω_] :=
  Module[{u, v, XingsByArmpits, bends, faces, p, A, is},
   u = Re[ω^(1/2)]; v = Re[ω]; (* so v=2u^2-1 *);
   XingsByArmpits =
    List @@ PD[K] /. x : X[i_, j_, k_, l_] :>
      If[PositiveQ[x], X+[-i, j, k, -l], X-[-j, k, l, -i]];
   bends = Times @@ XingsByArmpits /.
     _[X][a_, b_, c_, d_] :> p[a,-d] p[b,-a] p[c,-b] p[d,-c];
   faces = bends //. p[x__,y_] p[y_,z__] :> p[x,y,z];
   A = Table[0, Length@faces, Length@faces];
   Do[is = Position[faces, #][[1, 1]] & /@ List @@ x;
    A[[is, is]] += If[Head[x] === X+,
      {{v, u, 1, u}, {u, 1, u, 1}, {1, u, v, u}, {u, 1, u, 1}},
      -{{v, u, 1, u}, {u, 1, u, 1}, {1, u, v, u}, {u, 1, u, 1}}],
    {x, XingsByArmpits}];
   (MatrixSignature[A] - Writhe[K]) / 2];
```