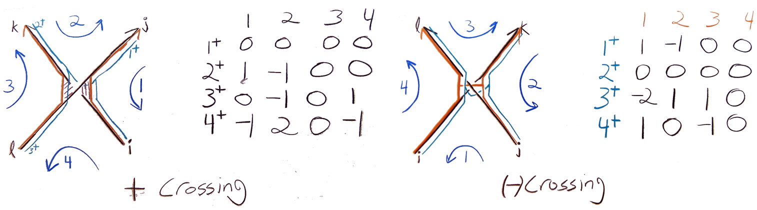
Dror Bar-Natan: Signatures

Initialization and Commons

```
Once[<< KnotTheory`]
```

```
MatrixSignature[M_?MatrixQ] :=
 Total@Sign@Eigenvalues@M
```

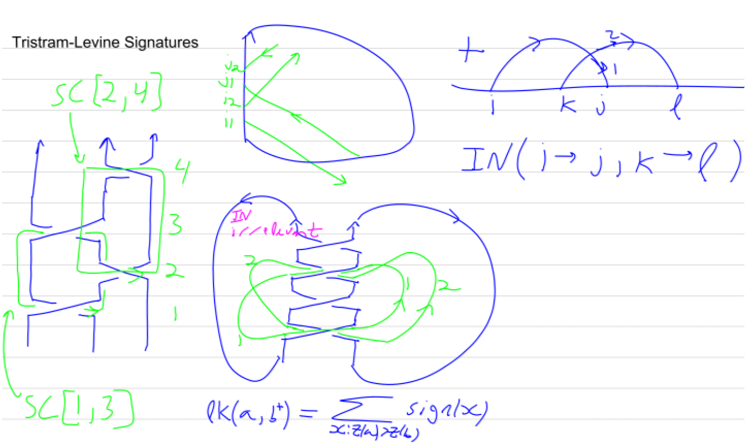
Knot Signatures as in KnotTheory`




```
TLS1[K_, ω_] := Module[{spd, a, s = 0, c, cs, A, is},
  spd = Times @@ PD[K] /.
    x_X :> If[PositiveQ@x, Xp, Xm] @@ x;
  cs = spd /. {
      Xp[i_, j_, k_, l_] :> a_{j,i}[++s] a_{k,-j}[++s]
        a_{-l,-k}[++s] a_{-i,l}[++s],
      Xm[i_, j_, k_, l_] :>
       a_{-j,i}[++s] a_{k,j}[++s] a_{l,-k}[++s] a_{-i,-l}[++s]
    } //. a_{i_,j_}[x__] a_{j_,k_}[y__] :> a_{i,k}[x, y] /.
    a_[x__] :> a[x];
  A = Table[0, Length@cs, Length@cs];
  Do[is = Position[cs, 4 i - #][[1, 1]] & /@
      {3, 2, 1, 0};
    A[[is, is]] += If[spd[[i, 0]] === Xp,
      ( 0  0  0  0 )   ( 1 -1  0  0 )
      ( 1 -1  0  0 )   ( 0  0  0  0 )
      ( 0 -1  0  1 ) , (-2  1  1  0 ) ],
      (-1  2  0 -1 )   ( 1  0 -1  0 )
    {i, Length[spd]}];
  Total[Sign[Select[
      Eigenvalues[(1 - ω) A + (1 - ω*) A^T],
      Abs[#] > 10^-6 &]]]];
```

To do. Alexander in this language.

Tristram-Levine for Braid Closures



IN[i, j, k, l] measures the upper-half-plane intersection number of the arrows $i \to j$ and $k \to l$.

```
χ[cond_] := If[TrueQ[cond], 1, 0];
IN[i_, j_, k_, l_] :=
 χ[i > l] + χ[j > k] - χ[i > k] - χ[j > l]
```

SC stands for "Simple Cycle".

```
SCs[β_BR] := Module[{n = β[[1]]}, Flatten@Table[
    SC @@@ Subsets[Flatten@Position[Abs[β[[2]]], k],
      {2}], {k, n - 1}] ]
```

lk[β, sc1, sc2] computes lk(sc1, sc2$^+$) in β.

```
lk[β_BR, SC[i_, j_], SC[k_, l_]] := Module[
  {n = β[[1]], s1 = Abs@β[[2, i]], s2 = Abs@β[[2, k]]},
  Which[
   s2 - s1 == 1, IN[i, j, l, k],
   s1 == s2, χ[β[[2, i]] > 0] (χ[i == l] - χ[i == k]) +
    χ[β[[2, j]] > 0] (χ[j == k] - χ[j == l]) -
    IN[i + 0.1, j + 0.1, k, l],
   True, 0] ]
```

SM for Seifert Matrix.

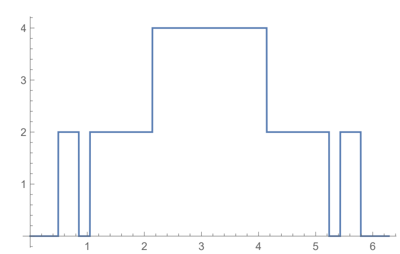
```
SM[β_BR] := SM[β] = Module[{n = β[[1]], col, H1},
   H1 = Flatten@Table[
      col = Flatten@Position[Abs[β[[2]]], k];
      SC[First@col, #] & /@ Rest[col],
      {k, n - 1}];
   Table[lk[β, sc1, sc2], {sc1, H1}, {sc2, H1}] ];
```

```
AlexFromSM[K_] :=
 Module[{A = SM@BR@K}, Det[t A - A^T]]
```

```
SignatureFromSM[K_] :=
 Module[{A = SM@BR@K}, MatrixSignature[A + A^T]]
```

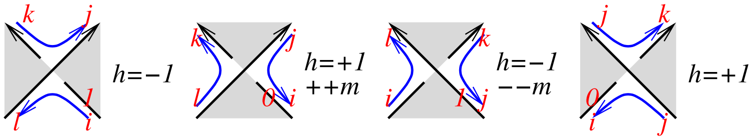
```
TLSFromSM[K_, ω_] :=
 Module[{A = SM@BR@K},
  MatrixSignature[(1 - ω) A + (1 - ω*) A^T]]
```

```
Plot[TLSFromSM[Knot@"K12a422", e^(i t)], {t, 0, 2 π}]
```



Knot Signatures Using the Goeritz Matrix

Formulas follow Gordon-Litherland. For checkerboard colouring, the region to the right of an odd-numbered arc is declared to be black.




```
GoeritzSignature[K_] :=
  Module[{m = 0, a, c = 0, ds, cs, is, A},
   ds = List @@ PD[K] /.
     x : X[i_, j_, k_, l_] :→ If[PositiveQ@x,
       If[OddQ@i, {a_{-i,l}[++c] a_{k,-j}[++c], -1},
        {a_{j,i}[++c] a_{-l,-k}[++c], ++m; 1}],
       If[OddQ@i, {a_{-i,-l}[++c] a_{k,j}[++c], --m; -1},
        {a_{-j,i}[++c] a_{l,-k}[++c], 1}]];
   cs =
    Times @@ ds⟦All, 1⟧ //.
      a_{i_,j_}[x__] a_{j_,k_}[y__] :→ a_{i,k}[x, y] /.
     a__[x__] :→ a[x];
   A = Table[0, Length@cs, Length@cs];
   Do[is = Position[cs, 2 i - #]⟦1, 1⟧ & /@ {1, 0};
    A⟦is, is⟧ += ds⟦i, 2⟧ {{1, -1}, {-1, 1}},
    {i, Length[ds]} ];
   MatrixSignature[A] - m ];
```

To do. Tristram-Levine and Alexander in this language.