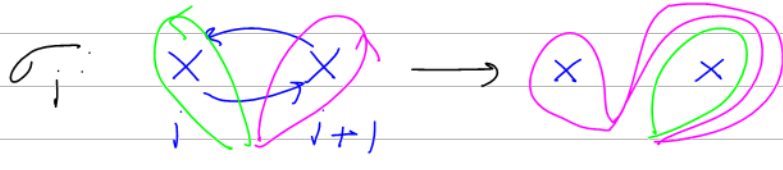


$$H_1(\tilde{D}_n) \rightarrow H_1(\tilde{D}_n, \tilde{b}) \xrightarrow{\cong} H_0(\tilde{b}) = \mathbb{Z}[t^{\pm 1}]$$

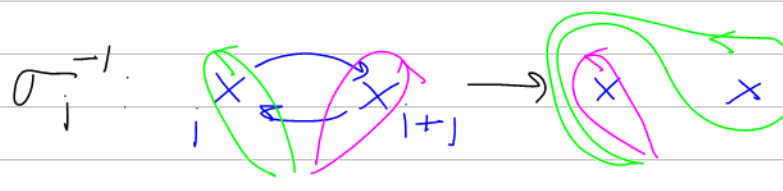
$$\tilde{\gamma}_i \rightarrow \tilde{\beta}_{i+1} - \tilde{\beta}_i \quad \tilde{\beta}_i \rightarrow t-1$$

$$\sum_{i \leq j} \tilde{\gamma}_j \longleftarrow \tilde{\beta}_j$$



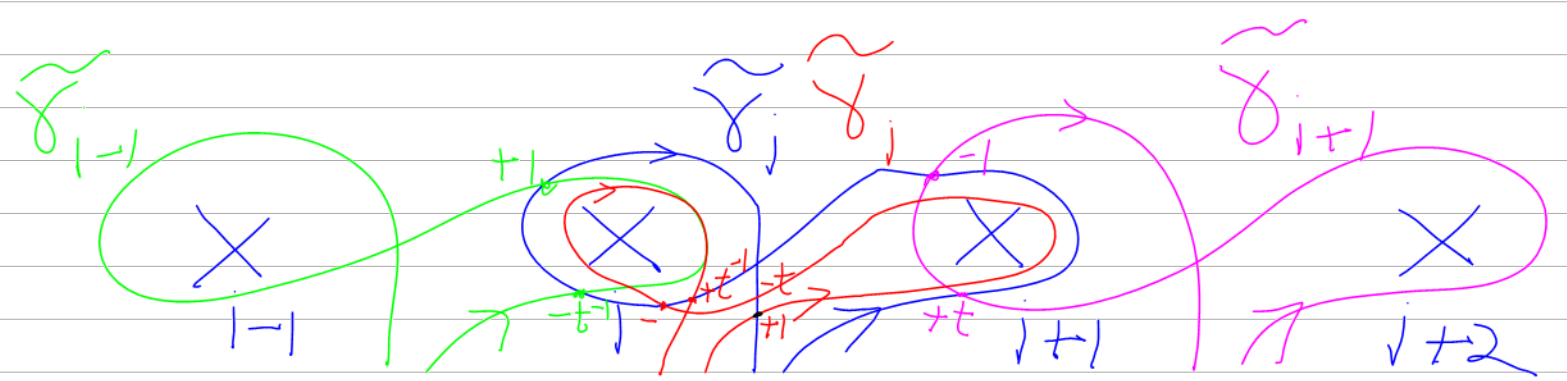
$$\tilde{\beta}_i \mapsto \tilde{\beta}_{i+1}$$

$$\tilde{\beta}_{i+1} \mapsto t\tilde{\beta}_i + (1-t)\tilde{\beta}_{i+1}$$



$$\tilde{\beta}_i \mapsto t^{-1}\tilde{\beta}_{i+1} + (1-t^{-1})\tilde{\beta}_i$$

$$\tilde{\beta}_{i+1} \mapsto \tilde{\beta}_i$$



$$\langle \tilde{\gamma}_i, \tilde{\gamma}_{i-1} \rangle = 1 - t^{-1}$$

$$\langle \tilde{\gamma}_i, \tilde{\gamma}_{i+1} \rangle = t - 1$$

$$\langle \tilde{\gamma}_i, \tilde{\gamma}_i \rangle = 1 - t + t^{-1} - 1$$

$$\gamma_{i-1} = \begin{array}{c} \times \quad \times \quad \times \quad \times \\ \circlearrowleft \quad \circlearrowright \quad \circlearrowleft \quad \circlearrowright \\ \downarrow \quad \uparrow \quad \downarrow \quad \uparrow \\ \times \quad \times \quad \times \quad \times \end{array} \xrightarrow{\beta_i} \begin{array}{c} \times \quad \times \quad \times \quad \times \\ \circlearrowleft \quad \circlearrowright \quad \circlearrowleft \quad \circlearrowright \\ \downarrow \quad \uparrow \quad \downarrow \quad \uparrow \\ \times \quad \times \quad \times \quad \times \end{array} = \gamma_{i-1} - t\gamma_i$$

$$\gamma_i = \begin{array}{c} \times \quad \times \quad \times \quad \times \\ \circlearrowleft \quad \circlearrowright \quad \circlearrowleft \quad \circlearrowright \\ \downarrow \quad \uparrow \quad \downarrow \quad \uparrow \\ \times \quad \times \quad \times \quad \times \end{array} \xrightarrow{\beta_i} \begin{array}{c} \times \quad \times \quad \times \quad \times \\ \circlearrowleft \quad \circlearrowright \quad \circlearrowleft \quad \circlearrowright \\ \downarrow \quad \uparrow \quad \downarrow \quad \uparrow \\ \times \quad \times \quad \times \quad \times \end{array} =$$

$$\gamma_{i+1} = \begin{array}{c} \times \quad \times \quad \times \quad \times \\ \circlearrowleft \quad \circlearrowright \quad \circlearrowleft \quad \circlearrowright \\ \downarrow \quad \uparrow \quad \downarrow \quad \uparrow \\ \times \quad \times \quad \times \quad \times \end{array} \xrightarrow{\beta_i} \begin{array}{c} \times \quad \times \quad \times \quad \times \\ \circlearrowleft \quad \circlearrowright \quad \circlearrowleft \quad \circlearrowright \\ \downarrow \quad \uparrow \quad \downarrow \quad \uparrow \\ \times \quad \times \quad \times \quad \times \end{array} =$$