Pensieve header: An FxF degenerate-Gaussian tangle formalism for tangles. Continues AlexanderSS.nb here and Signature.nb at pensieve://Talks/Budapest-2311.

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Semi-Seifert"];
Once[<< KnotTheory`]
```

```
Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at http://katlas.org/wiki/KnotTheory.
```

Integration

As in Talks/Beijing-2407

In[∘]:=
```
CF[ω_. ℰ_𝔼] := CF[ω] CF /@ ℰ;
CF[ℰ_List] := CF /@ ℰ;
CF[q_] := Expand@Collect[q, ϵ | (p | x | π | ξ)_, F] /. F → Factor;
```

In[∘]:=
```
𝔼 /: 𝔼[A_] 𝔼[B_] := 𝔼[A + B];
```

In[∘]:=
```
$π = Identity;
```

In[∘]:=
```
Unprotect[Integrate];
∫ω_. 𝔼[L_] ⅆ(vs_List) := Module[{n, L0, Q, Δ, G, Z0, Z, λ, DZ, DDZ, FZ, a, b},
  n = Length@vs; L0 = L /. ϵ → 0;
  Q = Table[(-∂_{vs⟦a⟧,vs⟦b⟧} L0) /. Thread[vs → 0] /. (p | x)__ → 0, {a, n}, {b, n}];
  If[(Δ = Det[Q]) == 0, Return@"Degenerate Q!"];
  Z = Z0 = CF@$π[L + vs.Q.vs / 2]; G = Inverse[Q];
  FixedPoint[(DZ = Table[∂_v Z, {v, vs}];
    DDZ = Table[∂_u DZ, {u, vs}];
    FZ = Sum[G⟦a, b⟧ (DDZ⟦a, b⟧ + DZ⟦a⟧ DZ⟦b⟧), {a, n}, {b, n}] / 2;
    Z = CF[Z0 + ∫_0^λ $π[FZ] ⅆλ]) &, Z];
  PowerExpand@Factor[ω Δ^(-1/2)] 𝔼[CF[Z /. λ → 1 /. Thread[vs → 0]]]];
Protect[Integrate];
```

(Alt) In[∘]:=

$$\int \mathbb{E}\left[-\mu x^2/2 + i\,\xi\,x\right] \,ⅆ\{x\}$$

(Alt) Out[∘]=

$$\frac{\mathbb{E}\left[-\frac{\xi^2}{2\,\mu}\right]}{\sqrt{\mu}}$$

(Alt) In[ ]:=

$$\text{FofG} = \int \mathbb{E}\left[-\mu\,(x-a)^2/2 + i\,\xi\,x\right] d\{x\}$$

(Alt) Out[ ]=

$$\frac{\mathbb{E}\left[\frac{i\,(2\,a\,\mu+i\,\xi)\,\xi}{2\,\mu}\right]}{\sqrt{\mu}}$$

(Alt) In[ ]:=

$$\int \text{FofG}\,\mathbb{E}[-i\,\xi\,x]\,d\{\xi\}$$

(Alt) Out[ ]=

$$\mathbb{E}\left[-\frac{1}{2}\,(a-x)^2\,\mu\right]$$

(Alt) In[ ]:=

$$L = -\frac{1}{2}\{x_1, x_2\}.\begin{pmatrix} a & b \\ b & c \end{pmatrix}.\{x_1, x_2\} + \{\xi_1, \xi_2\}.\{x_1, x_2\};$$

$$\text{Z12} = \int \mathbb{E}[L]\,d\{x_1, x_2\}$$

(Alt) Out[ ]=

$$\frac{\mathbb{E}\left[\frac{c\,\xi_1^2}{2\,(-b^2+a\,c)} + \frac{b\,\xi_1\,\xi_2}{b^2-a\,c} + \frac{a\,\xi_2^2}{2\,(-b^2+a\,c)}\right]}{\sqrt{-b^2+a\,c}}$$

(Alt) In[ ]:=

$$\left\{\text{Z1} = \int \mathbb{E}[L]\,d\{x_1\},\ \text{Z12} == \int \text{Z1}\,d\{x_2\}\right\}$$

(Alt) Out[ ]=

$$\left\{\frac{\mathbb{E}\left[-\frac{(-b^2+a\,c)\,x_2^2}{2\,a} - \frac{b\,x_2\,\xi_1}{a} + \frac{\xi_1^2}{2\,a} + x_2\,\xi_2\right]}{\sqrt{a}},\ \text{True}\right\}$$

(Alt) In[ ]:=

$$\$\pi = \text{Normal}\left[\# + O[\epsilon]^{13}\right]\&;\ \int \mathbb{E}\left[-\phi^2/2 + \epsilon\,\phi^3/6\right] d\{\phi\}$$

(Alt) Out[ ]=

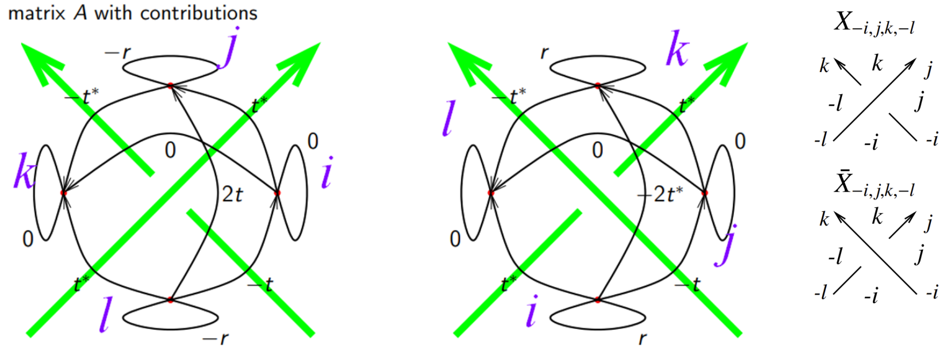
$$\mathbb{E}\left[\frac{5\,\epsilon^2}{24} + \frac{5\,\epsilon^4}{16} + \frac{1105\,\epsilon^6}{1152} + \frac{565\,\epsilon^8}{128} + \frac{82\,825\,\epsilon^{10}}{3072} + \frac{19\,675\,\epsilon^{12}}{96}\right]$$

The Planar Algebra

http://drorbn.net/cms21

Bedlewo for Mathematicians.

For a knot K and a complex unit ω set $t=1-\omega$, $r=2\Re(t)$, make an $F\times F$ matrix A with contributions



(conjugate if going against the flow) and output $\sigma(A)$.

```
In[∘]:= SetAttributes[{B, σ}, Orderless];
        CF[b_B] := RotateLeft[#, First@Ordering[#] - 1] & /@ DeleteCases[b, {}]
```

```
In[∘]:= CF[Σ_b_[σ_, q_]] := Σ_CF[b][σ, CF[q]];
```

SSA for Semi-Seifert Alexander.

```
In[∘]:= SSA[c : X[i_, j_, k_, l_]] := SSA@If[PositiveQ[c], X_{-i,j,k,-l}, X̄_{-j,k,l,-i}];
        SSA[(c : X | X̄)_{i_,j_,k_,l_}] := Module[{m},
          m = If[c === X, {{-1, -1, 2, 0}, {0, 0, 0, 0}, {0, 1, -1, 0}, {1, 0, -1, 0}},
                          {{1, -1, 0, 0}, {0, 0, 0, 0}, {-2, 1, 1, 0}, {1, 0, -1, 0}}];
          CF@Σ_B[{i,j,k,l}][σ[i → j, l → k], 𝔼[{p_i, p_j, p_k, p_l}.(t m - m^T / t).{x_i, x_j, x_k, x_l}]]]
```

```
In[∘]:= SSA[X[1, 2, 3, 4]]
```

Out[∘]=

$$\Sigma_{B[\{-4,-1,2,3\}]}\Big[\sigma[-4\to 3,\ -1\to 2],\ \mathbb{E}\Big[-\frac{p_{-1}x_{-4}}{t}+\frac{p_3x_{-4}}{t}+t\,p_{-4}x_{-1}-\frac{(-1+t)(1+t)\,p_{-1}x_{-1}}{t}+\frac{p_2x_{-1}}{t}-\frac{2p_3x_{-1}}{t}-t\,p_{-1}x_2+t\,p_3x_2-t\,p_{-4}x_3+2t\,p_{-1}x_3-\frac{p_2x_3}{t}-\frac{(-1+t)(1+t)\,p_3x_3}{t}\Big]\Big]$$

```
In[∘]:= SSA[X_{1,2,3,4}] /. t → 1
```

Out[∘]=

$$\Sigma_{B[\{1,2,3,4\}]}[\sigma[1\to 2,\ 4\to 3],\ \mathbb{E}[p_2x_1-2p_3x_1+p_4x_1-p_1x_2+p_3x_2+2p_1x_3-p_2x_3-p_4x_3-p_1x_4+p_3x_4]]$$

In[∘]:= $Q = p_2\,x_1 - 2\,p_3\,x_1 + p_4\,x_1 - p_1\,x_2 + p_3\,x_2 + 2\,p_1\,x_3 - p_2\,x_3 - p_4\,x_3 - p_1\,x_4 + p_3\,x_4$;
{Coefficient[Q, x_2], Coefficient[Q, p_2], Coefficient[Q, x_4], Coefficient[Q, p_4]}

Out[∘]=

$\{-p_1 + p_3,\ x_1 - x_3,\ -p_1 + p_3,\ x_1 - x_3\}$

In[∘]:= $\left\{\int \mathbb{E}[Q]\, d\{p_1, x_1\},\ \int \mathbb{E}[Q]\, d\{p_2, x_2\},\ \int \mathbb{E}[Q]\, d\{x_3\},\ \int \mathbb{E}[Q]\, d\{x_4\}\right\}$

Out[∘]=

{Degenerate Q!, Degenerate Q!, Degenerate Q!, Degenerate Q!}

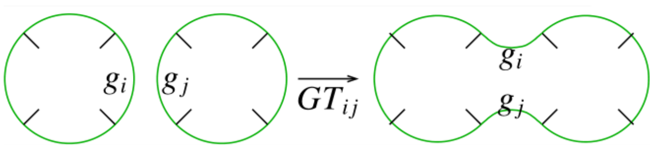
In[∘]:= SSA@$P_{i\_,j\_}$:= CF@$\Sigma_{B[\{i,j\}]}$[σ[$i \to j$], $\mathbb{E}$[0]]

In[∘]:= $\Sigma_{b1\_}$[$\sigma 1$_, $\mathcal{E}1$_] ⊕ $\Sigma_{b2\_}$[$\sigma 2$_, $\mathcal{E}2$_] ^:= CF@$\Sigma_{Join[b1,b2]}$[$\sigma 1 \cup \sigma 2$, $\mathcal{E}1\ \mathcal{E}2$];

In[∘]:= SSA[X[1, 2, 3, 4]] ⊕ SSA[X[5, 6, 7, 8]]

Out[∘]=

$$\Sigma_{B[\{-8,-5,6,7\},\{-4,-1,2,3\}]}\Big[\sigma[-8\to 7,\ -5\to 6,\ -4\to 3,\ -1\to 2],$$
$$\mathbb{E}\Big[-\frac{p_{-5}\,x_{-8}}{t} + \frac{p_7\,x_{-8}}{t} + t\,p_{-8}\,x_{-5} - \frac{(-1+t)\,(1+t)\,p_{-5}\,x_{-5}}{t} + \frac{p_6\,x_{-5}}{t} - \frac{2\,p_7\,x_{-5}}{t} - \frac{p_{-1}\,x_{-4}}{t} + \frac{p_3\,x_{-4}}{t} +$$
$$t\,p_{-4}\,x_{-1} - \frac{(-1+t)\,(1+t)\,p_{-1}\,x_{-1}}{t} + \frac{p_2\,x_{-1}}{t} - \frac{2\,p_3\,x_{-1}}{t} - t\,p_{-1}\,x_2 + t\,p_3\,x_2 - t\,p_{-4}\,x_3 + 2\,t\,p_{-1}\,x_3 - \frac{p_2\,x_3}{t} -$$
$$\frac{(-1+t)\,(1+t)\,p_3\,x_3}{t} - t\,p_{-5}\,x_6 + t\,p_7\,x_6 - t\,p_{-8}\,x_7 + 2\,t\,p_{-5}\,x_7 - \frac{p_6\,x_7}{t} - \frac{(-1+t)\,(1+t)\,p_7\,x_7}{t}\Big]\Big]$$



In[∘]:= $GT_{i\_,j\_}$@$\Sigma_{B[\{li\_\_\_,i\_,ri\_\_\_\},\{lj\_\_\_,j\_,rj\_\_\_\},bs\_\_\_]}$[$\sigma$_, $\mathcal{E}$_] :=
CF@$\Sigma_{B[\{ri,li,j,rj,lj,i\},bs]}$[$\sigma$, $\mathcal{E}$ /. {$p_{i|j} \to p_i + p_j$, $x_{i|j} \to x_i + x_j$}]

In[∘]:= (SSA[X[1, 2, 3, 4]] ⊕ SSA[X[5, 6, 7, 8]]) // $GT_{2,7}$

Out[∘]=

$$\Sigma_{B[\{-8,-5,6,2,3,-4,-1,7\}]}\Big[\sigma[-8\to 7,\ -5\to 6,\ -4\to 3,\ -1\to 2],$$
$$\mathbb{E}\Big[-\frac{p_{-5}\,x_{-8}}{t} + \frac{p_2\,x_{-8}}{t} + \frac{p_7\,x_{-8}}{t} + t\,p_{-8}\,x_{-5} - \frac{(-1+t)\,(1+t)\,p_{-5}\,x_{-5}}{t} - \frac{2\,p_2\,x_{-5}}{t} + \frac{p_6\,x_{-5}}{t} - \frac{2\,p_7\,x_{-5}}{t} -$$
$$\frac{p_{-1}\,x_{-4}}{t} + \frac{p_3\,x_{-4}}{t} + t\,p_{-4}\,x_{-1} - \frac{(-1+t)\,(1+t)\,p_{-1}\,x_{-1}}{t} + \frac{p_2\,x_{-1}}{t} - \frac{2\,p_3\,x_{-1}}{t} + \frac{p_7\,x_{-1}}{t} - t\,p_{-8}\,x_2 +$$
$$2\,t\,p_{-5}\,x_2 - t\,p_{-1}\,x_2 - \frac{(-1+t)\,(1+t)\,p_2\,x_2}{t} + t\,p_3\,x_2 - \frac{p_6\,x_2}{t} - \frac{(-1+t)\,(1+t)\,p_7\,x_2}{t} - t\,p_{-4}\,x_3 +$$
$$2\,t\,p_{-1}\,x_3 - \frac{p_2\,x_3}{t} - \frac{(-1+t)\,(1+t)\,p_3\,x_3}{t} - \frac{p_7\,x_3}{t} - t\,p_{-5}\,x_6 + t\,p_2\,x_6 + t\,p_7\,x_6 - t\,p_{-8}\,x_7 +$$
$$2\,t\,p_{-5}\,x_7 - t\,p_{-1}\,x_7 - \frac{(-1+t)\,(1+t)\,p_2\,x_7}{t} + t\,p_3\,x_7 - \frac{p_6\,x_7}{t} - \frac{(-1+t)\,(1+t)\,p_7\,x_7}{t}\Big]\Big]$$

In[∘]:=
```
Cordon_i_@Σ_B[{li___,i_,ri___},bs___][σ_, PQ[C_, q_]] :=
 Module[{ϕ = ∂_{γ_i} C, λ = ∂_{γ̄_i,γ_i} q, nσ = σ, nC, nq, p},
  {p} = FirstPosition[(# =!= 0) & /@ ϕ, True, {0}];
  {nC, nq} = Which[
    p > 0, {C, q} /. (γ_i → -C⟦p⟧ / ϕ⟦p⟧)^+ /. (γ_i → 0)^+,
    λ =!= 0, (nσ += sign[λ]; {C, q /. (γ_i → -(∂_{γ̄_i} q) / λ)^+ /. (γ_i → 0)^+}),
    λ === 0, {C ⋃ {∂_{γ̄_i} q}, q /. (γ_i → 0)^+}];
  CF@Σ_B[Most@{ri,li},bs][nσ, PQ[nC, nq] /. (γ_Last@{ri,li} → γ_First@{ri,li})^+] ]
```

In[∘]:=
```
c_{i_,j_}@t : Σ_B[{li___,i_,ri___},{___,j_,___},___][__] := t // GT_{j,First@{ri,li}} // Cordon_j
```

In[∘]:=
```
c_{i_,j_}@t : Σ_B[{___,i_,j_,___},___][__] := Cordon_j@t
c_{i_,j_}@t : Σ_B[{j_,___,i_},___][__] := Cordon_j@t
c_{i_,j_}@t : Σ_B[{___,j_,i_,___},___][__] := Cordon_i@t
c_{i_,j_}@t : Σ_B[{i_,___,j_},___][__] := Cordon_i@t
```

In[∘]:=
```
mc[ℰ_] := ℰ //.
  t : Σ_B[{___,i_,___},{___,j_,___},___][__] | Σ_B[{___,i_,j_,___},___][__] | Σ_B[{j_,___,i_},___][__] /;
   i + j == 0 :→ c_{i,j}@t
```