Pensieve header: Delta function games.

Integration

As in Talks/Beijing-2407

In[∘]:=
```
CF[ω_. ℰ_𝔼] := CF[ω] CF /@ ℰ;
CF[ℰ_List] := CF /@ ℰ;
CF[q_, vp_, h_] := Module[{H}, Expand@Collect[q, vp, H] /. H → h];
CF[q_, vp_] := CF[q, vp, Factor];
CF[q_] := CF[q, ϵ | (p | x | π | ξ)_ | (p | x | π | ξ)_+ | (p | x | π | ξ)_-];
```

In[∘]:=
```
𝔼 /: 𝔼[A_] 𝔼[B_] := 𝔼[A + B];
```

In[∘]:=
```
$π = Identity;
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In[∘]:=
```
Unprotect[Integrate];
∫ω_. 𝔼[L_] ⅆ(vs_List) := Module[{n, L0, Q, Δ, G, Z0, Z, λ, DZ, DDZ, FZ, a, b},
   n = Length@vs; L0 = L /. ϵ → 0;
   Q = Table[(-∂_{vs⟦a⟧,vs⟦b⟧} L0) /. Thread[vs → 0] /. (p | x)__ → 0, {a, n}, {b, n}];
   If[(Δ = Det[Q]) == 0, Return@"Degenerate Q!"];
   Z = Z0 = CF@$π[L + vs.Q.vs / 2]; G = Inverse[Q];
   FixedPoint[(DZ = Table[∂_v Z, {v, vs}];
      DDZ = Table[∂_u DZ, {u, vs}];
      FZ = Sum[G⟦a, b⟧ (DDZ⟦a, b⟧ + DZ⟦a⟧ DZ⟦b⟧), {a, n}, {b, n}] / 2;
      Z = CF[Z0 + ∫_0^λ $π[FZ] ⅆλ]) &, Z];
   PowerExpand@Factor[ω Δ^(-1/2)] 𝔼[CF[Z /. λ → 1 /. Thread[vs → 0]]]];
Protect[Integrate];
```

In[∘]:=
```
∫𝔼[- μ x^2 / 2 + ⅈ ξ x] ⅆ{x}
```

Out[∘]=

$$\frac{\mathbb{E}\left[-\frac{\xi^2}{2\,\mu}\right]}{\sqrt{\mu}}$$

In[∘]:=
```
FofG = ∫𝔼[- μ (x - a)^2 / 2 + ⅈ ξ x] ⅆ{x}
```

Out[∘]=

$$\frac{\mathbb{E}\left[\frac{\mathrm{i}\,(2\,a\,\mu+\mathrm{i}\,\xi)\,\xi}{2\,\mu}\right]}{\sqrt{\mu}}$$

In[∘]:= $\int \text{FofG}\, \mathbb{E}[-i\, \xi\, x]\, d\{\xi\}$

Out[∘]= $\mathbb{E}\left[-\frac{1}{2}(a-x)^2 \mu\right]$

In[∘]:= $L = -\frac{1}{2}\{x_1, x_2\}.\begin{pmatrix} a & b \\ b & c \end{pmatrix}.\{x_1, x_2\} + \{\xi_1, \xi_2\}.\{x_1, x_2\};$

$Z12 = \int \mathbb{E}[L]\, d\{x_1, x_2\}$

Out[∘]= $\frac{\mathbb{E}\left[\frac{c\,\xi_1^2}{2(-b^2+a\,c)} + \frac{b\,\xi_1\,\xi_2}{b^2-a\,c} + \frac{a\,\xi_2^2}{2(-b^2+a\,c)}\right]}{\sqrt{-b^2+a\,c}}$

In[∘]:= $\left\{Z1 = \int \mathbb{E}[L]\, d\{x_1\},\ Z12 == \int Z1\, d\{x_2\}\right\}$

Out[∘]= $\left\{\frac{\mathbb{E}\left[-\frac{(-b^2+a\,c)\,x_2^2}{2\,a} - \frac{b\,x_2\,\xi_1}{a} + \frac{\xi_1^2}{2\,a} + x_2\,\xi_2\right]}{\sqrt{a}},\ \text{True}\right\}$

In[∘]:= $\$\pi = \text{Normal}\left[\# + O[\epsilon]^{13}\right]\ \&;\ \int \mathbb{E}\left[-\phi^2/2 + \epsilon\,\phi^3/6\right] d\{\phi\}$

Out[∘]= $\mathbb{E}\left[\frac{5\,\epsilon^2}{24} + \frac{5\,\epsilon^4}{16} + \frac{1105\,\epsilon^6}{1152} + \frac{565\,\epsilon^8}{128} + \frac{82\,825\,\epsilon^{10}}{3072} + \frac{19\,675\,\epsilon^{12}}{96}\right]$

In[∘]:= $\left(\int \mathbb{E}\left[i\,x\,y + \lambda\,(x^2+y^2) + i\,\alpha\,y\right] d\{x, y\}\right)\ /.\ \lambda \to 0$

Out[∘]= $\mathbb{E}[0]$

In[∘]:= $\$\pi = \text{Normal}\left[\# + O[\epsilon]^{3}\right]\ \&;$

$\left(\int \mathbb{E}\left[i\,x\,y + \lambda\,(x^2+y^2) + \epsilon\,x^3 + i\,\alpha\,y\right] d\{x, y\}\right)\ /.\ \lambda \to 0$

Out[∘]= $\mathbb{E}\left[-\alpha^3\,\epsilon\right]$

In[∘]:= $\$\pi = \text{Normal}\left[\# + O[\epsilon]^{6}\right]\ \&;$

$\left(\int \mathbb{E}\left[i\,x\,y + \lambda\,(x^2+y^2) + \epsilon\,x^3 + i\,\alpha\,y\right] d\{x, y\}\right)\ /.\ \lambda \to 0$

Out[∘]= $\mathbb{E}\left[-\alpha^3\,\epsilon\right]$

In[∘]:= $\$\pi = \text{Normal}\left[\# + O[\epsilon]^{2}\right]\ \&;$

$\left(\int \mathbb{E}\left[i\,x\,y + \lambda\,(x^2+y^2) + \epsilon\,x^4 + i\,\alpha\,y\right] d\{x, y\}\right)\ /.\ \lambda \to 0$

Out[∘]= $\mathbb{E}\left[\alpha^4\,\epsilon\right]$

In[ ]:=
```
$π = Normal[# + O[ϵ]^3] &;
(∫𝔼[ⅈ x y + λ (x^2 + y^2) + ϵ x^4 + ⅈ α y] ⅆ{x, y}) /. λ → 0
```

Out[ ]=

$\mathbb{E}\left[\alpha^4 \epsilon\right]$