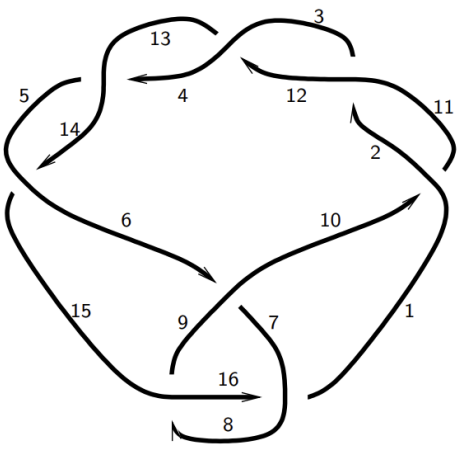
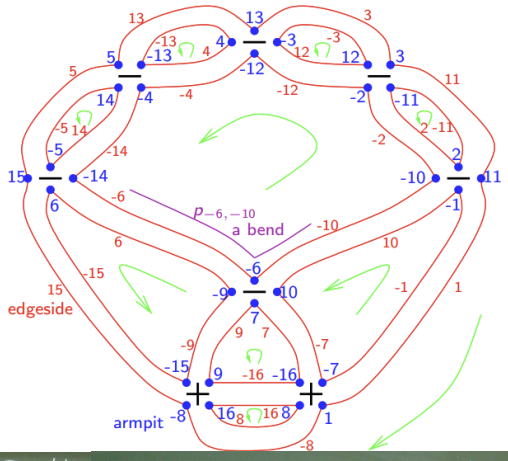
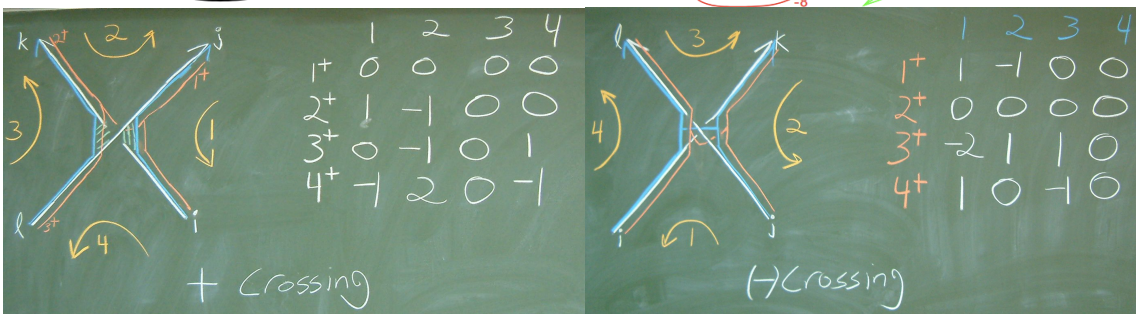
Pensieve header: A semi-Seifert formula for the Alexander polynomial. Continues AlexanderAsFxF.nb at pensieve://Projects/Signatures/.

In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Semi-Seifert"];
Once[<< KnotTheory`]
```

```
Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at http://katlas.org/wiki/KnotTheory.
```







SM for Seifert Matrix:

```
SM[K_] := Module[{XingsByArmpits, bends, faces, p, A, is, es, keeps},
  XingsByArmpits = List @@ PD[K] /.
    x : X[i_, j_, k_, l_] :> If[PositiveQ[x], X₊[-i, j, k, -l], X₋[-j, k, l, -i]];
  bends = Times @@ XingsByArmpits /. _[X][a_, b_, c_, d_] :> p_{a,-d} p_{b,-a} p_{c,-b} p_{d,-c};
  faces = bends //. p_{x__,y_} p_{y_,z__} :> p_{x,y,z};
  A = Table[0, Length@faces, Length@faces];
  Do[is = Position[faces, #][[1, 1]] & /@ List @@ x;
   A[[is, is]] += If[Head[x] === X₊,
     {{-1, -1, 2, 0}, {0, 0, 0, 0}, {0, 1, -1, 0}, {1, 0, -1, 0}},
     {{1, -1, 0, 0}, {0, 0, 0, 0}, {-2, 1, 1, 0}, {1, 0, -1, 0}}
    ],
   {x, XingsByArmpits}];
  A / 2]
```

In[•]:=
```
SM[Knot[3, 1]] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[•]:=
```
SSA[K_] := Module[{A, M, keeps},
  A = SM[K];
  M = Factor[T^(1/2) A - T^(-1/2) A^T];
  keeps = Complement[Range@Length@M, Union @@ FirstPosition[1] /@ NullSpace[M]];
  Factor@Det[M[[keeps, keeps]]]
 ]
Union@Table[Simplify[SSA[K] / Alexander[K][T]], {K, AllKnots[{3, 10}]}]
```

Out[•]=
```
{1}
```

In[•]:=
```
Plus[{{-1, -1, 2, 0}, {0, 0, 0, 0}, {0, 1, -1, 0}, {1, 0, -1, 0}},
  {{1, -1, 0, 0}, {0, 0, 0, 0}, {-2, 1, 1, 0}, {1, 0, -1, 0}}] / 2 // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix}$$

```
In[•]:= Plus[{{-1, -1, 2, 0}, {0, 0, 0, 0}, {0, 1, -1, 0}, {1, 0, -1, 0}}, -{{1, -1, 0, 0}, {0, 0, 0, 0}, {-2, 1, 1, 0}, {1, 0, -1, 0}}] / 2 // MatrixForm
```

```
Out[•]//MatrixForm=
( -1  0  1  0 )
(  0  0  0  0 )
(  1  0 -1  0 )
(  0  0  0  0 )
```

```
In[•]:= (A ↦ MatrixForm[A - A^T]) /@ {{{-1, -1, 2, 0}, {0, 0, 0, 0}, {0, 1, -1, 0}, {1, 0, -1, 0}}, {{1, -1, 0, 0}, {0, 0, 0, 0}, {-2, 1, 1, 0}, {1, 0, -1, 0}}}
```

```
Out[•]=
{ (  0 -1  2 -1 )   (  0 -1  2 -1 ) }
{ (  1  0 -1  0 ) , (  1  0 -1  0 ) }
{ ( -2  1  0  1 )   ( -2  1  0  1 ) }
{ (  1  0 -1  0 )   (  1  0 -1  0 ) }
```