Task. Define $\mathrm{Exp}_{m,i,k}[P]$ to compute $\mathbb{e}^{\mathbb{O}(P)}$ to ϵ^k in the using the $m_{i,i\to i}$ multiplication, where P is an ϵ-dependent near-docile element, giving the answer in $\mathbb{E}$-form.

Methodology. If $P_0 := P_{\epsilon=0}$ and $\mathbb{e}^{\lambda\,\mathbb{O}(P)} = \mathbb{O}\big(\mathbb{e}^{\lambda P_0}\,F(\lambda)\big)$, then $F(\lambda=0)=1$ and we have:

$$\mathbb{O}\big(\mathbb{e}^{\lambda P_0}(P_0\,F(\lambda)+\partial_\lambda F)\big) = \mathbb{O}\big(\partial_\lambda \mathbb{e}^{\lambda P_0}\,F(\lambda)\big) = \qquad .$$

$$\partial_\lambda \mathbb{O}\big(\mathbb{e}^{\lambda P_0}\,F(\lambda)\big) = \partial_\lambda \mathbb{e}^{\lambda\,\mathbb{O}(P)} = \mathbb{e}^{\lambda\,\mathbb{O}(P)}\,\mathbb{O}(P) = \mathbb{O}\big(\mathbb{e}^{\lambda P_0}\,F(\lambda)\big)\,\mathbb{O}(P)$$

This is a linear ODE for F. Setting inductively $F_k = F_{k-1} + \epsilon^k\,\varphi$ we find that $F_0 = 1$ and solve for φ.