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DeclareAlgebra[U_Symbol, opts_Rule] :=
Module[{gp, sr, g, cp, M, CE, pow, k = 0,
  gs = Generators /. {opts},
  cs = Centrals /. {opts} /. Centrals → {}},
(#U = U@#) & /@ gs;
gp = Alternatives @@ gs; gp = gp | gp_ (* gens *)
sr = Flatten@Table[{g → ++k, gi_ → {i, k}}, {g, gs}];
(* sorting → *)
cp = Alternatives @@ cs; (* cents *)
SetAttributes[M, HoldRest]; M[0, _] = 0;
M[a_, x_] := a x;
CE[ε_] := Collect[ε, _U, Expand] /. $trim;
Ui_ [ε_] := ε /. {t : cp ⇒ ti, u_U ⇒ (#i &) /@ u};
Ui_ [NCM[]] = pow[ε_, 0] = U@{} = 1U = U[];
B[U@(x_)i_, U@(y_)i_] := Ui@B[U@x, U@y];
B[U@(x_)i_, U@(y_)j_] /; i != j := 0;
B[U@y_, U@x_] := CE[-B[U@x, U@y]];
x_ ** (c_. 1U) := CE[c x]; (c_. 1U) ** x_ := CE[c x];
(a_. U[xx___, x_]) ** (b_. U[y_, yy___]) :=
If[OrderedQ[{x, y} /. sr],
  CE@M[a b /. $trim, U[xx, x, y, yy]],
  U@xx **
  CE@M[a b /. $trim, U@y ** U@x + B[U@x, U@y, $E]] **
  U@yy];
U@{c_. * (L : gp)^n_, r___} /; FreeQ[c, gp] :=
  CE[c U@Table[L, {n}] ** U@{r}];
U@{c_. * L : gp, r___} := CE[c U[L] ** U@{r}];
U@{c_, r___} /; FreeQ[c, gp] := CE[c U@{r}];
U@{L_Plus, r___} := CE[U@{#, r} & /@ L];
U@{L_, r___} := U@{Expand[L], r};
U[ε_NonCommutativeMultiply] := U /@ ε;
OU[specs___, poly_] := Module[{sp, null, vs, us},
  sp = Replace[{specs}, L_List ⇒ Lnull, {1}];
  vs = Join@@ (First /@ sp);
  us = Join@@ (sp /. L_s_ ⇒ (L /. x_i_ ⇒ xs));
  CE[Total[
    CoefficientRules[poly, vs] /. (p_ → c_) ⇒ c U@(us^p)
  ]] /. x_nnull ⇒ x];
OU[specs___, E[L_, Q_, P_]] :=
  OU[specs, SS@Normal[P e^{L+Q}]];
pow[ε_, n_] := pow[ε, n - 1] ** ε;
SU[ε_, ss___Rule] := CE@Total[
  CoefficientRules[ε, First /@ {ss}] /.
  (p_ → c_) ⇒
  c NCM @@ MapThread[pow, {Last /@ {ss}, p}]];
σrs___ [c_. * u_U] :=
  (c /. (t : cp)j_ ⇒ tj/.{rs}) U[List@@ (u /. v_j_ ⇒ vj/.{rs})];
mj_→k_ [c_. * u_U] :=
  CE[ ((c /. (t : cp)j → tk) DeleteCases[u, _j|k]) **
  U@@ Cases[u, w_j ⇒ wk] ** U@@ Cases[u, _k] ];
U /: c_. * u_U * v_U := CE[c u ** v];
Si_ [c_. * u_U] :=
  CE[ ((c /. Si[U, Centrals]) DeleteCases[u, _i]) **
  Ui[NCM @@ Reverse@Cases[u, x_i ⇒ S@U@x] ]];
Δi_→j_,k_ [c_. * u_U] :=
  CE[ ((c /. Δi_→j_,k[U, Centrals]) DeleteCases[u, _i]) **
  (NCM @@ Cases[u, x_i ⇒ σ1_→j,2_→k@Δ@U@x] /
  NCM[] → U[]) ]];

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