Theorem 2

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Theorem 2. *With the assumptions in Subsection 1.1.2, let us take $\{y_q : q \in Q\}$ to be a minimal set of generators for M as a two-sided F-module. Suppose the $\{y_q + I_F^3 : q \in Q\}$ are linearly independent in $(M + I_F^3)/I_F^3$. Then we have an isomorphism F^1:* "minimal R-moves"

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$$R^A_m \xrightarrow{\sim} \mathfrak{R}_m$$

as vector spaces over $\mathbb{Q}$. Moreover, $\partial^{Ind} = \partial_U \circ F^1$, and hence $\ker \partial^{Ind}$ consists of the syzygies of the quadratic algebra U.

Finally, $gr_I A$ is quadratic if and only if $F_{Syz} : \ker \partial_A \longrightarrow \ker \partial^{Ind}$ is surjective, i.e. iff all syzygies of U are 'covered' by the global syzygies $\ker \partial_A$.

Proof. Deferred to Subsection 3.4. $\square$

$$R^A_m = \sum I_A^{:p-1} : \left(\ker \mu_A : I_A : I_A \to I_A^2\right) : I_A^{:m-p-1}$$

$$\mathfrak{R}_m = \sum \left(\frac{I_A}{I_A^2}\right)^{\otimes p-1} \otimes_{\mathbb{Q}} \left(\ker \mu_{11} : \left(\frac{I_A}{I_A^2}\right)^{\otimes 2} \to \frac{I_A^2}{I_A^3}\right) \otimes_{\mathbb{Q}} \left(\frac{I_A}{I_A^2}\right)^{\otimes m-p-1}$$

$$F^1 : [a_1 : a_2 : \dots : \underbrace{a_p : a_{p+1}} : \dots : a_m]$$

$$\mapsto [a_1] \otimes [a_2] \otimes \dots \otimes [a_p] \otimes [a_{p+1}] \otimes \dots \otimes [a_m]$$

$m = 2$

$$\left(\ker \mu_A : I_A^{:2} \to I_A^2\right) \xrightarrow{\sim} \left(\ker \mu_{11} : \left(\frac{I_A}{I_A^2}\right)^{\otimes 2} \to \frac{I_A^2}{I_A^3}\right)$$

$$\sum a_i : b_i \longleftarrow\!\!| \sum [a_i] \otimes [b_i]$$

with $\sum a_i b_i \in I_A^3$