

The k be an algebra w/ aug ideal I ;

$$\hat{K} = \varprojlim K/I^m, \quad \text{gr}_I K = \bigoplus I^m/I^{m+1}$$

$$R := \ker(\mu: F/I^2 \otimes I/I^2 \rightarrow I^2/I^3)$$

the "quadratic approximation"

$$qK = T(I/I^2)/\langle R \rangle$$

$\text{gr}_I K$ & qK are both generated in deg 1, & have the same quadratic relations. When are they the same?

Always \exists surjection $qK \twoheadrightarrow \text{gr}_I K$.

An "expansion" is a filtered map $Z: \hat{K} \rightarrow qK$ s.t. $\text{gr} Z$ inverts that surjection.

If $K = \bigoplus PB_n$, $qK = A_n$, Z exists by the Kontsevich integral. $\Rightarrow qK \cong \text{gr}_I K$ as algebras.

$$PB_n := \frac{\langle R_{ij} \rangle_{1 \leq i \neq j \leq n}}{R_{III}, LS}$$

$$R_{III}: R_{ij} R_{ik} R_{jk} = R_{jk} R_{ik} R_{ij}$$

$$LS: [R_{ij}, R_{kl}] = 1.$$

$$I = \langle R_{ij} - 1 \rangle = \langle \bigoplus_{R_{ij}} \rangle = \langle \bar{R}_{ij} \rangle$$

$$R = \langle \bar{R}_{ij} : \text{Loc}, \text{GT}: [\bar{R}_{ij}, \bar{R}_{ik} + \bar{R}_{jk}] + [\bar{R}_{ik}, \bar{R}_{jk}] = 0 \rangle$$

So $qK = T\langle \bar{R}_{ij} \rangle / \text{Loc}, \text{GT}$

[BEER]: \nexists homomorphic Z . " PB_n isn't 1-formal".

Thm (Lee) $\text{gr}_I PB_n \cong qPB_n$ as graded vector spaces.

$$1, 1, \dots, T \otimes_K M, T \otimes_K (M-1) \quad \dots \quad 1, 1, \dots, 2$$

Let $M_n: I^{\otimes km} \rightarrow I^{\otimes k(n-1)}$ "multiply any two adjacent components"

By analogy w/ R , let

$$R^k := \ker(M_2: I^{\otimes k} \rightarrow I^{\otimes 2})$$

and

$$R_m^k := \bigoplus_j I^{\otimes j} \otimes R^k \otimes I^{\otimes (n-j-1)}$$

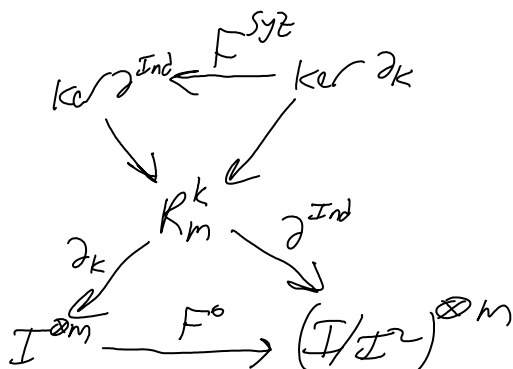
$$\exists \text{ map } \partial_k: R_m^k \rightarrow I^{\otimes m}$$

by construction, $\partial_k(R_m^k) \subset \ker \mu_m$.

Lemma Equality holds: $\partial_k(R_m^k) = \ker \mu_m$.

Consider $I^{\otimes k(m+1)} \xrightarrow{\mu} I^{\otimes km} \rightarrow \text{cok } \mu \cong \frac{I^{\otimes m}}{I \cdot I^{\otimes m}}$
 $\cong_{\text{dim}} (I/I^2)^{\otimes m}$

Consider



Thm K is quadratic iff F^{Syz} is surjective.

[in PB, this is Hatching's]

Modulo mild assumptions,

$$R_n^k \cong_{\mathbb{F}} R_m := \langle R \rangle_{\text{deg } m}$$

and $\partial^{\text{Ind}} = d_{\mu} \circ F^1$ w/ $d_{\mu}: R_m \rightarrow (I/I^2)^{\otimes m}$
 the obvious.

Thm 1.1.1.1 $\cong \dots$

Then A. $\ker d_n \cong \ker \partial^{Ind}$ is "relations among relations in \mathfrak{K} "

B. Can interpret $I^{\otimes m}$ as "n-singular R/B"
so $\ker \partial_k =$ "topological syzygies"

So we need to show the surjectivity of

"topological syzygies" $\xrightarrow{F^{Syz}}$ "combinatorial syzygies"

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