Finding \ker\delta

January-27-11
11:26 AM

Notation! $| = \otimes_{\mathbb{Q}}$, $\vdots = \otimes_A$

We'd like to immitate

$$\begin{array}{ccccccccc}
\cdots \xrightarrow{\delta} & \mathcal{K}^1_{n+1} & \xrightarrow{\delta} & \mathcal{K}^1_n & \xrightarrow{\delta} & \mathcal{K}^1_{n-1} & \xrightarrow{\delta} \cdots \\
& \downarrow b & & \downarrow b & & \downarrow b & \\
\cdots \xrightarrow{\delta} & \mathcal{K}_{n+1} & \xrightarrow{\delta} & \mathcal{K}_n & \xrightarrow{\delta} & \mathcal{K}_{n-1} & \xrightarrow{\delta} \cdots ,
\end{array}
\qquad
\begin{array}{ccccccc}
\mathcal{K}^1_n & \xrightarrow{\delta} & \mathcal{K}^1_{n-1} & \xrightarrow{\pi} & \bar{\mathcal{K}}^1_{n-1} & \longrightarrow & 0 \\
\downarrow b & & \downarrow b & & \downarrow \bar{b} & & \\
\mathcal{K}_n & \xrightarrow{\delta} & \mathcal{K}_{n-1} & \xrightarrow{\pi} & \bar{\mathcal{K}}_{n-1} & \longrightarrow & 0
\end{array}
\quad \text{(exact rows).}$$

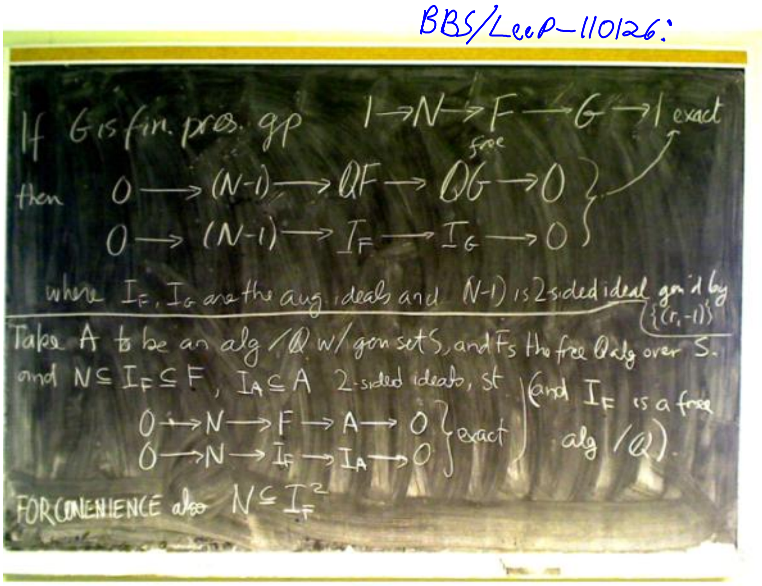
Pasted from <http://www.math.toronto.edu/~drorbn/papers/Species/1_4Hutchings_theory_integra.html>

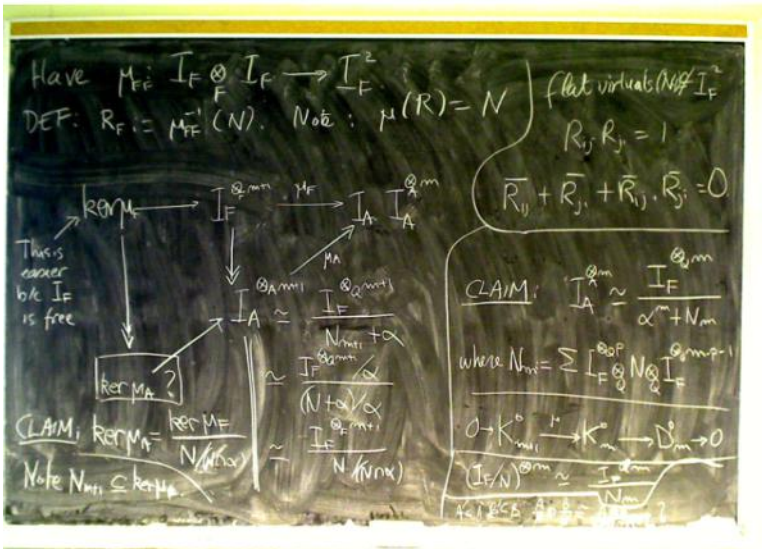
in the general (A, I) situation.

$$K_n = K^0_n := (\underbrace{I \vdots I \ \cdots \vdots I}_{n \text{ times}}) = I^{\vdots n}$$

$$\delta : I^{\vdots n} \to I^{\vdots n-1} \text{ by } \delta(\alpha_1 \vdots \cdots \vdots \alpha_n) = (\alpha_1\alpha_2 \vdots \alpha_3 \cdots \vdots \alpha_n)$$
$$= (\alpha_1 \vdots \alpha_2\alpha_3 \vdots \cdots) = \cdots$$

BBS/LeeP-110126:





NDS/Lect-110126:

If G is fin. pres. gp $\quad 1 \to N \to \underset{\text{free}}{F} \to G \to 1$ exact

then
$$\left.\begin{array}{l} 0 \to (N-1) \to \mathbb{Q}F \to \mathbb{Q}G \to 0 \\ 0 \to (N-1) \to I_F \to I_G \to 0 \end{array}\right\}$$

where I_F, I_G are the aug. ideals and $(N-1)$ is 2 sided ideal gen'd by $\{(r_i - 1)\}$

Take A to be an alg $/\mathbb{Q}$ w/ gen set S, and F_S the free $\mathbb{Q}$ alg over S.
and $N \subseteq I_F \subseteq F$, $I_A \subseteq A$ 2-sided ideals, st

$$\left.\begin{array}{l} 0 \to N \to F \to A \to 0 \\ 0 \to N \to I_F \to I_A \to 0 \end{array}\right\} \text{exact}$$

(and I_F is a free alg $/\mathbb{Q}$).

FOR CONVENIENCE also $N \subseteq I_F^2$

Have $\mu_{FF} : I_F \otimes_F I_F \to I_F^2$

DEF: $R_F := \mu_{FF}^{-1}(N)$. Note: $\mu(R) = N$

flat virtuals ($N \not\subseteq I_F^2$)

$$R_{ij} \cdot R_{ji} = 1$$

$$\bar{R}_{ij} + \bar{R}_{ji} + \bar{R}_{ij} \cdot \bar{R}_{ji} = 0$$

$$\begin{array}{ccccc} \ker \mu_F & \longrightarrow & I_F^{\otimes_F m+1} & \xrightarrow{\mu_F} & I_A \cdot I_A^{\otimes_A m} \\ \downarrow & & \downarrow & \nearrow{\scriptstyle \mu_A} & \\ \boxed{\ker \mu_A \ ?} & \longrightarrow & I_A^{\otimes_A m+1} & & \end{array}$$

This is easier b/c I_F is free

$$I_A^{\otimes_A m+1} \simeq \frac{I_F^{\otimes_Q m+1}}{N_{m+1} + \alpha} \simeq \frac{I_F^{\otimes_Q m+1}/\alpha}{(N+\alpha)/\alpha} \simeq \frac{I_F^{\otimes_F m+1}}{N/(N \cap \alpha)}$$

CLAIM: $\ker \mu_A = \dfrac{\ker \mu_F}{N/(N \cap \alpha)}$

Note $N_{m+1} \subseteq \ker \mu_F$

CLAIM: $I_A^{\otimes_A m} \simeq \dfrac{I_F^{\otimes_Q m}}{\alpha^m + N_m}$

where $N_m = \sum I_F^{\otimes_Q p} \otimes_Q N \otimes_Q I_F^{\otimes_Q m-p-1}$

$$0 \to K^\circ_{m+1} \xrightarrow{\mu} K^\circ_m \to D^\circ_m \to 0$$

$$(I_F/N)^{\otimes m} \simeq \frac{I_F^{\otimes m}}{N_m}$$

$A \triangleleft A$, $B \triangleleft B$ ... ?

$$\begin{array}{ccccccc} 0 \to \dfrac{N_m}{\alpha \cap N_m} & \longrightarrow & I_F^{\otimes_F m} & \longrightarrow & I_A^{\otimes_A m} & \simeq & \dfrac{I_F^{\otimes_F m}}{N_m/(\alpha \cap N)} \\ & & \cup\!| & & \cup\!| & & \\ & & I_F^2 \otimes_F I_F^{\otimes_F m-1} & \longrightarrow & I_A \cdot I_A^{\otimes_A m} & & \\ & & = I_F \cdot I_F^{\otimes_F m} & & & & \end{array}$$

b/c $N \subseteq I_F^2$

CLAIM: Surjective b/c $\mu_{FF}(R_F) = N$ hence iso

restric μ_{FF}

$R_{m+1} = \sum I_F^{\otimes_F p} \otimes_F R_F \otimes_F I_F^{\otimes_F m-p-1} \hookrightarrow I_F^{\otimes_F m+1}$

$\mu_{FF} \ \simeq$ b/c I_F is free

μ_F

HENCE:

$$I_A \cdot I_A^{\otimes_A m} \simeq \frac{I_F \cdot I_F^{\otimes_F m}}{N_m/(N_m \cap \alpha)} \simeq \frac{I_F^{\otimes_F m+1}}{R_{m+1}}$$

$$\Rightarrow \ker \mu_F = R_{m+1}$$