Compressing The Key

April-29-11
10:14 AM

Goal.

$\mu: \mathcal{A} \to \mathrm{gr}\, K$ is surjective. Find a quick criterion for it to be also injective.

Abstract Generalities. (K,I): an algebra and an "augmentation ideal" in it. $\hat{K} := \varprojlim K/I^m$ the "I-adic completion". $\mathrm{gr}_I K := \widehat{\bigoplus} I^m/I^{m+1}$ has a product μ, especially, $\mu_{11}\colon (C = I/I^2)^{\otimes 2} \to I^2/I^3$. The "quadratic approximation" $\mathcal{A}_I(K) := \widehat{FC}/\langle \ker \mu_{11} \rangle$ of K surjects using μ on $\mathrm{gr}\, K$.

Peter Lee

The Prized Object. A "homomorphic $\mathcal{A}$-expansion": a homomorphic filtered $Z\colon K \to \mathcal{A}$ inducing the identity on $I/I^2 = C$.

Dror's Dream. All interesting graded objects and equations, especially those around quantum groups, arise this way.

From the SwissKnots Hanout, http://www.math.toronto.edu/~drorbn/Talks/SwissKnots-1105/

Scheme? Define cubic, quartic, quintic, etc. approximations and inductively show the injectivity

$$\mathcal{A}^2 \to \mathcal{A}^3 \to \mathcal{A}^4 \to \dots \to \mathcal{A}^\infty = \mathrm{gr}\, K$$

$$\mathcal{A}^d = \mathcal{A}^{d-1}\Big/\left(\ker \mu_d\colon C^{\otimes d} \to I^d/I^{d+1}\right)$$

Need to show

$$\ker \mu_d \subset C \otimes \ker \mu_{d-1} + \ker \mu_{d-1} \otimes C$$

$a:b:c \to [a]\otimes[b]\otimes[c]$

$$\begin{array}{ccc} I^{:k} & \twoheadrightarrow & I^k/I^{k+1} \\ & \searrow \quad \nearrow & \\ & C^{\otimes k} & \end{array}$$

... here $[xb]=[b]$ for any x.

Peter's/Hutchings' key:

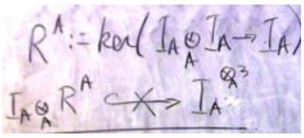
$$\frac{K^o_{m+1}}{\partial K'_{m+1}} \xrightarrow{1\text{-}1?} K^o_m/\partial K'_m$$

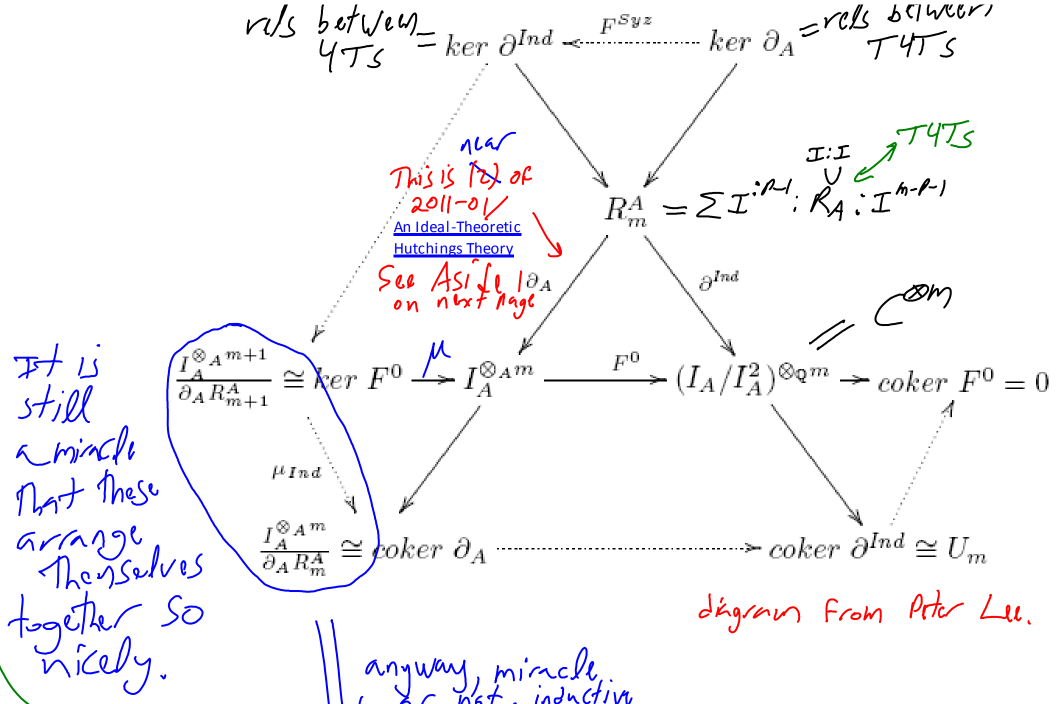
From 2011-03/

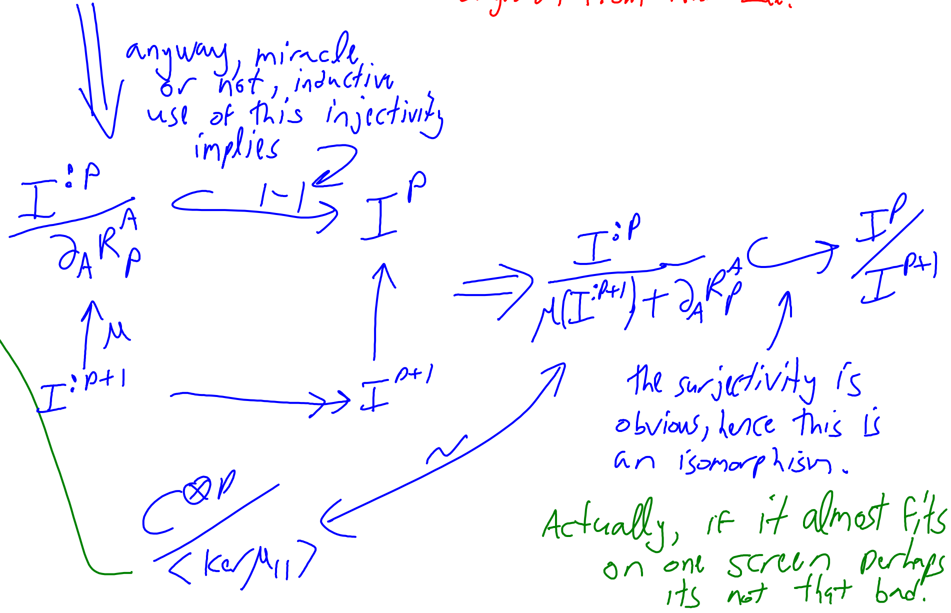
Peter's "Hexagon Lemma"

This I should interpret in a simpler language!

A problem:
BBS/LeeP-110421







Question. Is there some homology, or sequence of homologies, whose vanishing is the quadraticity condition? If so, the whole story might be the result of applying that homology functor to

$$0 \to (N, ?) \longrightarrow (F, I_F) \longrightarrow (A, I_A) \to 0.$$