

# A Resolution of PvBn

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10:31 AM

From <http://katlas.math.toronto.edu/drorbn/bbs/show?shot=LeeP-110721-181554.jpg>:

Claim ES:  $\bigoplus_{0 \leq i < j < m-2} (R_{m,i} \cap R_{m,j}) \rightarrow \bigoplus R_{m,i} \rightarrow V^{\otimes m} \rightarrow A_n^m \rightarrow 0$

↑  
exact

$\vec{A} = \frac{TV}{\langle R \rangle}$

$\bigoplus (R_{m,i} \cap R_{m,j}) \rightarrow \bigoplus R_{m,i} \rightarrow V^{\otimes m} \rightarrow 0$

$0 \rightarrow V^{\otimes m} \rightarrow \bigoplus R_{m,i}^* \rightarrow \bigoplus (R_{m,i} \cap R_{m,j})^*$

**CLAIM**  $0 \rightarrow A^{\otimes m} \rightarrow \bigoplus A^{\otimes i} \otimes A^{\otimes j} \otimes A^{\otimes m-i-j} \rightarrow \dots$

$0 \rightarrow [A^{\otimes m}]_m \xrightarrow{d} [A^{\otimes m}]_{m-1} \rightarrow [A^{\otimes m}]_{m-2} \rightarrow \dots$

$d_R(a_i \otimes \dots \otimes a_k) = a_i a_2 \otimes a_3 \otimes \dots \otimes a_k + \dots + \dots$

Claim  $R^* \simeq A_n^{\otimes 2} := \frac{V^* \otimes V^*}{R^*}$

$A^{\otimes 3} \simeq [(A \otimes V^*) \cap (V^* \otimes R)]^*$

$Q \otimes_B \dots$  to bar resolution of B:  $\dots \rightarrow B \otimes B^{\otimes 2} \rightarrow \dots$

- ① Put filtration  $\mathbb{Z}$  on  $A^1$ , induced by total ordering of monomials.
- ② Take ass. gr complex; it is the bar complex of  $\text{gr}_{\mathbb{Z}} A^1$ .
- ③  $\text{gr}_{\mathbb{Z}} A^1$  is quadratic, quotient of exterior alg.
- ④  $\text{gr}_{\mathbb{Z}} A^1$  has monomial relations; easy to show exactness.
- ⑤  $\text{gr}_{\mathbb{Z}} A^1$  complex exact  $\Rightarrow A^1$  cplx is exact.