Report on the paper "Goldman–Turaev formality from the Kontsevich integral"

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This paper presents a new approach to the formality problem for the Goldman–Turaev Lie bialgebra in genus zero, a fundamental structure defined on the space of loops of any surface. Building on the Kontsevich integral of links and tangles, the authors construct explicit "formality isomorphisms", i.e. isomorphisms between this Lie bialgebra (which carries a natural filtration arising from the I-adic filtration on the group algebra of the surface's fundamental group) and its associated graded structure. Their method relies on a clever three-dimensional description of the two operations, namely the Goldman bracket and the Turaev cobracket, together with a proof that the Kontsevich integral behaves well with respect to the tangle operations involved in these 3D descriptions.

This approach may be compared to the work of Massuyeau [Mas18], who also used the Kontsevich integral to provide the first proof of the formality of the Goldman–Turaev Lie bialgebra in genus zero. However, whereas Massuyeau's proof employed braids and the so-called "Fox derivatives" for the 3D description of the Goldman–Turaev operations, the authors here work with tangles and their quotient by a Conway-type skein relation, leading to the notion of "emergent tangles" (a simplified setting in which most knotting information disappears). The two approachess are therefore technically quite different, and the comparison might be an interesting subject.

The article stands out for its style of exposition, emphasizing that notion of "emergent tangles", while clearly presenting (in §2) the general conceptual method that led to their solution of this formality problem. All the necessary material is recalled in §3, and the new concepts (in particular, the above-mentioned "Conway quotient") are presented in §4. The bulk of the paper is located in §5: although the proofs are highly technical, with a

lot of commutative diagrams, the authors manage to make it rather easily understandable.

It should also be noted that the work places itself within the broader context of the Kashiwara–Vergne problem and its connection to the theory of Drinfeld associators, highlighting deep links between topology, Lie theory, and quantum algebra. Furthermore, several other papers closely related to the notion of "emergent tangles" have appeared around the same time (see [Kun25], [ANR24] and [Ren25]), which testifies to the vitality of the subject. In particular, the possibility of extending the results of this paper to surfaces of higher genus represents an important future direction, in connection with the higher-genus formulation of the Kashiwara–Vergne problem by Alekseev, Kawazumi, Kuno & Naef.

So, my overall assessment of this work is very positive, and I would highly recommend publication in *Quantum Topology* after revision of the paper.