

Pensieve header: Testing \hat{a} arrows.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1604"];
<< Local-160529.m
```

In the $U(T) \otimes U(H)$ conventions. Internal use symbols: {rr, pp}

Export

Canonical forms for $\delta_{\hat{a}\hat{a}}$'s

```
 $\delta_{\hat{a}\hat{a}}[f_, i_, j_, k_, l_] :=$ 
   $\delta_{aa}[f, i, j, k, l] - \delta_{aa}[b_i f, \zeta, j, k, l] - \delta_{aa}[b_k f, i, j, \zeta, l];$ 

Union[CF[UU[ $\delta_{aa}[f, i, j, k, l] - \delta_{aa}[f, i, l, k, j]$ ] /. Thread[{i, j, k, l} → #]] & /@
  Tuples[{1, 2}, 4]

{UU[0], UU[ $\delta_{aa}[-f b_1, \zeta, 1, 2, 2] + \delta_{aa}[f b_1, \zeta, 2, 2, 1] +$ 
   $\delta_{aa}[-f b_2, \zeta, 2, 1, 1] + \delta_{aa}[f b_2, \zeta, 1, 1, 2]$ ], UU[ $\delta_{aa}[-f b_1, \zeta, 2, 2, 1] +$ 
   $\delta_{aa}[f b_1, \zeta, 1, 2, 2] + \delta_{aa}[-f b_2, \zeta, 1, 1, 2] + \delta_{aa}[f b_2, \zeta, 2, 1, 1]$ ]}

Union[CF[UU[ $\delta_{\hat{a}\hat{a}}[f, i, j, k, l] - \delta_{\hat{a}\hat{a}}[f, i, l, k, j]$ ] /. Thread[{i, j, k, l} → #]] & /@
  Tuples[{1, 2, 3, 4}, 4]

{UU[0]}

Union[
  CF[UU[ $\delta_{\hat{a}\hat{a}}[f, i, j, k, l] - \delta_{\hat{a}\hat{a}}[f, \text{Min}@{i, k}, \text{Min}@{j, l}, \text{Max}@{i, k}, \text{Max}@{j, l}]$ ]] /.
    Thread[{i, j, k, l} → #]] & /@ Tuples[{1, 2, 3, 4}, 4]

{UU[0]}
```

Commuting heads within a $\delta_{\hat{a}\hat{a}}$

```
{CF[UU[ $\delta_{aa}[f, i, l, k, l] - \delta_{aa}[f, k, l, i, l]$ ]],
  CF[UU[ $\delta_{\hat{a}\hat{a}}[f, i, l, k, l] - \delta_{\hat{a}\hat{a}}[f, k, l, i, l]$ ]]}
{UU[ $\delta_a[-f b_i, k, l] + \delta_a[f b_k, i, l]$ ], UU[0]}
```

```
B = bb[1, 2, 3, 4];
```

```
B[UU@a[1, 1, 2], UU@a[1, 1, 3]]
```

```
UU[ $\delta_{aa}[-1, \zeta, 2, 1, 3] + \delta_{aa}[1, \zeta, 3, 1, 2]$ ]
```

```
UU@ $\delta_{\hat{a}\hat{a}}$ [1, 1, 2, 1, 3] // CF
```

```
UU[ $\delta_{aa}[1, 1, 2, 1, 3] + \delta_{aa}[-b_1, \zeta, 2, 1, 3] + \delta_{aa}[-b_1, \zeta, 3, 1, 2]$ ]
```

```
UU@ $\delta_{\hat{a}\hat{a}}$ [1, 1, 3, 1, 2] // CF
```

```
UU[ $\delta_{aa}[1, 1, 2, 1, 3] + \delta_{aa}[-b_1, \zeta, 2, 1, 3] + \delta_{aa}[-b_1, \zeta, 3, 1, 2]$ ]
```

Calculations for OneCo-1605

```

Clear[ca, ac];
ac[f_, i_, j_, k_] := ca[f, k, i, j] + Kδjk ao[f, i, j];
AutoCollecting /@ {c, ao, ca, aao};
ToOld = ReplaceAll[{c[f_, i_] => δa[f, ϕ, i],
  ao[f_, i_, j_] => δa[f, i, j] - δa[bi f, ϕ, j],
  ca[f_, i_, j_, k_] => δaa[f, ϕ, i, j, k],
  ac[f_, i_, j_, k_] => δaa[f, i, j, ϕ, k],
  aao[f_, i_, j_, k_, l_] => δaa[f, i, j, k, l]
}];
ToNew = ReplaceAll[{
  δa[f_, ϕ, j_] => c[f, j],
  δa[f_, i_, j_] => ao[f, i, j] + c[bi f, j],
  δaa[f_, ϕ, j_, k_, l_] => ca[f, j, k, l],
  δaa[f_, j_, k_, ϕ, l_] => ac[f, j, k, l],
  δaa[f_, i_, j_, k_, l_] => aao[f, i, j, k, l] + ca[bi f, j, k, l] + ac[bk f, i, j, l]
}];
UU@aao[f, i, j, k, l] // ToOld // ToNew
UU[aao[f, i, j, k, l]]
DeleteCases[ (# -> CF[# - ToOld@ToNew@#]) & /@ UUBasis[{1, 2}, {3, 4}, f],
  _ -> UU[0]]
{}

```

$$ac[f, i, j, k] = ca[f, k, i, j] + K\delta_{jk} ao[f, i, j]$$

```

UU@δaa[f, i, j, ϕ, k] // CF // ToNew
UU[ca[f, k, i, j]]
UU@δaa[f, i, j, ϕ, j] // CF // ToNew
UU[ao[f, i, j] + ca[f, j, i, j]]

```

NonCommutativeMultiply

```

t1 = ToNew[UU[ao[f, i, j] // ToOld] ** UU[a[g, k, l] // ToOld]]
UU[aao[f g, i, j, k, l] + ca[f g bk, l, i, j]]
UU[ca[f g bk, l, i, j] + aao[f g, i, j, k, l]]
UU[aao[f g, i, j, k, l] + ca[f g bk, l, i, j]]

```

```
t2 = ToNew[UU[ao[f, i, j] // ToOld] ** UU[a[g, j, l] // ToOld]]
```

```
UU[aao[f g, i, j, j, l] + ao[-f g bj, i, l] + ca[f g bj, l, i, j]]
```

```
t2 - t1
```

```
UU[aao[f g, i, j, j, l] - aao[f g, i, j, k, l] + ao[-f g bj, i, l] + ca[f g (bj - bk), l, i, j]]
```

tm, hm, hts, dm

```
δhb[f, z, ϕ, j]
```

```
δa[f, z, j] + δa[-f bz, ϕ, j]
```

```
UU@δhb[f, z, ϕ, j] // ToNew
```

```
UU[ao[f, z, j]]
```

```
UU@δhb[f, i, ϕ, y] // ToNew
```

```
UU[ao[f, i, y]]
```

```
UU[ca[f, y, j, x]] // ToOld // hm[x, y, z] // ToNew
```

```
UU[ao[f, j, z] + ca[f, z, j, z]]
```

```
UU[ca[f, x, j, y]] // ToOld // hm[x, y, z] // ToNew
```

```
UU[ca[f, z, j, z]]
```

```
UU[ca[f, x, j, x]] // ToOld // hm[x, y, z] // ToNew
```

```
UU[ca[f, z, j, z]]
```

```
UU[ca[f, y, j, y]] // ToOld // hm[x, y, z] // ToNew
```

```
UU[ca[f, z, j, z]]
```

```
UU[a[f[bx, by], x, y]] // hts[y, x] // ToNew
```

```
UU[a[f[bx, by], x, y] + ao[-f(1,0)[bx, by], x, y] +  
c[f[bx, by], y] + β[-f[bx, by] bx] + δβ[bx f(1,0)[bx, by]]]
```

```
UU[ao[f, x, y]] // ToOld // hts[y, x] // ToNew
```

```
UU[ao[f, x, y] + δβ[-f bx]]
```

```
UU@aao[f[bx, by], i, j, k, l] // ToOld // hts[y, x] // ToNew
```

```
UU[aao[f[bx, by], i, j, k, l]]
```

```
UU@aao[f[bx, by], x, y, k, l] // ToOld // hts[y, x] // ToNew
```

```
UU[aao[f[bx, by], k, l, x, y] + ao[-f[bx, by] bx, k, l]]
```

```
UU@aao[f[bx, by], x, j, k, y] // ToOld // hts[y, x] // ToNew
```

```
UU[aao[f[bx, by], k, j, x, y] + ao[-f[bx, by] bx, k, j]]
```

```

UU@aao[f[bx, by], i, y, x, l] // ToOld // hts[y, x] // ToNew
UU[aao[f[bx, by], i, l, x, y] + ao[-f[bx, by] bx, i, l]]

UU@aao[f[bx, by], i, j, x, y] // ToOld // hts[y, x] // ToNew
UU[aao[f[bx, by], i, j, x, y] + ao[-f[bx, by] bx, i, j]]

UU@aao[f[bx, by], x, y, k, y] // ToOld // hts[y, x] // ToNew
UU[aao[f[bx, by], k, y, x, y] + ao[-2 f[bx, by] bx, k, y]]

UU@aao[f[bx, by], i, y, x, y] // ToOld // hts[y, x] // ToNew
UU[aao[f[bx, by], i, y, x, y] + ao[-2 f[bx, by] bx, i, y]]

UU@aao[f[bx, by], x, y, x, l] // ToOld // hts[y, x] // ToNew
UU[aao[f[bx, by], x, l, x, y] + ao[-2 f[bx, by] bx, x, l]]

UU@aao[f[bx, by], x, j, x, y] // ToOld // hts[y, x] // ToNew
UU[aao[f[bx, by], x, j, x, y] + ao[-2 f[bx, by] bx, x, j]]

UU@aao[f[bx, by], x, y, x, y] // ToOld // hts[y, x] // ToNew
UU[aao[f[bx, by], x, y, x, y] + ao[-4 f[bx, by] bx, x, y] + δβ[2 f[bx, by] bx2]]

```

tb, hb, thb, htb, db, bb on {β, a, δβ, ao, aao}

```

tb[x][ToOld@UU@a[f[bx], x, j], ToOld@UU@β[g[bx]]] // ToNew
UU[ao[f[bx] g'[bx], x, j]]

tb[x][ToOld@UU@a[f[bx], x, j], ToOld@UU@a[g[bx], k, l]] // ToNew
UU[aao[f[bx] g'[bx], k, j, x, l] + ca[f[bx] bk g'[bx], l, x, j]]

tb[x][ToOld@UU@a[f[bx], x, j], ToOld@UU@a[g[bx], x, l]] // ToNew
UU[aao[-g[bx] f'[bx] + f[bx] g'[bx], x, j, x, l] +
  ca[-g[bx] (f[bx] + bx f'[bx]), j, x, l] + ca[f[bx] (g[bx] + bx g'[bx]), l, x, j]]

tb[x][ToOld@UU@a[f[bx], j, k], ToOld@UU@a[g, x, l]] // ToNew
UU[aao[-g f'[bx], j, k, x, l] + ca[-g bj f'[bx], k, x, l]]

tb[x][ToOld@UU@a[f[bx], x, k], ToOld@UU@a[g, x, l]] // ToNew
UU[aao[-g f'[bx], x, k, x, l] + ca[g f[bx], l, x, k] + ca[-g (f[bx] + bx f'[bx]), k, x, l]]

hb[y][ToOld@UU@a[f, i, y], ToOld@UU@c[g, y]] // ToNew
UU[ao[f g, i, y]]

hb[y][ToOld@UU@a[f, i, y], ToOld@UU@ao[g, j, y]] // ToNew
UU[ao[-f g bi, j, y]]

```

```

hb[y][ToOld@UU@a[f, i, y], ToOld@UU@ca[g, j, k, l]] // ToNew
UU[0]

hb[y][ToOld@UU@a[f, i, y], ToOld@UU@ca[g, y, k, l]] // ToNew
UU[aao[f g, i, l, k, y] + ca[f g b_k, l, i, y]]

hb[y][ToOld@UU@a[f, i, y], ToOld@UU@ca[g, j, k, y]] // ToNew
UU[ca[-f g b_i, j, k, y] + ca[f g b_k, j, i, y]]

hb[y][ToOld@UU@a[f, i, y], ToOld@UU@ca[g, y, k, y]] // ToNew
UU[aao[f g, i, y, k, y] + ao[f g b_k, i, y] + ca[-f g b_i, y, k, y] + ca[2 f g b_k, y, i, y]]

hb[y][ToOld@UU@a[f, i, y], ToOld@UU@aao[g, j, y, l, m]] // ToNew
UU[aao[-f g b_i, j, m, l, y] + ca[-f g b_j b_l, m, i, y]]

hb[y][ToOld@UU@a[f, i, y], ToOld@UU@aao[g, j, k, l, y]] // ToNew
UU[aao[-f g b_i, j, k, l, y] + ca[-f g b_j b_l, k, i, y]]

hb[y][ToOld@UU@a[f, i, y], ToOld@UU@aao[g, j, y, l, y]] // ToNew
UU[aao[-2 f g b_i, j, y, l, y] + ao[-f g b_j b_l, i, y] + ca[-2 f g b_j b_l, y, i, y]]

```

thb

```

thb[x, y][ToOld@UU@a[f[b_x], i, j], ToOld@UU@a[g, k, l]] // ToNew
UU[0]

t1 = thb[x, y][ToOld@UU@a[f[b_x], i, j], ToOld@UU@a[g, k, y]] // ToNew
UU[aao[g f'[b_x], i, j, k, y] + ca[g b_i f'[b_x], j, k, y]]

t2 = thb[x, y][ToOld@UU@a[f[b_x], x, j], ToOld@UU@a[g, k, y]] // ToNew
UU[a[-g f[b_x] b_k, x, j] + a[g f[b_x] b_x, k, j] + aao[g f'[b_x], k, j, x, y] +
  ao[-g b_x f'[b_x], k, j] + ca[-g f[b_x], y, k, j] + ca[g (f[b_x] + b_x f'[b_x]), j, k, y]]

t2 - t1
UU[a[-g f[b_x] b_k, x, j] + a[g f[b_x] b_x, k, j] +
  aao[-g f'[b_x], i, j, k, y] + aao[g f'[b_x], k, j, x, y] + ao[-g b_x f'[b_x], k, j] +
  ca[-g f[b_x], y, k, j] + ca[g (f[b_x] + (-b_i + b_x) f'[b_x]), j, k, y]]

CF[t2 - t1 /. i -> x]
UU[a[-g f[b_x] b_k, x, j] + a[g f[b_x] b_x, k, j] +
  aao[-g f'[b_x], x, j, k, y] + aao[g f'[b_x], k, j, x, y] +
  ao[-g b_x f'[b_x], k, j] + ca[-g f[b_x], y, k, j] + ca[g f[b_x], j, k, y]]

```

```

thb[x, y][ToOld@UU@a[f, x, j], ToOld@UU@c[g, y]] // ToNew
UU[ao[-f g, x, j]]

thb[x, y][ToOld@UU@a[f, x, j], ToOld@UU@ao[g, k, y]] // ToNew
UU[ao[f g bx, k, j]]

thb[x, y][ToOld@UU@a[f, x, j], ToOld@UU@ca[g, k, l, m]] // ToNew
UU[0]

t1 = thb[x, y][ToOld@UU@a[f, x, j], ToOld@UU@ca[g, y, l, m]] // ToNew
UU[aa[-f g, l, j, x, m] + ca[-f g b1, m, x, j]]

t2 = thb[x, y][ToOld@UU@a[f, x, j], ToOld@UU@ca[g, k, l, y]] // ToNew
UU[ca[-f g b1, k, x, j] + ca[f g bx, k, l, j]]

t3 = thb[x, y][ToOld@UU@a[f, x, j], ToOld@UU@ca[g, y, l, y]] // ToNew
UU[aa[-f g, l, j, x, y] + ao[-f g b1, x, j] + ca[-2 f g b1, y, x, j] + ca[f g bx, y, l, j]]

CF[(t3 - t1 - t2) /. {k | m → y}]
UU[ao[-f g b1, x, j]]

thb[x, y][ToOld@UU@a[f, x, j], ToOld@UU@aao[g, k, l, m, n]] // ToNew
UU[0]

t1 = thb[x, y][ToOld@UU@a[f, x, j], ToOld@UU@aao[g, k, y, m, n]] // ToNew
UU[aa[f g bx, k, j, m, n] + ca[f g bk bm, n, x, j]]

t2 = thb[x, y][ToOld@UU@a[f, x, j], ToOld@UU@aao[g, k, l, m, y]] // ToNew
UU[aa[f g bx, k, j, m, l] + ca[f g bk bm, l, x, j]]

t3 = thb[x, y][ToOld@UU@a[f, x, j], ToOld@UU@aao[g, k, y, m, y]] // ToNew
UU[aa[2 f g bx, k, j, m, y] + ao[f g bk bm, x, j] + ca[2 f g bk bm, y, x, j]]

CF[(t3 - t1 - t2) /. {l | n → y}]
UU[ao[f g bk bm, x, j]]

thb[x, y][ToOld@UU@ao[f, x, j], ToOld@UU@a[g, k, y]] // ToNew
UU[ao[-f g bk, x, j] + ao[f g bx, k, j]]

thb[x, y][ToOld@UU@ca[f, m, x, j], ToOld@UU@a[g, k, y]] // ToNew
UU[ca[-f g bk, m, x, j] + ca[f g bx, m, k, j]]

thb[x, y][ToOld@UU@aao[f, i, j, m, n], ToOld@UU@a[g, k, y]] // ToNew
UU[0]

```

```

t1 = thb[x, y][ToOld@UU@aao[f, x, j, m, n], ToOld@UU@a[g, k, y]] // ToNew
UU[aao[-f g bk, m, j, x, n] + aao[f g bx, k, j, m, n]]

t2 = thb[x, y][ToOld@UU@aao[f, i, j, x, n], ToOld@UU@a[g, k, y]] // ToNew
UU[aao[-f g bk, i, j, x, n] + aao[f g bx, i, j, k, n]]

t3 = thb[x, y][ToOld@UU@aao[f, x, j, x, n], ToOld@UU@a[g, k, y]] // ToNew
UU[aao[-2 f g bk, x, j, x, n] + aao[2 f g bx, k, j, x, n]]

CF[t3 - t1 - t2 /. i | m → x]
UU[aao[-f g bx, x, j, k, n] + aao[f g bx, k, j, x, n]]

```

ct (contract)

```

ct[h, t][ToOld@UU@a[f, i, h], ToOld@UU@ao[g, t, j]] // ToNew
UU[ao[f g, i, j]]

ct[h, t][ToOld@UU@a[f, i, h], ToOld@UU@ao[g[bt], j, k]] // ToNew
UU[ao[f bi g'[0], j, k]]

ct[h, t][ToOld@UU@ao[f, i, h], ToOld@UU@β[g[bt]]] // ToNew
UU[0]

pp[a[f, i, h], ao[g, j, k]] //
ToNew[ct[h, t][ToOld@UU@#[[1]], ToOld@UU@(#[[2]] /. g → g[bt]]] &
UU[ao[f bi g'[0], j, k]]

pp[ao[f, i, h], a[g, j, k]] //
ToNew[ct[h, t][ToOld@UU@#[[1]], ToOld@UU@(#[[2]] /. g → g[bt]]] &
UU[0]

pp[aao[f, i, h, j, k], β[g]] //
ToNew[ct[h, t][ToOld@UU@#[[1]], ToOld@UU@(#[[2]] /. g → g[bt]]] &
UU[c[-f bi bj g'[0], k]]

pp[aao[f, i, h, j, k], a[g, l, m]] //
ToNew[ct[h, t][ToOld@UU@#[[1]], ToOld@UU@(#[[2]] /. g → g[bt]]] &
UU[ca[-f bi bj g'[0], k, l, m]]

pp[aao[f, i, j, k, h], β[g]] //
ToNew[ct[h, t][ToOld@UU@#[[1]], ToOld@UU@(#[[2]] /. g → g[bt]]] &
UU[c[-f bi bk g'[0], j]]

```

```

pp[aao[f, i, j, k, h], a[g, l, m]] //
  ToNew[ct[h, t][ToOld@UU#[1], ToOld@UU(#[2] /. g -> g[b_t])]] &
UU[ca[-f b_i b_k g'[0], j, l, m]]

pp[c[f, h], a[g, j, k]] //
  ToNew[ct[h, t][ToOld@UU#[1], ToOld@UU(#[2] /. g -> g[b_t])]] &
UU[ao[f g'[0], j, k] + c[f b_j g'[0], k]]

pp[ca[f, h, i, j], beta[g]] //
  ToNew[ct[h, t][ToOld@UU#[1], ToOld@UU(#[2] /. g -> g[b_t])]] &
UU[ao[f g'[0], i, j] + c[f b_i g'[0], j]]

pp[ca[f, h, i, j], a[g, k, l]] //
  ToNew[ct[h, t][ToOld@UU#[1], ToOld@UU(#[2] /. g -> g[b_t])]] &
UU[aao[f g'[0], i, j, k, l] + ca[f b_i g'[0], j, k, l] + ca[f b_k g'[0], l, i, j]]

```