

Pensieve header: Counting classical braids by computing  $|\bar{\Gamma}|$  of all classical proud braids.

(Alt) In[ ]:=

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OU";
<< "OU-Programs.m"
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.  
Read more at <http://katlas.org/wiki/KnotTheory>.

## 2

(Alt) In[ ]:=

```
Timing@CountBs [2, 0]
```

(Alt) Out[ ]=

```
{0., 1}
```

(Alt) In[ ]:=

```
Timing@CountBs [2, 1]
```

(Alt) Out[ ]=

```
{0., 3}
```

(Alt) In[ ]:=

```
Timing@CountBs [2, 2]
```

(Alt) Out[ ]=

```
{0., 5}
```

(Alt) In[ ]:=

```
Timing@CountBs [2, 3]
```

(Alt) Out[ ]=

```
{0., 7}
```

## 3

(Alt) In[ ]:=

```
Timing@CountBs [3, 0]
```

(Alt) Out[ ]=

```
{0., 1}
```

(Alt) In[ ]:=

```
Timing@CountBs [3, 1]
```

(Alt) Out[ ]=

```
{0., 5}
```

(Alt) In[ ]:=

```
Timing@CountBs [3, 2]
```

(Alt) Out[ ]=

```
{0.015625, 17}
```

(Alt) In[ ]:=

```
Timing@CountBs [3, 3]
```

(Alt) Out[ ]=

```
{0.046875, 47}
```

```

(Alt) In[ ]:=
      Timing@CountBs [3, 4]
(Alt) Out[ ]=
      {0.203125, 115}

(Alt) In[ ]:=
      Timing@CountBs [3, 5]
(Alt) Out[ ]=
      {1.07813, 263}

(Alt) In[ ]:=
      Timing@CountBs [3, 6]
(Alt) Out[ ]=
      {4.89063, 577}

(Alt) In[ ]:=
      Timing@CountBs [3, 7]
(Alt) Out[ ]=
      {27.9219, 1233}

(Alt) In[ ]:=
      Timing@CountBs [3, 8]
(Alt) Out[ ]=
      {155.781, 2589}

(Alt) In[ ]:=
      Timing@CountBs [3, 9]
(Alt) Out[ ]=
      {901.359, 5371}

(Alt) In[ ]:=
      Timing@CountBs [3, 10]
(Alt) Out[ ]=
      {5687.67, 11047}

(Alt) In[ ]:=
      Timing@CountBs [3, 11]
(Alt) Out[ ]=
      {36763.5, 22579}

(Alt) In[ ]:=
      Timing@CountBs [3, 12]
(Alt) Out[ ]=
      {253568., 45933}

(Alt) In[ ]:=
      f3[m] =
      FullSimplify[m // FindSequenceFunction@{1, 5, 17, 47, 115, 263, 577, 1233, 2589, 5371}]
(Alt) Out[ ]=
      
$$1 + 3 \times 2^{1+m} + \left(-3 - \frac{7}{\sqrt{5}}\right) \left(\frac{1}{2} (1 + \sqrt{5})\right)^m + \frac{1}{5} \left(\frac{1}{2} (-1 + \sqrt{5})\right)^m (-15 + 7\sqrt{5}) e^{i m \pi} - 2 m$$


```

```

(Alt) In[ ]:=
  Expand[f3[m] /. m -> 11]
(Alt) Out[ ]=
  11 047

(Alt) In[ ]:=
  Expand[f3[m] /. m -> 12]
(Alt) Out[ ]=
  22 579

(Alt) In[ ]:=
  Table[12 x 2^m - 2 Fibonacci[m + 5] - 2 m - 1, {m, 0, 12}]
(Alt) Out[ ]=
  {1, 5, 17, 47, 115, 263, 577, 1233, 2589, 5371, 11 047, 22 579, 45 933}

(Alt) In[ ]:=
  Table[12 x 2^m - 2 Fibonacci[m + 5] - 2 m - 1, {m, 0, 15}] // FindLinearRecurrence
(Alt) Out[ ]=
  {5, -8, 3, 3, -2}

(Alt) In[ ]:=
  FindGeneratingFunction[Table[12 x 2^m - 2 Fibonacci[m + 5] - 2 m - 1, {m, 0, 15}], x] // Factor
(Alt) Out[ ]=
  - (1 + x) (-1 + x - x^2 + 2 x^3)
  -----
  (-1 + x)^2 (-1 + 2 x) (-1 + x + x^2)

4

(Alt) In[ ]:=
  Timing@CountBs[4, 0]
(Alt) Out[ ]=
  {0., 1}

(Alt) In[ ]:=
  Timing@CountBs[4, 1]
(Alt) Out[ ]=
  {0., 7}

(Alt) In[ ]:=
  Timing@CountBs[4, 2]
(Alt) Out[ ]=
  {0.03125, 33}

(Alt) In[ ]:=
  Timing@CountBs[4, 3]
(Alt) Out[ ]=
  {0.109375, 131}

(Alt) In[ ]:=
  Timing@CountBs[4, 4]
(Alt) Out[ ]=
  {0.5, 469}

```

(Alt) In[]:=  
**Timing@CountBs [4, 5]**

(Alt) Out[]=  
{5.21875, 1579}

(Alt) In[]:=  
**Timing@CountBs [4, 6]**

(Alt) Out[]=  
{36.4375, 5121}

(Alt) In[]:=  
**Timing@CountBs [4, 7]**

(Alt) Out[]=  
{284.844, 16219}

(Alt) In[]:=  
**Timing@CountBs [4, 8]**

(Alt) Out[]=  
{3601.2, 50581}

(Alt) In[]:=  
**Timing@CountBs [4, 9]**

(Alt) Out[]=  
{21730.2, 156127}

## 5

(Alt) In[]:=  
**Timing@CountBs [5, 0]**

(Alt) Out[]=  
{0., 1}

(Alt) In[]:=  
**Timing@CountBs [5, 1]**

(Alt) Out[]=  
{0., 9}

(Alt) In[]:=  
**Timing@CountBs [5, 2]**

(Alt) Out[]=  
{0.03125, 53}

(Alt) In[]:=  
**Timing@CountBs [5, 3]**

(Alt) Out[]=  
{0.15625, 259}

(Alt) In[]:=  
**Timing@CountBs [5, 4]**

(Alt) Out[]=  
{1.35938, 1143}

(Alt) In[]:=  
**Timing@CountBs [5, 5]**

(Alt) Out[]=  
{11.8281, 4743}

(Alt) In[]:=  
**Timing@CountBs [5, 6]**

(Alt) Out[]=  
{111.109, 18941}

(Alt) In[]:=  
**Timing@CountBs [5, 7]**

(Alt) Out[]=  
{1546.03, 73817}

(Alt) In[]:=  
**Timing@CountBs [5, 8]**

(Alt) Out[]=  
{9350.73, 283165}

(Alt) In[]:=  
**Timing@CountBs [5, 9]**

(Alt) Out[]=  
{72404.5, 1074963}

## 6

(Alt) In[]:=  
**Timing@CountBs [6, 0]**

(Alt) Out[]=  
{0., 1}

(Alt) In[]:=  
**Timing@CountBs [6, 1]**

(Alt) Out[]=  
{0., 11}

(Alt) In[]:=  
**Timing@CountBs [6, 2]**

(Alt) Out[]=  
{0.015625, 77}

(Alt) In[]:=  
**Timing@CountBs [6, 3]**

(Alt) Out[]=  
{0.34375, 439}

(Alt) In[]:=  
**Timing@CountBs [6, 4]**

(Alt) Out[]=  
{3.09375, 2233}

(Alt) In[]:=  
**Timing@CountBs [6, 5]**

(Alt) Out[]=  
{28.0938, 10 603}

(Alt) In[]:=  
**Timing@CountBs [6, 6]**

(Alt) Out[]=  
{375.469, 48 209}

(Alt) In[]:=  
**Timing@CountBs [6, 7]**

(Alt) Out[]=  
{2419.14, 213 119}

(Alt) In[]:=  
**Timing@CountBs [6, 8]**

(Alt) Out[]=  
{21 731.1, 924 865}

(Alt) In[]:=  
**Timing@CountBs [6, 9]**

(Alt) Out[]=  
{196 519., 3 964 411}

## 7

(Alt) In[]:=  
**Timing@CountBs [7, 0]**

(Alt) Out[]=  
{0., 1}

(Alt) In[]:=  
**Timing@CountBs [7, 1]**

(Alt) Out[]=  
{0., 13}

(Alt) In[]:=  
**Timing@CountBs [7, 2]**

(Alt) Out[]=  
{0.015625, 105}

(Alt) In[]:=  
**Timing@CountBs [7, 3]**

(Alt) Out[]=  
{0.484375, 679}

(Alt) In[]:=  
**Timing@CountBs [7, 4]**

(Alt) Out[]=  
{4.65625, 3867}

```
(Alt) In[ ]:=  
    Timing@CountBs [7, 5]
```

```
(Alt) Out[ ]=  
    {44.8281, 20351}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [7, 6]
```

```
(Alt) Out[ ]=  
    {534.875, 101765}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [7, 7]
```

```
(Alt) Out[ ]=  
    {5441.44, 491893}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [7, 8]
```

```
(Alt) Out[ ]=  
    {60274.8, 2323785}
```

## 8

```
(Alt) In[ ]:=  
    Timing@CountBs [8, 0]
```

```
(Alt) Out[ ]=  
    {0., 1}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [8, 1]
```

```
(Alt) Out[ ]=  
    {0., 15}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [8, 2]
```

```
(Alt) Out[ ]=  
    {0.0625, 137}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [8, 3]
```

```
(Alt) Out[ ]=  
    {0.71875, 987}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [8, 4]
```

```
(Alt) Out[ ]=  
    {7.96875, 6189}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [8, 5]
```

```
(Alt) Out[ ]=  
    {81.4531, 35515}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [8, 6]
```

```
(Alt) Out[ ]=  
    {1114.48, 192 081}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [8, 7]
```

```
(Alt) Out[ ]=  
    {7451.81, 997 547}
```

## 9

```
(Alt) In[ ]:=  
    Timing@CountBs [9, 0]
```

```
(Alt) Out[ ]=  
    {0.015625, 1}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [9, 1]
```

```
(Alt) Out[ ]=  
    {0., 17}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [9, 2]
```

```
(Alt) Out[ ]=  
    {0.125, 173}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [9, 3]
```

```
(Alt) Out[ ]=  
    {1.3125, 1371}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [9, 4]
```

```
(Alt) Out[ ]=  
    {13.6875, 9359}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [9, 5]
```

```
(Alt) Out[ ]=  
    {175.094, 57 959}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [9, 6]
```

```
(Alt) Out[ ]=  
    {1388.78, 335 773}
```

```
(Alt) In[ ]:=  
    Timing@CountBs [9, 7]
```

```
(Alt) Out[ ]=  
    {12 015.6, 1 855 937}
```



**n***(Alt) In[ ]:=***Expand[n - 1 // FindSequenceFunction@{5, 17, 33, 53, 77}]***(Alt) Out[ ]:=*

$$-7 + 2n + 2n^2$$

*(Alt) In[ ]:=***Expand[n - 2 // FindSequenceFunction@{47, 131, 259, 439, 679, 987, 1371}]***(Alt) Out[ ]:=*

$$-21 - \frac{22n}{3} + 6n^2 + \frac{4n^3}{3}$$

*(Alt) In[ ]:=*

$$-21 - \frac{22n}{3} + 6n^2 + \frac{4n^3}{3} /. n \rightarrow 3$$

*(Alt) Out[ ]:=*

47