$\mathcal{W} : \mathcal{A}^w(\uparrow_n) \to \widehat{\mathcal{U}(I\mathfrak{g})^{\otimes n}}$

$\mathcal{U}(I\mathfrak{g})$ is graded by the # of $\mathfrak{g}^*$ terms

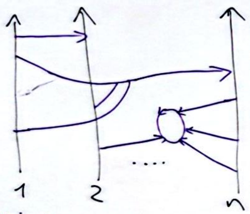
$\mathfrak{g}$ arbitrary Lie alg
finite dim'l over $\Bbbk$ char 0

$\mathfrak{g}^*$ abelian

$I\mathfrak{g} = \mathfrak{g} \ltimes \mathfrak{g}^* \qquad \mathfrak{g} \circlearrowright \mathfrak{g}^*$

$[(x_1, \varphi_1), (x_2, \varphi_2)] = ([x_1, x_2],\ x_1 \cdot \varphi_2 - x_2 \cdot \varphi_1)$

$\mathcal{A}^w(\uparrow_n)$



uni-trivalent
2-in-1-out
cyclic ori

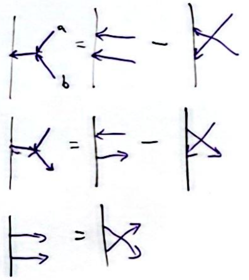
Key to $\mathcal{W}$ is structure constants

[illegible] $\mathfrak{g}^* \otimes \mathfrak{g}$

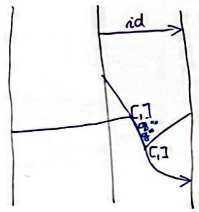
$id \in \mathfrak{g}^* \otimes \mathfrak{g}$

STU

TC

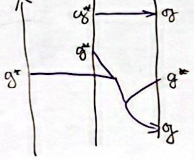


$D \in \mathcal{A}^w(\uparrow_n)$



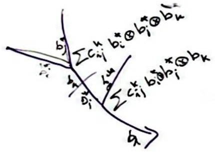
contract
$\mathfrak{g}^*$ w/ $\mathfrak{g}$
using eval
$\varphi(g)$

$\sum c$



$\xrightarrow[\text{off}]{\text{read}} \mathcal{U}(I\mathfrak{g})^{\otimes n}$

here $n = 3$



degree completion

arrow diagrams on n strands $\longrightarrow \widehat{(\mathcal{U} I\mathfrak{g})^{\otimes n}}$

degree preserving
map of algebras

Descends to a map of graded complete algebras

$\mathcal{A}^w(\uparrow_n) \to \widehat{\mathcal{U}(I\mathfrak{g})^{\otimes n}}$

STU $\quad [a, b] = ab - ba$

TC $\quad [\varphi_1, \varphi_2] = 0$ if $\varphi_1, \varphi_2 \in \mathfrak{g}^*$

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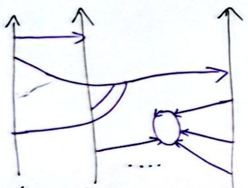
$\mathcal{A}^w(\uparrow_n)$



Uni-trivalent

2-in-1-out
cyclic ori

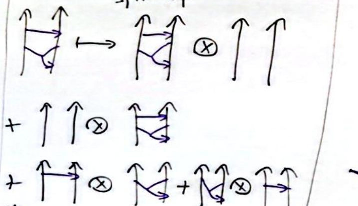
Key to $\mathcal{W}$ is structure constants

$id \in \mathfrak{g}^* \otimes \mathfrak{g}$

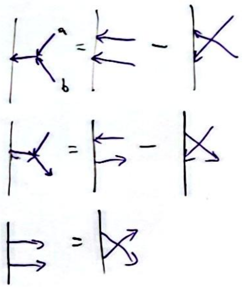
makes $\mathcal{A}^w(\uparrow_n)$ a Hopf alg.

$\Box: \mathcal{A}^w(\uparrow_n) \to \mathcal{A}^w(\uparrow_n) \otimes \mathcal{A}^w(\uparrow_n)$

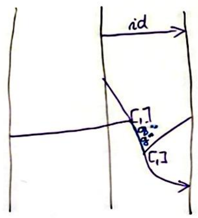
"split up"



this does not interact w/ $\mathcal{W}$

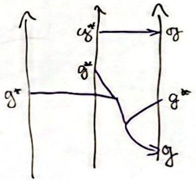


$D \in \mathcal{A}^w(\uparrow_n)$



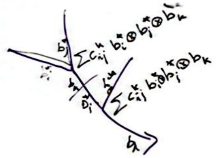
$\xrightarrow{\text{contract}}$ $\mathfrak{g}^*$ w/ $\mathfrak{g}$ using eval $\varphi(g)$

$\sum c$



$\xrightarrow[\text{off}]{\text{read}} \mathcal{U}(I\mathfrak{g})^{\otimes n}$

here $n=3$



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$\Delta_i: \mathcal{A}^w(\uparrow_n) \to \mathcal{A}^w(\uparrow_{n+1})$

"Coproduct 2"

$$\begin{tikzcd}
\mathcal{A}^w(\uparrow_n) \arrow[r, "\Delta_i"] \arrow[d, "\mathcal{W}"'] \arrow[dr, phantom, "\circlearrowright"] & \mathcal{A}^w(\uparrow_{n+1}) \arrow[d, "\mathcal{W}"] \\
(\mathcal{U}I\mathfrak{g})^{\otimes n} \arrow[r, "\Delta_i"] & \mathcal{U}(I\mathfrak{g})^{\otimes n+1}
\end{tikzcd}$$

Compatible

Meta-Hopf Algebras: $\{\mathcal{T}_S\}$

$$\{\mathcal{A}^w_S = \langle \text{arrow diagrams on strands } i, j, k \rangle / \text{STU, TC}\}$$

If H is a Hopf algebra

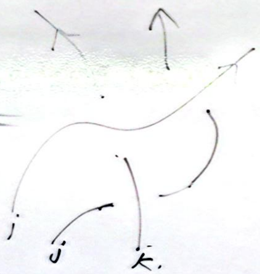
$$\{H_S := H^{\otimes S}\}$$

$$\begin{array}{ccc} \{\mathcal{T}^v_S\} & \xrightarrow{Z} & \{\mathcal{U}_\hbar(Ig)_S\} \\ \downarrow & & \\ \{\mathcal{T}^v_S\} \xrightarrow{Z^w} & \{\mathcal{A}^w_S\} \xrightarrow{W} & \{\mathcal{U}(Ig)_S\} \end{array}$$

Morphism of meta-monads but not of meta-Hopf algebras.

is a morphism of meta-Hopf algebras.

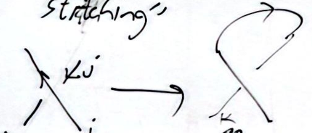
$\mathcal{T}^v_S =$



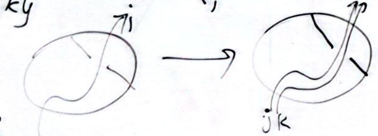
$\mathcal{T}^v_{S_1} \times \mathcal{T}^v_{S_2} \to \mathcal{T}^v_{S_1 \sqcup S_2}$

$m^{ij}_k : \mathcal{T}_{S \sqcup \{i,j\}} \longrightarrow \mathcal{T}_{S \sqcup \{k\}}$ "strand stitching"

$\Delta^i_{jk} : \mathcal{T}_{S \sqcup \{i\}} \longrightarrow \mathcal{T}_{S \sqcup \{j,k\}}$



"strand doubling"



σ^i_j : strand renaming

S_i : reverse strand i

ϵ_i : m_i : $T \to T$ $|_i$

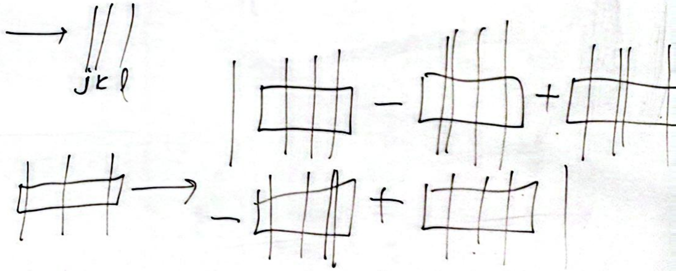
Axioms of a Hopf-Algebra

$i/j/o/k$

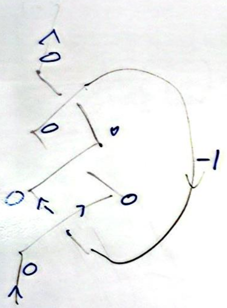
$m^{ij}_t /\!/ m^{tk}_l = m^{jk}_t /\!/ m^{it}_l$ Associativity

coassociativity:

$\Delta^i_{jt} /\!/ \Delta^t_{kl} = \Delta^i_{tl} /\!/ \Delta^t_{jk}$: $|_i \longrightarrow |||_{jkl}$



$\vdots$



Meta-Hopf Algebras: $\{{}^{\sigma}T_S\}$

$\{A^w_S = \langle \ldots \rangle / STU, TC\}$

If H is a Hopf algebra

$\{H_S := H^{\otimes S}\}$

$\{{}^{\sigma}T^{rv}_S\} \xrightarrow{Z_{Ig}} \{U_\hbar(Ig)_S\}$

$\{{}^{\sigma}T^{rv}_S\} \xrightarrow{Z^{rw}} \{A^w_S\}$

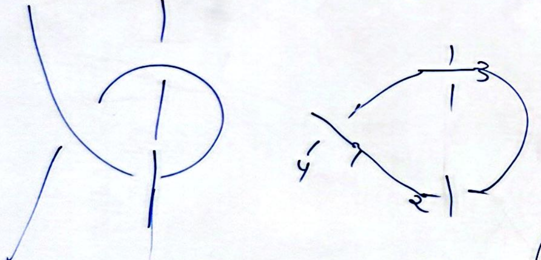
$\{{}^{\sigma}T^{v}_S\} \xrightarrow{Z^{w}} \{A^w_S\} \xrightarrow{W_g} \{U(Ig)_S\}$

|| as monoids

Morphism of meta-monoids but not of meta-Hopf algebras.

is a morphism of meta-Hopf algebras.

$\sigma T^{rv} = MM\langle \ldots \rangle / R1, R2, R3, R3c, R3b$



$\sigma T^{rv} = MM\langle \ldots$, pivot $\rangle$

$= R$

$C = C = \ldots$

$\rho = | \ldots$ R2b,c R3b,c

$\Delta(R) \neq R^{23}R^{13}$

$q\Delta_1(R) = R^{23}R^{13}$

$m^{12}_3 R = 1$

$W(\exp(\cdot)) \neq 1$

$U_\hbar(Ig) = (U(Ig), m, q\Delta(w) = V_g\Delta(w)V_g^{-1})$

where $V_g = W_g(V)$

$Z_{Ig}(\ldots) = W(\underbrace{e^{\mapsto}}_{R})$

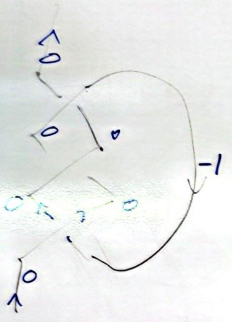
$V = \ldots + \ldots$ satisfies KV1

$Z_{Ig}(C) = W(e^{\mapsto a})$

$\frac{1}{2} \mapsto$

$q\Delta(x) = x\otimes 1 + 1\otimes x$

$q\Delta(\varphi) = \varphi\otimes 1 + 1\otimes\varphi +$



Meta-Hopf Al

IF

$m_0 : Sg \otimes Sg \longrightarrow Sg$

$F_\hbar : Sg \otimes Sg \to$ 

$m_\hbar = m_0 \circ F_\hbar$

$$\frac{dm_\hbar}{d\hbar} = m_0 \circ F_\hbar \circ \left(F_\hbar^{-1} \circ \frac{dF_\hbar}{d\hbar} \right)$$

d

$M_0 : (Sg \otimes \Lambda g^*) \otimes (Sg \otimes \Lambda g^*) \longrightarrow (Sg \otimes \Lambda g^*)$

$(M_\hbar = M_0 \circ (F_\hbar \otimes 1),\ d)$

$$\frac{dM_\hbar}{d\hbar} = \left[\cancel{d},\ M_\hbar \circ \left(1 \otimes F_\hbar^{-1} \frac{dF_\hbar}{d\hbar} \right) \right]$$

$[d,$

$$\begin{tikzcd}
\{\sigma T^{rv}_S\} \arrow[rr, "Z_{Ig}"] \arrow[d] \arrow[dr, "Z^{rw}"] & & \{U_\hbar(Ig)_S\} \\
\{\sigma T^{v}_S\} \arrow[r, "Z^{w}"] & \{A^w_S\} \arrow[r, "W_g"] & \{U(Ig)_S\}
\end{tikzcd}$$

|| as monoids

Morphism of meta-monads of meta-Hopf algebras. is a morphism of meta-Hopf algebras.

$1) = (U(Ig), m, q\Delta(w) = V_g \Delta(w) V_g^{-1}$

where $V_g = W_g(V)$

$\overbrace{W(e^{\mapsto})}^{R}$

$W(e^{\mapsto a})$

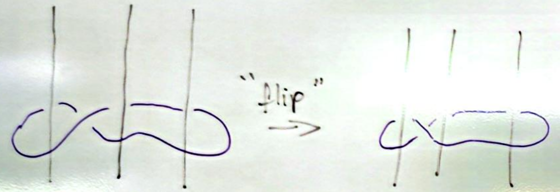
$V = \underbrace{\text{[tangle]} + \ldots.}_{\text{satisfies KV1}}$ 

$\frac{1}{2} \mapsto$

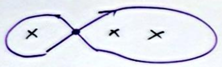
$q\Delta(x) = x \otimes 1 + 1 \otimes x$

$q\Delta(\varphi) = \varphi \otimes 1 + 1 \otimes \varphi +$

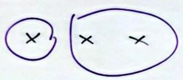
1



for each crossing you get a Tyraev-like sum

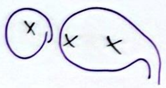


HOMFLY/ Alexander



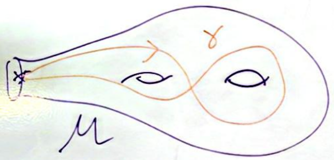
This is symmetric !!

then when you resolve you get



then anti-symmetrise by hand

$? \quad A = \int_\Sigma \mathrm{Tr} \frac{\delta}{\delta a(x)} \cdot \frac{\delta}{\delta a^*(x)}$



$\mathcal{M}$

$\parallel$

$\mathrm{Hom}(\pi_1, IG) \cong IG$?

$\rho \in$ $IG \ni$

$W_\gamma(\rho) = \rho(\gamma) \quad \mu(\gamma) = \gamma' \otimes |\gamma''|$

$A \subset \mathrm{Fun}(\mathcal{M})$

$A(W_\gamma) = W_{\gamma'} W_{|\gamma''|}$

$$\begin{tikzcd} \{\sigma T^{rv}_S\} \arrow[rr, "Z_{I}"] \arrow[d] \arrow[dr, "Z^{rw}"] & & \{U_\hbar(Ig)_S\} \\ \{\sigma T^{v}_S\} \arrow[r, "Z^w"] & \{A^w_S\} \arrow[r, "W_g"] & \{U(Ig)_S\} \end{tikzcd}$$

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$\overbrace{\iota/(e^{\mapsto}}^{R})$

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$V = \underbrace{\ \ \ + \dots}_{\text{satisfies KV!}}$

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