

Older Stuff

```

hta[x_, y_, z_][μ[ω_, λ_]] := Module[{λx, Adλx},
  λx = MakeLieSeries[D[λ, h[x]]];
  Adλx = LieMorphism[{LW[y] → Ad[ScaleLieSeries[-1, λx]][LW[z]]}];
  μ[
    AddCWSeries[StableApply[Adλx, ω], JA[LW[y], λx]],
    Collect[λ, _h, StableApply[Adλx, #] &]
  ]
];
hta[x_, y_][μ[ω_, λ_]] :=
  μ[ω, λ] // hta[x, y, <"z">] // LieMorphism[{LW["z"] → LW[y]}];
dm[x_, y_, z_][μ_] := μ // hta[y, x] // tm[x, y, z] // hm[x, y, z];
Rp[x_, y_] := μ[MakeCWSeries[0], h[y] MakeLieSeries[<x>]];
Rm[x_, y_] := μ[MakeCWSeries[0], h[y] MakeLieSeries[-<x>]];

n = $SeriesShowDegree = $SeriesCompareDegree = 4;
Print /@ {
  μ0 = Randomμ[3, 2, 2],
  μ0 // hta[1, 1],
  μ1 = ReplacePart[μ0, 1 → MakeCWSeries[0]],
  μ1 // hta[1, 1]
};

μ[CWS[-2 CW[1], -CW[12], CW[112] - 2 CW[122], 0],
  h[1] LS[2 <1> - 2 <2>, -<12>, <112> - <122>, 0] + h[2] LS[<1> - 2 <2>, 0, <122>, 0]]
μ[CWS[0, -2 CW[12], CW[112] - 8 CW[122], 3 CW[1112] + 3 CW[1122] + CW[1222]],
  h[1] LS[2 <1> - 2 <2>, 3 <12>, -<112> + <122>, -4 <1112>/3 + <1122>/3 - 4 <1222>/3] +
  h[2] LS[<1> - 2 <2>, 2 <12>, -<112> + 3 <122>, -2 <1112>/3 - 4 <1122>/3 + 10 <1222>/3]]
μ[CWS[0, 0, 0, 0],
  h[1] LS[2 <1> - 2 <2>, -<12>, <112> - <122>, 0] + h[2] LS[<1> - 2 <2>, 0, <122>, 0]]
μ[CWS[2 CW[1], -CW[12], 2 CW[112] - 2 CW[122], CW[1112] - CW[1122]/2 + 2 CW[1212] + CW[1222]],
  h[1] LS[2 <1> - 2 <2>, 3 <12>, -<112> + <122>, -4 <1112>/3 + <1122>/3 - 4 <1222>/3] +
  h[2] LS[<1> - 2 <2>, 2 <12>, -<112> + 3 <122>, -2 <1112>/3 - 4 <1122>/3 + 10 <1222>/3]]

```

Testing tm

```

n = $SeriesShowDegree = $SeriesCompareDegree = 3;
Print /@ {μ0 = Randomμ[n, 4, 1],
  μ0 // tm[1, 2, 2],
  μ0 // tm[2, 3, 3],
  t1 = μ0 // tm[1, 2, 2] // tm[2, 3, 3],
  t2 = μ0 // tm[2, 3, 3] // tm[1, 3, 3],
  t1 ≡ t2
};

μ[CWS[2 CW[1] - CW[2] + CW[4], 2 CW[12] - CW[13] + 2 CW[14] + 2 CW[23] - 2 CW[24] + CW[34],
  -CW[112] - 2 CW[113] - 2 CW[114] + CW[124] - 2 CW[132] + CW[133] + 2 CW[134] -
  2 CW[144] + 2 CW[223] - CW[224] - CW[233] - CW[243] + 2 CW[244] + 2 CW[334]],

h[1] LS[-2 ⟨1⟩ - 2 ⟨2⟩ + 2 ⟨3⟩, -⟨12⟩ + ⟨14⟩ - ⟨23⟩ + 2 ⟨24⟩, 2 ⟨112⟩ + ⟨114⟩ + ⟨122⟩ +
  ⟨123⟩ - ⟨124⟩ - ⟨142⟩ - ⟨143⟩ + ⟨144⟩ - ⟨223⟩ + 2 ⟨224⟩ + 2 ⟨233⟩ + ⟨234⟩ - ⟨243⟩]],

μ[CWS[CW[2] + CW[4], 2 CW[22] + CW[23] + CW[34],
  -CW[222] - 2 CW[223] - 2 CW[224] + 2 CW[234] - CW[243] + 2 CW[334]],

h[1] LS[-4 ⟨2⟩ + 2 ⟨3⟩, -⟨23⟩ + 3 ⟨24⟩, 3 ⟨224⟩ + 2 ⟨233⟩ + ⟨234⟩ - 2 ⟨243⟩ + ⟨244⟩]],

μ[CWS[2 CW[1] - CW[3] + CW[4], CW[13] + 2 CW[14] + 2 CW[33] - CW[34],
  -3 CW[113] - 2 CW[114] - CW[133] + 3 CW[134] - 2 CW[144] + CW[333] + 2 CW[344]],

h[1] LS[-2 ⟨1⟩, -⟨13⟩ + ⟨14⟩ + 2 ⟨34⟩, 2 ⟨113⟩ + ⟨114⟩ + ⟨133⟩ - ⟨134⟩ - 2 ⟨143⟩ + ⟨144⟩ + 4 ⟨334⟩]],

μ[CWS[CW[3] + CW[4], 3 CW[33] + CW[34], -3 CW[333] + CW[334]],

h[1] LS[-2 ⟨3⟩, 3 ⟨34⟩, 6 ⟨334⟩ + ⟨344⟩]],

μ[CWS[CW[3] + CW[4], 3 CW[33] + CW[34], -3 CW[333] + CW[334]],

h[1] LS[-2 ⟨3⟩, 3 ⟨34⟩, 6 ⟨334⟩ + ⟨344⟩]],

True

t1 = μ0 // tm[1, 2, 2] // tm[2, 3, 3] // InputForm
μ[CWSeries[LieMorphismOnCWSeries$1866], h[1]*LieSeries[LieMorphismOnLieSeries$1868]];

```

Testing hm

```

Print /@ {μ0 = Randomμ[4, 2, 3],
  μ0 // hm[1, 2, 2],
  t1 = μ0 // hm[1, 2, 2] // hm[2, 3, 3],
  t2 = μ0 // hm[2, 3, 3] // hm[1, 3, 3],
  t1 ≡ t2
};

```

$$\begin{aligned} & \mu[\text{CWS}[2 \text{CW}[1] - \text{CW}[2], 0, -2 \text{CW}[122]], h[1] \text{LS}[2 \langle 1 \rangle - \langle 2 \rangle, 2 \langle 12 \rangle, 2 \langle 112 \rangle - 2 \langle 122 \rangle] + \\ & h[2] \text{LS}[\langle 2 \rangle, 2 \langle 12 \rangle, -2 \langle 112 \rangle + \langle 122 \rangle] + h[3] \text{LS}[\langle 1 \rangle - 2 \langle 2 \rangle, -\langle 12 \rangle, -\langle 112 \rangle - \langle 122 \rangle]] \\ & \mu[\text{CWS}[2 \text{CW}[1] - \text{CW}[2], 0, -2 \text{CW}[122]], \\ & h[2] \text{LS}\left[2 \langle 1 \rangle, 5 \langle 12 \rangle, \frac{7 \langle 112 \rangle}{3} + \frac{4 \langle 122 \rangle}{3}\right] + h[3] \text{LS}[\langle 1 \rangle - 2 \langle 2 \rangle, -\langle 12 \rangle, -\langle 112 \rangle - \langle 122 \rangle]] \\ & \mu[\text{CWS}[2 \text{CW}[1] - \text{CW}[2], 0, -2 \text{CW}[122]], h[3] \text{LS}\left[3 \langle 1 \rangle - 2 \langle 2 \rangle, 2 \langle 12 \rangle, -\frac{5 \langle 112 \rangle}{2} - 4 \langle 122 \rangle\right]] \\ & \mu[\text{CWS}[2 \text{CW}[1] - \text{CW}[2], 0, -2 \text{CW}[122]], h[3] \text{LS}\left[3 \langle 1 \rangle - 2 \langle 2 \rangle, 2 \langle 12 \rangle, -\frac{5 \langle 112 \rangle}{2} - 4 \langle 122 \rangle\right]] \end{aligned}$$

True

Testing hta

```

n = $SeriesShowDegree = $SeriesCompareDegree = 4;
Print /@ {μ0 = Randomμ[3, 2, 2], μ0 // hta[1, 1]};

μ[CWS[-CW[1] - 2 CW[2], CW[12], -CW[112], 0],
h[1] LS[⟨1⟩ + ⟨2⟩, 0, -2 ⟨112⟩ - 2 ⟨122⟩, 0] + h[2] LS[2 ⟨1⟩ - 2 ⟨2⟩, -2 ⟨12⟩, 0, 0]]

μ[CWS[-2 CW[2], 3 CW[12], 5 CW[112], 11 CW[122],
-23 CW[1112], 7 CW[1122], 29 CW[1212], 23 CW[1222],
24, 12, 12, 24],
h[1] LS[⟨1⟩ + ⟨2⟩, -⟨12⟩, -3 ⟨112⟩, 3 ⟨122⟩, 11 ⟨1112⟩, 10 ⟨1122⟩, 11 ⟨1222⟩],
h[2] LS[2 ⟨1⟩ - 2 ⟨2⟩, -4 ⟨12⟩, ⟨112⟩ + 3 ⟨122⟩, 11 ⟨1112⟩, 5 ⟨1122⟩, 4 ⟨1222⟩]]]

Print /@ {μ0 = Randomμ[n, 3, 2],
t1 = μ0 // hta[1, 1] // hta[1, 2] // tm[1, 2, 1],
t2 = μ0 // tm[1, 2, 1] // hta[1, 1],
t1 ≡ t2
};

```

$$\begin{aligned} & \mu[\text{CWS}[-\text{CW}[1] + 2 \text{CW}[2], -2 \text{CW}[13] - 2 \text{CW}[23], \\ & 2 \text{CW}[112] + 2 \text{CW}[122] + \text{CW}[123] - 2 \text{CW}[132] + 2 \text{CW}[133] - 2 \text{CW}[233], \\ & \text{CW}[1112] + \text{CW}[1113] + 2 \text{CW}[1122] - \text{CW}[1133] + \text{CW}[1213] - \text{CW}[1222] - 2 \text{CW}[1223] - \\ & \text{CW}[1233] + \text{CW}[1322] + \text{CW}[1323] + 2 \text{CW}[1333] + 2 \text{CW}[2223] + 2 \text{CW}[2233]], \\ & h[1] \text{LS}[\langle 1 \rangle - \langle 3 \rangle, \langle 12 \rangle - \langle 13 \rangle + \langle 23 \rangle, 2 \langle 112 \rangle + \langle 113 \rangle - 2 \langle 122 \rangle - \langle 133 \rangle - \langle 233 \rangle, \\ & 2 \langle 1112 \rangle + \langle 1113 \rangle + \langle 1122 \rangle + 2 \langle 1123 \rangle - 2 \langle 1132 \rangle + \langle 1133 \rangle + 2 \langle 1222 \rangle - \langle 1223 \rangle - \\ & 2 \langle 1232 \rangle + 2 \langle 1233 \rangle + \langle 1322 \rangle + 2 \langle 1323 \rangle + 2 \langle 1332 \rangle - \langle 1333 \rangle - \langle 2233 \rangle + 2 \langle 2333 \rangle] + \\ & h[2] \text{LS}[-2 \langle 1 \rangle + \langle 2 \rangle, \langle 12 \rangle + 2 \langle 23 \rangle, -2 \langle 113 \rangle + \langle 122 \rangle + 2 \langle 123 \rangle - 2 \langle 132 \rangle - \langle 133 \rangle + \langle 223 \rangle, \\ & -\langle 1113 \rangle + \langle 1122 \rangle + 2 \langle 1123 \rangle - \langle 1132 \rangle + \langle 1133 \rangle - \langle 1222 \rangle + 2 \langle 1223 \rangle - \\ & 2 \langle 1232 \rangle + 2 \langle 1233 \rangle + \langle 1323 \rangle + 2 \langle 1332 \rangle + 2 \langle 1333 \rangle + 2 \langle 2223 \rangle - 2 \langle 2333 \rangle]] \\ & \mu\left[\text{CWS}\left[2 \text{CW}[1], -\frac{9 \text{CW}[13]}{2}, 4 \text{CW}[111] - \frac{11 \text{CW}[113]}{6} - \frac{11 \text{CW}[133]}{6},\right. \\ & \left.2 \text{CW}[1111] - \frac{85 \text{CW}[1113]}{24} + \frac{35 \text{CW}[1133]}{12} - \frac{7 \text{CW}[1313]}{12} + \frac{47 \text{CW}[1333]}{24}\right], \\ & h[1] \text{LS}\left[\langle 1 \rangle - \langle 3 \rangle, \langle 13 \rangle, \frac{\langle 113 \rangle}{2} - \frac{3 \langle 133 \rangle}{2}, \frac{37 \langle 1113 \rangle}{6} + \frac{7 \langle 1133 \rangle}{3} - \frac{5 \langle 1333 \rangle}{6}\right] + \\ & h[2] \text{LS}\left[-\langle 1 \rangle, \langle 13 \rangle, \frac{7 \langle 113 \rangle}{2} + \frac{\langle 133 \rangle}{2}, \frac{53 \langle 1113 \rangle}{6} + \frac{5 \langle 1133 \rangle}{3} - \frac{\langle 1333 \rangle}{6}\right]] \\ & \mu\left[\text{CWS}\left[2 \text{CW}[1], -\frac{9 \text{CW}[13]}{2}, 4 \text{CW}[111] - \frac{11 \text{CW}[113]}{6} - \frac{11 \text{CW}[133]}{6},\right. \\ & \left.2 \text{CW}[1111] - \frac{85 \text{CW}[1113]}{24} + \frac{35 \text{CW}[1133]}{12} - \frac{7 \text{CW}[1313]}{12} + \frac{47 \text{CW}[1333]}{24}\right], \\ & h[1] \text{LS}\left[\langle 1 \rangle - \langle 3 \rangle, \langle 13 \rangle, \frac{\langle 113 \rangle}{2} - \frac{3 \langle 133 \rangle}{2}, \frac{37 \langle 1113 \rangle}{6} + \frac{7 \langle 1133 \rangle}{3} - \frac{5 \langle 1333 \rangle}{6}\right] + \\ & h[2] \text{LS}\left[-\langle 1 \rangle, \langle 13 \rangle, \frac{7 \langle 113 \rangle}{2} + \frac{\langle 133 \rangle}{2}, \frac{53 \langle 1113 \rangle}{6} + \frac{5 \langle 1133 \rangle}{3} - \frac{\langle 1333 \rangle}{6}\right]\right] \end{aligned}$$

True

```

n = $SeriesShowDegree = $SeriesCompareDegree = 5;
Print /@ {μ0 = ReplacePart[Randomμ[n, 2, 3], 1 → MakeCWSeries[0]],
t1 = μ0 // hta[1, 1] // hta[2, 1] // hm[1, 2, 1],
t2 = μ0 // hm[1, 2, 1] // hta[1, 1],
t1 ≈ t2
};

```

$$\begin{aligned}
& \mu[\text{CWS}[0, 0, 0, 0, 0], h[1] \text{LS}[-2 \langle 1 \rangle + 2 \langle 2 \rangle, 0, 2 \langle 122 \rangle, \\
& \quad 2 \langle 1112 \rangle + 2 \langle 1122 \rangle - 2 \langle 1222 \rangle, -\langle 11112 \rangle - 2 \langle 11122 \rangle - \langle 11222 \rangle - 2 \langle 12222 \rangle] + \\
& \quad h[2] \text{LS}[-\langle 1 \rangle + 2 \langle 2 \rangle, -2 \langle 12 \rangle, \langle 112 \rangle - \langle 122 \rangle, -2 \langle 1112 \rangle + 2 \langle 1122 \rangle + 2 \langle 1222 \rangle, \\
& \quad -\langle 11112 \rangle - \langle 11122 \rangle - 2 \langle 11212 \rangle + 2 \langle 12222 \rangle] + \\
& \quad h[3] \text{LS}[0, \langle 12 \rangle, 2 \langle 122 \rangle, -\langle 1122 \rangle - \langle 1222 \rangle, \langle 11112 \rangle + 2 \langle 11122 \rangle - \langle 11212 \rangle + \langle 11222 \rangle]] \\
& \mu \left[\text{CWS} \left[-3 \text{CW}[1], -3 \text{CW}[12], -\frac{14 \text{CW}[112]}{3} + \text{CW}[122], \right. \right. \\
& \quad \left. \left. -\frac{43 \text{CW}[1112]}{12} + \frac{61 \text{CW}[1122]}{6} - \frac{34 \text{CW}[1212]}{3} + \frac{8 \text{CW}[1222]}{3}, \right. \right. \\
& \quad \left. \left. -\frac{187 \text{CW}[11112]}{60} + \frac{454 \text{CW}[11122]}{15} - \frac{551 \text{CW}[11212]}{15} - \frac{14 \text{CW}[11222]}{3} + 18 \text{CW}[12122] - \frac{26 \text{CW}[12222]}{15} \right], \right. \\
& \quad h[3] \text{LS}[0, \langle 12 \rangle, -2 \langle 122 \rangle, -4 \langle 1122 \rangle - \langle 1222 \rangle, \\
& \quad \langle 11112 \rangle - \frac{8 \langle 11122 \rangle}{3} + \frac{11 \langle 11212 \rangle}{3} + 10 \langle 11222 \rangle + 9 \langle 12122 \rangle + \frac{28 \langle 12222 \rangle}{3}] + \\
& \quad h[1] \text{LS} \left[-3 \langle 1 \rangle + 4 \langle 2 \rangle, 9 \langle 12 \rangle, \frac{73 \langle 112 \rangle}{6} - 9 \langle 122 \rangle, \frac{77 \langle 1112 \rangle}{6} - \frac{63 \langle 1122 \rangle}{2} - \frac{2 \langle 1222 \rangle}{3}, \right. \\
& \quad \left. \frac{277 \langle 11112 \rangle}{24} - \frac{1309 \langle 11122 \rangle}{20} + \frac{163 \langle 11212 \rangle}{20} + \frac{379 \langle 11222 \rangle}{45} + \frac{2237 \langle 12122 \rangle}{90} + 6 \langle 12222 \rangle \right] \\
& \mu \left[\text{CWS} \left[-3 \text{CW}[1], -3 \text{CW}[12], -\frac{14 \text{CW}[112]}{3} + \text{CW}[122], \right. \right. \\
& \quad \left. \left. -\frac{43 \text{CW}[1112]}{12} + \frac{61 \text{CW}[1122]}{6} - \frac{34 \text{CW}[1212]}{3} + \frac{8 \text{CW}[1222]}{3}, \right. \right. \\
& \quad \left. \left. -\frac{187 \text{CW}[11112]}{60} + \frac{454 \text{CW}[11122]}{15} - \frac{551 \text{CW}[11212]}{15} - \frac{14 \text{CW}[11222]}{3} + 18 \text{CW}[12122] - \frac{26 \text{CW}[12222]}{15} \right], \right. \\
& \quad h[1] \text{LS} \left[-3 \langle 1 \rangle + 4 \langle 2 \rangle, 9 \langle 12 \rangle, \frac{73 \langle 112 \rangle}{6} - 9 \langle 122 \rangle, \frac{77 \langle 1112 \rangle}{6} - \frac{63 \langle 1122 \rangle}{2} - \frac{2 \langle 1222 \rangle}{3}, \right. \\
& \quad \left. \frac{277 \langle 11112 \rangle}{24} - \frac{1309 \langle 11122 \rangle}{20} + \frac{163 \langle 11212 \rangle}{20} + \frac{379 \langle 11222 \rangle}{45} + \frac{2237 \langle 12122 \rangle}{90} + 6 \langle 12222 \rangle \right] + \\
& \quad h[3] \text{LS}[0, \langle 12 \rangle, -2 \langle 122 \rangle, -4 \langle 1122 \rangle - \langle 1222 \rangle, \\
& \quad \langle 11112 \rangle - \frac{8 \langle 11122 \rangle}{3} + \frac{11 \langle 11212 \rangle}{3} + 10 \langle 11222 \rangle + 9 \langle 12122 \rangle + \frac{28 \langle 12222 \rangle}{3}] \]
\end{aligned}$$

True

Testing dm

```

$SeriesShowDegree = 3;
$SeriesCompareDegree = n = 5;
Timing[Print /@ {μ0 = Randomμ[n, 4, 4],
  t1 = μ0 // dm[1, 2, 1] // dm[1, 3, 1],
  t2 = μ0 // dm[2, 3, 2] // dm[1, 2, 1],
  t1 ≈ t2
}];
```

```

 $\mu[\text{CWS}[-2 \text{CW}[1] + 2 \text{CW}[2] - \text{CW}[3], -\text{CW}[12] - 2 \text{CW}[23] + \text{CW}[24],$ 
 $-\text{CW}[112] + \text{CW}[114] + \text{CW}[122] + 2 \text{CW}[123] + \text{CW}[133] - \text{CW}[134] + \text{CW}[142] - \text{CW}[143] -$ 
 $\text{CW}[144] + 2 \text{CW}[224] - \text{CW}[233] - 2 \text{CW}[234] + \text{CW}[244] - 2 \text{CW}[334] - \text{CW}[344]],$ 
 $h[1] \text{LS}[-2 \langle 2 \rangle - 2 \langle 4 \rangle, 2 \langle 13 \rangle + 2 \langle 14 \rangle + \langle 23 \rangle - 2 \langle 24 \rangle - \langle 34 \rangle, -2 \langle 114 \rangle + 2 \langle 122 \rangle - \langle 123 \rangle - \langle 124 \rangle +$ 
 $2 \langle 132 \rangle + 2 \langle 133 \rangle + 2 \langle 134 \rangle + \langle 142 \rangle + 2 \langle 224 \rangle + 2 \langle 233 \rangle - 2 \langle 234 \rangle - 2 \langle 244 \rangle + 2 \langle 334 \rangle - 2 \langle 344 \rangle] +$ 
 $h[2] \text{LS}[2 \langle 1 \rangle + 2 \langle 2 \rangle + \langle 3 \rangle + 2 \langle 4 \rangle, \langle 12 \rangle - 2 \langle 13 \rangle - 2 \langle 23 \rangle + \langle 24 \rangle - \langle 34 \rangle, 2 \langle 113 \rangle - 2 \langle 114 \rangle - \langle 122 \rangle +$ 
 $\langle 133 \rangle + \langle 134 \rangle - \langle 142 \rangle + 2 \langle 143 \rangle - \langle 223 \rangle + 2 \langle 224 \rangle + \langle 234 \rangle + \langle 243 \rangle - 2 \langle 244 \rangle + \langle 334 \rangle - 2 \langle 344 \rangle] +$ 
 $h[3] \text{LS}[\langle 1 \rangle - 2 \langle 2 \rangle - 2 \langle 3 \rangle - \langle 4 \rangle, \langle 12 \rangle + \langle 13 \rangle + 2 \langle 14 \rangle - 2 \langle 23 \rangle - 2 \langle 24 \rangle - 2 \langle 34 \rangle,$ 
 $- \langle 112 \rangle - \langle 113 \rangle - \langle 122 \rangle + \langle 132 \rangle + \langle 134 \rangle + 2 \langle 142 \rangle + 2 \langle 144 \rangle +$ 
 $2 \langle 223 \rangle - 2 \langle 224 \rangle + 2 \langle 233 \rangle + 2 \langle 234 \rangle - 2 \langle 243 \rangle + \langle 244 \rangle + \langle 344 \rangle] +$ 
 $h[4] \text{LS}[-2 \langle 2 \rangle + \langle 3 \rangle + 2 \langle 4 \rangle, \langle 13 \rangle - 2 \langle 14 \rangle - \langle 24 \rangle - \langle 34 \rangle,$ 
 $2 \langle 112 \rangle - \langle 113 \rangle + 2 \langle 114 \rangle + \langle 122 \rangle - 2 \langle 123 \rangle - \langle 124 \rangle - \langle 132 \rangle + \langle 133 \rangle - \langle 134 \rangle +$ 
 $\langle 142 \rangle - \langle 223 \rangle + 2 \langle 224 \rangle - \langle 233 \rangle - \langle 234 \rangle - 2 \langle 243 \rangle + \langle 244 \rangle + 2 \langle 334 \rangle + 2 \langle 344 \rangle]]$ 
 $\mu[\text{CWS}\left[0, 4 \text{CW}[11] + \frac{11 \text{CW}[14]}{2}, -\frac{7 \text{CW}[111]}{3} + \frac{7 \text{CW}[114]}{6} + \frac{\text{CW}[144]}{6}\right],$ 
 $h[4] \text{LS}[-\langle 1 \rangle + 2 \langle 4 \rangle, -6 \langle 14 \rangle, -2 \langle 114 \rangle + 3 \langle 144 \rangle] +$ 
 $h[1] \text{LS}\left[-\langle 4 \rangle, -\frac{13 \langle 14 \rangle}{2}, \frac{17 \langle 114 \rangle}{2} - \frac{45 \langle 144 \rangle}{4}\right]]$ 
 $\mu[\text{CWS}\left[0, 4 \text{CW}[11] + \frac{11 \text{CW}[14]}{2}, -\frac{7 \text{CW}[111]}{3} + \frac{7 \text{CW}[114]}{6} + \frac{\text{CW}[144]}{6}\right],$ 
 $h[4] \text{LS}[-\langle 1 \rangle + 2 \langle 4 \rangle, -6 \langle 14 \rangle, -2 \langle 114 \rangle + 3 \langle 144 \rangle] +$ 
 $h[1] \text{LS}\left[-\langle 4 \rangle, -\frac{13 \langle 14 \rangle}{2}, \frac{17 \langle 114 \rangle}{2} - \frac{45 \langle 144 \rangle}{4}\right]]$ 

```

True

{43.93, Null}

Testing R-Moves

```

(Rp[1, 2] Rm[3, 4]) // dm[1, 3, 1] // dm[2, 4, 2]
 $\mu[\text{CWS}[0, 0, 0], h[1] \text{LS}[0, 0, 0] + h[2] \text{LS}[0, 0, 0]]$ 

(Rp[1, 2] Rm[3, 4]) // dm[1, 3, 1] // dm[4, 2, 2]
 $\mu[\text{CWS}[0, 0, 0], h[1] \text{LS}[0, 0, 0] + h[2] \text{LS}[0, 0, 0]]$ 

t1 = Rp[1, 2] Rp[4, 3] Rp[5, 6] // dm[1, 4, 1] // dm[2, 5, 2] // dm[3, 6, 3]
 $\mu[\text{CWS}[0, 0, 0],$ 
 $h[1] \text{LS}[0, 0, 0] + h[2] \text{LS}[\langle 1 \rangle, 0, 0] + h[3] \text{LS}\left[\langle 1 \rangle + \langle 2 \rangle, \frac{\langle 12 \rangle}{2}, \frac{\langle 112 \rangle}{12} + \frac{\langle 122 \rangle}{12}\right]]$ 

t2 = Rp[2, 3] Rp[1, 4] Rp[5, 6] // dm[1, 5, 1] // dm[2, 6, 2] // dm[3, 4, 3]
 $\mu[\text{CWS}[0, 0, 0],$ 
 $h[1] \text{LS}[0, 0, 0] + h[2] \text{LS}[\langle 1 \rangle, 0, 0] + h[3] \text{LS}\left[\langle 1 \rangle + \langle 2 \rangle, \frac{\langle 12 \rangle}{2}, \frac{\langle 112 \rangle}{12} + \frac{\langle 122 \rangle}{12}\right]]$ 

```

```
{t1[7], t2[7], t1[7] == t2[7]}

{μ[0, h[3] (⟨1111112⟩ - ⟨1111122⟩ + ⟨1111212⟩ + ⟨1111222⟩ + ⟨1112112⟩ + ⟨1112122⟩ +
⟨1112212⟩ + ⟨1112222⟩ + ⟨1121122⟩ - ⟨1121212⟩ + 13 ⟨1121222⟩ + ⟨1122122⟩ -
1260 3780 2016 5040 15120 10080
⟨1122212⟩ - ⟨1122222⟩ + ⟨1212122⟩ - ⟨1212222⟩ - ⟨1221222⟩ + ⟨1222222⟩ )],
μ[0, h[3] (⟨1111112⟩ - ⟨1111122⟩ + ⟨1111212⟩ + ⟨1111222⟩ + ⟨1112112⟩ + ⟨1112122⟩ +
⟨1112212⟩ + ⟨1112222⟩ + ⟨1121122⟩ - ⟨1121212⟩ + 13 ⟨1121222⟩ + ⟨1122122⟩ -
1260 3780 2016 5040 15120 10080
⟨1122212⟩ - ⟨1122222⟩ + ⟨1212122⟩ - ⟨1212222⟩ - ⟨1221222⟩ + ⟨1222222⟩ )], True]}
```

8|7

```
Print /@ {
μ0 = Rm[12, 1] Rm[2, 7] Rm[8, 3] Rm[4, 11] Rp[16, 5] Rp[6, 13] Rp[14, 9] Rp[10, 15],
Do[μ0 = μ0 // dm[1, k, 1], {k, 2, 16}]; μ0
}

μ[CWS[0, 0, 0],
h[1] LS[-⟨b⟩, 0, 0] + h[7] LS[-⟨2⟩, 0, 0] + h[3] LS[-⟨8⟩, 0, 0] + h[11] LS[-⟨4⟩, 0, 0] +
h[5] LS[⟨f⟩, 0, 0] + h[13] LS[⟨6⟩, 0, 0] + h[9] LS[⟨d⟩, 0, 0] + h[15] LS[⟨0⟩, 0, 0]]
μ[CWS[0, -CW[11], 0], h[1] LS[0, 0, 0]]
{Null, Null}

$RecursionLimit = 2^16;
μ0[{4}]
μ[CWS[0, -CW[11], 0, -31 CW[1111]/12], h[1] LS[0, 0, 0, 0]]

μ0[{6}]
μ[CWS[0, -CW[11], 0, -31 CW[1111]/12, 0, -1351 CW[111111]/360], h[1] LS[0, 0, 0, 0, 0, 0]]

μ0[{8}]
μ[CWS[0, -CW[11], 0, -31 CW[1111]/12, 0, -1351 CW[111111]/360, 0, -123271 CW[11111111]/20160],
h[1] LS[0, 0, 0, 0, 0, 0, 0, 0]]

Print /@ {β = -1 - 4 x + 8 x^2 - 11 x^3 + 8 x^4 - 4 x^5 + x^6/x^3 /. x → e^x,
Series[β, {x, 0, 8}],
Series[Log[β], {x, 0, 9}]
};
```

$$\begin{aligned} & -e^{-3x} \left(1 - 4 e^x + 8 e^{2x} - 11 e^{3x} + 8 e^{4x} - 4 e^{5x} + e^{6x} \right) \\ & 1 - x^2 - \frac{25 x^4}{12} - \frac{481 x^6}{360} - \frac{1109 x^8}{4032} + O[x]^9 \\ & -x^2 - \frac{31 x^4}{12} - \frac{1351 x^6}{360} - \frac{123271 x^8}{20160} + O[x]^{10} \end{aligned}$$