Pensieve header: Finding the A2 $d=1$ invariant using undetermined coefficients.

Searching for $Q + pxx + \epsilon(ppx + 1 + px + ppxx)$ solutions.

Initialization

(Alt) In[ ]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank"];
Once[<< KnotTheory`; << Rot.m];
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Loading Rot.m from http://drorbn.net/AP/Projects/HigherRank to compute rotation numbers.

In[ ]:=
```
Features[Knot[8, 17]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[ ]=

Features[18,

(Alt) In[ ]:=
```
𝒮 = {x__, p__};
q[s_, i_, j_] := Sum[
    x_{ν,i} (p_{ν,i⁺} - p_{ν,i}) + x_{ν,j} (p_{ν,j⁺} - p_{ν,j}) + (T_ν^s - 1) x_{ν,i} (p_{ν,i⁺} - p_{ν,j⁺}),
    {ν, 3}];
ℒ[X_{i_,j_}[s_]] :=
   T_3^s 𝔼[q[s, i, j] + B^-1 r_0[s, i, j] + ϵ B r_1[s, i, j] + ϵ r_42[s, i, j] + O[ϵ]^2];
(*γ1[φ_,k_]:=φ(3/2-x_{1,k}p_{1,k}-x_{2,k}p_{2,k}-x_{3,k}p_{3,k});*)
ℒ[C_k_[0]] := 𝔼[Sum[x_{ν,k} (p_{ν,k⁺} - p_{ν,k}), {ν, 3}] + O[ϵ]^2];
ℒ[C_k_[φ_]] :=
```

In[ ]:=
```
vs;
```

Out[ ]=

Sequence[p₁,ᵢ, p₂,ᵢ, p₃,ᵢ, x₁,ᵢ, x₂,ᵢ, x₃,ᵢ]

The Various Terms

The pxx Terms (r_0)

(Alt) In[ ]:=
```
κ = 0;
r_0[1, i_, j_] := Evaluate[Sum[
    a_{++κ} p_{3,k3} x_{1,k1} x_{2,k2},
    {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
```

(Alt) Out[ ]=

$a_1 p_{3,i} x_{1,i} x_{2,i} + a_2 p_{3,j} x_{1,i} x_{2,i} + a_5 p_{3,i} x_{1,j} x_{2,i} + a_6 p_{3,j} x_{1,j} x_{2,i} +$

(Alt) In[ ]:=
```
κ = 0;
r_0[-1, i_, j_] := Evaluate[Sum[
    d_{++κ} p_{3,k3} x_{1,k1} x_{2,k2},
    {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
```

(Alt) Out[ ]=

$d_1 p_{3,i} x_{1,i} x_{2,i} + d_2 p_{3,j} x_{1,i} x_{2,i} + d_5 p_{3,i} x_{1,j} x_{2,i} + d_6 p_{3,j} x_{1,j} x_{2,i} +$

The ppx Terms (r_1)

(Alt) In[ ]:=

```
κ = 0;
r1[1, i_, j_] := Evaluate[Sum[
    b++κ x3,k3 p1,k1 p2,k2,
```

(Alt) Out[ ]=

$b_1\, p_{1,i}\, p_{2,i}\, x_{3,i} + b_5\, p_{1,j}\, p_{2,i}\, x_{3,i} + b_3\, p_{1,i}\, p_{2,j}\, x_{3,i} + b_7\, p_{1,j}\, p_{2,j}\, x_{3,i} +$

(Alt) In[ ]:=

```
κ = 0;
r1[-1, i_, j_] := Evaluate[Sum[
    e++κ x3,k3 p1,k1 p2,k2,
```

(Alt) Out[ ]=

$e_1\, p_{1,i}\, p_{2,i}\, x_{3,i} + e_5\, p_{1,j}\, p_{2,i}\, x_{3,i} + e_3\, p_{1,i}\, p_{2,j}\, x_{3,i} + e_7\, p_{1,j}\, p_{2,j}\, x_{3,i} +$

The ppxx Terms (r_{42})

(Alt) In[ ]:=

```
κ = 0;
Short[r42[1, i_, j_] = Evaluate[Plus[
    Sum[
      c++κ xν1,k1 pν1,k2 xν2,k3 pν2,k4,
      {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}, {k4, {i, j}}, {ν1, 2}, {ν2, ν1, 3}
    ],
    Sum[
```

(Alt) Out[ ]//Short=

$c_{92} + c_{81}\, p_{1,i}\, x_{1,i} + \text{<<89>>} + c_{60}\, p_{2,j}\, p_{3,j}\, x_{2,j}\, x_{3,j} + c_{90}\, p_{2,j}\, p_{3,j}\, x_{2,j}\, x_{3,j}$

(Alt) In[ ]:=

```
κ = 0;
Short[r42[-1, i_, j_] = Evaluate[Plus[
    Sum[
      f++κ xν1,k1 pν1,k2 xν2,k3 pν2,k4,
      {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}, {k4, {i, j}}, {ν1, 2}, {ν2, ν1, 3}
    ],
    Sum[
```

(Alt) Out[ ]//Short=

$f_{92} + f_{81}\, p_{1,i}\, x_{1,i} + \text{<<89>>} + f_{60}\, p_{2,j}\, p_{3,j}\, x_{2,j}\, x_{3,j} + f_{90}\, p_{2,j}\, p_{3,j}\, x_{2,j}\, x_{3,j}$

The γ Terms (γ_0, γ_1, γ_{42})

(Alt) In[ ]:=

```
κ = 0;
γ0[1, k_] := Evaluate[g++κ p3,k x1,k x2,k];
γ1[1, k_] := Evaluate[g++κ x3,k p1,k p2,k];
γ42[1, k_] := Evaluate[Plus[
    Sum[g++κ xν,k pν,k, {ν, 3}],
```

(Alt) Out[ ]=

$\{g_1\, p_{3,k}\, x_{1,k}\, x_{2,k},\ g_1\, p_{3,k}\, x_{1,k}\, x_{2,k},\ g_3\, p_{1,k}\, x_{1,k} + g_6\, p_{1,k}^2\, x_{1,k}^2 + g_4\, p_{2,k}\, x_{2,k} +$

(Alt) In[ ]:=

```
κ = 0;
γ0[-1, k_] := Evaluate[h++κ p3,k x1,k x2,k];
γ1[-1, k_] := Evaluate[h++κ x3,k p1,k p2,k];
γ42[-1, k_] := Evaluate[Plus[
    Sum[h... x... p..., {γ, 3}],
```

(Alt) Out[ ]=

$\{h_1\, p_{3,k}\, x_{1,k}\, x_{2,k},\ h_1\, p_{3,k}\, x_{1,k}\, x_{2,k},\ h_3\, p_{1,k}\, x_{1,k} + h_6\, p_{1,k}^2\, x_{1,k}^2 + h_4\, p_{2,k}\, x_{2,k} +$

Reidemeister 3b

(Alt) In[ ]:=

```
Timing[{LeftR3b} =
  Cases[∫ℱ[i, j, k] × ℒ /@ (X_{i,j}[1] X_{i⁺,k}[1] X_{j⁺,k⁺}[1]) d{vs_i, vs_j, vs_k, vs_{i⁺}, vs_{j⁺}, vs_{k⁺}},
```

(Alt) Out[ ]=

$\{4.40625,$
$\{\epsilon\text{Series}[T_1^2\, p_{1,2+i}\, \pi_{1,i} - (-1 + T_1)\, T_1\, p_{1,2+j}\, \pi_{1,i} + (1 - T_1)\, p_{1,2+k}\, \pi_{1,i} + T_1\, p_{1,2+j}\, \pi_{1,j} + (1 - T_1)\, p_{1,2+k}\, \pi_{1,j} + p_{1,2+k}\, \pi_{1,k} +$
$T_2^2\, p_{2,2+i}\, \pi_{2,i} - (-1 + T_2)\, T_2\, p_{2,2+j}\, \pi_{2,i} + (1 - T_2)\, p_{2,2+k}\, \pi_{2,i} + \langle\!\langle 36 \rangle\!\rangle +$
$\frac{a_7\, T_3\, p_{3,2+i}\, \pi_{1,k}\, \pi_{2,k}}{B} + \frac{a_7\, T_3\, p_{3,2+j}\, \pi_{1,k}\, \pi_{2,k}}{B} - \frac{2\,(-a_7 - a_8 + a_7\, T_3)\, p_{3,2+k}\, \pi_{1,k}\, \pi_{2,k}}{B} + T_3^2\, p_{3,2+i}\, \pi_{3,i} -$
$(-1 + T_3)\, T_3\, p_{3,2+j}\, \pi_{3,i} + (1 - T_3)\, p_{3,2+k}\, \pi_{3,i} + T_3\, p_{3,2+j}\, \pi_{3,j} + (1 - T_3)\, p_{3,2+k}\, \pi_{3,j} + p_{3,2+k}\, \pi_{3,k},$
$3\, (a_1\, b_1 + a_2\, b_2 + a_3\, b_3 + a_4\, b_4 + a_5\, b_5 + a_6\, b_6 + a_7\, b_7 + a_8\, b_8 + 2\, c_1 + c_2 + c_3 + 2\, c_4 + c_5 + \langle\!\langle 13 \rangle\!\rangle + c_{65} + 2\, c_{76} + c_{77} +$
$c_{78} + 2\, c_{79} + c_{80} + c_{81} + c_{82} + c_{83} + c_{90} + c_{91} + c_{92} + c_{93}) + \langle\!\langle 495 \rangle\!\rangle + \langle\!\langle 1 \rangle\!\rangle]\}\}$

Full expression not available (original memory size: 4.1 MB)

(Alt) In[ ]:=

```
Timing[{RightR3b} =
  Cases[∫ℱ[i, j, k] × ℒ /@ (X_{j,k}[1] X_{i,k⁺}[1] X_{i⁺,j⁺}[1]) d{vs_i, vs_j, vs_k, vs_{i⁺}, vs_{j⁺}, vs_{k⁺}},
   𝔼[ℰ_] ⧴ ℰ, ∞];]
```

(Alt) Out[ ]=

{4.98438, Null}

(Alt) In[ ]:=

```
Short[eqn = CF[LeftR3b[[1]] - RightR3b[[1]]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r0[1, i, j], a_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :→ (c == 0), 3]
{sol} = Solve[eqns, vars]
```

(Alt) Out[ ]//Short=

$$\frac{\left(\ll 27\gg + a_7 T_1^2 T_2^2 T_3\right) p_{3,\ll 1\gg}\, \pi_{\ll 1\gg}\, \pi_{2,i}}{B} - \frac{(\ll 1\gg) \ll 2\gg \pi_{\ll 1\gg}}{B} - \frac{\ll 1\gg}{B} + \ll 30\gg + \frac{a_7 (-1+T_3) \ll 3\gg \pi_{2,k}}{B}$$

(Alt) Out[ ]=

$\{p_{3,2+i}, p_{3,2+j}, p_{3,2+k}, \pi_{1,i}, \pi_{1,j}, \pi_{1,k}, \pi_{2,i}, \pi_{2,j}, \pi_{2,k}\}$

(Alt) Out[ ]=

$\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$

(Alt) Out[ ]//Short=

$$\left\{-\frac{a_3 T_1 T_2 T_3}{B} + \frac{a_3 T_2 T_3^2}{B} == 0, \frac{a_3 T_1 T_3}{B} - \frac{a_3 T_3^2}{B} == 0, -\frac{a_5 T_1 T_2 T_3}{B} + \frac{a_5 T_1 T_3^2}{B} == 0, \ll 20\gg, \frac{a_7 T_2}{B} + \frac{a_8 T_2}{B} - \frac{a_7 T_2^2}{B} - \frac{a_8 T_2^2}{B} - \frac{a_7 T_2 T_3}{B} + \frac{a_7 T_2^2 T_3}{B} == 0, -\frac{a_7}{B} - \frac{a_8}{B} + \frac{a_7 T_2}{B} + \frac{a_8 T_2}{B} + \frac{a_7 T_3}{B} - \frac{a_7 T_2 T_3}{B} == 0\right\}$$

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]=

$$\left\{\left\{a_2 \to -\frac{a_1 (T_1 T_2 - T_3 + T_1 T_3 + T_2 T_3 - 2 T_1 T_2 T_3)}{T_1 T_2 - T_3}, a_3 \to 0, a_4 \to -\frac{a_1 (-T_3 + T_1 T_3)}{T_1 T_2 - T_3}, a_5 \to 0, a_6 \to -\frac{a_1 (-T_3 + T_2 T_3)}{T_1 T_2 - T_3}, a_7 \to 0, a_8 \to 0\right\}\right\}$$

(Alt) In[ ]:=

```
sol /. (v_ → val_) :→ (v = CF[val]);
r0[1, i, j]
```

(Alt) Out[ ]=

$$a_1 p_{3,i} x_{1,i} x_{2,i} + \frac{a_1 (-T_1 T_2 + T_3 - T_1 T_3 - T_2 T_3 + 2 T_1 T_2 T_3) p_{3,j} x_{1,i} x_{2,i}}{T_1 T_2 - T_3} - \frac{a_1 (-1 + T_2) T_3 p_{3,j} x_{1,j} x_{2,i}}{T_1 T_2 - T_3} - \frac{a_1 (-1 + T_1) T_3 p_{3,j} x_{1,i} x_{2,j}}{T_1 T_2 - T_3}$$

(Alt) In[ ]:=

```
Short[eqn = CF[Coefficient[
    LeftR3b[[2]] - RightR3b[[2]] /. v : (π | p)__ :> μ v,
    μ^3
  ]], 5]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r_1[1, i, j], b_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0), 3]
{sol} = Solve[eqns, vars]
```

(Alt) Out[ ]//Short=

$$B T_2 (b_5 T_1 T_2 - b_1 T_3 - b_5 T_3 + b_1 T_1 T_3) p_{1,2+j} p_{2,2+i} \pi_{3,i} - B T_2 (b_5 T_1 T_2 - b_1 T_3 - b_5 T_3 + b_1 T_1 T_3) p_{1,2+k} p_{2,2+i} \pi_{3,i} + \ll 31 \gg + B (-1 + T_1) T_1 (-b_2 - b_4 + b_2 T_2) p_{1,2+i} p_{2,2+k} \pi_{3,k} - B (-1 + T_1) T_1 (-b_2 - b_4 + b_2 T_2) p_{1,2+j} p_{2,2+k} \pi_{3,k}$$

(Alt) Out[ ]=

$$\{p_{1,2+i}, p_{1,2+j}, p_{1,2+k}, p_{2,2+i}, p_{2,2+j}, p_{2,2+k}, \pi_{3,i}, \pi_{3,j}, \pi_{3,k}\}$$

(Alt) Out[ ]=

$$\{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8\}$$

(Alt) Out[ ]//Short=

$$\{-B b_2 T_1 T_2 T_3 + B b_2 T_1^2 T_2^2 T_3 == 0, \ll 23 \gg, -B b_6 T_1 - B b_8 T_1 - B b_4 T_2 - B b_8 T_2 + B b_4 T_1 T_2 + B b_6 T_1 T_2 + B b_8 T_1 T_2 + B b_2 T_3 + B b_4 T_3 + B b_6 T_3 + B b_8 T_3 - B b_2 T_1 T_3 - B b_4 T_1 T_3 - B b_2 T_2 T_3 - B b_6 T_2 T_3 + B b_2 T_1 T_2 T_3 == 0\}$$

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]=

$$\left\{\left\{b_2 \to 0, b_3 \to -\frac{b_1 (-T_3 + T_2 T_3)}{T_1 T_2 - T_3}, b_4 \to 0, b_5 \to -\frac{b_1 (-T_3 + T_1 T_3)}{T_1 T_2 - T_3}, b_6 \to 0, b_7 \to -\frac{b_1 (T_1 T_2 + T_3 - T_1 T_3 - T_2 T_3)}{T_1 T_2 - T_3}, b_8 \to 0\right\}\right\}$$

(Alt) In[ ]:=

```
sol /. (v_ → val_) :> (v = CF[val]);
r_1[1, i, j]
```

(Alt) Out[ ]=

$$b_1 p_{1,i} p_{2,i} x_{3,i} - \frac{b_1 (-1 + T_1) T_3 p_{1,j} p_{2,i} x_{3,i}}{T_1 T_2 - T_3} - \frac{b_1 (-1 + T_2) T_3 p_{1,i} p_{2,j} x_{3,i}}{T_1 T_2 - T_3} - \frac{b_1 (T_1 T_2 + T_3 - T_1 T_3 - T_2 T_3) p_{1,j} p_{2,j} x_{3,i}}{T_1 T_2 - T_3}$$

(Alt) In[ ]:=

```
Short[eqn = CF[LeftR3b[[2]] - RightR3b[[2]]], 5]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r42[1, i, j], c_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :→ (c == 0), 3]
Short[{sol} = Solve[eqns, vars]]
```

(Alt) Out[ ]//Short=

$$-\left(\left(2 c_{11}+2 c_{41}+c_{42}+c_{43}+2 c_{56}+c_{57}+c_{58}+2 c_{71}+c_{87}\right)\left(-1+T_1\right) T_1^2 p_{1,2+j} \pi_{1,i}\right)-\frac{1}{T_1 T_2-T_3}$$
$$\left(-1+T_1\right)\left(a_1 b_1 T_1 T_2+4 c_1 T_1 T_2+c_2 T_1 T_2+c_3 T_1 T_2+\ll 137\gg+c_{58} T_1^2 T_3+2 c_{71} T_1^2 T_3+c_{87} T_1^2 T_3\right)$$
$$p_{1\ll 1\gg}\ll 1\gg \pi_{1,i}+2 c_1\left(-1+T_1\right) T_1^3 p_{1,2+i} p_{1,2+j} \pi_{1,i}^2+$$
$$\ll 477\gg+\left(-1+T_2\right) T_2\left(-c_{55}-c_{60}+c_{55} T_3\right) p_{2,2+i} p_{3,2+k} \pi_{2,k} \pi_{3,k}-$$
$$\left(-1+T_2\right) T_2\left(-c_{55}-c_{60}+c_{55} T_3\right) p_{2,2+j} p_{3,2+k} \pi_{2,k} \pi_{3,k}$$

(Alt) Out[ ]=

$$\{p_{1,2+i}, p_{1,2+j}, p_{1,2+k}, p_{2,2+i}, p_{2,2+j}, p_{2,2+k}, p_{3,2+i}, p_{3,2+j}, p_{3,2+k}, \pi_{1,i}, \pi_{1,j}, \pi_{1,k}, \pi_{2,i}, \pi_{2,j}, \pi_{2,k}, \pi_{3,i}, \pi_{3,j}, \pi_{3,k}\}$$

(Alt) Out[ ]=

$$\{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12}, c_{13}, c_{14}, c_{15}, c_{16}, c_{17}, c_{18}, c_{19}, c_{20}, c_{21}, c_{22}, c_{23}, c_{24}, c_{25}, c_{26}, c_{27}, c_{28}, c_{29}, c_{30}, c_{31}, c_{32}, c_{33}, c_{34}, c_{35}, c_{36}, c_{37}, c_{38}, c_{39}, c_{40}, c_{41}, c_{42}, c_{43}, c_{44}, c_{45}, c_{46}, c_{47}, c_{48}, c_{49}, c_{50}, c_{51}, c_{52}, c_{53}, c_{54}, c_{55}, c_{56}, c_{57}, c_{58}, c_{59}, c_{60}, c_{61}, c_{62}, c_{63}, c_{64}, c_{65}, c_{66}, c_{67}, c_{68}, c_{69}, c_{70}, c_{71}, c_{72}, c_{73}, c_{74}, c_{75}, c_{76}, c_{77}, c_{78}, c_{79}, c_{80}, c_{81}, c_{82}, c_{83}, c_{84}, c_{85}, c_{86}, c_{87}, c_{88}, c_{89}, c_{90}, c_{91}, c_{92}, c_{93}\}$$

(Alt) Out[ ]//Short=

$$\{-c_{11} T_1^4-c_{41} T_1^4+c_{11} T_1^5+c_{41} T_1^5==0,\ c_{11} T_1^3+c_{41} T_1^3-c_{11} T_1^4-c_{41} T_1^4==0,\ \ll 315\gg,$$
$$c_{13} T_3+c_{15} T_3+c_{73} T_3+c_{75} T_3+c_{89} T_3-c_{13} T_3^2-c_{15} T_3^2-c_{73} T_3^2-c_{75} T_3^2-c_{89} T_3^2==0\}$$

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]//Short=

$$\left\{\left\{c_1 \to 0,\ \ll 60\gg,\ c_{92} \to -\frac{\left(-2 a_1 b_1+a_1 \ll 1\gg T_1+a_1 b_1 T_2\right) \ll 1\gg}{\left(T_1 \ll 1\gg-\ll 1\gg\right)^2}-\frac{\ll 1\gg \ll 1\gg \ll 1\gg}{\ll 1\gg}-\frac{c_{86}\left(\ll 1\gg\right)}{\ll 1\gg}\right\}\right\}$$

(Alt) In[ ]:=

```
sol /. (v_ → val_) :→ (v = CF[val]);
```

(Alt) In[]:=

```
Short[CF[r42[1, i, j]], 20]
```

(Alt) Out[]//Short=

$$\begin{aligned}
&c_{93} + c_{81} p_{1,i} x_{1,i} + c_{84} p_{1,j} x_{1,i} + (c_6 + c_{21})\, p_{1,i} p_{1,j} x_{1,i}^2 + \\
&\frac{1}{2} (-1 + T_1)\, (2 c_6 + 2 c_{21} + c_{16} T_1 + c_{31} T_1 + c_{46} T_1 + c_{61} T_1)\, p_{1,j}^2 x_{1,i}^2 - \\
&\frac{(c_{81} + c_{84})\, p_{1,j} x_{1,j}}{T_1} + (c_{16} + c_{31} + c_{46} + c_{61})\, p_{1,i} p_{1,j} x_{1,i} x_{1,j} + \\
&\frac{1}{2} (-2 c_6 - c_{16} - 2 c_{21} - c_{31} - c_{46} - c_{61} - c_{16} T_1 - c_{31} T_1 - c_{46} T_1 - c_{61} T_1)\, p_{1,j}^2 x_{1,i} x_{1,j} + \\
&c_{82} p_{2,i} x_{2,i} + \ll 34 \gg + \frac{(\ll 22 \gg + a_1 b_1 \ll 1 \gg T_3^3)\, p_{\ll 1 \gg} \ll 1 \gg \ll 1 \gg \ll 1 \gg x_{3,i}}{(T_1 T_2 - T_3)^2} + \\
&\frac{(-c_{10} T_1^2 T_2^2 + c_{25} T_1^2 T_2^2 + c_{10} T_1^2 T_2^3 + \ll 15 \gg + a_1 b_1 T_3^3 - c_{25} T_3^3 - a_1 b_1 T_2 T_3^3)\, p_{2,j} p_{3,i} x_{2,j} x_{3,i}}{(-1 + T_2)\, (T_1 T_2 - T_3)^2\, (-1 + T_3)} - \\
&\frac{a_1 b_1 (-1 + T_1)\, T_3^2 p_{2,i} p_{3,j} x_{2,j} x_{3,i}}{(T_1 T_2 - T_3)^2} - \frac{1}{(-1 + T_2)\, (T_1 T_2 - T_3)^2} \\
&\quad (-c_{25} T_1^2 T_2^2 - a_1 b_1 T_1 T_2 T_3 + 2 c_{25} T_1 T_2 T_3 + a_1 b_1 T_1 T_2^2 T_3 + c_{25} T_1^2 T_2^2 T_3 - c_{25} T_3^2 + a_1 b_1 T_1 T_3^2 + a_1 b_1 T_2 T_3^2 - \\
&\qquad a_1 b_1 T_1 T_2 T_3^2 - 2 c_{25} T_1 T_2 T_3^2 - a_1 b_1 T_2^2 T_3^2 - a_1 b_1 T_3^3 + c_{25} T_3^3 + a_1 b_1 T_2 T_3^3)\, p_{2,j} p_{3,j} x_{2,j} x_{3,i} - \\
&\frac{1}{(T_1 T_2 - T_3)^2 T_3} (c_{83} T_1^2 T_2^2 + c_{86} T_1^2 T_2^2 - 2 c_{83} T_1 T_2 T_3 - 2 c_{86} T_1 T_2 T_3 - 2 a_1 b_1 T_3^2 + c_{83} T_3^2 + \\
&\quad c_{86} T_3^2 + a_1 b_1 T_1 T_3^2 + a_1 b_1 T_2 T_3^2)\, p_{3,j} x_{3,j} - \frac{1}{(-1 + T_1)\, (T_1 T_2 - T_3)^2\, (-1 + T_3)} \\
&\quad (-c_8 T_1^2 T_2^2 + c_{23} T_1^2 T_2^2 + c_8 T_1^3 T_2^2 + 2 c_8 T_1 T_2 T_3 - 2 c_{23} T_1 T_2 T_3 - 2 c_8 T_1^2 T_2 T_3 - c_{23} T_1^2 T_2^2 T_3 - \\
&\qquad a_1 b_1 T_3^2 - c_8 T_3^2 + c_{23} T_3^2 + a_1 b_1 T_1 T_3^2 + c_8 T_1 T_3^2 + 2 c_{23} T_1 T_2 T_3^2 + a_1 b_1 T_3^3 - c_{23} T_3^3 - a_1 b_1 T_1 T_3^3) \\
&\quad p_{1,i} p_{3,j} x_{1,i} x_{3,j} - \frac{c_8 (-1 + T_1)\, p_{1,j} p_{3,j} x_{1,i} x_{3,j}}{-1 + T_3} - \frac{1}{(-1 + T_2)\, (T_1 T_2 - T_3)^2\, (-1 + T_3)} \\
&\quad (-c_{10} T_1^2 T_2^2 + c_{25} T_1^2 T_2^2 + c_{10} T_1^2 T_2^3 + 2 c_{10} T_1 T_2 T_3 - 2 c_{25} T_1 T_2 T_3 - 2 c_{10} T_1 T_2^2 T_3 - c_{25} T_1^2 T_2^2 T_3 - \\
&\qquad a_1 b_1 T_3^2 - c_{10} T_3^2 + c_{25} T_3^2 + a_1 b_1 T_2 T_3^2 + c_{10} T_2 T_3^2 + 2 c_{25} T_1 T_2 T_3^2 + a_1 b_1 T_3^3 - c_{25} T_3^3 - a_1 b_1 T_2 T_3^3) \\
&\quad p_{2,i} p_{3,j} x_{2,i} x_{3,j} - \frac{c_{10} (-1 + T_2)\, p_{2,j} p_{3,j} x_{2,i} x_{3,j}}{-1 + T_3}
\end{aligned}$$

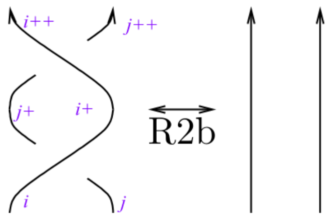
(Alt) In[]:=

```
CF[LeftR3b - RightR3b]
```

(Alt) Out[]=

```
∈Series[0, 0]
```

Reidemeister 2b



Territory: $(x \mid p)_{(i|j)^+}$, $x_{i|j}$.

(Alt) In[#]:=

```
Timing[Short[LeftR2b = (∫𝓕[i, j] × ℒ /@ (X_{i,j}[1] X_{i+,j+}[-1]) ⅆ{vs_i, vs_j, vs_{i+}, vs_{j+}})⟦1⟧
]]
```

(Alt) Out[#]=

$\{0.9375, \in\text{Series}[p_{1,2+i}\,\pi_{1,i} + p_{1,2+j}\,\pi_{1,j} + p_{2,2+i}\,\pi_{2,i} + \ll 13\gg + p_{3,2+i}\,\pi_{3,i} + p_{3,2+j}\,\pi_{3,j},\ \frac{\ll 1\gg}{\ll 1\gg} + \ll 103\gg + \frac{\ll 1\gg}{\ll 1\gg}]\}$

(Alt) In[#]:=

```
RightR2b = ∈Series[p_{1,2+i} π_{1,i} + p_{1,2+j} π_{1,j} + p_{2,2+i} π_{2,i} + p_{2,2+j} π_{2,j} + p_{3,2+i} π_{3,i} + p_{3,2+j} π_{3,j}, 0]
```

(Alt) Out[#]=

$\in\text{Series}[p_{1,2+i}\,\pi_{1,i} + p_{1,2+j}\,\pi_{1,j} + p_{2,2+i}\,\pi_{2,i} + p_{2,2+j}\,\pi_{2,j} + p_{3,2+i}\,\pi_{3,i} + p_{3,2+j}\,\pi_{3,j},\ 0]$

(Alt) In[#]:=

```
Short[eqn = CF[LeftR2b⟦1⟧ - RightR2b⟦1⟧]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r_0[-1, i, j], d_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :→ (c == 0), 3]
{sol} = Solve[eqns, vars]
```

(Alt) Out[#]//Short=

$$\frac{(d_7 + d_3 T_1 - d_7 T_1 + \ll 8\gg + d_7 T_1 T_2 + a_1 T_3)\ \ll 2\gg\ \pi_{\ll 1\gg}}{B\,T_3} + \ll 10\gg + \frac{\ll 1\gg}{B\,T_3}$$

(Alt) Out[#]=

$\{p_{3,2+i},\ p_{3,2+j},\ \pi_{1,i},\ \pi_{1,j},\ \pi_{2,i},\ \pi_{2,j}\}$

(Alt) Out[#]=

$\{d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8\}$

(Alt) Out[#]//Short=

$$\Big\{\frac{a_1}{B} + \frac{d_7}{B\,T_3} + \frac{d_3 T_1}{B\,T_3} - \frac{d_7 T_1}{B\,T_3} + \frac{d_5 T_2}{B\,T_3} - \frac{d_7 T_2}{B\,T_3} + \frac{d_1 T_1 T_2}{B\,T_3} - \frac{d_3 T_1 T_2}{B\,T_3} - \frac{d_5 T_1 T_2}{B\,T_3} + \frac{d_7 T_1 T_2}{B\,T_3} == 0,$$
$$\frac{d_7}{B\,T_3} + \frac{d_3 T_1}{B\,T_3} - \frac{d_7 T_1}{B\,T_3} == 0,\ \ll 5\gg,\ \frac{d_7}{B} + \frac{d_8}{B} - \frac{d_7}{B\,T_3} == 0\Big\}$$

(Alt) Out[#]=

$$\Big\{\Big\{d_1 \to -\frac{a_1 T_3}{T_1 T_2},\ d_2 \to -\frac{-2 a_1 T_3 + a_1 T_1 T_3 + a_1 T_2 T_3 - a_1 T_1 T_2 T_3 + a_1 T_3^2}{T_1 T_2 (T_1 T_2 - T_3)},$$
$$d_3 \to 0,\ d_4 \to -\frac{a_1 T_3 - a_1 T_1 T_3}{T_1 (T_1 T_2 - T_3)},\ d_5 \to 0,\ d_6 \to -\frac{a_1 T_3 - a_1 T_2 T_3}{T_2 (T_1 T_2 - T_3)},\ d_7 \to 0,\ d_8 \to 0\Big\}\Big\}$$

(Alt) In[]:=

```
sol /. (v_ → val_) :→ (v = CF[val]);
r0[-1, i, j]
```

(Alt) Out[]=

$$-\frac{a_1 T_3 p_{3,i} x_{1,i} x_{2,i}}{T_1 T_2}+\frac{a_1 (2-T_1-T_2+T_1 T_2-T_3) T_3 p_{3,j} x_{1,i} x_{2,i}}{T_1 T_2 (T_1 T_2-T_3)}+\frac{a_1 (-1+T_2) T_3 p_{3,j} x_{1,j} x_{2,i}}{T_2 (T_1 T_2-T_3)}+\frac{a_1 (-1+T_1) T_3 p_{3,j} x_{1,i} x_{2,j}}{T_1 (T_1 T_2-T_3)}$$

(Alt) In[]:=

```
Short[eqn = CF[LeftR2b[[2]] - RightR2b[[2]]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r1[-1, i, j] + r42[-1, i, j], e_ | f_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :→ (c == 0), 3]
Short[{sol} = Solve[eqns, vars]]
```

(Alt) Out[]//Short=

$$\ll 125\gg+\frac{(\ll 1\gg)\ \ll 4\gg}{T_1 \ll 1\gg\ \ll 1\gg\ \ll 1\gg\ T_3}$$

(Alt) Out[]=

$\{p_{1,2+i}, p_{1,2+j}, p_{2,2+i}, p_{2,2+j}, p_{3,2+i}, p_{3,2+j}, \pi_{1,i}, \pi_{1,j}, \pi_{2,i}, \pi_{2,j}, \pi_{3,i}, \pi_{3,j}\}$

(Alt) Out[]=

$\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_{12}, f_{13}, f_{14}, f_{15}, f_{16}, f_{17}, f_{18}, f_{19}, f_{20}, f_{21}, f_{22}, f_{23}, f_{24}, f_{25}, f_{26}, f_{27}, f_{28}, f_{29}, f_{30}, f_{31}, f_{32}, f_{33}, f_{34}, f_{35}, f_{36}, f_{37}, f_{38}, f_{39}, f_{40}, f_{41}, f_{42}, f_{43}, f_{44}, f_{45}, f_{46}, f_{47}, f_{48}, f_{49}, f_{50}, f_{51}, f_{52}, f_{53}, f_{54}, f_{55}, f_{56}, f_{57}, f_{58}, f_{59}, f_{60}, f_{61}, f_{62}, f_{63}, f_{64}, f_{65}, f_{66}, f_{67}, f_{68}, f_{69}, f_{70}, f_{71}, f_{72}, f_{73}, f_{74}, f_{75}, f_{76}, f_{77}, f_{78}, f_{79}, f_{80}, f_{81}, f_{82}, f_{83}, f_{84}, f_{85}, f_{86}, f_{87}, f_{88}, f_{89}, f_{90}, f_{91}, f_{92}, f_{93}\}$

(Alt) Out[]//Short=

$$\left\{f_1-f_{11}-f_{41}+f_{51}+\frac{f_{51}}{T_1^2}+\frac{f_{11}}{T_1}+\frac{f_{41}}{T_1}-\frac{2 f_{51}}{T_1}==0,\ \frac{2 f_{51}}{T_1^2}+\frac{f_{11}}{T_1}+\frac{f_{41}}{T_1}-\frac{2 f_{51}}{T_1}==0,\ \ll 83\gg,\right.$$
$$\frac{2 c_{83} T_1 T_2}{(1-T_1)(1-\ll 1\gg)\ \ll 1\gg\ (\ll 1\gg\ \ll 1\gg\ (\ll 1\gg-\ll 1\gg))^2}+\frac{\ll 1\gg}{\ll 1\gg}+\ll 682\gg==0,$$
$$\left.\frac{2 c_{83} T_1 T_2}{(T_1 T_2-T_3)^2}+\ll 238\gg+\frac{a_1 e_2 T_3^3}{T_1 T_2 (T_1 T_2-T_3)^2}==0\right\}$$

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[]//Short=

$$\left\{\left\{e_1\to-\frac{b_1 T_1 T_2}{T_3},\ e_2\to 0,\ \ll 84\gg,\ f_{93}\to -c_{93}\right\}\right\}$$

(Alt) In[]:=

```
sol /. (v_ → val_) :→ (v = CF[val]);
```

(Alt) In[ ]:=

```
r1[-1, i, j]
Short[CF[r42[-1, i, j]], 5]
```

(Alt) Out[ ]=

$$-\frac{b_1 T_1 T_2 p_{1,i} p_{2,i} x_{3,i}}{T_3}+\frac{b_1 (-1+T_1) T_1 T_2^2 p_{1,j} p_{2,i} x_{3,i}}{(T_1 T_2-T_3) T_3}+\frac{b_1 T_1^2 (-1+T_2) T_2 p_{1,i} p_{2,j} x_{3,i}}{(T_1 T_2-T_3) T_3}-\frac{b_1 T_1 T_2 (-T_1-T_2+T_1 T_2+T_3) p_{1,j} p_{2,j} x_{3,i}}{(T_1 T_2-T_3) T_3}$$

(Alt) Out[ ]//Short=

$$-c_{93}-c_{81} p_{1,i} x_{1,i}+\ll 53\gg+\frac{\left(c_{25} T_1^2 T_2^2-2 c_{25} T_1 T_2 T_3-a_1 b_1 T_3^2+c_{25} T_3^2+a_1 b_1 T_2 T_3^2\right) p_{2,j} p_{3,j} x_{2,i} x_{3,j}}{T_2 (T_1 T_2-T_3)^2}$$

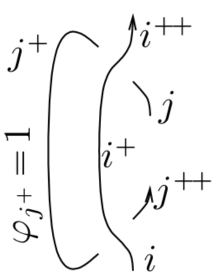
(Alt) In[ ]:=

```
CF[LeftR2b - RightR2b]
```

(Alt) Out[ ]=

```
ϵSeries[0, 0]
```

Reidemeister 2c



(Alt) In[ ]:=

```
Timing[Short[{LeftR2c} = Cases[
    ∫ℱ[i, j] × ℒ /@ (X_{i+1,j}[1] X_{i,j+2}[-1] C_{j+1}[1]) 𝕕{vs_i, vs_j, vs_{i+}, vs_{j+}, vs_{j+2}}, 𝔼[ℰ_] ⧴ ℰ]
]]
```

(Alt) Out[ ]=

{0.359375, {ϵSeries[
$p_{1,2+i}\,\pi_{1,i}+p_{1,3+j}\,\pi_{1,j}+p_{2,2+i}\,\pi_{2,i}+\ll 7\gg+p_{3,2+i}\,\pi_{3,i}+p_{3,3+j}\,\pi_{3,j},\ g_1 g_2+\ll 50\gg+\ll 1\gg$]}}

(Alt) In[]:=

```
Timing[Short[{RightR2c} =
  Cases[∫𝓕[i, j] × ℒ /@ (C_i[0] C_{i+1}[0] C_j[0] C_{j+1}[1] C_{j+2}[0]) ⅆ{vs_i, vs_j, vs_{i⁺}, vs_{j⁺}, vs_{j+2}},
   𝔼[ℰ_] :→ ℰ]
 ]]
```

(Alt) Out[]=

$\{0., \{\epsilon\text{Series}[p_{1,2+i}\,\pi_{1,i} + p_{1,3+j}\,\pi_{1,j} + p_{2,2+i}\,\pi_{2,i} + p_{2,\ll 1\gg}\,\pi_{\ll 1\gg} + \frac{\ll 1\gg}{B} + p_{3,2+i}\,\pi_{3,i} + p_{3,3+j}\,\pi_{3,j},\ \ll 1\gg]\}\}$

(Alt) In[]:=

```
Short[eqn = CF[LeftR2c〚1〛 - RightR2c〚1〛]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[γ_0[1, k], g_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :→ (c == 0), 3]
{sol} = Solve[eqns, vars]
```

(Alt) Out[]//Short=

$\frac{g_1\,(-1+T_1)\,(-1+T_2)\,p_{3\,\ll 1\gg\,\ll 1\gg}\,\pi_{1,i}\,\pi_{2,i}}{B\,T_1\,T_2} - \frac{g_1\,\ll 3\gg\,\pi_{\ll 1\gg}}{B\,T_2} - \frac{g_1\,\ll 3\gg\,\pi_{2,j}}{B\,T_1}$

(Alt) Out[]=

$\{p_{3,3+j},\ \pi_{1,i},\ \pi_{1,j},\ \pi_{2,i},\ \pi_{2,j}\}$

(Alt) Out[]=

$\{g_1\}$

(Alt) Out[]//Short=

$\left\{\frac{g_1}{B} - \frac{g_1}{B\,T_1} - \frac{g_1}{B\,T_2} + \frac{g_1}{B\,T_1\,T_2} == 0,\ -\frac{g_1}{B} + \frac{g_1}{B\,T_1} == 0,\ -\frac{g_1}{B} + \frac{g_1}{B\,T_2} == 0\right\}$

(Alt) Out[]=

$\{\{g_1 \to 0\}\}$

(Alt) In[]:=

```
sol /. (v_ → val_) :→ (v = CF[val]);
γ_0[1, k]
```

(Alt) Out[]=

0

(Alt) In[ ]:=

```
Short[eqn = CF[LeftR2c[[2]] - RightR2c[[2]]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[γ1[1, k] + γ42[1, k], g_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0), 3]
Short[{sol} = Solve[eqns, vars]]
```

(Alt) Out[ ]//Short=

$$\frac{(c_{16} + c_{31} + c_{46} + c_{61} - g_3 - 4\, g_6 - g_7 - g_8) \ll 2 \gg \pi_{\ll 1 \gg}}{T_1} + \ll 28 \gg$$

(Alt) Out[ ]=

$\{p_{1,3+j}, p_{2,3+j}, p_{3,3+j}, \pi_{1,i}, \pi_{1,j}, \pi_{2,i}, \pi_{2,j}, \pi_{3,i}, \pi_{3,j}\}$

(Alt) Out[ ]=

$\{g_2, g_3, g_4, g_5, g_6, g_7, g_8, g_9, g_{10}\}$

(Alt) Out[ ]//Short=

$$\Big\{g_6 + \frac{g_6}{T_1^2} - \frac{2\, g_6}{T_1} == 0,\ -2\, g_6 + \frac{2\, g_6}{T_1} == 0,\ \ll 13 \gg,$$
$$c_{19} + c_{34} + c_{49} + c_{64} - g_4 - g_7 - 4\, g_9 - g_{10} - \frac{c_{19}}{T_2} - \frac{c_{34}}{T_2} - \frac{c_{49}}{T_2} - \frac{c_{64}}{T_2} + \frac{g_4}{T_2} + \frac{g_7}{T_2} + \frac{4\, g_9}{T_2} + \frac{g_{10}}{T_2} == 0,$$
$$-g_5 - g_8 - g_{10} + \frac{g_5}{T_3} + \frac{g_8}{T_3} + \frac{g_{10}}{T_3} == 0\Big\}$$

(Alt) Out[ ]//Short=

$$\{\{g_2 \to 0,\ g_3 \to c_{16} + c_{31} + c_{46} + c_{61},$$
$$g_4 \to c_{19} + c_{34} + c_{49} + c_{64},\ g_5 \to 0,\ g_6 \to 0,\ g_7 \to 0,\ g_8 \to 0,\ g_9 \to 0,\ g_{10} \to 0\}\}$$

(Alt) In[ ]:=

```
sol /. (v_ → val_) :> (v = CF[val]);
```

(Alt) In[ ]:=

```
γ1[1, k]
Short[CF[γ42[1, k]], 5]
```

(Alt) Out[ ]=

0

(Alt) Out[ ]//Short=

$(c_{16} + c_{31} + c_{46} + c_{61})\, p_{1,k}\, x_{1,k} + (c_{19} + c_{34} + c_{49} + c_{64})\, p_{2,k}\, x_{2,k}$

(Alt) In[ ]:=

```
CF[LeftR2c - RightR2c]
```

(Alt) Out[ ]=

∈Series[0, 0]

$C_k[1]$ and $C_k[-1]$ are inverses

(Alt) In[ ]:=

```
Timing[Short[{LeftCC} = Cases[{∫ℱ[k] × ℒ /@ (C_k[1] C_{k+1}[-1]) ⅆ{vs_k, vs_{k⁺}}}, 𝔼[ℰ_] ⧴ ℰ]
]]
```

(Alt) Out[ ]=

$$\left\{0.,\ \left\{\in\text{Series}\left[p_{1,2+k}\,\pi_{1,k}+p_{2,2+k}\,\pi_{2,k}+\frac{h_1\,p_{\ll 1\gg}\ \ll 1\gg\ \pi_{2,k}}{B}+p_{3,2+k}\,\pi_{3,k},\ c_{16}+\ll 24\gg+(h_1\,h_2+h_{10})\ \ll 3\gg\ \pi_{\ll 1\gg}\right]\right\}\right\}$$

(Alt) In[ ]:=

```
Timing[Short[{RightCC} = Cases[{∫ℱ[k] × ℒ /@ (C_k[0] C_{k+1}[0]) ⅆ{vs_k, vs_{k⁺}}}, 𝔼[ℰ_] ⧴ ℰ]
]]
```

(Alt) Out[ ]=

$$\{0.,\ \{\in\text{Series}[p_{1,2+k}\,\pi_{1,k}+p_{2,2+k}\,\pi_{2,k}+p_{3,2+k}\,\pi_{3,k},\ 0]\}\}$$

(Alt) In[ ]:=

```
Short[eqn = CF[LeftCC⟦1⟧ - RightCC⟦1⟧]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[γ_0[-1, k], h_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) ⧴ (c == 0), 3]
{sol} = Solve[eqns, vars]
```

(Alt) Out[ ]//Short=

$$\frac{h_1\,p_{3,2+k}\,\pi_{1,k}\,\pi_{2,k}}{B}$$

(Alt) Out[ ]=

$$\{p_{3,2+k},\ \pi_{1,k},\ \pi_{2,k}\}$$

(Alt) Out[ ]=

$$\{h_1\}$$

(Alt) Out[ ]//Short=

$$\left\{\frac{h_1}{B}==0\right\}$$

(Alt) Out[ ]=

$$\{\{h_1\to 0\}\}$$

(Alt) In[ ]:=

```
sol /. (v_ → val_) ⧴ (v = CF[val]);
γ_0[-1, k]
```

(Alt) Out[ ]=

0

(Alt) In[ ]:=
```
Short[eqn = CF[LeftCC⟦2⟧ - RightCC⟦2⟧]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[γ1[-1, k] + γ42[-1, k], h_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) ⧴ (c == 0), 3]
Short[{sol} = Solve[eqns, vars]]
```

(Alt) Out[ ]//Short=

$c_{16} + c_{19} + c_{31} + \ll 20 \gg + h_8\, p_{1,2+k}\, p_{3,2+k}\, \pi_{1,k}\, \pi_{3,k} + h_{10}\, p_{2,2+k}\, p_{3,2+k}\, \pi_{2,k}\, \pi_{3,k}$

(Alt) Out[ ]=

$\{p_{1,2+k},\ p_{2,2+k},\ p_{3,2+k},\ \pi_{1,k},\ \pi_{2,k},\ \pi_{3,k}\}$

(Alt) Out[ ]=

$\{h_2,\ h_3,\ h_4,\ h_5,\ h_6,\ h_7,\ h_8,\ h_9,\ h_{10}\}$

(Alt) Out[ ]//Short=

$\{h_6 == 0,\ h_7 == 0,\ B\, h_2 == 0,\ h_8 == 0,\ c_{16} + c_{31} + c_{46} + c_{61} + h_3 + 4 h_6 + h_7 + h_8 == 0,$
$h_9 == 0,\ h_{10} == 0,\ c_{19} + c_{34} + c_{49} + c_{64} + h_4 + h_7 + 4 h_9 + h_{10} == 0,\ h_5 + h_8 + h_{10} == 0,$
$c_{16} + c_{19} + c_{31} + c_{34} + c_{46} + c_{49} + c_{61} + c_{64} + h_3 + h_4 + h_5 + 2 h_6 + h_7 + h_8 + 2 h_9 + h_{10} == 0\}$

(Alt) Out[ ]//Short=

$\{\{h_2 \to 0,\ h_3 \to -c_{16} - c_{31} - c_{46} - c_{61},$
$h_4 \to -c_{19} - c_{34} - c_{49} - c_{64},\ h_5 \to 0,\ h_6 \to 0,\ h_7 \to 0,\ h_8 \to 0,\ h_9 \to 0,\ h_{10} \to 0\}\}$

(Alt) In[ ]:=
```
sol /. (v_ → val_) ⧴ (v = CF[val]);
```

(Alt) In[ ]:=
```
γ1[-1, k]
Short[CF[γ42[-1, k]], 5]
```

(Alt) Out[ ]=

0

(Alt) Out[ ]//Short=

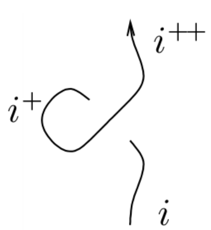
$(-c_{16} - c_{31} - c_{46} - c_{61})\, p_{1,k}\, x_{1,k} + (-c_{19} - c_{34} - c_{49} - c_{64})\, p_{2,k}\, x_{2,k}$

(Alt) In[ ]:=
```
CF[LeftCC - RightCC]
```

(Alt) Out[ ]=

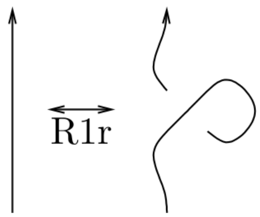
𝔼Series[0, 0]

Invariance Under R1l



(Alt) In[]:=
```
{LeftR1l} = Cases[{∫𝓕[i] × ℒ /@ (X_{i+2,i}[1] C_{i+1}[1]) d{vs_i, vs_{i⁺}, vs_{i+2}}}, 𝔼[ℰ_] ⧴ ℰ, ∞]
```
(Alt) Out[]=
```
{∊Series[p_{1,3+i} π_{1,i} + p_{2,3+i} π_{2,i} + p_{3,3+i} π_{3,i}, c_93]}
```
(Alt) In[]:=
```
{RightR1l} = Cases[{∫𝓕[i] × ℒ /@ (C_i[0] C_{i+1}[0] C_{i+2}[0]) d{vs_i, vs_{i⁺}, vs_{i+2}}}, 𝔼[ℰ_] ⧴ ℰ, ∞]
```
(Alt) Out[]=
```
{∊Series[p_{1,3+i} π_{1,i} + p_{2,3+i} π_{2,i} + p_{3,3+i} π_{3,i}, 0]}
```
(Alt) In[]:=
```
LeftR1l〚1〛 == RightR1l〚1〛
```
(Alt) Out[]=
```
True
```
(Alt) In[]:=
```
Short[eqn = CF[LeftR1l〚2〛 - RightR1l〚2〛]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[eqn, (c | d | e | f | g | h)_, ∞]
Short[eqns = If[cvs === {},
    {eqn == 0},
    CoefficientRules[eqn, cvs] /. (_ → c_) ⧴ (c == 0)
  ], 3]
{sol} = Solve[eqns, vars]
```
(Alt) Out[]//Short=
```
c_93
```
(Alt) Out[]=
```
{}
```
(Alt) Out[]=
```
{}
```
(Alt) Out[]//Short=
```
{c_93 == 0}
```
(Alt) Out[]=
```
{{c_93 → 0}}
```
(Alt) In[]:=
```
sol /. (v_ → val_) ⧴ (v = CF[val]);
```
(Alt) In[]:=
```
CF[LeftR1l - RightR1l]
```
(Alt) Out[]=
```
∊Series[0, 0]
```

Invariance Under R1r



(Alt) In[◦]:=

$$\{\textbf{LeftR1r}\} = \textbf{Cases}\left[\left\{\int \mathcal{F}[\mathtt{i}] \times \mathcal{L} \;/@\; (X_{i,i+2}[1]\; C_{i+1}[-1])\; d\{vs_i,\, vs_{i^+},\, vs_{i+2}\}\right\},\; \mathbb{E}[\mathcal{E}\_] \mapsto \mathcal{E},\; \infty\right]$$

(Alt) Out[◦]=

$$\Big\{\in\text{Series}\Big[p_{1,3+i}\,\pi_{1,i} + p_{2,3+i}\,\pi_{2,i} + p_{3,3+i}\,\pi_{3,i},$$
$$\frac{1}{T_1 T_2 (T_1 T_2 - T_3)^2 T_3} \big(-c_{83} T_1^3 T_2^3 - c_{86} T_1^3 T_2^3 + 2 c_{83} T_1^2 T_2^2 T_3 + 2 c_{86} T_1^2 T_2^2 T_3 - c_{82} T_1^3 T_2^2 T_3 - c_{85} T_1^3 T_2^2 T_3 -$$
$$c_{81} T_1^2 T_2^3 T_3 - c_{84} T_1^2 T_2^3 T_3 + c_{81} T_1^3 T_2^3 T_3 + c_{82} T_1^3 T_2^3 T_3 + c_{83} T_1^3 T_2^3 T_3 + 2 a_1 b_1 T_1 T_2 T_3^2 - c_{83} T_1 T_2 T_3^2 -$$
$$c_{86} T_1 T_2 T_3^2 - a_1 b_1 T_1^2 T_2 T_3^2 + 2 c_{82} T_1^2 T_2 T_3^2 + 2 c_{85} T_1^2 T_2 T_3^2 - a_1 b_1 T_1 T_2^2 T_3^2 + 2 c_{81} T_1 T_2^2 T_3^2 +$$
$$2 c_{84} T_1 T_2^2 T_3^2 + a_1 b_1 T_1^2 T_2^2 T_3^2 - 2 c_{81} T_1^2 T_2^2 T_3^2 - 2 c_{82} T_1^2 T_2^2 T_3^2 - 2 c_{83} T_1^2 T_2^2 T_3^2 - c_{82} T_1 T_3^3 -$$
$$c_{85} T_1 T_3^3 - c_{81} T_2 T_3^3 - c_{84} T_2 T_3^3 - a_1 b_1 T_1 T_2 T_3^3 + c_{81} T_1 T_2 T_3^3 + c_{82} T_1 T_2 T_3^3 + c_{83} T_1 T_2 T_3^3\big)\Big]\Big\}$$

(Alt) In[◦]:=

$$\{\textbf{RightR1r}\} = \textbf{Cases}\left[\left\{\int \mathcal{F}[\mathtt{i}] \times \mathcal{L} \;/@\; (C_i[0]\; C_{i+1}[0]\; C_{i+2}[0])\; d\{vs_i,\, vs_{i^+},\, vs_{i+2}\}\right\},\; \mathbb{E}[\mathcal{E}\_] \mapsto \mathcal{E},\; \infty\right]$$

(Alt) Out[◦]=

$$\{\in\text{Series}[p_{1,3+i}\,\pi_{1,i} + p_{2,3+i}\,\pi_{2,i} + p_{3,3+i}\,\pi_{3,i},\; 0]\}$$

(Alt) In[◦]:=

LeftR1r[[1]] == **RightR1r**[[1]]

(Alt) Out[◦]=

True

(Alt) In[ ]:=

```
Short[eqn = CF[LeftR1r⟦2⟧ - RightR1r⟦2⟧]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[eqn, (c | d | e | f | g | h)_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) ⧴ (c == 0), 3]
{sol} = Solve[eqns, vars]
```

(Alt) Out[ ]//Short=

$$\frac{-c_{83}\,T_1^3\,T_2^3 - c_{86}\ll 1\gg T_2^3 + \ll 45\gg + c_{83}\,T_1\,T_2\,T_3^3}{T_1\,T_2\,(T_1\ll 1\gg - \ll 1\gg)^2\,T_3}$$

(Alt) Out[ ]=

{ }

(Alt) Out[ ]=

$\{c_{81}, c_{82}, c_{83}, c_{84}, c_{85}, c_{86}\}$

(Alt) Out[ ]//Short=

$$\left\{\frac{-c_{83}\,T_1^3\,T_2^3 - c_{86}\,T_1^3\,T_2^3 + 2\,c_{83}\,T_1^2\,T_2^2\,T_3 + \ll 42\gg + c_{81}\,T_1\,T_2\,T_3^3 + c_{82}\,T_1\,T_2\,T_3^3 + c_{83}\,T_1\,T_2\,T_3^3}{T_1\,T_2\,(T_1\,T_2 - T_3)^2\,T_3} == 0\right\}$$

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]=

$$\Bigg\{\Bigg\{c_{86} \to -\frac{c_{84}\,T_3}{T_1} - \frac{c_{85}\,T_3}{T_2} - \frac{c_{83}\left(\frac{2\,T_1\,T_2}{(T_1\,T_2-T_3)^2} + \frac{T_1^2\,T_2^2}{(T_1\,T_2-T_3)^2} - \frac{T_1^2\,T_2^2}{(T_1\,T_2-T_3)^2\,T_3} - \frac{T_3}{(T_1\,T_2-T_3)^2} - \frac{2\,T_1\,T_2\,T_3}{(T_1\,T_2-T_3)^2} + \frac{T_3^2}{(T_1\,T_2-T_3)^2}\right)}{\frac{2\,T_1\,T_2}{(T_1\,T_2-T_3)^2} - \frac{T_1^2\,T_2^2}{(T_1\,T_2-T_3)^2\,T_3} - \frac{T_3}{(T_1\,T_2-T_3)^2}} -$$
$$\frac{\frac{2\,a_1\,b_1\,T_3}{(T_1\,T_2-T_3)^2} - \frac{a_1\,b_1\,T_1\,T_3}{(T_1\,T_2-T_3)^2} - \frac{a_1\,b_1\,T_2\,T_3}{(T_1\,T_2-T_3)^2} + \frac{a_1\,b_1\,T_1\,T_2\,T_3}{(T_1\,T_2-T_3)^2} - \frac{a_1\,b_1\,T_3^2}{(T_1\,T_2-T_3)^2}}{\frac{2\,T_1\,T_2}{(T_1\,T_2-T_3)^2} - \frac{T_1^2\,T_2^2}{(T_1\,T_2-T_3)^2\,T_3} - \frac{T_3}{(T_1\,T_2-T_3)^2}} -$$
$$\frac{c_{81}\left(-\frac{T_1\,T_2^2}{(T_1\,T_2-T_3)^2} + \frac{T_1^2\,T_2^2}{(T_1\,T_2-T_3)^2} + \frac{2\,T_2\,T_3}{(T_1\,T_2-T_3)^2} - \frac{2\,T_1\,T_2\,T_3}{(T_1\,T_2-T_3)^2} + \frac{T_3^2}{(T_1\,T_2-T_3)^2} - \frac{T_3^2}{T_1\,(T_1\,T_2-T_3)^2}\right)}{\frac{2\,T_1\,T_2}{(T_1\,T_2-T_3)^2} - \frac{T_1^2\,T_2^2}{(T_1\,T_2-T_3)^2\,T_3} - \frac{T_3}{(T_1\,T_2-T_3)^2}} -$$
$$\frac{c_{82}\left(-\frac{T_1^2\,T_2}{(T_1\,T_2-T_3)^2} + \frac{T_1^2\,T_2^2}{(T_1\,T_2-T_3)^2} + \frac{2\,T_1\,T_3}{(T_1\,T_2-T_3)^2} - \frac{2\,T_1\,T_2\,T_3}{(T_1\,T_2-T_3)^2} + \frac{T_3^2}{(T_1\,T_2-T_3)^2} - \frac{T_3^2}{T_2\,(T_1\,T_2-T_3)^2}\right)}{\frac{2\,T_1\,T_2}{(T_1\,T_2-T_3)^2} - \frac{T_1^2\,T_2^2}{(T_1\,T_2-T_3)^2\,T_3} - \frac{T_3}{(T_1\,T_2-T_3)^2}}\Bigg\}\Bigg\}$$

(Alt) In[ ]:=

```
sol /. (v_ → val_) ⧴ (v = CF[val]);
```

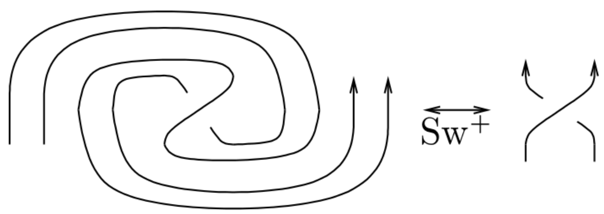
(Alt) In[ ]:=

```
CF[LeftR1r - RightR1r]
```

(Alt) Out[ ]=

∊Series[0, 0]

Invariance Under Sw



(Alt) In[]:=
```
Timing[Short[{LeftSw} = Cases[{∫ℱ[i, j] × ℒ /@ (X_{i+1,j+1}[1] C_i[-1] C_j[-1] C_{i+2}[1] C_{j+2}[1])
        ⅆ{vs_i, vs_j, vs_{i+}, vs_{j+}, vs_{i+2}, vs_{j+2}}}, 𝔼[ℰ_] ⧴ ℰ, ∞]
]]
```
(Alt) Out[]=
{0.03125, {∊Series[$T_1 p_{1,3+i} \pi_{1,i} + (1 - T_1) p_{1,3+j} \pi_{1,i}$ + ≪12≫ + $p_{3,3+j} \pi_{3,j}$, $c_{16} + c_{19}$ + ≪76≫]}}

(Alt) In[]:=
```
Timing[Short[{RightSw} = Cases[{∫ℱ[i, j] × ℒ /@ (X_{i+1,j+1}[1] C_i[0] C_j[0] C_{i+2}[0] C_{j+2}[0])
        ⅆ{vs_i, vs_j, vs_{i+}, vs_{j+}, vs_{i+2}, vs_{j+2}}}, 𝔼[ℰ_] ⧴ ℰ, ∞]
]]
```
(Alt) Out[]=
{0.046875, {∊Series[$T_1 p_{1,3+i} \pi_{1,i} + (1 - T_1) p_{1,3+j} \pi_{1,i}$ + ≪12≫ + $p_{3,3+j} \pi_{3,j}$, $c_{16} + c_{19}$ + ≪76≫]}}

(Alt) In[]:=
```
LeftSw == RightSw
```
(Alt) Out[]=
True

The Solution

(Alt) In[]:=
```
Union@Cases[ℒ@X_{i,j}[1], (a | b | c | d | e | f | g | h)_, ∞]
```
(Alt) Out[]=
{a_1, b_1, c_6, c_7, c_8, c_9, c_{10}, c_{16}, c_{19}, c_{21}, c_{22}, c_{23}, c_{24},
c_{25}, c_{31}, c_{34}, c_{36}, c_{39}, c_{46}, c_{49}, c_{61}, c_{64}, c_{81}, c_{82}, c_{83}, c_{84}, c_{85}}

Some Knots

(Alt) In[]:=
```
tab1 = Last /@ Table[K = Knot[n, 1];
    Echo@Timing[K → ∫ℒ[K] ⅆ vs[K]],
    {n, 3, 10}];
```

KnotTheory: Loading precomputed data in PD4Knots`.

» {2.9375,

Knot[3, 1] → −(((ⅈ $T_1^3 T_2^3$ 𝔼[∊Series[0, $-\frac{1}{T_1 (1 - T_1 + T_1^2)^2 T_2 (1 - T_2 + T_2^2)^2 (1 - T_3 + T_3^2)}$ ($c_{81} T_1 T_2 + c_{82} T_1 T_2$ +

$c_{84} T_1 T_2 + c_{85} T_1 T_2 - c_{16} T_1^2 T_2 - c_{31} T_1^2 T_2 - c_{46} T_1^2 T_2 - c_{61} T_1^2 T_2 - 4 c_{81} T_1^2 T_2 - 2 c_{82} T_1^2 T_2 -$
$3 c_{84} T_1^2 T_2 - 2 c_{85} T_1^2 T_2 + 2 c_{16} T_1^3 T_2 + 2 c_{31} T_1^3 T_2 + 2 c_{46} T_1^3 T_2 + 2 c_{61} T_1^3 T_2 + 6 c_{81} T_1^3 T_2 + 3 c_{82} T_1^3 T_2 +$
$3 c_{84} T_1^3 T_2 + 3 c_{85} T_1^3 T_2 - 3 c_{16} T_1^4 T_2 - 3 c_{31} T_1^4 T_2 - 3 c_{46} T_1^4 T_2 - 3 c_{61} T_1^4 T_2 - 5 c_{81} T_1^4 T_2 - 2 c_{82} T_1^4 T_2 -$
$2 c_{84} T_1^4 T_2 - 2 c_{85} T_1^4 T_2 + 2 c_{16} T_1^5 T_2 + 2 c_{31} T_1^5 T_2 + 2 c_{46} T_1^5 T_2 + 2 c_{61} T_1^5 T_2 + 2 c_{81} T_1^5 T_2 + c_{82} T_1^5 T_2 +$
$c_{85} T_1^5 T_2 - c_{19} T_1 T_2^2 - c_{34} T_1 T_2^2 - c_{49} T_1 T_2^2 - c_{64} T_1 T_2^2 - 2 c_{81} T_1 T_2^2 - 4 c_{82} T_1 T_2^2 - 2 c_{84} T_1 T_2^2 -$
$3 c_{85} T_1 T_2^2 + 2 c_{16} T_1^2 T_2^2 + 2 c_{19} T_1^2 T_2^2 + 2 c_{31} T_1^2 T_2^2 + 2 c_{34} T_1^2 T_2^2 + 2 c_{46} T_1^2 T_2^2 + 2 c_{49} T_1^2 T_2^2 + 2 c_{61} T_1^2 T_2^2 +$
$2 c_{64} T_1^2 T_2^2 + 8 c_{81} T_1^2 T_2^2 + 8 c_{82} T_1^2 T_2^2 + 6 c_{84} T_1^2 T_2^2 + 6 c_{85} T_1^2 T_2^2 - 4 c_{16} T_1^3 T_2^2 - 3 c_{19} T_1^3 T_2^2 - 4 c_{31} T_1^3 T_2^2 -$
$3 c_{34} T_1^3 T_2^2 - 4 c_{46} T_1^3 T_2^2 - 3 c_{49} T_1^3 T_2^2 - 4 c_{61} T_1^3 T_2^2 - 3 c_{64} T_1^3 T_2^2 - 12 c_{81} T_1^3 T_2^2 - 12 c_{82} T_1^3 T_2^2 -$
$6 c_{84} T_1^3 T_2^2 - 9 c_{85} T_1^3 T_2^2 + 6 c_{16} T_1^4 T_2^2 + 2 c_{19} T_1^4 T_2^2 + 6 c_{31} T_1^4 T_2^2 + 2 c_{34} T_1^4 T_2^2 + 6 c_{46} T_1^4 T_2^2 + 2 c_{49} T_1^4 T_2^2 +$
$6 c_{61} T_1^4 T_2^2 + 2 c_{64} T_1^4 T_2^2 + 10 c_{81} T_1^4 T_2^2 + 8 c_{82} T_1^4 T_2^2 + 4 c_{84} T_1^4 T_2^2 + 6 c_{85} T_1^4 T_2^2 - 4 c_{16} T_1^5 T_2^2 - c_{19} T_1^5 T_2^2 -$
$4 c_{31} T_1^5 T_2^2 - c_{34} T_1^5 T_2^2 - 4 c_{46} T_1^5 T_2^2 - c_{49} T_1^5 T_2^2 - 4 c_{61} T_1^5 T_2^2 - c_{64} T_1^5 T_2^2 - 4 c_{81} T_1^5 T_2^2 - 4 c_{82} T_1^5 T_2^2 -$
$3 c_{85} T_1^5 T_2^2 + 2 c_{19} T_1 T_2^3 + 2 c_{34} T_1 T_2^3 + 2 c_{49} T_1 T_2^3 + 2 c_{64} T_1 T_2^3 + 3 c_{81} T_1 T_2^3 + 6 c_{82} T_1 T_2^3 + 3 c_{84} T_1 T_2^3 +$
$3 c_{85} T_1 T_2^3 - 3 c_{16} T_1^2 T_2^3 - 4 c_{19} T_1^2 T_2^3 - 3 c_{31} T_1^2 T_2^3 - 4 c_{34} T_1^2 T_2^3 - 3 c_{46} T_1^2 T_2^3 - 4 c_{49} T_1^2 T_2^3 - 3 c_{61} T_1^2 T_2^3 -$
$4 c_{64} T_1^2 T_2^3 - 12 c_{81} T_1^2 T_2^3 - 12 c_{82} T_1^2 T_2^3 - 9 c_{84} T_1^2 T_2^3 - 6 c_{85} T_1^2 T_2^3 + 6 c_{16} T_1^3 T_2^3 + 6 c_{19} T_1^3 T_2^3 +$
$6 c_{31} T_1^3 T_2^3 + 6 c_{34} T_1^3 T_2^3 + 6 c_{46} T_1^3 T_2^3 + 6 c_{49} T_1^3 T_2^3 + 6 c_{61} T_1^3 T_2^3 + 6 c_{64} T_1^3 T_2^3 + 18 c_{81} T_1^3 T_2^3 +$
$18 c_{82} T_1^3 T_2^3 + 9 c_{84} T_1^3 T_2^3 + 9 c_{85} T_1^3 T_2^3 - 9 c_{16} T_1^4 T_2^3 - 4 c_{19} T_1^4 T_2^3 - 9 c_{31} T_1^4 T_2^3 - 4 c_{34} T_1^4 T_2^3 -$
$9 c_{46} T_1^4 T_2^3 - 4 c_{49} T_1^4 T_2^3 - 9 c_{61} T_1^4 T_2^3 - 4 c_{64} T_1^4 T_2^3 - 15 c_{81} T_1^4 T_2^3 - 12 c_{82} T_1^4 T_2^3 - 6 c_{84} T_1^4 T_2^3 -$
$6 c_{85} T_1^4 T_2^3 + 6 c_{16} T_1^5 T_2^3 + 2 c_{19} T_1^5 T_2^3 + 6 c_{31} T_1^5 T_2^3 + 2 c_{34} T_1^5 T_2^3 + 6 c_{46} T_1^5 T_2^3 + 2 c_{49} T_1^5 T_2^3 + 6 c_{61} T_1^5 T_2^3 +$
$2 c_{64} T_1^5 T_2^3 + 6 c_{81} T_1^5 T_2^3 + 6 c_{82} T_1^5 T_2^3 + 3 c_{85} T_1^5 T_2^3 - 3 c_{19} T_1 T_2^4 - 3 c_{34} T_1 T_2^4 - 3 c_{49} T_1 T_2^4 - 3 c_{64} T_1 T_2^4 -$
$2 c_{81} T_1 T_2^4 - 5 c_{82} T_1 T_2^4 - 2 c_{84} T_1 T_2^4 - 2 c_{85} T_1 T_2^4 + 2 c_{16} T_1^2 T_2^4 + 6 c_{19} T_1^2 T_2^4 + 2 c_{31} T_1^2 T_2^4 + 6 c_{34} T_1^2 T_2^4 +$
$2 c_{46} T_1^2 T_2^4 + 6 c_{49} T_1^2 T_2^4 + 2 c_{61} T_1^2 T_2^4 + 6 c_{64} T_1^2 T_2^4 + 8 c_{81} T_1^2 T_2^4 + 10 c_{82} T_1^2 T_2^4 + 6 c_{84} T_1^2 T_2^4 +$
$4 c_{85} T_1^2 T_2^4 - 4 c_{16} T_1^3 T_2^4 - 9 c_{19} T_1^3 T_2^4 - 4 c_{31} T_1^3 T_2^4 - 9 c_{34} T_1^3 T_2^4 - 4 c_{46} T_1^3 T_2^4 - 9 c_{49} T_1^3 T_2^4 - 4 c_{61} T_1^3 T_2^4 -$
$9 c_{64} T_1^3 T_2^4 - 12 c_{81} T_1^3 T_2^4 - 15 c_{82} T_1^3 T_2^4 - 6 c_{84} T_1^3 T_2^4 - 6 c_{85} T_1^3 T_2^4 + 6 c_{16} T_1^4 T_2^4 + 6 c_{19} T_1^4 T_2^4 +$
$6 c_{31} T_1^4 T_2^4 + 6 c_{34} T_1^4 T_2^4 + 6 c_{46} T_1^4 T_2^4 + 6 c_{49} T_1^4 T_2^4 + 6 c_{61} T_1^4 T_2^4 + 6 c_{64} T_1^4 T_2^4 + 10 c_{81} T_1^4 T_2^4 +$
$10 c_{82} T_1^4 T_2^4 + 4 c_{84} T_1^4 T_2^4 + 4 c_{85} T_1^4 T_2^4 - 4 c_{16} T_1^5 T_2^4 - 3 c_{19} T_1^5 T_2^4 - 4 c_{31} T_1^5 T_2^4 - 3 c_{34} T_1^5 T_2^4 -$
$4 c_{46} T_1^5 T_2^4 - 3 c_{49} T_1^5 T_2^4 - 4 c_{61} T_1^5 T_2^4 - 3 c_{64} T_1^5 T_2^4 - 4 c_{81} T_1^5 T_2^4 - 5 c_{82} T_1^5 T_2^4 - 2 c_{85} T_1^5 T_2^4 + 2 c_{19} T_1 T_2^5 +$
$2 c_{34} T_1 T_2^5 + 2 c_{49} T_1 T_2^5 + 2 c_{64} T_1 T_2^5 + c_{81} T_1 T_2^5 + 2 c_{82} T_1 T_2^5 + c_{84} T_1 T_2^5 - c_{16} T_1^2 T_2^5 - 4 c_{19} T_1^2 T_2^5 -$
$c_{31} T_1^2 T_2^5 - 4 c_{34} T_1^2 T_2^5 - c_{46} T_1^2 T_2^5 - 4 c_{49} T_1^2 T_2^5 - c_{61} T_1^2 T_2^5 - 4 c_{64} T_1^2 T_2^5 - 4 c_{81} T_1^2 T_2^5 - 4 c_{82} T_1^2 T_2^5 -$
$3 c_{84} T_1^2 T_2^5 + 2 c_{16} T_1^3 T_2^5 + 6 c_{19} T_1^3 T_2^5 + 2 c_{31} T_1^3 T_2^5 + 6 c_{34} T_1^3 T_2^5 + 2 c_{46} T_1^3 T_2^5 + 6 c_{49} T_1^3 T_2^5 + 2 c_{61} T_1^3 T_2^5 +$
$6 c_{64} T_1^3 T_2^5 + 6 c_{81} T_1^3 T_2^5 + 6 c_{82} T_1^3 T_2^5 + 3 c_{84} T_1^3 T_2^5 - 3 c_{16} T_1^4 T_2^5 - 4 c_{19} T_1^4 T_2^5 - 3 c_{31} T_1^4 T_2^5 - 4 c_{34} T_1^4 T_2^5 -$
$3 c_{46} T_1^4 T_2^5 - 4 c_{49} T_1^4 T_2^5 - 3 c_{61} T_1^4 T_2^5 - 4 c_{64} T_1^4 T_2^5 - 5 c_{81} T_1^4 T_2^5 - 4 c_{82} T_1^4 T_2^5 - 2 c_{84} T_1^4 T_2^5 +$
$2 c_{16} T_1^5 T_2^5 + 2 c_{19} T_1^5 T_2^5 + 2 c_{31} T_1^5 T_2^5 + 2 c_{34} T_1^5 T_2^5 + 2 c_{46} T_1^5 T_2^5 + 2 c_{49} T_1^5 T_2^5 + 2 c_{61} T_1^5 T_2^5 +$
$2 c_{64} T_1^5 T_2^5 + 2 c_{81} T_1^5 T_2^5 + 2 c_{82} T_1^5 T_2^5 - c_{82} T_1 T_3 - c_{85} T_1 T_3 + 2 c_{82} T_1^2 T_3 + 2 c_{85} T_1^2 T_3 - 3 c_{82} T_1^3 T_3 -$
$3 c_{85} T_1^3 T_3 + 2 c_{82} T_1^4 T_3 + 2 c_{85} T_1^4 T_3 - c_{82} T_1^5 T_3 - c_{85} T_1^5 T_3 - c_{81} T_2 T_3 - c_{84} T_2 T_3 + 2 c_{81} T_1 T_2 T_3 +$
$2 c_{82} T_1 T_2 T_3 + c_{84} T_1 T_2 T_3 + c_{85} T_1 T_2 T_3 + c_{16} T_1^2 T_2 T_3 + c_{31} T_1^2 T_2 T_3 + c_{46} T_1^2 T_2 T_3 + c_{61} T_1^2 T_2 T_3 -$
$c_{81} T_1^2 T_2 T_3 - 4 c_{82} T_1^2 T_2 T_3 - 2 c_{85} T_1^2 T_2 T_3 - 2 c_{16} T_1^3 T_2 T_3 - 2 c_{31} T_1^3 T_2 T_3 - 2 c_{46} T_1^3 T_2 T_3 -$
$2 c_{61} T_1^3 T_2 T_3 - c_{81} T_1^3 T_2 T_3 + 6 c_{82} T_1^3 T_2 T_3 - c_{84} T_1^3 T_2 T_3 + 3 c_{85} T_1^3 T_2 T_3 + 3 c_{16} T_1^4 T_2 T_3 +$
$3 c_{31} T_1^4 T_2 T_3 + 3 c_{46} T_1^4 T_2 T_3 + 3 c_{61} T_1^4 T_2 T_3 + 2 c_{81} T_1^4 T_2 T_3 - 4 c_{82} T_1^4 T_2 T_3 + c_{84} T_1^4 T_2 T_3 -$
$2 c_{85} T_1^4 T_2 T_3 - 2 c_{16} T_1^5 T_2 T_3 - 2 c_{31} T_1^5 T_2 T_3 - 2 c_{46} T_1^5 T_2 T_3 - 2 c_{61} T_1^5 T_2 T_3 - c_{81} T_1^5 T_2 T_3 +$
$2 c_{82} T_1^5 T_2 T_3 + c_{85} T_1^5 T_2 T_3 + 2 c_{81} T_2^2 T_3 + 2 c_{84} T_2^2 T_3 + c_{19} T_1 T_2^2 T_3 + c_{34} T_1 T_2^2 T_3 + c_{49} T_1 T_2^2 T_3 +$
$c_{64} T_1 T_2^2 T_3 - 4 c_{81} T_1 T_2^2 T_3 - c_{82} T_1 T_2^2 T_3 - 2 c_{84} T_1 T_2^2 T_3 - 2 c_{16} T_1^2 T_2^2 T_3 - 2 c_{19} T_1^2 T_2^2 T_3 -$
$2 c_{31} T_1^2 T_2^2 T_3 - 2 c_{34} T_1^2 T_2^2 T_3 - 2 c_{46} T_1^2 T_2^2 T_3 - 2 c_{49} T_1^2 T_2^2 T_3 - 2 c_{61} T_1^2 T_2^2 T_3 - 2 c_{64} T_1^2 T_2^2 T_3 +$
$2 c_{81} T_1^2 T_2^2 T_3 + 2 c_{82} T_1^2 T_2^2 T_3 + 4 c_{16} T_1^3 T_2^2 T_3 + 3 c_{19} T_1^3 T_2^2 T_3 + 4 c_{31} T_1^3 T_2^2 T_3 + 3 c_{34} T_1^3 T_2^2 T_3 +$
$4 c_{46} T_1^3 T_2^2 T_3 + 3 c_{49} T_1^3 T_2^2 T_3 + 4 c_{61} T_1^3 T_2^2 T_3 + 3 c_{64} T_1^3 T_2^2 T_3 + 2 c_{81} T_1^3 T_2^2 T_3 - 3 c_{82} T_1^3 T_2^2 T_3 +$
$2 c_{84} T_1^3 T_2^2 T_3 - 6 c_{16} T_1^4 T_2^2 T_3 - 2 c_{19} T_1^4 T_2^2 T_3 - 6 c_{31} T_1^4 T_2^2 T_3 - 2 c_{34} T_1^4 T_2^2 T_3 - 6 c_{46} T_1^4 T_2^2 T_3 -$
$2 c_{49} T_1^4 T_2^2 T_3 - 6 c_{61} T_1^4 T_2^2 T_3 - 2 c_{64} T_1^4 T_2^2 T_3 - 4 c_{81} T_1^4 T_2^2 T_3 + 2 c_{82} T_1^4 T_2^2 T_3 - 2 c_{84} T_1^4 T_2^2 T_3 +$
$4 c_{16} T_1^5 T_2^2 T_3 + c_{19} T_1^5 T_2^2 T_3 + 4 c_{31} T_1^5 T_2^2 T_3 + c_{34} T_1^5 T_2^2 T_3 + 4 c_{46} T_1^5 T_2^2 T_3 + c_{49} T_1^5 T_2^2 T_3 +$
$4 c_{61} T_1^5 T_2^2 T_3 + c_{64} T_1^5 T_2^2 T_3 + 2 c_{81} T_1^5 T_2^2 T_3 - c_{82} T_1^5 T_2^2 T_3 - 3 c_{81} T_2^3 T_3 - 3 c_{84} T_2^3 T_3 - 2 c_{19} T_1 T_2^3 T_3 -$
»
$2 c_{34} T_1 T_2^3 T_3 - 2 c_{49} T_1 T_2^3 T_3 - 2 c_{64} T_1 T_2^3 T_3 + 6 c_{81} T_1 T_2^3 T_3 - c_{82} T_1 T_2^3 T_3 + 3 c_{84} T_1 T_2^3 T_3 -$
$c_{85} T_1 T_2^3 T_3 + 3 c_{16} T_1^2 T_2^3 T_3 + 4 c_{19} T_1^2 T_2^3 T_3 + 3 c_{31} T_1^2 T_2^3 T_3 + 4 c_{34} T_1^2 T_2^3 T_3 + 3 c_{46} T_1^2 T_2^3 T_3 +$

$4 c_{49} T_1^2 T_2^3 T_3 + 3 c_{61} T_1^2 T_2^3 T_3 + 4 c_{64} T_1^2 T_2^3 T_3 - 3 c_{81} T_1^2 T_2^3 T_3 + 2 c_{82} T_1^2 T_2^3 T_3 + 2 c_{85} T_1^2 T_2^3 T_3 -$
$6 c_{16} T_1^3 T_2^3 T_3 - 6 c_{19} T_1^3 T_2^3 T_3 - 6 c_{31} T_1^3 T_2^3 T_3 - 6 c_{34} T_1^3 T_2^3 T_3 - 6 c_{46} T_1^3 T_2^3 T_3 - 6 c_{49} T_1^3 T_2^3 T_3 -$
$6 c_{61} T_1^3 T_2^3 T_3 - 6 c_{64} T_1^3 T_2^3 T_3 - 3 c_{81} T_1^3 T_2^3 T_3 - 3 c_{82} T_1^3 T_2^3 T_3 - 3 c_{84} T_1^3 T_2^3 T_3 - 3 c_{85} T_1^3 T_2^3 T_3 +$
$9 c_{16} T_1^4 T_2^3 T_3 + 4 c_{19} T_1^4 T_2^3 T_3 + 9 c_{31} T_1^4 T_2^3 T_3 + 4 c_{34} T_1^4 T_2^3 T_3 + 9 c_{46} T_1^4 T_2^3 T_3 + 4 c_{49} T_1^4 T_2^3 T_3 +$
$9 c_{61} T_1^4 T_2^3 T_3 + 4 c_{64} T_1^4 T_2^3 T_3 + 6 c_{81} T_1^4 T_2^3 T_3 + 2 c_{82} T_1^4 T_2^3 T_3 + 3 c_{84} T_1^4 T_2^3 T_3 + 2 c_{85} T_1^4 T_2^3 T_3 -$
$6 c_{16} T_1^5 T_2^3 T_3 - 2 c_{19} T_1^5 T_2^3 T_3 - 6 c_{31} T_1^5 T_2^3 T_3 - 2 c_{34} T_1^5 T_2^3 T_3 - 6 c_{46} T_1^5 T_2^3 T_3 - 2 c_{49} T_1^5 T_2^3 T_3 -$
$6 c_{61} T_1^5 T_2^3 T_3 - 2 c_{64} T_1^5 T_2^3 T_3 - 3 c_{81} T_1^5 T_2^3 T_3 - c_{82} T_1^5 T_2^3 T_3 - c_{85} T_1^5 T_2^3 T_3 + 2 c_{81} T_2^4 T_3 + 2 c_{84} T_2^4 T_3 +$
$3 c_{19} T_1 T_2^4 T_3 + 3 c_{34} T_1 T_2^4 T_3 + 3 c_{49} T_1 T_2^4 T_3 + 3 c_{64} T_1 T_2^4 T_3 - 4 c_{81} T_1 T_2^4 T_3 + 2 c_{82} T_1 T_2^4 T_3 -$
$2 c_{84} T_1 T_2^4 T_3 + c_{85} T_1 T_2^4 T_3 - 2 c_{16} T_1^2 T_2^4 T_3 - 6 c_{19} T_1^2 T_2^4 T_3 - 2 c_{31} T_1^2 T_2^4 T_3 - 6 c_{34} T_1^2 T_2^4 T_3 -$
$2 c_{46} T_1^2 T_2^4 T_3 - 6 c_{49} T_1^2 T_2^4 T_3 - 2 c_{61} T_1^2 T_2^4 T_3 - 6 c_{64} T_1^2 T_2^4 T_3 + 2 c_{81} T_1^2 T_2^4 T_3 - 4 c_{82} T_1^2 T_2^4 T_3 -$
$2 c_{85} T_1^2 T_2^4 T_3 + 4 c_{16} T_1^3 T_2^4 T_3 + 9 c_{19} T_1^3 T_2^4 T_3 + 4 c_{31} T_1^3 T_2^4 T_3 + 9 c_{34} T_1^3 T_2^4 T_3 + 4 c_{46} T_1^3 T_2^4 T_3 +$
$9 c_{49} T_1^3 T_2^4 T_3 + 4 c_{61} T_1^3 T_2^4 T_3 + 9 c_{64} T_1^3 T_2^4 T_3 + 2 c_{81} T_1^3 T_2^4 T_3 + 6 c_{82} T_1^3 T_2^4 T_3 + 2 c_{84} T_1^3 T_2^4 T_3 +$
$3 c_{85} T_1^3 T_2^4 T_3 - 6 c_{16} T_1^4 T_2^4 T_3 - 6 c_{19} T_1^4 T_2^4 T_3 - 6 c_{31} T_1^4 T_2^4 T_3 - 6 c_{34} T_1^4 T_2^4 T_3 - 6 c_{46} T_1^4 T_2^4 T_3 -$
$6 c_{49} T_1^4 T_2^4 T_3 - 6 c_{61} T_1^4 T_2^4 T_3 - 6 c_{64} T_1^4 T_2^4 T_3 - 4 c_{81} T_1^4 T_2^4 T_3 - 4 c_{82} T_1^4 T_2^4 T_3 - 2 c_{84} T_1^4 T_2^4 T_3 -$
$2 c_{85} T_1^4 T_2^4 T_3 + 4 c_{16} T_1^5 T_2^4 T_3 + 3 c_{19} T_1^5 T_2^4 T_3 + 4 c_{31} T_1^5 T_2^4 T_3 + 3 c_{34} T_1^5 T_2^4 T_3 + 4 c_{46} T_1^5 T_2^4 T_3 +$
$3 c_{49} T_1^5 T_2^4 T_3 + 4 c_{61} T_1^5 T_2^4 T_3 + 3 c_{64} T_1^5 T_2^4 T_3 + 2 c_{81} T_1^5 T_2^4 T_3 + 2 c_{82} T_1^5 T_2^4 T_3 + c_{85} T_1^5 T_2^4 T_3 -$
$c_{81} T_2^5 T_3 - c_{84} T_2^5 T_3 - 2 c_{19} T_1 T_2^5 T_3 - 2 c_{34} T_1 T_2^5 T_3 - 2 c_{49} T_1 T_2^5 T_3 - 2 c_{64} T_1 T_2^5 T_3 + 2 c_{81} T_1 T_2^5 T_3 -$
$c_{82} T_1 T_2^5 T_3 + c_{84} T_1 T_2^5 T_3 + c_{16} T_1^2 T_2^5 T_3 + 4 c_{19} T_1^2 T_2^5 T_3 + c_{31} T_1^2 T_2^5 T_3 + 4 c_{34} T_1^2 T_2^5 T_3 + c_{46} T_1^2 T_2^5 T_3 +$
$4 c_{49} T_1^2 T_2^5 T_3 + c_{61} T_1^2 T_2^5 T_3 + 4 c_{64} T_1^2 T_2^5 T_3 - c_{81} T_1^2 T_2^5 T_3 + 2 c_{82} T_1^2 T_2^5 T_3 - 2 c_{16} T_1^3 T_2^5 T_3 -$
$6 c_{19} T_1^3 T_2^5 T_3 - 2 c_{31} T_1^3 T_2^5 T_3 - 6 c_{34} T_1^3 T_2^5 T_3 - 2 c_{46} T_1^3 T_2^5 T_3 - 6 c_{49} T_1^3 T_2^5 T_3 - 2 c_{61} T_1^3 T_2^5 T_3 -$
$6 c_{64} T_1^3 T_2^5 T_3 - c_{81} T_1^3 T_2^5 T_3 - 3 c_{82} T_1^3 T_2^5 T_3 - c_{84} T_1^3 T_2^5 T_3 + 3 c_{16} T_1^4 T_2^5 T_3 + 4 c_{19} T_1^4 T_2^5 T_3 +$
$3 c_{31} T_1^4 T_2^5 T_3 + 4 c_{34} T_1^4 T_2^5 T_3 + 3 c_{46} T_1^4 T_2^5 T_3 + 4 c_{49} T_1^4 T_2^5 T_3 + 3 c_{61} T_1^4 T_2^5 T_3 + 4 c_{64} T_1^4 T_2^5 T_3 +$
$2 c_{81} T_1^4 T_2^5 T_3 + 2 c_{82} T_1^4 T_2^5 T_3 + c_{84} T_1^4 T_2^5 T_3 - 2 c_{16} T_1^5 T_2^5 T_3 - 2 c_{19} T_1^5 T_2^5 T_3 - 2 c_{31} T_1^5 T_2^5 T_3 -$
$2 c_{34} T_1^5 T_2^5 T_3 - 2 c_{46} T_1^5 T_2^5 T_3 - 2 c_{49} T_1^5 T_2^5 T_3 - 2 c_{61} T_1^5 T_2^5 T_3 - 2 c_{64} T_1^5 T_2^5 T_3 - c_{81} T_1^5 T_2^5 T_3 -$
$c_{82} T_1^5 T_2^5 T_3 + 2 c_{82} T_1 T_3^2 + 2 c_{85} T_1 T_3^2 - 4 c_{82} T_1^2 T_3^2 - 4 c_{85} T_1^2 T_3^2 + 6 c_{82} T_1^3 T_3^2 + 6 c_{85} T_1^3 T_3^2 -$
$4 c_{82} T_1^4 T_3^2 - 4 c_{85} T_1^4 T_3^2 + 2 c_{82} T_1^5 T_3^2 + 2 c_{85} T_1^5 T_3^2 + 2 c_{81} T_2 T_3^2 + 2 c_{84} T_2 T_3^2 - 5 c_{81} T_1 T_2 T_3^2 -$
$5 c_{82} T_1 T_2 T_3^2 - 3 c_{84} T_1 T_2 T_3^2 - 3 c_{85} T_1 T_2 T_3^2 - c_{16} T_1^2 T_2 T_3^2 - c_{31} T_1^2 T_2 T_3^2 - c_{46} T_1^2 T_2 T_3^2 -$
$c_{61} T_1^2 T_2 T_3^2 + 6 c_{81} T_1^2 T_2 T_3^2 + 10 c_{82} T_1^2 T_2 T_3^2 + 3 c_{84} T_1^2 T_2 T_3^2 + 6 c_{85} T_1^2 T_2 T_3^2 + 2 c_{16} T_1^3 T_2 T_3^2 +$
$2 c_{31} T_1^3 T_2 T_3^2 + 2 c_{46} T_1^3 T_2 T_3^2 + 2 c_{61} T_1^3 T_2 T_3^2 - 4 c_{81} T_1^3 T_2 T_3^2 - 15 c_{82} T_1^3 T_2 T_3^2 - c_{84} T_1^3 T_2 T_3^2 -$
$9 c_{85} T_1^3 T_2 T_3^2 - 3 c_{16} T_1^4 T_2 T_3^2 - 3 c_{31} T_1^4 T_2 T_3^2 - 3 c_{46} T_1^4 T_2 T_3^2 - 3 c_{61} T_1^4 T_2 T_3^2 + c_{81} T_1^4 T_2 T_3^2 +$
$10 c_{82} T_1^4 T_2 T_3^2 + 6 c_{85} T_1^4 T_2 T_3^2 + 2 c_{16} T_1^5 T_2 T_3^2 + 2 c_{31} T_1^5 T_2 T_3^2 + 2 c_{46} T_1^5 T_2 T_3^2 + 2 c_{61} T_1^5 T_2 T_3^2 -$
$5 c_{82} T_1^5 T_2 T_3^2 - 3 c_{85} T_1^5 T_2 T_3^2 - 4 c_{81} T_2^2 T_3^2 - 4 c_{84} T_2^2 T_3^2 - c_{19} T_1 T_2^2 T_3^2 - c_{34} T_1 T_2^2 T_3^2 - c_{49} T_1 T_2^2 T_3^2 -$
$c_{64} T_1 T_2^2 T_3^2 + 10 c_{81} T_1 T_2^2 T_3^2 + 6 c_{82} T_1 T_2^2 T_3^2 + 6 c_{84} T_1 T_2^2 T_3^2 + 3 c_{85} T_1 T_2^2 T_3^2 + 2 c_{16} T_1^2 T_2^2 T_3^2 +$
$2 c_{19} T_1^2 T_2^2 T_3^2 + 2 c_{31} T_1^2 T_2^2 T_3^2 + 2 c_{34} T_1^2 T_2^2 T_3^2 + 2 c_{46} T_1^2 T_2^2 T_3^2 + 2 c_{49} T_1^2 T_2^2 T_3^2 + 2 c_{61} T_1^2 T_2^2 T_3^2 +$
$2 c_{64} T_1^2 T_2^2 T_3^2 - 12 c_{81} T_1^2 T_2^2 T_3^2 - 12 c_{82} T_1^2 T_2^2 T_3^2 - 6 c_{84} T_1^2 T_2^2 T_3^2 - 6 c_{85} T_1^2 T_2^2 T_3^2 - 4 c_{16} T_1^3 T_2^2 T_3^2 -$
$3 c_{19} T_1^3 T_2^2 T_3^2 - 4 c_{31} T_1^3 T_2^2 T_3^2 - 3 c_{34} T_1^3 T_2^2 T_3^2 - 4 c_{46} T_1^3 T_2^2 T_3^2 - 3 c_{49} T_1^3 T_2^2 T_3^2 - 4 c_{61} T_1^3 T_2^2 T_3^2 -$
$3 c_{64} T_1^3 T_2^2 T_3^2 + 8 c_{81} T_1^3 T_2^2 T_3^2 + 18 c_{82} T_1^3 T_2^2 T_3^2 + 2 c_{84} T_1^3 T_2^2 T_3^2 + 9 c_{85} T_1^3 T_2^2 T_3^2 + 6 c_{16} T_1^4 T_2^2 T_3^2 +$
$2 c_{19} T_1^4 T_2^2 T_3^2 + 6 c_{31} T_1^4 T_2^2 T_3^2 + 2 c_{34} T_1^4 T_2^2 T_3^2 + 6 c_{46} T_1^4 T_2^2 T_3^2 + 2 c_{49} T_1^4 T_2^2 T_3^2 + 6 c_{61} T_1^4 T_2^2 T_3^2 +$
$2 c_{64} T_1^4 T_2^2 T_3^2 - 2 c_{81} T_1^4 T_2^2 T_3^2 - 12 c_{82} T_1^4 T_2^2 T_3^2 - 6 c_{85} T_1^4 T_2^2 T_3^2 - 4 c_{16} T_1^5 T_2^2 T_3^2 - c_{19} T_1^5 T_2^2 T_3^2 -$
$4 c_{31} T_1^5 T_2^2 T_3^2 - c_{34} T_1^5 T_2^2 T_3^2 - 4 c_{46} T_1^5 T_2^2 T_3^2 - c_{49} T_1^5 T_2^2 T_3^2 - 4 c_{61} T_1^5 T_2^2 T_3^2 - c_{64} T_1^5 T_2^2 T_3^2 +$
$6 c_{82} T_1^5 T_2^2 T_3^2 + 3 c_{85} T_1^5 T_2^2 T_3^2 + 6 c_{81} T_2^3 T_3^2 + 6 c_{84} T_2^3 T_3^2 + 2 c_{19} T_1 T_2^3 T_3^2 + 2 c_{34} T_1 T_2^3 T_3^2 +$
$2 c_{49} T_1 T_2^3 T_3^2 + 2 c_{64} T_1 T_2^3 T_3^2 - 15 c_{81} T_1 T_2^3 T_3^2 - 4 c_{82} T_1 T_2^3 T_3^2 - 9 c_{84} T_1 T_2^3 T_3^2 - c_{85} T_1 T_2^3 T_3^2 -$
$3 c_{16} T_1^2 T_2^3 T_3^2 - 4 c_{19} T_1^2 T_2^3 T_3^2 - 3 c_{31} T_1^2 T_2^3 T_3^2 - 4 c_{34} T_1^2 T_2^3 T_3^2 - 3 c_{46} T_1^2 T_2^3 T_3^2 - 4 c_{49} T_1^2 T_2^3 T_3^2 -$
$3 c_{61} T_1^2 T_2^3 T_3^2 - 4 c_{64} T_1^2 T_2^3 T_3^2 + 18 c_{81} T_1^2 T_2^3 T_3^2 + 8 c_{82} T_1^2 T_2^3 T_3^2 + 9 c_{84} T_1^2 T_2^3 T_3^2 + 2 c_{85} T_1^2 T_2^3 T_3^2 +$
$6 c_{16} T_1^3 T_2^3 T_3^2 + 6 c_{19} T_1^3 T_2^3 T_3^2 + 6 c_{31} T_1^3 T_2^3 T_3^2 + 6 c_{34} T_1^3 T_2^3 T_3^2 + 6 c_{46} T_1^3 T_2^3 T_3^2 + 6 c_{49} T_1^3 T_2^3 T_3^2 +$
$6 c_{61} T_1^3 T_2^3 T_3^2 + 6 c_{64} T_1^3 T_2^3 T_3^2 - 12 c_{81} T_1^3 T_2^3 T_3^2 - 12 c_{82} T_1^3 T_2^3 T_3^2 - 3 c_{84} T_1^3 T_2^3 T_3^2 - 3 c_{85} T_1^3 T_2^3 T_3^2 -$
$9 c_{16} T_1^4 T_2^3 T_3^2 - 4 c_{19} T_1^4 T_2^3 T_3^2 - 9 c_{31} T_1^4 T_2^3 T_3^2 - 4 c_{34} T_1^4 T_2^3 T_3^2 - 9 c_{46} T_1^4 T_2^3 T_3^2 - 4 c_{49} T_1^4 T_2^3 T_3^2 -$
$9 c_{61} T_1^4 T_2^3 T_3^2 - 4 c_{64} T_1^4 T_2^3 T_3^2 + 3 c_{81} T_1^4 T_2^3 T_3^2 + 8 c_{82} T_1^4 T_2^3 T_3^2 + 2 c_{85} T_1^4 T_2^3 T_3^2 + 6 c_{16} T_1^5 T_2^3 T_3^2 +$
$2 c_{19} T_1^5 T_2^3 T_3^2 + 6 c_{31} T_1^5 T_2^3 T_3^2 + 2 c_{34} T_1^5 T_2^3 T_3^2 + 6 c_{46} T_1^5 T_2^3 T_3^2 + 2 c_{49} T_1^5 T_2^3 T_3^2 + 6 c_{61} T_1^5 T_2^3 T_3^2 +$
$2 c_{64} T_1^5 T_2^3 T_3^2 - 4 c_{82} T_1^5 T_2^3 T_3^2 - c_{85} T_1^5 T_2^3 T_3^2 - 4 c_{81} T_2^4 T_3^2 - 4 c_{84} T_2^4 T_3^2 - 3 c_{19} T_1 T_2^4 T_3^2 - 3 c_{34} T_1 T_2^4 T_3^2 -$

$$3 c_{49} T_1 T_2^4 T_3^2 - 3 c_{64} T_1 T_2^4 T_3^2 + 10 c_{81} T_1 T_2^4 T_3^2 + c_{82} T_1 T_2^4 T_3^2 + 6 c_{84} T_1 T_2^4 T_3^2 + 2 c_{16} T_1^2 T_2^4 T_3^2 +$$
$$6 c_{19} T_1^2 T_2^4 T_3^2 + 2 c_{31} T_1^2 T_2^4 T_3^2 + 6 c_{34} T_1^2 T_2^4 T_3^2 + 2 c_{46} T_1^2 T_2^4 T_3^2 + 6 c_{49} T_1^2 T_2^4 T_3^2 + 2 c_{61} T_1^2 T_2^4 T_3^2 +$$
$$6 c_{64} T_1^2 T_2^4 T_3^2 - 12 c_{81} T_1^2 T_2^4 T_3^2 - 2 c_{82} T_1^2 T_2^4 T_3^2 - 6 c_{84} T_1^2 T_2^4 T_3^2 - 4 c_{16} T_1^3 T_2^4 T_3^2 - 9 c_{19} T_1^3 T_2^4 T_3^2 -$$
$$4 c_{31} T_1^3 T_2^4 T_3^2 - 9 c_{34} T_1^3 T_2^4 T_3^2 - 4 c_{46} T_1^3 T_2^4 T_3^2 - 9 c_{49} T_1^3 T_2^4 T_3^2 - 4 c_{61} T_1^3 T_2^4 T_3^2 - 9 c_{64} T_1^3 T_2^4 T_3^2 +$$
$$8 c_{81} T_1^3 T_2^4 T_3^2 + 3 c_{82} T_1^3 T_2^4 T_3^2 + 2 c_{84} T_1^3 T_2^4 T_3^2 + 6 c_{16} T_1^4 T_2^4 T_3^2 + 6 c_{19} T_1^4 T_2^4 T_3^2 + 6 c_{31} T_1^4 T_2^4 T_3^2 +$$
$$6 c_{34} T_1^4 T_2^4 T_3^2 + 6 c_{46} T_1^4 T_2^4 T_3^2 + 6 c_{49} T_1^4 T_2^4 T_3^2 + 6 c_{61} T_1^4 T_2^4 T_3^2 + 6 c_{64} T_1^4 T_2^4 T_3^2 - 2 c_{81} T_1^4 T_2^4 T_3^2 -$$
$$2 c_{82} T_1^4 T_2^4 T_3^2 - 4 c_{16} T_1^5 T_2^4 T_3^2 - 3 c_{19} T_1^5 T_2^4 T_3^2 - 4 c_{31} T_1^5 T_2^4 T_3^2 - 3 c_{34} T_1^5 T_2^4 T_3^2 - 4 c_{46} T_1^5 T_2^4 T_3^2 -$$
$$3 c_{49} T_1^5 T_2^4 T_3^2 - 4 c_{61} T_1^5 T_2^4 T_3^2 - 3 c_{64} T_1^5 T_2^4 T_3^2 + c_{82} T_1^5 T_2^4 T_3^2 + 2 c_{81} T_2^5 T_3^2 + 2 c_{84} T_2^5 T_3^2 + 2 c_{19} T_1 T_2^5 T_3^2 +$$
$$2 c_{34} T_1 T_2^5 T_3^2 + 2 c_{49} T_1 T_2^5 T_3^2 + 2 c_{64} T_1 T_2^5 T_3^2 - 5 c_{81} T_1 T_2^5 T_3^2 - 3 c_{84} T_1 T_2^5 T_3^2 - c_{16} T_1^2 T_2^5 T_3^2 -$$
$$4 c_{19} T_1^2 T_2^5 T_3^2 - c_{31} T_1^2 T_2^5 T_3^2 - 4 c_{34} T_1^2 T_2^5 T_3^2 - c_{46} T_1^2 T_2^5 T_3^2 - 4 c_{49} T_1^2 T_2^5 T_3^2 - c_{61} T_1^2 T_2^5 T_3^2 -$$
$$4 c_{64} T_1^2 T_2^5 T_3^2 + 6 c_{81} T_1^2 T_2^5 T_3^2 + 3 c_{84} T_1^2 T_2^5 T_3^2 + 2 c_{16} T_1^3 T_2^5 T_3^2 + 6 c_{19} T_1^3 T_2^5 T_3^2 + 2 c_{31} T_1^3 T_2^5 T_3^2 +$$
$$6 c_{34} T_1^3 T_2^5 T_3^2 + 2 c_{46} T_1^3 T_2^5 T_3^2 + 6 c_{49} T_1^3 T_2^5 T_3^2 + 2 c_{61} T_1^3 T_2^5 T_3^2 + 6 c_{64} T_1^3 T_2^5 T_3^2 - 4 c_{81} T_1^3 T_2^5 T_3^2 -$$
$$c_{84} T_1^3 T_2^5 T_3^2 - 3 c_{16} T_1^4 T_2^5 T_3^2 - 4 c_{19} T_1^4 T_2^5 T_3^2 - 3 c_{31} T_1^4 T_2^5 T_3^2 - 4 c_{34} T_1^4 T_2^5 T_3^2 - 3 c_{46} T_1^4 T_2^5 T_3^2 -$$
$$4 c_{49} T_1^4 T_2^5 T_3^2 - 3 c_{61} T_1^4 T_2^5 T_3^2 - 4 c_{64} T_1^4 T_2^5 T_3^2 + c_{81} T_1^4 T_2^5 T_3^2 + 2 c_{16} T_1^5 T_2^5 T_3^2 + 2 c_{19} T_1^5 T_2^5 T_3^2 +$$
$$2 c_{31} T_1^5 T_2^5 T_3^2 + 2 c_{34} T_1^5 T_2^5 T_3^2 + 2 c_{46} T_1^5 T_2^5 T_3^2 + 2 c_{49} T_1^5 T_2^5 T_3^2 + 2 c_{61} T_1^5 T_2^5 T_3^2 + 2 c_{64} T_1^5 T_2^5 T_3^2 \Big) \Big] \Big] \Bigg) \Big/$$

$$\Big(\big(1 - T_1 + T_1^2\big) \big(1 - T_2 + T_2^2\big) T_3 \big(1 - T_3 + T_3^2\big) \Big) \Bigg) \Big\}$$

» $\{2.9375,$

$$\text{Knot}[4,1] \to \Bigg(T_1^2\,T_2^2\,\mathbb{E}\Big[\in\text{Series}\Big[0,\ -\frac{1}{T_1\,(1-3\,T_1+T_1^2)\,T_2\,(1-3\,T_2+T_2^2)\,(1-3\,T_3+T_3^2)}\,\big(-c_{16}\,T_1\,T_2-c_{19}\,T_1\,T_2-$$
$$c_{31}\,T_1\,T_2-c_{34}\,T_1\,T_2-c_{46}\,T_1\,T_2-c_{49}\,T_1\,T_2-c_{61}\,T_1\,T_2-c_{64}\,T_1\,T_2+3\,c_{81}\,T_1\,T_2+3\,c_{82}\,T_1\,T_2+3\,c_{84}\,T_1\,T_2+$$
$$3\,c_{85}\,T_1\,T_2+3\,c_{19}\,T_1^2\,T_2+3\,c_{34}\,T_1^2\,T_2+3\,c_{49}\,T_1^2\,T_2+3\,c_{64}\,T_1^2\,T_2-5\,c_{81}\,T_1^2\,T_2-9\,c_{82}\,T_1^2\,T_2-2\,c_{84}\,T_1^2\,T_2-$$
$$9\,c_{85}\,T_1^2\,T_2+c_{16}\,T_1^3\,T_2-c_{19}\,T_1^3\,T_2+c_{31}\,T_1^3\,T_2-c_{34}\,T_1^3\,T_2+c_{46}\,T_1^3\,T_2-c_{49}\,T_1^3\,T_2+c_{61}\,T_1^3\,T_2-c_{64}\,T_1^3\,T_2+$$
$$2\,c_{81}\,T_1^3\,T_2+3\,c_{82}\,T_1^3\,T_2+3\,c_{85}\,T_1^3\,T_2+3\,c_{16}\,T_1\,T_2^2+3\,c_{31}\,T_1\,T_2^2+3\,c_{46}\,T_1\,T_2^2+3\,c_{61}\,T_1\,T_2^2-9\,c_{81}\,T_1\,T_2^2-$$
$$5\,c_{82}\,T_1\,T_2^2-9\,c_{84}\,T_1\,T_2^2-2\,c_{85}\,T_1\,T_2^2+15\,c_{81}\,T_1^2\,T_2^2+15\,c_{82}\,T_1^2\,T_2^2+6\,c_{84}\,T_1^2\,T_2^2+6\,c_{85}\,T_1^2\,T_2^2-3\,c_{16}\,T_1^3\,T_2^2-$$
$$3\,c_{31}\,T_1^3\,T_2^2-3\,c_{46}\,T_1^3\,T_2^2-3\,c_{61}\,T_1^3\,T_2^2-6\,c_{81}\,T_1^3\,T_2^2-5\,c_{82}\,T_1^3\,T_2^2-2\,c_{85}\,T_1^3\,T_2^2-c_{16}\,T_1\,T_2^3+c_{19}\,T_1\,T_2^3-$$
$$c_{31}\,T_1\,T_2^3+c_{34}\,T_1\,T_2^3-c_{46}\,T_1\,T_2^3+c_{49}\,T_1\,T_2^3-c_{61}\,T_1\,T_2^3+c_{64}\,T_1\,T_2^3+3\,c_{81}\,T_1\,T_2^3+2\,c_{82}\,T_1\,T_2^3+3\,c_{84}\,T_1\,T_2^3-$$
$$3\,c_{19}\,T_1^2\,T_2^3-3\,c_{34}\,T_1^2\,T_2^3-3\,c_{49}\,T_1^2\,T_2^3-3\,c_{64}\,T_1^2\,T_2^3-5\,c_{81}\,T_1^2\,T_2^3-6\,c_{82}\,T_1^2\,T_2^3-2\,c_{84}\,T_1^2\,T_2^3+c_{16}\,T_1^3\,T_2^3+$$
$$c_{19}\,T_1^3\,T_2^3+c_{31}\,T_1^3\,T_2^3+c_{34}\,T_1^3\,T_2^3+c_{46}\,T_1^3\,T_2^3+c_{49}\,T_1^3\,T_2^3+c_{61}\,T_1^3\,T_2^3+c_{64}\,T_1^3\,T_2^3+2\,c_{81}\,T_1^3\,T_2^3+2\,c_{82}\,T_1^3\,T_2^3-$$
$$3\,c_{82}\,T_1\,T_3-3\,c_{85}\,T_1\,T_3+9\,c_{82}\,T_1^2\,T_3+9\,c_{85}\,T_1^2\,T_3-3\,c_{82}\,T_1^3\,T_3-3\,c_{85}\,T_1^3\,T_3-3\,c_{81}\,T_2\,T_3-3\,c_{84}\,T_2\,T_3+$$
$$3\,c_{16}\,T_1\,T_2\,T_3+3\,c_{19}\,T_1\,T_2\,T_3+3\,c_{31}\,T_1\,T_2\,T_3+3\,c_{34}\,T_1\,T_2\,T_3+3\,c_{46}\,T_1\,T_2\,T_3+3\,c_{49}\,T_1\,T_2\,T_3+$$
$$3\,c_{61}\,T_1\,T_2\,T_3+3\,c_{64}\,T_1\,T_2\,T_3+3\,c_{81}\,T_1\,T_2\,T_3+3\,c_{82}\,T_1\,T_2\,T_3-9\,c_{19}\,T_1^2\,T_2\,T_3-9\,c_{34}\,T_1^2\,T_2\,T_3-$$
$$9\,c_{49}\,T_1^2\,T_2\,T_3-9\,c_{64}\,T_1^2\,T_2\,T_3+3\,c_{81}\,T_1^2\,T_2\,T_3-9\,c_{82}\,T_1^2\,T_2\,T_3+3\,c_{84}\,T_1^2\,T_2\,T_3-3\,c_{16}\,T_1^3\,T_2\,T_3+$$
$$3\,c_{19}\,T_1^3\,T_2\,T_3-3\,c_{31}\,T_1^3\,T_2\,T_3+3\,c_{34}\,T_1^3\,T_2\,T_3-3\,c_{46}\,T_1^3\,T_2\,T_3+3\,c_{49}\,T_1^3\,T_2\,T_3-3\,c_{61}\,T_1^3\,T_2\,T_3+$$
$$3\,c_{64}\,T_1^3\,T_2\,T_3-3\,c_{81}\,T_1^3\,T_2\,T_3+3\,c_{82}\,T_1^3\,T_2\,T_3+9\,c_{81}\,T_2^2\,T_3+9\,c_{84}\,T_2^2\,T_3-9\,c_{16}\,T_1\,T_2^2\,T_3-9\,c_{31}\,T_1\,T_2^2\,T_3-$$
$$9\,c_{46}\,T_1\,T_2^2\,T_3-9\,c_{61}\,T_1\,T_2^2\,T_3-9\,c_{81}\,T_1\,T_2^2\,T_3+3\,c_{82}\,T_1\,T_2^2\,T_3+3\,c_{85}\,T_1\,T_2^2\,T_3-9\,c_{81}\,T_1^2\,T_2^2\,T_3-$$
$$9\,c_{82}\,T_1^2\,T_2^2\,T_3-9\,c_{84}\,T_1^2\,T_2^2\,T_3-9\,c_{85}\,T_1^2\,T_2^2\,T_3+9\,c_{16}\,T_1^3\,T_2^2\,T_3+9\,c_{31}\,T_1^3\,T_2^2\,T_3+9\,c_{46}\,T_1^3\,T_2^2\,T_3+$$
$$9\,c_{61}\,T_1^3\,T_2^2\,T_3+9\,c_{81}\,T_1^3\,T_2^2\,T_3+3\,c_{82}\,T_1^3\,T_2^2\,T_3+3\,c_{85}\,T_1^3\,T_2^2\,T_3-3\,c_{81}\,T_2^3\,T_3-3\,c_{84}\,T_2^3\,T_3+3\,c_{16}\,T_1\,T_2^3\,T_3-$$
$$3\,c_{19}\,T_1\,T_2^3\,T_3+3\,c_{31}\,T_1\,T_2^3\,T_3-3\,c_{34}\,T_1\,T_2^3\,T_3+3\,c_{46}\,T_1\,T_2^3\,T_3-3\,c_{49}\,T_1\,T_2^3\,T_3+3\,c_{61}\,T_1\,T_2^3\,T_3-$$
$$3\,c_{64}\,T_1\,T_2^3\,T_3+3\,c_{81}\,T_1\,T_2^3\,T_3-3\,c_{82}\,T_1\,T_2^3\,T_3+9\,c_{19}\,T_1^2\,T_2^3\,T_3+9\,c_{34}\,T_1^2\,T_2^3\,T_3+9\,c_{49}\,T_1^2\,T_2^3\,T_3+$$
$$9\,c_{64}\,T_1^2\,T_2^3\,T_3+3\,c_{81}\,T_1^2\,T_2^3\,T_3+9\,c_{82}\,T_1^2\,T_2^3\,T_3+3\,c_{84}\,T_1^2\,T_2^3\,T_3-3\,c_{16}\,T_1^3\,T_2^3\,T_3-3\,c_{19}\,T_1^3\,T_2^3\,T_3-$$
$$3\,c_{31}\,T_1^3\,T_2^3\,T_3-3\,c_{34}\,T_1^3\,T_2^3\,T_3-3\,c_{46}\,T_1^3\,T_2^3\,T_3-3\,c_{49}\,T_1^3\,T_2^3\,T_3-3\,c_{61}\,T_1^3\,T_2^3\,T_3-3\,c_{64}\,T_1^3\,T_2^3\,T_3-$$
$$3\,c_{81}\,T_1^3\,T_2^3\,T_3-3\,c_{82}\,T_1^3\,T_2^3\,T_3+2\,c_{82}\,T_1\,T_3^2+2\,c_{85}\,T_1\,T_3^2-6\,c_{82}\,T_1^2\,T_3^2-6\,c_{85}\,T_1^2\,T_3^2+2\,c_{82}\,T_1^3\,T_3^2+$$
$$2\,c_{85}\,T_1^3\,T_3^2+2\,c_{81}\,T_2\,T_3^2+2\,c_{84}\,T_2\,T_3^2-c_{16}\,T_1\,T_2\,T_3^2-c_{19}\,T_1\,T_2\,T_3^2-c_{31}\,T_1\,T_2\,T_3^2-c_{34}\,T_1\,T_2\,T_3^2-$$
$$c_{46}\,T_1\,T_2\,T_3^2-c_{49}\,T_1\,T_2\,T_3^2-c_{61}\,T_1\,T_2\,T_3^2-c_{64}\,T_1\,T_2\,T_3^2-5\,c_{81}\,T_1\,T_2\,T_3^2-5\,c_{82}\,T_1\,T_2\,T_3^2-3\,c_{84}\,T_1\,T_2\,T_3^2-$$
$$3\,c_{85}\,T_1\,T_2\,T_3^2+3\,c_{19}\,T_1^2\,T_2\,T_3^2+3\,c_{34}\,T_1^2\,T_2\,T_3^2+3\,c_{49}\,T_1^2\,T_2\,T_3^2+3\,c_{64}\,T_1^2\,T_2\,T_3^2+3\,c_{81}\,T_1^2\,T_2\,T_3^2+$$
$$15\,c_{82}\,T_1^2\,T_2\,T_3^2+9\,c_{85}\,T_1^2\,T_2\,T_3^2+c_{16}\,T_1^3\,T_2\,T_3^2-c_{19}\,T_1^3\,T_2\,T_3^2+c_{31}\,T_1^3\,T_2\,T_3^2-c_{34}\,T_1^3\,T_2\,T_3^2+c_{46}\,T_1^3\,T_2\,T_3^2-$$
$$c_{49}\,T_1^3\,T_2\,T_3^2+c_{61}\,T_1^3\,T_2\,T_3^2-c_{64}\,T_1^3\,T_2\,T_3^2-5\,c_{82}\,T_1^3\,T_2\,T_3^2-3\,c_{85}\,T_1^3\,T_2\,T_3^2-6\,c_{81}\,T_2^2\,T_3^2-6\,c_{84}\,T_2^2\,T_3^2+$$
$$3\,c_{16}\,T_1\,T_2^2\,T_3^2+3\,c_{31}\,T_1\,T_2^2\,T_3^2+3\,c_{46}\,T_1\,T_2^2\,T_3^2+3\,c_{61}\,T_1\,T_2^2\,T_3^2+15\,c_{81}\,T_1\,T_2^2\,T_3^2+3\,c_{82}\,T_1\,T_2^2\,T_3^2+$$
$$9\,c_{84}\,T_1\,T_2^2\,T_3^2-9\,c_{81}\,T_1^2\,T_2^2\,T_3^2-9\,c_{82}\,T_1^2\,T_2^2\,T_3^2-3\,c_{16}\,T_1^3\,T_2^2\,T_3^2-3\,c_{31}\,T_1^3\,T_2^2\,T_3^2-3\,c_{46}\,T_1^3\,T_2^2\,T_3^2-$$
$$3\,c_{61}\,T_1^3\,T_2^2\,T_3^2+3\,c_{82}\,T_1^3\,T_2^2\,T_3^2+2\,c_{81}\,T_2^3\,T_3^2+2\,c_{84}\,T_2^3\,T_3^2-c_{16}\,T_1\,T_2^3\,T_3^2+c_{19}\,T_1\,T_2^3\,T_3^2-c_{31}\,T_1\,T_2^3\,T_3^2+$$
$$c_{34}\,T_1\,T_2^3\,T_3^2-c_{46}\,T_1\,T_2^3\,T_3^2+c_{49}\,T_1\,T_2^3\,T_3^2-c_{61}\,T_1\,T_2^3\,T_3^2+c_{64}\,T_1\,T_2^3\,T_3^2-5\,c_{81}\,T_1\,T_2^3\,T_3^2-3\,c_{84}\,T_1\,T_2^3\,T_3^2-$$
$$3\,c_{19}\,T_1^2\,T_2^3\,T_3^2-3\,c_{34}\,T_1^2\,T_2^3\,T_3^2-3\,c_{49}\,T_1^2\,T_2^3\,T_3^2-3\,c_{64}\,T_1^2\,T_2^3\,T_3^2+3\,c_{81}\,T_1^2\,T_2^3\,T_3^2+c_{16}\,T_1^3\,T_2^3\,T_3^2+$$
$$c_{19}\,T_1^3\,T_2^3\,T_3^2+c_{31}\,T_1^3\,T_2^3\,T_3^2+c_{34}\,T_1^3\,T_2^3\,T_3^2+c_{46}\,T_1^3\,T_2^3\,T_3^2+c_{49}\,T_1^3\,T_2^3\,T_3^2+c_{61}\,T_1^3\,T_2^3\,T_3^2+c_{64}\,T_1^3\,T_2^3\,T_3^2\big)\Big]\Big]\Bigg)\Big/$$
$$\big((1-3\,T_1+T_1^2)\,(1-3\,T_2+T_2^2)\,(1-3\,T_3+T_3^2)\big)\}$$

(Alt) Out[*]=

$Aborted

In[*]:= **tab1[[1]]**

(Alt) In[ ]:=

```
K = Knot["K11n34"]; Conway = ∫ℒ[K] ⅆ vs[K]
```

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

(Alt) Out[ ]=

$$-i\, \mathbb{E}\left[\epsilon\text{Series}\left[0, \frac{1}{T_1^3 T_2^3} 2 \left(c_{19} T_1^3 + c_{34} T_1^3 + c_{49} T_1^3 + c_{64} T_1^3 - 2 c_{19} T_1^3 T_2 - 2 c_{34} T_1^3 T_2 - 2 c_{49} T_1^3 T_2 - 2 c_{64} T_1^3 T_2 + c_{19} T_1^3 T_2^2 + c_{34} T_1^3 T_2^2 + c_{49} T_1^3 T_2^2 + c_{64} T_1^3 T_2^2 + c_{16} T_2^3 + c_{31} T_2^3 + c_{46} T_2^3 + c_{61} T_2^3 - 2 c_{16} T_1 T_2^3 - 2 c_{31} T_1 T_2^3 - 2 c_{46} T_1 T_2^3 - 2 c_{61} T_1 T_2^3 + c_{16} T_1^2 T_2^3 + c_{31} T_1^2 T_2^3 + c_{46} T_1^2 T_2^3 + c_{61} T_1^2 T_2^3 + c_{16} T_1^4 T_2^3 + c_{31} T_1^4 T_2^3 + c_{46} T_1^4 T_2^3 + c_{61} T_1^4 T_2^3 - 2 c_{16} T_1^5 T_2^3 - 2 c_{31} T_1^5 T_2^3 - 2 c_{46} T_1^5 T_2^3 - 2 c_{61} T_1^5 T_2^3 + c_{16} T_1^6 T_2^3 + c_{31} T_1^6 T_2^3 + c_{46} T_1^6 T_2^3 + c_{61} T_1^6 T_2^3 + c_{19} T_1^3 T_2^4 + c_{34} T_1^3 T_2^4 + c_{49} T_1^3 T_2^4 + c_{64} T_1^3 T_2^4 - 2 c_{19} T_1^3 T_2^5 - 2 c_{34} T_1^3 T_2^5 - 2 c_{49} T_1^3 T_2^5 - 2 c_{64} T_1^3 T_2^5 + c_{19} T_1^3 T_2^6 + c_{34} T_1^3 T_2^6 + c_{49} T_1^3 T_2^6 + c_{64} T_1^3 T_2^6\right)\right]\right]$$

(Alt) In[ ]:=

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K = Knot["K11n42"]; KT = ∫ℒ[K] ⅆ vs[K]
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(Alt) Out[ ]=

$$-\frac{1}{T_3} i\, T_1 T_2\, \mathbb{E}\left[\epsilon\text{Series}\left[0, \frac{1}{T_1^3 T_2^3} 2 \left(c_{19} T_1^3 + c_{34} T_1^3 + c_{49} T_1^3 + c_{64} T_1^3 - 2 c_{19} T_1^3 T_2 - 2 c_{34} T_1^3 T_2 - 2 c_{49} T_1^3 T_2 - 2 c_{64} T_1^3 T_2 + c_{19} T_1^3 T_2^2 + c_{34} T_1^3 T_2^2 + c_{49} T_1^3 T_2^2 + c_{64} T_1^3 T_2^2 + c_{16} T_2^3 + c_{31} T_2^3 + c_{46} T_2^3 + c_{61} T_2^3 - 2 c_{16} T_1 T_2^3 - 2 c_{31} T_1 T_2^3 - 2 c_{46} T_1 T_2^3 - 2 c_{61} T_1 T_2^3 + c_{16} T_1^2 T_2^3 + c_{31} T_1^2 T_2^3 + c_{46} T_1^2 T_2^3 + c_{61} T_1^2 T_2^3 + c_{16} T_1^4 T_2^3 + c_{31} T_1^4 T_2^3 + c_{46} T_1^4 T_2^3 + c_{61} T_1^4 T_2^3 - 2 c_{16} T_1^5 T_2^3 - 2 c_{31} T_1^5 T_2^3 - 2 c_{46} T_1^5 T_2^3 - 2 c_{61} T_1^5 T_2^3 + c_{16} T_1^6 T_2^3 + c_{31} T_1^6 T_2^3 + c_{46} T_1^6 T_2^3 + c_{61} T_1^6 T_2^3 + c_{19} T_1^3 T_2^4 + c_{34} T_1^3 T_2^4 + c_{49} T_1^3 T_2^4 + c_{64} T_1^3 T_2^4 - 2 c_{19} T_1^3 T_2^5 - 2 c_{34} T_1^3 T_2^5 - 2 c_{49} T_1^3 T_2^5 - 2 c_{64} T_1^3 T_2^5 + c_{19} T_1^3 T_2^6 + c_{34} T_1^3 T_2^6 + c_{49} T_1^3 T_2^6 + c_{64} T_1^3 T_2^6\right)\right]\right]$$

(Alt) In[ ]:=

$$\left(\frac{1}{T_1^3 T_2^3} 2 \left(c_{19} T_1^3 + c_{34} T_1^3 + c_{49} T_1^3 + c_{64} T_1^3 - 2 c_{19} T_1^3 T_2 - 2 c_{34} T_1^3 T_2 - 2 c_{49} T_1^3 T_2 - 2 c_{64} T_1^3 T_2 + c_{19} T_1^3 T_2^2 + c_{34} T_1^3 T_2^2 + c_{49} T_1^3 T_2^2 + c_{64} T_1^3 T_2^2 + c_{16} T_2^3 + c_{31} T_2^3 + c_{46} T_2^3 + c_{61} T_2^3 - 2 c_{16} T_1 T_2^3 - 2 c_{31} T_1 T_2^3 - 2 c_{46} T_1 T_2^3 - 2 c_{61} T_1 T_2^3 + c_{16} T_1^2 T_2^3 + c_{31} T_1^2 T_2^3 + c_{46} T_1^2 T_2^3 + c_{61} T_1^2 T_2^3 + c_{16} T_1^4 T_2^3 + c_{31} T_1^4 T_2^3 + c_{46} T_1^4 T_2^3 + c_{61} T_1^4 T_2^3 - 2 c_{16} T_1^5 T_2^3 - 2 c_{31} T_1^5 T_2^3 - 2 c_{46} T_1^5 T_2^3 - 2 c_{61} T_1^5 T_2^3 + c_{16} T_1^6 T_2^3 + c_{31} T_1^6 T_2^3 + c_{46} T_1^6 T_2^3 + c_{61} T_1^6 T_2^3 + c_{19} T_1^3 T_2^4 + c_{34} T_1^3 T_2^4 + c_{49} T_1^3 T_2^4 + c_{64} T_1^3 T_2^4 - 2 c_{19} T_1^3 T_2^5 - 2 c_{34} T_1^3 T_2^5 - 2 c_{49} T_1^3 T_2^5 - 2 c_{64} T_1^3 T_2^5 + c_{19} T_1^3 T_2^6 + c_{34} T_1^3 T_2^6 + c_{49} T_1^3 T_2^6 + c_{64} T_1^3 T_2^6\right)\right) ==$$

$$\left(\frac{1}{T_1^3 T_2^3} 2 \left(c_{19} T_1^3 + c_{34} T_1^3 + c_{49} T_1^3 + c_{64} T_1^3 - 2 c_{19} T_1^3 T_2 - 2 c_{34} T_1^3 T_2 - 2 c_{49} T_1^3 T_2 - 2 c_{64} T_1^3 T_2 + c_{19} T_1^3 T_2^2 + c_{34} T_1^3 T_2^2 + c_{49} T_1^3 T_2^2 + c_{64} T_1^3 T_2^2 + c_{16} T_2^3 + c_{31} T_2^3 + c_{46} T_2^3 + c_{61} T_2^3 - 2 c_{16} T_1 T_2^3 - 2 c_{31} T_1 T_2^3 - 2 c_{46} T_1 T_2^3 - 2 c_{61} T_1 T_2^3 + c_{16} T_1^2 T_2^3 + c_{31} T_1^2 T_2^3 + c_{46} T_1^2 T_2^3 + c_{61} T_1^2 T_2^3 + c_{16} T_1^4 T_2^3 + c_{31} T_1^4 T_2^3 + c_{46} T_1^4 T_2^3 + c_{61} T_1^4 T_2^3 - 2 c_{16} T_1^5 T_2^3 - 2 c_{31} T_1^5 T_2^3 - 2 c_{46} T_1^5 T_2^3 - 2 c_{61} T_1^5 T_2^3 + c_{16} T_1^6 T_2^3 + c_{31} T_1^6 T_2^3 + c_{46} T_1^6 T_2^3 + c_{61} T_1^6 T_2^3 + c_{19} T_1^3 T_2^4 + c_{34} T_1^3 T_2^4 + c_{49} T_1^3 T_2^4 + c_{64} T_1^3 T_2^4 - 2 c_{19} T_1^3 T_2^5 - 2 c_{34} T_1^3 T_2^5 - 2 c_{49} T_1^3 T_2^5 - 2 c_{64} T_1^3 T_2^5 + c_{19} T_1^3 T_2^6 + c_{34} T_1^3 T_2^6 + c_{49} T_1^3 T_2^6 + c_{64} T_1^3 T_2^6\right)\right)$$

(Alt) Out[ ]=

True