

Pensieve header: Finding the  $A_2$   $\mathcal{S}d=1$  invariant using undetermined coefficients.

Searching for  $Q + p_{xx} + \epsilon(ppx + 1 + px + ppx)$  solutions.

## Initialization

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank"];
Once[<< KnotTheory` ; << Rot.m];
<< FormalGaussianIntegration.m;
i_+ := i + 1;
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

Loading Rot.m from <http://drorbn.net/AP/Projects/HigherRank> to compute rotation numbers.

```
In[*]:= Features[Knot[8, 17]]
```

 KnotTheory: Loading precomputed data in PD4Knots`.

```
Out[*]=
```

```
Features[18,
C6[-1] C14[-1] X1,7[1] X3,9[-1] X5,13[-1] X8,16[1] X10,4[-1] X12,18[1] X15,2[-1] X17,11[1]]
```

```
In[*]:= T3 = T1 T2;
S = {x_, p_};
q[s_, i_, j_] := Sum[
  xv,i (pv,i+ - pv,i) + xv,j (pv,j+ - pv,j) + (T3^s - 1) xv,i (pv,i+ - pv,j+),
  {v, 3}];
L[Xi_,j_[s_]] :=
  T3^s E[q[s, i, j] + B^-1 r0[s, i, j] + E B r1[s, i, j] + E r42[s, i, j] + O[epsilon]^2];
(* gamma1[phi_, k_] := phi (3/2 - X1,k p1,k - X2,k p2,k - X3,k p3,k); *)
L[Ck_[0]] := E[Sum[xv,k (pv,k+ - pv,k), {v, 3}] + O[epsilon]^2];
L[Ck_[phi_]] :=
  T3^phi E[Sum[xv,k (pv,k+ - pv,k), {v, 3}] + B^-1 gamma0[phi, k] + E B gamma1[phi, k] + E gamma42[phi, k] + O[epsilon]^2];
ps_i := Sequence[p1,i, p2,i, p3,i];
xs_i := Sequence[x1,i, x2,i, x3,i];
vs_i := Sequence[ps_i, xs_i];
F[is___] := E[Sum[pi,i pv,i, {i, {is}}, {v, 3}]];
L[K_] := CF[L/@Features[K][[2]]];
vs[K_] := Union@@Table[{vs_i}, {i, Features[K][[1]]}]
```

```
In[*]:= VS_i
```

```
Out[*]=
```

```
Sequence[p1,i, p2,i, p3,i, x1,i, x2,i, x3,i]
```

## The Various Terms ( $r_0$ )

### The pxx Terms ( $r_0$ )

```
In[*]:= x = 0;
r0[1, i_, j_] := Evaluate[Sum[
  a++k p3,k3 x1,k1 x2,k2,
  {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
]];
r0[1, i, j]
```

Out[\*]=

$$a_1 p_{3,i} x_{1,i} x_{2,i} + a_2 p_{3,j} x_{1,i} x_{2,i} + a_5 p_{3,i} x_{1,j} x_{2,i} + a_6 p_{3,j} x_{1,j} x_{2,i} +$$

$$a_3 p_{3,i} x_{1,i} x_{2,j} + a_4 p_{3,j} x_{1,i} x_{2,j} + a_7 p_{3,i} x_{1,j} x_{2,j} + a_8 p_{3,j} x_{1,j} x_{2,j}$$

```
In[*]:= x = 0;
r0[-1, i_, j_] := Evaluate[Sum[
  d++k p3,k3 x1,k1 x2,k2,
  {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
]];
r0[-1, i, j]
```

Out[\*]=

$$d_1 p_{3,i} x_{1,i} x_{2,i} + d_2 p_{3,j} x_{1,i} x_{2,i} + d_5 p_{3,i} x_{1,j} x_{2,i} + d_6 p_{3,j} x_{1,j} x_{2,i} +$$

$$d_3 p_{3,i} x_{1,i} x_{2,j} + d_4 p_{3,j} x_{1,i} x_{2,j} + d_7 p_{3,i} x_{1,j} x_{2,j} + d_8 p_{3,j} x_{1,j} x_{2,j}$$

### The pxx Terms ( $r_1$ )

```
In[*]:= x = 0;
r1[1, i_, j_] := Evaluate[Sum[
  b++k x3,k3 p1,k1 p2,k2,
  {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
]];
r1[1, i, j]
```

Out[\*]=

$$b_1 p_{1,i} p_{2,i} x_{3,i} + b_5 p_{1,j} p_{2,i} x_{3,i} + b_3 p_{1,i} p_{2,j} x_{3,i} + b_7 p_{1,j} p_{2,j} x_{3,i} +$$

$$b_2 p_{1,i} p_{2,i} x_{3,j} + b_6 p_{1,j} p_{2,i} x_{3,j} + b_4 p_{1,i} p_{2,j} x_{3,j} + b_8 p_{1,j} p_{2,j} x_{3,j}$$

```
In[*]:= x = 0;
r1[-1, i_, j_] := Evaluate[Sum[
  e++k x3,k3 p1,k1 p2,k2,
  {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
]];
r1[-1, i, j]
```

Out[\*]=

$$e_1 p_{1,i} p_{2,i} x_{3,i} + e_5 p_{1,j} p_{2,i} x_{3,i} + e_3 p_{1,i} p_{2,j} x_{3,i} + e_7 p_{1,j} p_{2,j} x_{3,i} +$$

$$e_2 p_{1,i} p_{2,i} x_{3,j} + e_6 p_{1,j} p_{2,i} x_{3,j} + e_4 p_{1,i} p_{2,j} x_{3,j} + e_8 p_{1,j} p_{2,j} x_{3,j}$$

## The ppx Terms ( $r_{42}$ )

```
In[*]:= x = 0;
Short[r42[1, i_, j_] = Evaluate[Plus[
  Sum[
    C++x Xv1,k1 Pv1,k2 Xv2,k3 Pv2,k4,
    {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}, {k4, {i, j}}, {v1, 2}, {v2, v1, 3}
  ],
  Sum[
    C++x Xv,k1 Pv,k2,
    {k1, {i, j}}, {k2, {i, j}}, {v, 3}
  ],
  C++x
]]]
```

```
Out[*]//Short=
C93 + C81 p1,i X1,i + C84 p1,j X1,i + C1 p1,i^2 X1,i + <<86>> +
C75 p2,j p3,i X2,j X3,j + C60 p2,i p3,j X2,j X3,j + C80 p2,j p3,j X2,j X3,j
```

```
In[*]:= x = 0;
Short[r42[-1, i_, j_] = Evaluate[Plus[
  Sum[
    f++x Xv1,k1 Pv1,k2 Xv2,k3 Pv2,k4,
    {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}, {k4, {i, j}}, {v1, 2}, {v2, v1, 3}
  ],
  Sum[
    f++x Xv,k1 Pv,k2,
    {k1, {i, j}}, {k2, {i, j}}, {v, 3}
  ],
  f++x
]]]
```

```
Out[*]//Short=
f93 + f81 p1,i X1,i + f84 p1,j X1,i + f1 p1,i^2 X1,i + <<86>> +
f75 p2,j p3,i X2,j X3,j + f60 p2,i p3,j X2,j X3,j + f80 p2,j p3,j X2,j X3,j
```

### The $\gamma$ Terms ( $\gamma_0, \gamma_1, \gamma_{42}$ )

```
In[*]:=  $\kappa = 0$ ;
 $\gamma_0[1, k_] := Evaluate[g_{++\kappa} p_{3,k} x_{1,k} x_{2,k}];$ 
 $\gamma_1[1, k_] := Evaluate[g_{++\kappa} x_{3,k} p_{1,k} p_{2,k}];$ 
 $\gamma_{42}[1, k_] := Evaluate[Plus[
  Sum[g_{++\kappa} x_{v,k} p_{v,k}, \{v, 3\}],
  Sum[g_{++\kappa} x_{v1,k} p_{v1,k} x_{v2,k} p_{v2,k}, \{v1, 2\}, \{v2, v1, 3\}]
]];
 $\{\gamma_0[1, k], \gamma_0[1, k], \gamma_{42}[1, k]\}$$ 
```

Out[\*]=

$$\{g_1 p_{3,k} x_{1,k} x_{2,k}, g_1 p_{3,k} x_{1,k} x_{2,k}, g_3 p_{1,k} x_{1,k} + g_6 p_{1,k}^2 x_{1,k}^2 + g_4 p_{2,k} x_{2,k} + g_7 p_{1,k} p_{2,k} x_{1,k} x_{2,k} + g_9 p_{2,k}^2 x_{2,k}^2 + g_5 p_{3,k} x_{3,k} + g_8 p_{1,k} p_{3,k} x_{1,k} x_{3,k} + g_{10} p_{2,k} p_{3,k} x_{2,k} x_{3,k}\}$$

```
In[*]:=  $\kappa = 0$ ;
 $\gamma_0[-1, k_] := Evaluate[h_{++\kappa} p_{3,k} x_{1,k} x_{2,k}];$ 
 $\gamma_1[-1, k_] := Evaluate[h_{++\kappa} x_{3,k} p_{1,k} p_{2,k}];$ 
 $\gamma_{42}[-1, k_] := Evaluate[Plus[
  Sum[h_{++\kappa} x_{v,k} p_{v,k}, \{v, 3\}],
  Sum[h_{++\kappa} x_{v1,k} p_{v1,k} x_{v2,k} p_{v2,k}, \{v1, 2\}, \{v2, v1, 3\}]
]];
 $\{\gamma_0[-1, k], \gamma_0[-1, k], \gamma_{42}[-1, k]\}$$ 
```

Out[\*]=

$$\{h_1 p_{3,k} x_{1,k} x_{2,k}, h_1 p_{3,k} x_{1,k} x_{2,k}, h_3 p_{1,k} x_{1,k} + h_6 p_{1,k}^2 x_{1,k}^2 + h_4 p_{2,k} x_{2,k} + h_7 p_{1,k} p_{2,k} x_{1,k} x_{2,k} + h_9 p_{2,k}^2 x_{2,k}^2 + h_5 p_{3,k} x_{3,k} + h_8 p_{1,k} p_{3,k} x_{1,k} x_{3,k} + h_{10} p_{2,k} p_{3,k} x_{2,k} x_{3,k}\}$$

### Reidemeister 3b

```
In[*]:= Timing[ {LeftR3b} =
Cases[ [  $\int \mathcal{F}[i, j, k] \times \mathcal{L} / @ (X_{i,j}[1] X_{i^+,k}[1] X_{j^+,k^+}[1]) d\{VS_i, VS_j, VS_k, VS_{i^+}, VS_{j^+}, VS_{k^+}\}$ ,
E[ $\mathcal{E}_-$ ]  $\rightarrow \mathcal{E}, \infty$  ] ] ]
```

Out[\*]=

$$\{54.3438, \{ \in Series[ T_1^2 p_{1,2+i} \pi_{1,i} - (-1 + T_1) T_1 p_{1,2+j} \pi_{1,i} + (1 - T_1) p_{1,2+k} \pi_{1,i} + T_1 p_{1,2+j} \pi_{1,j} + (1 - T_1) p_{1,2+k} \pi_{1,j} + p_{1,2+k} \pi_{1,k} + T_2^2 p_{2,2+i} \pi_{2,i} - (-1 + T_2) T_2 p_{2,2+j} \pi_{2,i} + (1 - T_2) p_{2,2+k} \pi_{2,i} + \dots 33 \dots + \frac{a_3 T_1 T_2 p_{3,2+j} \pi_{1,j} \pi_{2,k}}{B} - \frac{(-a_3 - a_4 + a_3 T_1 T_2) p_{3,2+k} \pi_{1,j} \pi_{2,k}}{B} + \frac{a_7 T_1 T_2 p_{3,2+i} \pi_{1,k} \pi_{2,k}}{B} + \frac{a_7 T_1 T_2 p_{3,2+j} \pi_{1,k} \pi_{2,k}}{B} - \frac{2(-a_7 - a_8 + a_7 T_1 T_2) p_{3,2+k} \pi_{1,k} \pi_{2,k}}{B} + T_1^2 T_2^2 p_{3,2+i} \pi_{3,i} - T_1 T_2 (-1 + T_1 T_2) p_{3,2+j} \pi_{3,i} + (1 - T_1 T_2) p_{3,2+k} \pi_{3,i} + T_1 T_2 p_{3,2+j} \pi_{3,j} + (1 - T_1 T_2) p_{3,2+k} \pi_{3,j} + p_{3,2+k} \pi_{3,k}, 3(a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4 + a_5 b_5 + a_6 b_6 + a_7 b_7 + a_8 b_8 + 2 C_1 + C_2 + C_3 + 2 C_4 + C_5 + C_{16} + C_{17} + C_{18} + C_{19} + C_{20} + C_{31} + C_{34} + C_{46} + C_{49} + C_{61} + C_{62} + C_{63} + C_{64} + C_{65} + 2 C_{76} + C_{77} + C_{78} + 2 C_{79} + C_{80} + C_{81} + C_{82} + C_{83} + C_{90} + C_{91} + C_{92} + C_{93}) + \dots 498 \dots + (2 a_3 b_2 + 2 a_4 b_2 + a_7 b_2 + a_8 b_2 + 2 a_3 b_4 + 2 a_4 b_4 + a_7 b_4 + a_8 b_4 + 3 a_7 b_6 + 3 a_8 b_6 + \dots 45 \dots + a_7 b_2 T_1^2 T_2 + a_7 b_4 T_1^2 T_2 + 2 a_3 b_2 T_1 T_2^2 + a_7 b_2 T_1 T_2^2 + 3 a_7 b_6 T_1 T_2^2 + 2 c_{55} T_1 T_2^2 - a_7 b_2 T_1^2 T_2^2) p_2, \dots 1 \dots p_{\dots 1 \dots} \pi_{\dots 1 \dots} \pi_{3,k} ] ] \}$$

Full expression not available (original memory size: 3.8 MB)

```
In[*]:= Timing[ {RightR3b} =
Cases[ \int \mathcal{F}[i, j, k] \times \mathcal{L} / @ (X_{j,k}[1] X_{i,k^*}[1] X_{i^*,j^*}[1]) \mathcal{d} \{vs_i, vs_j, vs_k, vs_{i^*}, vs_{j^*}, vs_{k^*}\},
E[\mathcal{E}_-] \Rightarrow \mathcal{E}, \infty ] ; ]
```

Out[\*]= {43.4531, Null}

```
In[*]:= Short[eqn = CF[LeftR3b[[1]] - RightR3b[[1]]]
cvs = Union@Cases[eqn, p_ | \pi_, \infty]
vars = Union@Cases[r_0[1, i, j], a_, \infty]
Short[eqns = CoefficientRules[eqn, cvs] /. (\_ \to c_) \Rightarrow (c == 0), 3]
{sol} = Solve[eqns, vars]
```

Out[\*]//Short=

$$\frac{T_1 T_2 (-a_1 T_1 + \langle\langle 20 \rangle\rangle + a_7 T_1^2 T_2^2) p_{3,2+j} \pi_{1,i} \pi_{2,i}}{B} -$$

$$\frac{(\langle\langle 60 \rangle\rangle + a_7 T_1^3 T_2^3) \langle\langle 1 \rangle\rangle \langle\langle 1 \rangle\rangle \langle\langle 1 \rangle\rangle \pi_{\langle\langle 1 \rangle\rangle}}{B} + \langle\langle 32 \rangle\rangle + \frac{a_7 T_1 T_2 (-1 + T_1 T_2) p_{3,2+j} \pi_{1,k} \pi_{2,k}}{B}$$

Out[\*]= {p\_{3,2+i}, p\_{3,2+j}, p\_{3,2+k}, \pi\_{1,i}, \pi\_{1,j}, \pi\_{1,k}, \pi\_{2,i}, \pi\_{2,j}, \pi\_{2,k}}

Out[\*]= {a\_1, a\_2, a\_3, a\_4, a\_5, a\_6, a\_7, a\_8}

Out[\*]//Short=

$$\left\{ -\frac{a_3 T_1^2 T_2^2}{B} + \frac{a_3 T_1^2 T_2^3}{B} == 0, \frac{a_3 T_1^2 T_2}{B} - \frac{a_3 T_1^2 T_2^2}{B} == 0, \right.$$

$$\left. -\frac{a_5 T_1^2 T_2^2}{B} + \frac{a_5 T_1^3 T_2^2}{B} == 0, -\frac{a_7 T_1^2 T_2^2}{B} + \frac{a_7 T_1^3 T_2^2}{B} + \frac{a_7 T_1^2 T_2^3}{B} - \frac{a_7 T_1^3 T_2^3}{B} == 0, \right.$$

$$\langle\langle 18 \rangle\rangle, -\frac{a_7}{B} - \frac{a_8}{B} + \frac{a_7 T_1}{B} + \frac{a_8 T_1}{B} + \frac{a_7 T_1 T_2}{B} - \frac{a_7 T_1^2 T_2}{B} == 0,$$

$$\frac{a_7 T_2}{B} + \frac{a_8 T_2}{B} - \frac{a_7 T_2^2}{B} - \frac{a_8 T_2^2}{B} - \frac{a_7 T_1 T_2^2}{B} + \frac{a_7 T_1 T_2^3}{B} == 0,$$

$$\left. -\frac{a_7}{B} - \frac{a_8}{B} + \frac{a_7 T_2}{B} + \frac{a_8 T_2}{B} + \frac{a_7 T_1 T_2}{B} - \frac{a_7 T_1 T_2^2}{B} == 0 \right\}$$

Solve: Equations may not give solutions for all "solve" variables.

Out[\*]= { {a\_1 \to 0, a\_3 \to 0, a\_5 \to 0, a\_6 \to -\frac{a\_2}{T\_1} - \frac{a\_4 T\_2}{T\_1}, a\_7 \to 0, a\_8 \to 0} }

```
In[*]:= sol /. (v_ \to val_) \Rightarrow (v = CF[val]);
r_0[1, i, j]
```

Out[\*]=

$$a_2 p_{3,j} x_{1,i} x_{2,i} - \frac{(a_2 + a_4 T_2) p_{3,j} x_{1,j} x_{2,i}}{T_1} + a_4 p_{3,j} x_{1,i} x_{2,j}$$

```

In[*]:= Short[eqn = CF[Coefficient[
    LeftR3b[[2]] - RightR3b[[2]] /. v : (π | p) __ => μ v,
    μ^3
  ]], 5]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r1[1, i, j], b_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ -> c_) => (c == 0), 3]
{sol} = Solve[eqns, vars]

Out[*]//Short=
B b1 (-1 + T1) T1 T2^2 p1,2+j p2,2+i π3,i -
B b1 (-1 + T1) T1 T2^2 p1,2+k p2,2+i π3,i + B b1 T1^2 (-1 + T2) T2 p1,2+i p2,2+j π3,i -
B T1 T2 (-b1 T1 - b1 T2 + 2 b1 T1 T2 - b2 T1 T2 + b2 T1^2 T2^2) p1,2+j p2,2+j π3,i + <<23>> +
B b2 T1^2 (-1 + T2) T2 p1,2+i p2,2+j π3,k - B b2 (-1 + T1) T1 (-1 + T2) T2 p1,2+j p2,2+j π3,k -
B (-b2 - b6 + b2 T1) (-1 + T2) T2 p1,2+k p2,2+j π3,k +
B (-1 + T1) T1 (-b2 - b4 + b2 T2) p1,2+i p2,2+k π3,k - B (-1 + T1) T1 (-b2 - b4 + b2 T2) p1,2+j p2,2+k π3,k

Out[*]=
{p1,2+i, p1,2+j, p1,2+k, p2,2+i, p2,2+j, p2,2+k, π3,i, π3,j, π3,k}

Out[*]=
{b1, b2, b3, b4, b5, b6, b7, b8}

Out[*]//Short=
{-B b2 T1^2 T2^2 + B b2 T1^3 T2^3 == 0, B b2 T1 T2 - B b2 T1^2 T2^2 == 0, <<22>>,
 -B b6 T1 - B b8 T1 - B b4 T2 - B b8 T2 + B b2 T1 T2 + 2 B b4 T1 T2 + 2 B b6 T1 T2 +
 2 B b8 T1 T2 - B b2 T1^2 T2 - B b4 T1^2 T2 - B b2 T1 T2^2 - B b6 T1 T2^2 + B b2 T1^2 T2^2 == 0}

Solve: Equations may not give solutions for all "solve" variables. ⓘ

Out[*]=
{{b1 -> 0, b2 -> 0, b4 -> 0, b6 -> 0, b7 -> -b3 - b5, b8 -> 0}}

In[*]:= sol /. (v_ -> val_) => (v = CF[val]);
r1[1, i, j]

Out[*]=
b5 p1,j p2,i x3,i + b3 p1,i p2,j x3,i + (-b3 - b5) p1,j p2,j x3,i

```

```
In[*]:= Short[eqn = CF[LeftR3b[[2]] - RightR3b[[2]], 5]
cvs = Union@Cases[eqn, p_ | π_, ∞]
vars = Union@Cases[r42[1, i, j], c_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ -> c_) :-> (c == 0), 3]
Short[{sol} = Solve[eqns, vars]]
```

Out[\*]//Short=

$$\begin{aligned}
 & - \left( (2 c_{11} + 2 c_{41} + c_{42} + c_{43} + 2 c_{56} + c_{57} + c_{58} + 2 c_{71} + c_{87}) (-1 + T_1) T_1^2 p_{1,2+j} \pi_{1,i} \right) - \\
 & (-1 + T_1) \left( 4 c_1 + c_2 + c_3 + 2 c_6 + c_{16} + c_{17} + c_{18} + 2 c_{21} + c_{22} + c_{23} + c_{31} + 2 c_{36} + c_{37} + c_{38} + \right. \\
 & \quad c_{46} + c_{61} + 2 c_{66} + c_{81} + c_{84} + 2 c_{11} T_1 + c_{16} T_1 + c_{31} T_1 + 2 c_{41} T_1 + c_{42} T_1 + c_{43} T_1 + c_{46} T_1 + \\
 & \quad 2 c_{56} T_1 + c_{57} T_1 + c_{58} T_1 + c_{61} T_1 + c_{62} T_1 + c_{63} T_1 + 2 c_{71} T_1 + 4 c_{76} T_1 + c_{77} T_1 + c_{78} T_1 + c_{87} T_1 + \\
 & \quad \left. c_{90} T_1 - 2 c_{11} T_1^2 - 2 c_{41} T_1^2 - c_{42} T_1^2 - c_{43} T_1^2 - 2 c_{56} T_1^2 - c_{57} T_1^2 - c_{58} T_1^2 - 2 c_{71} T_1^2 - c_{87} T_1^2 \right) \\
 & p_{1,2+k} \pi_{1,i} + 2 c_1 (-1 + T_1) T_1^3 p_{1,2+i} p_{1,2+j} \pi_{1,i}^2 + \ll 473 \gg + \\
 & (-1 + T_2) T_2 (-c_{55} - c_{60} + c_{55} T_1 T_2) p_{2,2+i} p_{3,2+k} \pi_{2,k} \pi_{3,k} - \\
 & (-1 + T_2) T_2 (-c_{55} - c_{60} + c_{55} T_1 T_2) p_{2,2+j} p_{3,2+k} \pi_{2,k} \pi_{3,k}
 \end{aligned}$$

Out[\*]=

```
{p1,2+i, p1,2+j, p1,2+k, p2,2+i, p2,2+j, p2,2+k, p3,2+i,
p3,2+j, p3,2+k, π1,i, π1,j, π1,k, π2,i, π2,j, π2,k, π3,i, π3,j, π3,k}
```

Out[\*]=

```
{c1, c2, c3, c4, c5, c6, c7, c8, c9, c10, c11, c12, c13, c14, c15, c16, c17, c18, c19, c20,
c21, c22, c23, c24, c25, c26, c27, c28, c29, c30, c31, c32, c33, c34, c35, c36, c37, c38, c39,
c40, c41, c42, c43, c44, c45, c46, c47, c48, c49, c50, c51, c52, c53, c54, c55, c56, c57,
c58, c59, c60, c61, c62, c63, c64, c65, c66, c67, c68, c69, c70, c71, c72, c73, c74, c75,
c76, c77, c78, c79, c80, c81, c82, c83, c84, c85, c86, c87, c88, c89, c90, c91, c92, c93}
```

Out[\*]//Short=

$$\begin{aligned}
 & \{-c_{11} T_1^4 - c_{41} T_1^4 + c_{11} T_1^5 + c_{41} T_1^5 == 0, \\
 & c_{11} T_1^3 + c_{41} T_1^3 - c_{11} T_1^4 - c_{41} T_1^4 == 0, \ll 315 \gg, c_{13} T_1 T_2 + c_{15} T_1 T_2 + c_{73} T_1 T_2 + \\
 & c_{75} T_1 T_2 + c_{89} T_1 T_2 - c_{13} T_1^2 T_2^2 - c_{15} T_1^2 T_2^2 - c_{73} T_1^2 T_2^2 - c_{75} T_1^2 T_2^2 - c_{89} T_1^2 T_2^2 == 0\}
 \end{aligned}$$

Solve: Equations may not give solutions for all "solve" variables.

Out[\*]//Short=

$$\left\{ \left\{ c_1 \rightarrow 0, c_2 \rightarrow 0, c_3 \rightarrow 0, \ll 56 \gg, c_{90} \rightarrow -\frac{\ll 1 \gg}{\ll 1 \gg} - \ll 1 \gg, c_{91} \rightarrow -\frac{c_{82}}{T_2} - \frac{c_{85}}{T_2}, \right. \right. \\
 \left. \left. c_{92} \rightarrow -\frac{c_{83}}{T_1 T_2} - \frac{c_{86}}{T_1 T_2} - \frac{-a_2 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 - \ll 1 \gg - a_4 b_3 T_1 T_2 + a_4 b_5 T_1 T_2}{T_1^2 T_2 (-1 + T_1 T_2)} \right\} \right\}$$

```
In[*]:= sol /. (v_ -> val_) :-> (v = CF[val]);
```

In[\*]:= Short[CF[r<sub>42</sub>[1, i, j]], 20]

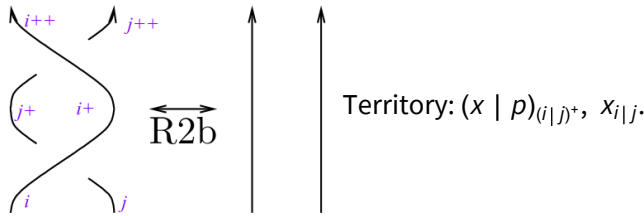
Out[\*]//Short=

$$\begin{aligned}
 & c_{93} + c_{81} p_{1,i} x_{1,i} + c_{84} p_{1,j} x_{1,i} + (c_6 + c_{21}) p_{1,i} p_{1,j} x_{1,i}^2 + \\
 & \frac{1}{2} (-1 + T_1) (2 c_6 + 2 c_{21} + c_{16} T_1 + c_{31} T_1 + c_{46} T_1 + c_{61} T_1) p_{1,j}^2 x_{1,i}^2 - \\
 & \frac{(c_{81} + c_{84}) p_{1,j} x_{1,j}}{T_1} + (c_{16} + c_{31} + c_{46} + c_{61}) p_{1,i} p_{1,j} x_{1,i} x_{1,j} + \\
 & \frac{1}{2} (-2 c_6 - c_{16} - 2 c_{21} - c_{31} - c_{46} - c_{61} - c_{16} T_1 - c_{31} T_1 - c_{46} T_1 - c_{61} T_1) p_{1,j}^2 x_{1,i} x_{1,j} + \\
 & c_{82} p_{2,i} x_{2,i} + c_{85} p_{2,j} x_{2,i} + c_{22} p_{1,j} p_{2,i} x_{1,i} x_{2,i} + c_7 p_{1,i} p_{2,j} x_{1,i} x_{2,i} + \\
 & \frac{1}{-1 + T_1 T_2} (a_2 b_3 + c_7 + c_{22} - c_7 T_1 + a_4 b_3 T_2 + a_2 b_5 T_2 - a_4 b_5 T_2 - c_{22} T_2 - \\
 & a_4 b_3 T_1 T_2 - c_7 T_1 T_2 - c_{22} T_1 T_2 + c_7 T_1^2 T_2 + a_4 b_5 T_2^2 + c_{22} T_1 T_2^2) p_{1,j} p_{2,j} x_{1,i} x_{2,i} + \\
 & \ll 18 \gg + \frac{b_3 (a_2 + a_4 T_2) p_{1,i} p_{3,j} x_{1,j} x_{3,i}}{T_1 (-1 + T_2)} - \frac{1}{(-1 + T_1) T_1 (-1 + T_2)} \\
 & (-a_2 b_3 + a_2 b_3 T_1 + a_2 b_5 T_1 + c_{23} T_1 - a_4 b_3 T_2 + a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + a_4 b_5 T_1 T_2 - c_{23} T_1 T_2 - c_{23} T_1^2 T_2 - \\
 & a_4 b_5 T_1 T_2^2 + c_{23} T_1^2 T_2^2) p_{1,j} p_{3,j} x_{1,j} x_{3,i} + c_{25} p_{2,j} p_{3,i} x_{2,i} x_{3,i} + c_{10} p_{2,i} p_{3,j} x_{2,i} x_{3,i} - \frac{1}{(-1 + T_1) T_1} \\
 & (-a_2 b_3 + a_2 b_3 T_1 - c_{10} T_1 - c_{25} T_1 + c_{10} T_1^2 + c_{25} T_1^2 - a_4 b_3 T_2 - a_2 b_5 T_2 + 2 a_4 b_3 T_1 T_2 + a_2 b_5 T_1 T_2 - \\
 & a_4 b_5 T_1 T_2 + c_{10} T_1 T_2 - a_4 b_3 T_1^2 T_2 - c_{10} T_1^2 T_2 + c_{25} T_1^2 T_2 - c_{25} T_1^3 T_2 - a_4 b_5 T_2^2 + 2 a_4 b_5 T_1 T_2^2) \\
 & p_{2,j} p_{3,j} x_{2,i} x_{3,i} + \frac{1}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} \\
 & (-a_2 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 + a_2 b_5 T_1 - a_4 b_5 T_1 + c_{10} T_1 - c_{25} T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 - c_{10} T_1^2 + c_{25} T_1^2 + \\
 & a_2 b_5 T_2 - a_4 b_5 T_2 - a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + 3 a_4 b_5 T_1 T_2 - c_{10} T_1 T_2 + a_4 b_3 T_1^2 T_2 + c_{10} T_1^2 T_2 + \\
 & c_{25} T_1^2 T_2 - c_{25} T_1^3 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2) p_{2,j} p_{3,i} x_{2,j} x_{3,i} - \frac{a_4 b_5 p_{2,i} p_{3,j} x_{2,j} x_{3,i}}{-1 + T_1} + \\
 & \frac{(-a_4 b_5 - c_{25} + c_{25} T_1 + a_4 b_3 T_2 + a_4 b_5 T_2 - a_4 b_3 T_1 T_2 + c_{25} T_1 T_2 - c_{25} T_1^2 T_2) p_{2,j} p_{3,j} x_{2,j} x_{3,i}}{(-1 + T_1) (-1 + T_2)} - \\
 & \frac{1}{T_1^2 T_2 (-1 + T_1 T_2)} \\
 & (-a_2 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 - c_{83} T_1 - c_{86} T_1 - a_4 b_5 T_2 - a_4 b_3 T_1 T_2 + a_4 b_5 T_1 T_2 + c_{83} T_1^2 T_2 + c_{86} T_1^2 T_2) \\
 & p_{3,j} x_{3,j} - \frac{(a_2 b_5 - a_4 b_5 - c_8 + c_{23} + c_8 T_1 + a_4 b_5 T_2 - c_{23} T_1 T_2) p_{1,i} p_{3,j} x_{1,i} x_{3,j}}{(-1 + T_1) (-1 + T_1 T_2)} - \\
 & \frac{(-a_2 b_3 + c_8 + a_2 b_3 T_1 - a_4 b_3 T_1 - c_8 T_1 - a_4 b_3 T_2 - c_8 T_2 + 2 a_4 b_3 T_1 T_2 + c_8 T_1 T_2) p_{1,j} p_{3,j} x_{1,i} x_{3,j}}{(-1 + T_2) (-1 + T_1 T_2)} - \\
 & \frac{(a_2 b_3 - c_{10} T_1 + c_{25} T_1 + a_4 b_3 T_2 - a_4 b_3 T_1 T_2 + c_{10} T_1 T_2 - c_{25} T_1^2 T_2) p_{2,i} p_{3,j} x_{2,i} x_{3,j}}{T_1 (-1 + T_2) (-1 + T_1 T_2)} - \\
 & \frac{1}{(-1 + T_1) T_1 (-1 + T_1 T_2)} \\
 & (c_{10} T_1 - c_{10} T_1^2 + a_2 b_5 T_2 - a_2 b_5 T_1 T_2 + a_4 b_5 T_1 T_2 - c_{10} T_1 T_2 + c_{10} T_1^2 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2) \\
 & p_{2,j} p_{3,j} x_{2,i} x_{3,j}
 \end{aligned}$$



```
In[*]:= CF[Leftr3b - RightR3b]
Out[*]=
eSeries[0, 0]
```

### Reidemeister 2b



```
In[*]:= Timing[Short[Leftr2b = (∫ F[i, j] × L / @ (Xi,j[1] Xi+,j+[-1]) d{vsi, vsj, vsi+, vsj+})][1]]
Out[*]=
```

$$\left\{ 3.60938, \text{eSeries} \left[ p_{1,2+i} \pi_{1,i} + p_{1,2+j} \pi_{1,j} + p_{2,2+i} \pi_{2,i} + \ll 10 \gg + p_{3,2+i} \pi_{3,i} + p_{3,2+j} \pi_{3,j}, \right. \right. \\ \left. \left. \frac{\ll 352 \gg + f_{93} T_1^3 T_2^2}{(-1 + T_1) T_1^2 (-1 + T_2) T_2} + \ll 98 \gg + \frac{(\ll 1 \gg) \ll 3 \gg \pi_{\ll 1 \gg}}{T_1 T_2^2} \right] \right\}$$

```
In[*]:= RightR2b = eSeries[p1,2+i π1,i + p1,2+j π1,j + p2,2+i π2,i + p2,2+j π2,j + p3,2+i π3,i + p3,2+j π3,j, 0]
Out[*]=
eSeries[p1,2+i π1,i + p1,2+j π1,j + p2,2+i π2,i + p2,2+j π2,j + p3,2+i π3,i + p3,2+j π3,j, 0]
```

```
In[*]:= Short[eqn = CF[LeftR2b[[1]] - RightR2b[[1]]]
cvs = Union@Cases[eqn, p_ | π_, ∞]
vars = Union@Cases[r_[-1, i, j], d_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ -> c_) :-> (c == 0), 3]
{sol} = Solve[eqns, vars]
```

Out[\*]//Short=

$$\frac{(d_7 + d_3 T_1 - d_7 T_1 + d_5 T_2 - d_7 T_2 + d_1 T_1 T_2 - d_3 T_1 T_2 - d_5 T_1 T_2 + d_7 T_1 T_2) \ll 2 \gg \pi_{\ll 1 \gg}}{B T_1 T_2} + \frac{(-d_7 + d_7 T_1 T_2 + d_8 T_1 T_2) \ll 2 \gg \pi_{\ll 1 \gg}}{B T_1 T_2} + \frac{\ll 1 \gg \ll 3 \gg}{B T_1 T_2} + \frac{\ll 5 \gg}{\ll 1 \gg}$$

Out[\*]=

$$\{p_{3,2+i}, p_{3,2+j}, \pi_{1,i}, \pi_{1,j}, \pi_{2,i}, \pi_{2,j}\}$$

Out[\*]=

$$\{d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8\}$$

Out[\*]//Short=

$$\left\{ \begin{aligned} \frac{d_1}{B} - \frac{d_3}{B} - \frac{d_5}{B} + \frac{d_7}{B} + \frac{d_5}{B T_1} - \frac{d_7}{B T_1} + \frac{d_3}{B T_2} - \frac{d_7}{B T_2} + \frac{d_7}{B T_1 T_2} &= 0, \frac{d_3}{B T_2} - \frac{d_7}{B T_2} + \frac{d_7}{B T_1 T_2} = 0, \\ \frac{d_5}{B T_1} - \frac{d_7}{B T_1} + \frac{d_7}{B T_1 T_2} &= 0, \frac{d_7}{B T_1 T_2} = 0, \frac{a_2}{B} - \frac{d_1}{B} + \frac{d_3}{B} + \ll 32 \gg + \frac{d_7 T_1 T_2}{B} + \frac{d_8 T_1 T_2}{B} = 0, \\ \frac{a_4}{B} + \frac{d_7}{B} + \frac{d_8}{B} + \frac{d_3 T_1}{B} + \frac{d_4 T_1}{B} - \frac{d_7 T_1}{B} - \frac{d_8 T_1}{B} - \frac{d_3}{B T_2} + \frac{d_7}{B T_2} - \frac{d_7}{B T_1 T_2} &= 0, \\ \frac{d_7}{B} + \frac{d_8}{B} - \frac{a_2}{B T_1} - \frac{d_5}{B T_1} + \frac{d_7}{B T_1} - \frac{d_7}{B T_1 T_2} + \frac{d_5 T_2}{B} + \frac{d_6 T_2}{B} - \frac{d_7 T_2}{B} - \frac{d_8 T_2}{B} - \frac{a_4 T_2}{B T_1} &= 0, \\ \frac{d_7}{B} + \frac{d_8}{B} - \frac{d_7}{B T_1 T_2} &= 0 \end{aligned} \right\}$$

Out[\*]=

$$\left\{ \left\{ d_1 \rightarrow 0, d_2 \rightarrow -\frac{a_2 - a_4 T_1 + a_4 T_2}{T_1^2 T_2}, d_3 \rightarrow 0, d_4 \rightarrow -\frac{a_4}{T_1}, d_5 \rightarrow 0, d_6 \rightarrow -\frac{-a_2 - a_4 T_2}{T_1 T_2}, d_7 \rightarrow 0, d_8 \rightarrow 0 \right\} \right\}$$

```
In[*]:= sol /. (v_ -> val_) :-> (v = CF[val]);
r_[-1, i, j]
```

Out[\*]=

$$\frac{(-a_2 + a_4 T_1 - a_4 T_2) p_{3,j} x_{1,i} x_{2,i}}{T_1^2 T_2} + \frac{(a_2 + a_4 T_2) p_{3,j} x_{1,j} x_{2,i}}{T_1 T_2} - \frac{a_4 p_{3,j} x_{1,i} x_{2,j}}{T_1}$$

```
In[*]:= Short[eqn = CF[LeftR2b[[2]] - RightR2b[[2]]]
cvs = Union@Cases[eqn, p_ | π_, ∞]
vars = Union@Cases[r1[-1, i, j] + r42[-1, i, j], e_ | f_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ -> c_) -> (c == 0), 3]
Short[{sol} = Solve[eqns, vars]]
```

Out[\*]//Short=

$$\llcorner 107 \gg + \frac{\llcorner 1 \gg}{\llcorner 1 \gg \llcorner 1 \gg} + \frac{(f_{55} + \llcorner 16 \gg + f_{80} T_1 T_2^2) \llcorner 3 \gg \pi_{\llcorner 1 \gg}}{T_1 T_2^2}$$

Out[\*]=

{p1,2+i, p1,2+j, p2,2+i, p2,2+j, p3,2+i, p3,2+j, π1,i, π1,j, π2,i, π2,j, π3,i, π3,j}

Out[\*]=

{e1, e2, e3, e4, e5, e6, e7, e8, f1, f2, f3, f4, f5, f6, f7, f8, f9, f10, f11, f12, f13, f14, f15, f16, f17, f18, f19, f20, f21, f22, f23, f24, f25, f26, f27, f28, f29, f30, f31, f32, f33, f34, f35, f36, f37, f38, f39, f40, f41, f42, f43, f44, f45, f46, f47, f48, f49, f50, f51, f52, f53, f54, f55, f56, f57, f58, f59, f60, f61, f62, f63, f64, f65, f66, f67, f68, f69, f70, f71, f72, f73, f74, f75, f76, f77, f78, f79, f80, f81, f82, f83, f84, f85, f86, f87, f88, f89, f90, f91, f92, f93}

Out[\*]//Short=

$$\left\{ \begin{aligned} & f_1 - f_{11} - f_{41} + f_{51} + \frac{f_{51}}{T_1^2} + \frac{f_{11}}{T_1} + \frac{f_{41}}{T_1} - \frac{2 f_{51}}{T_1} = 0, \\ & \frac{2 f_{51}}{T_1^2} + \frac{f_{11}}{T_1} + \frac{\llcorner 1 \gg}{\llcorner 1 \gg} - \frac{2 f_{51}}{T_1} = 0, \llcorner 83 \gg, \llcorner 1 \gg = 0, \\ & \frac{2 a_4 b_3}{(1 - T_1)(1 - T_2)} + \frac{c_{16}}{(1 - T_1)(1 - T_2)} + \frac{c_{19}}{(1 - T_1)(1 - T_2)} + \frac{c_{31}}{(1 - T_1)(1 - T_2)} + \frac{c_{34}}{(1 - T_1)(1 - T_2)} + \\ & \frac{c_{46}}{(1 - T_1)(1 - T_2)} + \llcorner 327 \gg + \frac{f_{82} T_1 T_2}{(1 - T_1)(1 - T_2)} + \frac{f_{83} T_1 T_2}{(1 - T_1)(1 - T_2)} + \\ & \frac{f_{90} T_1 T_2}{(1 - T_1)(1 - T_2)} + \frac{f_{91} T_1 T_2}{(1 - T_1)(1 - T_2)} + \frac{f_{92} T_1 T_2}{(1 - T_1)(1 - T_2)} + \frac{f_{93} T_1 T_2}{(1 - T_1)(1 - T_2)} = 0 \end{aligned} \right\}$$

 Solve: Equations may not give solutions for all "solve" variables. 

Out[\*]//Short=

{{e1 -> 0, e2 -> 0, \llcorner 84 \gg, f93 -> -c93}}

```
In[*]:= sol /. (v_ -> val_) -> (v = CF[val]);
```

```
In[*]:= r1[-1, i, j]
Short[CF[r42[-1, i, j]], 5]
```

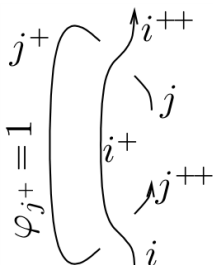
$$\text{Out[*]} = -\frac{b_5 p_{1,j} p_{2,i} x_{3,i}}{T_1} - \frac{b_3 p_{1,i} p_{2,j} x_{3,i}}{T_2} + \frac{(b_3 T_1 + b_5 T_2) p_{1,j} p_{2,j} x_{3,i}}{T_1 T_2}$$

$$\begin{aligned} \text{Out[*]//Short} = & -c_{93} - c_{81} p_{1,i} x_{1,i} + \ll 54 \gg + \\ & \frac{(a_2 b_3 - c_{10} T_1 + c_{25} T_1 + a_4 b_3 T_2 - a_4 b_3 T_1 T_2 + c_{10} T_1 T_2 - c_{25} T_1^2 T_2) p_{2,i} p_{3,j} x_{2,i} x_{3,j}}{T_1 (-1 + T_2) (-1 + T_1 T_2)} + \\ & \left( (a_2 b_3 - a_2 b_3 T_1 + c_{25} T_1 - c_{25} T_1^2 + a_4 b_3 T_2 + a_2 b_5 T_2 - 2 a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + a_4 b_5 T_1 T_2 + a_4 b_3 T_1^2 T_2 - \right. \\ & \left. c_{25} T_1^2 T_2 + c_{25} T_1^3 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2) p_{2,j} p_{3,j} x_{2,i} x_{3,j} \right) / ((-1 + T_1) T_1 T_2 (-1 + T_1 T_2)) \end{aligned}$$

```
In[*]:= CF[LeftR2b - RightR2b]
```

```
Out[*]=
Series[0, 0]
```

### Reidemeister 2c



```
In[*]:= Timing[ Short[ {LeftR2c} = Cases [
  Integrate[ F[i, j] x L / @ (X_{i+1,j}[1] X_{i,j+2}[-1] C_{j+1}[1]) d[ {vs_i, vs_j, vs_{i+}, vs_{j+}, vs_{j+2}}, E[ e_- ] => e ]
]] ]
```

$$\text{Out[*]} = \left\{ 4.29688, \left\{ \text{Series} \left[ p_{1,2+i} \pi_{1,i} + p_{1,3+j} \pi_{1,j} + p_{2,2+i} \pi_{2,i} + \frac{g_1 (-1 + T_1) \ll 3 \gg \pi_{2,i}}{B T_1 T_2} - \frac{\ll 1 \gg}{B \ll 1 \gg \ll 1 \gg} + \ll 1 \gg - \frac{g_1 \ll 3 \gg \pi_{\ll 1 \gg}}{B T_1} + \frac{g_1 p_{3,3+j} \pi_{1,j} \pi_{2,j}}{B} + p_{3,2+i} \pi_{3,i} + p_{3,3+j} \pi_{3,j}, \ll 1 \gg \right] \right\} \right\}$$

```
In[*]:= Timing [ Short [ { RightR2c } =
Cases [ ∫  $\mathcal{F}[i, j] \times \mathcal{L} / @ (C_i[\theta] C_{i+1}[\theta] C_j[\theta] C_{j+1}[1] C_{j+2}[\theta]) \mathbb{d} \{vs_i, vs_j, vs_{i+}, vs_{j+}, vs_{j+2}\},$ 
E [  $\mathcal{E}_-$  ]  $\Rightarrow \mathcal{E}$  ]
]]
```

```
Out[*]= { 0.125,
{ ∈Series [  $p_{1,2+i} \pi_{1,i} + p_{1,3+j} \pi_{1,j} + p_{2,2+i} \pi_{2,i} + p_{2,3+j} \pi_{2,j} + \frac{g_1 p_{3,3+j} \pi_{1,j} \pi_{2,j}}{B} + p_{3,2+i} \pi_{3,i} + p_{3,3+j} \pi_{3,j},$ 
 $g_1 g_2 + \ll 16 \gg + (g_1 g_2 + g_{10}) p_2, \ll 1 \gg p_{\ll 1 \gg} \pi_2 \ll 1 \gg \ll 1 \gg \pi_{3,j} ] \} \}$ 
```

```
In[*]:= Short [ eqn = CF [ LeftR2c [[1]] - RightR2c [[1]] ]
cvs = Union@Cases [ eqn, p__ |  $\pi_{-}, \infty$  ]
vars = Union@Cases [  $\gamma_0[1, k], g_-, \infty$  ]
Short [ eqns = CoefficientRules [ eqn, cvs ] /. ( _  $\rightarrow c_-$  )  $\Rightarrow (c = 0), 3$  ]
{ sol } = Solve [ eqns, vars ]
```

```
Out[*]//Short=  $\frac{g_1 (-1 + T_1) (-1 + T_2) p_{3,3+j} \pi_{1,i} \pi_{2,i}}{B T_1 T_2} - \frac{g_1 (-1 + T_2) p_{3, \ll 1 \gg} \pi_{1,j} \pi_{2,i}}{B T_2} - \frac{g_1 (-1 + T_1) p_{3,3+j} \pi_{1,i} \pi_{2,j}}{B T_1}$ 
```

```
Out[*]= { p_{3,3+j},  $\pi_{1,i}, \pi_{1,j}, \pi_{2,i}, \pi_{2,j}$  }
```

```
Out[*]= { g_1 }
```

```
Out[*]//Short= {  $\frac{g_1}{B} - \frac{g_1}{B T_1} - \frac{g_1}{B T_2} + \frac{g_1}{B T_1 T_2} == 0, -\frac{g_1}{B} + \frac{g_1}{B T_1} == 0, -\frac{g_1}{B} + \frac{g_1}{B T_2} == 0$  }
```

```
Out[*]= { { g_1  $\rightarrow 0$  } }
```

```
In[*]:= sol /. ( v_  $\rightarrow val_-$  )  $\Rightarrow (v = CF [ val ] ) ;$ 
 $\gamma_0[1, k]$ 
```

```
Out[*]= 0
```

```
In[*]:= Short[eqn = CF[LeftR2c[[2]] - RightR2c[[2]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[γ1[1, k] + γ42[1, k], g_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ -> c_) :-> (c == 0), 3]
Short[{sol} = Solve[eqns, vars]]
```

Out[\*]//Short=

$$\frac{(c_{16} + c_{31} + c_{46} + c_{61} - g_3 - 4 g_6 - g_7 - g_8) (-1 + T_1) p_{1,3+j} \pi_{1,i}}{T_1} + \frac{g_6 \langle\langle 1 \rangle\rangle^2 \langle\langle 1 \rangle\rangle \pi_{\langle\langle 1 \rangle\rangle}^2 - \langle\langle 1 \rangle\rangle}{T_1^2} - \frac{\langle\langle 1 \rangle\rangle}{T_1} + \langle\langle 22 \rangle\rangle$$

Out[\*]=

```
{p1,3+j, p2,3+j, p3,3+j, π1,i, π1,j, π2,i, π2,j, π3,i, π3,j}
```

Out[\*]=

```
{g2, g3, g4, g5, g6, g7, g8, g9, g10}
```

Out[\*]//Short=

$$\left\{ g_6 + \frac{g_6}{T_1^2} - \frac{2 g_6}{T_1} = 0, -2 g_6 + \frac{2 g_6}{T_1} = 0, \langle\langle 13 \rangle\rangle, c_{19} + \langle\langle 22 \rangle\rangle + \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle} = 0, \right.$$

$$\frac{2 a_4 b_3}{(1 - T_1) (1 - T_2)} + \frac{a_4 b_3}{(1 - T_1) T_1^2 (1 - T_2)} + \frac{a_2 b_5}{(1 - T_1) T_1^2 (1 - T_2)} - \frac{a_4 b_5}{(1 - T_1) T_1^2 (1 - T_2)} -$$

$$\frac{3 a_4 b_3}{(1 - T_1) T_1 (1 - T_2)} - \frac{a_2 b_5}{(1 - T_1) T_1 (1 - T_2)} + \langle\langle 33 \rangle\rangle + \frac{g_{10} T_2}{(1 - T_1) (1 - T_2)} + \frac{a_4 b_5 T_2}{(1 - T_1) T_1^2 (1 - T_2)} -$$

$$\left. \frac{2 a_4 b_5 T_2}{(1 - T_1) T_1 (1 - T_2)} - \frac{g_5 T_1 T_2}{(1 - T_1) (1 - T_2)} - \frac{g_8 T_1 T_2}{(1 - T_1) (1 - T_2)} - \frac{g_{10} T_1 T_2}{(1 - T_1) (1 - T_2)} = 0 \right\}$$

Out[\*]//Short=

$$\left\{ \left\{ g_2 \rightarrow 0, g_3 \rightarrow c_{16} + c_{31} + c_{46} + c_{61}, g_4 \rightarrow c_{19} + c_{34} + c_{49} + c_{64}, \right. \right.$$

$$\left. g_5 \rightarrow -\frac{-a_2 b_3 + a_2 b_5 + \langle\langle 22 \rangle\rangle + 2 a_4 b_5 T_1 T_2^2}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)}, g_6 \rightarrow 0, g_7 \rightarrow 0, g_8 \rightarrow 0, g_9 \rightarrow 0, g_{10} \rightarrow 0 \right\}$$

```
In[*]:= sol /. (v_ -> val_) :-> (v = CF[val]);
```

```
In[*]:= γ1[1, k]
Short[CF[γ42[1, k]], 5]
```

Out[\*]=

```
0
```

Out[\*]//Short=

$$\frac{(c_{16} + c_{31} + c_{46} + c_{61}) p_{1,k} x_{1,k} + (c_{19} + c_{34} + c_{49} + c_{64}) p_{2,k} x_{2,k} + (-b_3 + b_5 + b_3 T_1 - b_5 T_2) (-a_2 + a_2 T_1 - a_4 T_1 - a_4 T_2 + 2 a_4 T_1 T_2) p_{3,k} x_{3,k}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)}$$

```
In[*]:= CF[LeftR2c - RightR2c]
```

Out[\*]=

```
Series[0, 0]
```

## C<sub>k</sub>[1] and C<sub>k</sub>[-1] are inverses

```
In[*]:= Timing [ Short [ { LeftCC } = Cases [ { {  $\int \mathcal{F}[k] \times \mathcal{L} / @ (C_k[1] C_{k+1}[-1]) \mathbb{d} \{ \mathbf{vS}_k, \mathbf{vS}_{k^*} \} \}, \mathbb{E}[\mathcal{E}_-] \Rightarrow \mathcal{E} \}$  } ] ] ]
```

Out[\*]=

$$\left\{ 0.109375, \left\{ \in \text{Series} \left[ p_{1,2+k} \pi_{1,k} + p_{2,2+k} \pi_{2,k} + \frac{h_1 p_{3,2+k} \pi_{1,k} \pi_{2,k}}{B} + p_{3,2+k} \pi_{3,k}, \right. \right. \\ \left. \left. \frac{\ll 177 \gg + h_{10} T_1^3 T_2^2}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} + \ll 8 \gg + (h_1 h_2 + h_{10}) p_{2, \ll 1 \gg} \ll 1 \gg \pi_{\ll 1 \gg} \pi_{3,k} \right] \right\} \right\}$$

```
In[*]:= Timing [ Short [ { RightCC } = Cases [ { {  $\int \mathcal{F}[k] \times \mathcal{L} / @ (C_k[0] C_{k+1}[0]) \mathbb{d} \{ \mathbf{vS}_k, \mathbf{vS}_{k^*} \} \}, \mathbb{E}[\mathcal{E}_-] \Rightarrow \mathcal{E} \}$  } ] ] ]
```

Out[\*]=

$$\{ 0.015625, \{ \in \text{Series} [ p_{1,2+k} \pi_{1,k} + p_{2,2+k} \pi_{2,k} + p_{3,2+k} \pi_{3,k}, 0 ] \} \}$$

```
In[*]:= Short [ eqn = CF [ LeftCC[[1]] - RightCC[[1]] ]
cvs = Union@Cases [ eqn, p__ |  $\pi_{__}$ ,  $\infty$  ]
vars = Union@Cases [  $\gamma_0[-1, k]$ , h_,  $\infty$  ]
Short [ eqns = CoefficientRules [ eqn, cvs ] /. ( _ -> c_ ) => ( c == 0 ), 3 ]
{ sol } = Solve [ eqns, vars ]
```

Out[\*]//Short=

$$\frac{h_1 p_{3,2+k} \pi_{1,k} \pi_{2,k}}{B}$$

Out[\*]=

$$\{ p_{3,2+k}, \pi_{1,k}, \pi_{2,k} \}$$

Out[\*]=

$$\{ h_1 \}$$

Out[\*]//Short=

$$\left\{ \frac{h_1}{B} == 0 \right\}$$

Out[\*]=

$$\{ \{ h_1 \rightarrow 0 \} \}$$

```
In[*]:= sol /. ( v_ -> val_ ) => ( v = CF [ val ] );
 $\gamma_0[-1, k]$ 
```

Out[\*]=

$$0$$

```

In[*]:= Short[eqn = CF[LeftCC[[2]] - RightCC[[2]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[γ1[-1, k] + γ42[-1, k], h_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ -> c_) :-> (c == 0), 3]
Short[{sol} = Solve[eqns, vars]]

Out[*]//Short=

$$\frac{a_2 b_3 - a_2 b_5 - 2 a_2 b_3 T_1 + \ll 161 \gg + h_8 T_1^3 T_2^2 + 2 h_9 T_1^3 T_2^2 + h_{10} T_1^3 T_2^2}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} + \ll 8 \gg + h_{10} p_{2,2+k} p_{3,2+k} \pi_{2,k} \pi_{3,k}$$


Out[*]=
{p1,2+k, p2,2+k, p3,2+k, π1,k, π2,k, π3,k}

Out[*]=
{h2, h3, h4, h5, h6, h7, h8, h9, h10}

Out[*]//Short=
{h6 == 0, h7 == 0, B h2 == 0, h8 == 0, c16 + c31 + <<4>> + h7 + h8 == 0, h9 == 0,
h10 == 0, c19 + c34 + c49 + c64 + h4 + h7 + 4 h9 + h10 == 0,  $\frac{2 a_2 b_3}{(1 - T_1) (1 - T_2) (1 - T_1 T_2)} -$ 
 $\frac{a_4 b_3}{(1 - T_1) (1 - T_2) (1 - T_1 T_2)} - \frac{a_2 b_5}{(1 - T_1) \ll 1 \gg \ll 1 \gg (\ll 1 \gg)}$  + <<48>> == 0,
 $\frac{2 a_2 b_3}{(1 - T_1) (1 - T_2) (1 - T_1 T_2)} - \frac{a_4 b_3}{(1 - T_1) (1 - T_2) (1 - T_1 T_2)} - \frac{a_2 b_5}{(1 - T_1) (1 - T_2) (1 - T_1 T_2)} +$ 
 $\frac{a_4 b_5}{(1 - T_1) (1 - T_2) (1 - T_1 T_2)} + \frac{c_{16}}{(1 - T_1) (1 - T_2) (1 - T_1 T_2)} + \frac{\ll 1 \gg}{\ll 1 \gg} + \ll 162 \gg == 0$ }

Out[*]//Short=
{{h2 -> 0, h3 -> -c16 - c31 - c46 - c61, h4 -> -c19 - c34 - c49 - c64,
h5 -> - $\frac{\ll 22 \gg + \ll 1 \gg - 2 a_4 b_5 T_1 T_2^2}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)}$ , h6 -> 0, h7 -> 0, h8 -> 0, h9 -> 0, h10 -> 0}}

In[*]:= sol /. (v_ -> val_) :-> (v = CF[val]);

In[*]:= γ1[-1, k]
Short[CF[γ42[-1, k]], 5]

Out[*]=
0

Out[*]//Short=

$$\frac{(-c_{16} - c_{31} - c_{46} - c_{61}) p_{1,k} x_{1,k} + (-c_{19} - c_{34} - c_{49} - c_{64}) p_{2,k} x_{2,k} - (-b_3 + b_5 + b_3 T_1 - b_5 T_2) (-a_2 + a_2 T_1 - a_4 T_1 - a_4 T_2 + 2 a_4 T_1 T_2) p_{3,k} x_{3,k}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)}$$

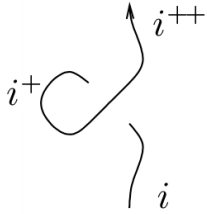

In[*]:= CF[LeftCC - RightCC]

Out[*]=
Series[0, 0]

```

## Invariance Under R1





```

In[*]:= {LeftR11} = Cases[{{∫ ℱ[i] × ℒ /@ (X_{i+2,i}[1] C_{i+1}[1]) d{v_{s_i}, v_{s_{i+}}, v_{s_{i+2}}}}, E[ℰ_] := ℰ, ∞]}
Out[*]:=
{∈Series[p_{1,3+i} π_{1,i} + p_{2,3+i} π_{2,i} + p_{3,3+i} π_{3,i}, c_{93}]}

In[*]:= {RightR11} = Cases[{{∫ ℱ[i] × ℒ /@ (C_i[0] C_{i+1}[0] C_{i+2}[0]) d{v_{s_i}, v_{s_{i+}}, v_{s_{i+2}}}}, E[ℰ_] := ℰ, ∞]}
Out[*]:=
{∈Series[p_{1,3+i} π_{1,i} + p_{2,3+i} π_{2,i} + p_{3,3+i} π_{3,i}, 0]}

In[*]:= LeftR11[[1]] == RightR11[[1]]
Out[*]:=
True

In[*]:= Short[eqn = CF[LeftR11[[2]] - RightR11[[2]]]
cvs = Union@Cases[eqn, p_ | π_, ∞]
vars = Union@Cases[eqn, (c | d | e | f | g | h)_ , ∞]
Short[eqns = If[cvs === {},
{eqn == 0},
CoefficientRules[eqn, cvs] /. (_ -> c_) := (c == 0)
], 3]
{sol} = Solve[eqns, vars]
Out[*]//Short=
c_{93}

Out[*]=
{}

Out[*]=
{}

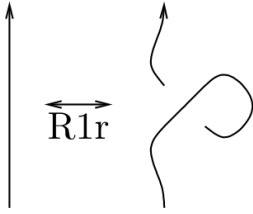
Out[*]//Short=
{c_{93} == 0}

Out[*]=
{{c_{93} -> 0}}

In[*]:= sol /. (v_ -> val_) := (v = CF[val]);

In[*]:= CF[LeftR11 - RightR11]
Out[*]=
∈Series[0, 0]
    
```

## Invariance Under R1r



`In[*]:= {LeftR1r} = Cases [ { { Integrate [ F[i] * L / @ (Xi[i+2][1] Ci[i+1][-1]) d {vs[i], vs[i+1], vs[i+2]} , E[ ] := E, infinity ]`

`Out[*]=`

$$\left\{ \in \text{Series} \left[ p_{1,3+i} \pi_{1,i} + p_{2,3+i} \pi_{2,i} + p_{3,3+i} \pi_{3,i}, \right. \right. \\ \left. \frac{1}{T_1^2 T_2 (-1 + T_1 T_2)} \left( a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 + c_{83} T_1 + c_{86} T_1 + c_{82} T_1^2 + c_{85} T_1^2 + a_4 b_5 T_2 + \right. \right. \\ \left. \left. a_4 b_3 T_1 T_2 - a_4 b_5 T_1 T_2 + c_{81} T_1 T_2 + c_{84} T_1 T_2 - c_{81} T_1^2 T_2 - c_{82} T_1^2 T_2 - 2 c_{83} T_1^2 T_2 - \right. \right. \\ \left. \left. c_{86} T_1^2 T_2 - c_{82} T_1^3 T_2 - c_{85} T_1^3 T_2 - c_{81} T_1^2 T_2^2 - c_{84} T_1^2 T_2^2 + c_{81} T_1^3 T_2^2 + c_{82} T_1^3 T_2^2 + c_{83} T_1^3 T_2^2 \right) \right] \left. \right\}$$

`In[*]:= {RightR1r} = Cases [ { { Integrate [ F[i] * L / @ (Ci[0] Ci[i+1][0] Ci[i+2][0]) d {vs[i], vs[i+1], vs[i+2]} , E[ ] := E, infinity ]`

`Out[*]=`

$$\left\{ \in \text{Series} \left[ p_{1,3+i} \pi_{1,i} + p_{2,3+i} \pi_{2,i} + p_{3,3+i} \pi_{3,i}, 0 \right] \right\}$$

`In[*]:= LeftR1r[[1]] == RightR1r[[1]]`

`Out[*]=`

True

```
In[ ]:= Short[eqn = CF[LeftR1r[[2]] - RightR1r[[2]]]
cvs = Union@Cases[eqn, p_ | π_, ∞]
vars = Union@Cases[eqn, (c | d | e | f | g | h)_ , ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ -> c_) :-> (c == 0), 3]
{sol} = Solve[eqns, vars]
```

Out[ ]//Short=

$$\frac{a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 + \ll 26 \gg + c_{81} T_1^3 T_2^2 + c_{82} T_1^3 T_2^2 + c_{83} T_1^3 T_2^2}{T_1^2 T_2 (-1 + T_1 T_2)}$$

Out[ ]=

```
{ }
```

Out[ ]=

```
{ c81, c82, c83, c84, c85, c86 }
```

Out[ ]//Short=

$$\left\{ \frac{1}{T_1^2 T_2 (-1 + T_1 T_2)} (a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 + c_{83} T_1 + c_{86} T_1 + c_{82} T_1^2 + c_{85} T_1^2 + a_4 b_5 T_2 + \ll 21 \gg + c_{81} T_1^3 T_2^2 + c_{82} T_1^3 T_2^2 + c_{83} T_1^3 T_2^2) == 0 \right\}$$

Solve: Equations may not give solutions for all "solve" variables.

Out[ ]=

$$\left\{ \left\{ c_{86} \rightarrow -c_{85} T_1 - c_{84} T_2 - c_{83} (1 - T_1 T_2) - c_{82} (T_1 - T_1 T_2) - c_{81} (T_2 - T_1 T_2) - \frac{-a_2 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 - a_4 b_5 T_2 - a_4 b_3 T_1 T_2 + a_4 b_5 T_1 T_2}{T_1 (-1 + T_1 T_2)} \right\} \right\}$$

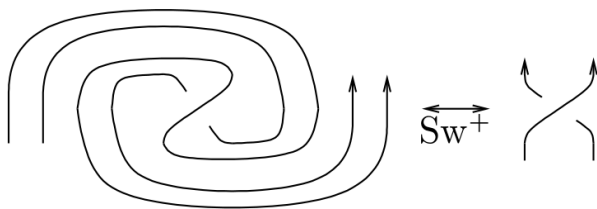
```
In[ ]:= sol /. (v_ -> val_) :-> (v = CF[val]);
```

```
In[ ]:= CF[LeftR1r - RightR1r]
```

Out[ ]=

```
εSeries[0, 0]
```

### Invariance Under Sw



```
In[*]:= Timing[ Short[ {LeftSw} = Cases[ [ { ∫ [i, j] × ℒ / @ (Xi+1,j+1[1] Ci[-1] Cj[-1] Ci+2[1] Cj+2[1])
    d[ {vsi, vsj, vsi+, vsj+, vsi+2, vsj+2} ], E[ ℰ ] := ℰ, ∞ ]
]]
```

```
Out[*]= {1.32813,
  {Series[ T1 p1,3+i π1,i + (1 - T1) p1,3+j π1,i + p1,3+j π1,j + T2 p2,3+i π2,i + <<6>> + T1 T2 p3,3+i π3,i +
  (1 - T1 T2) p3,3+j π3,i + p3,3+j π3,j, <<1>> / <<1>> + <<56>> + <<1>> / <<1>> ] ] }
```

```
In[*]:= Timing[ Short[ {RightSw} = Cases[ [ { ∫ [i, j] × ℒ / @ (Xi+1,j+1[1] Ci[0] Cj[0] Ci+2[0] Cj+2[0])
    d[ {vsi, vsj, vsi+, vsj+, vsi+2, vsj+2} ], E[ ℰ ] := ℰ, ∞ ]
]]
```

```
Out[*]= {1.57813,
  {Series[ T1 p1,3+i π1,i + (1 - T1) p1,3+j π1,i + p1,3+j π1,j + T2 p2,3+i π2,i + <<6>> + T1 T2 p3,3+i π3,i +
  (1 - T1 T2) p3,3+j π3,i + p3,3+j π3,j, <<1>> / <<1>> + <<56>> + <<1>> / <<1>> ] ] }
```

```
In[*]:= LeftSw == RightSw
```

```
Out[*]= True
```

## The Solution

```
In[*]:= Union@Cases[ ℒ@Xi,j[1], (a | b | c | d | e | f | g | h)_, ∞ ]
```

```
Out[*]= {a2, a4, b3, b5, c6, c7, c8, c9, c10, c16, c19, c21, c22, c23,
  c24, c25, c31, c34, c36, c39, c46, c49, c61, c64, c81, c82, c83, c84, c85}
```

```
In[*]:= LeafCount@CF[ ℒ@Xi,j[1] /. {c16|19|31|34|46|49|61|64|81|82|84|85 → 0, b3|5 → 1, a2|4 → 1, B → 1} // .
  {c21 → -c6, c22 → -c7, c24 → -c9, c7 → -1 / T1, c8|10|23|25|83 → 0} ]
```

```
Out[*]= 1310
```

```
In[*]:= LeafCount@CF[ ℒ@Xi,j[1] /. {c16|19|31|34|46|49|61|64|81|82|84|85 → 0, b3|5 → 1, a2|4 → 1, B → 1} // .
  {c21 → -c6, c22 → -c7, c24 → -c9, c7 → 0, c8|10|23|25|83 → 0} ]
```

```
Out[*]= 1205
```

In[\*]:= CF[ $\mathcal{L}@X_{i,j}[1]$ ] /. { $c_{16|19|31|34|46|49|61|64|81|82|84|85} \rightarrow 0$ ,  $b_{3|5} \rightarrow 1$ ,  $a_{2|4} \rightarrow 1$ ,  $B \rightarrow 1$ } // .  
 { $C_{21} \rightarrow -C_6$ ,  $C_{22} \rightarrow -C_7$ ,  $C_{24} \rightarrow -C_9$ ,  $C_7 \rightarrow 0$ ,  $C_{8|10|23|25|83} \rightarrow 0$ }

Out[\*]=

$$T_1 T_2 \mathbb{E} \left[ \text{Series} \left[ -p_{1,i} x_{1,i} + T_1 p_{1,1+i} x_{1,i} + (1 - T_1) p_{1,1+j} x_{1,i} - p_{1,j} x_{1,j} + p_{1,1+j} x_{1,j} - p_{2,i} x_{2,i} + \right. \right. \\
T_2 p_{2,1+i} x_{2,i} + (1 - T_2) p_{2,1+j} x_{2,i} + p_{3,j} x_{1,i} x_{2,i} - \frac{(1 + T_2) p_{3,j} x_{1,j} x_{2,i}}{T_1} - p_{2,j} x_{2,j} + p_{2,1+j} x_{2,j} + \\
p_{3,j} x_{1,i} x_{2,j} - p_{3,i} x_{3,i} + T_1 T_2 p_{3,1+i} x_{3,i} + (1 - T_1 T_2) p_{3,1+j} x_{3,i} - p_{3,j} x_{3,j} + p_{3,1+j} x_{3,j}, \\
\left. \frac{(-1 - T_2 + T_1 T_2 - T_2^2) p_{1,j} p_{2,j} x_{1,i} x_{2,i}}{-1 + T_1 T_2} - \frac{(T_1 - T_2) (-1 - T_2 + T_1 T_2) p_{1,j} p_{2,i} x_{1,j} x_{2,i}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} - \right. \\
\frac{(1 + T_2) p_{1,j} p_{2,j} x_{1,j} x_{2,i}}{(-1 + T_1) T_1} - \frac{(T_1 - T_2) T_2 p_{1,i} p_{2,j} x_{1,i} x_{2,j}}{(-1 + T_1) (-1 + T_2) (-1 + T_1 T_2)} + \frac{p_{1,j} p_{2,j} x_{1,i} x_{2,j}}{-1 + T_2} + \\
p_{1,j} p_{2,i} x_{3,i} + p_{1,i} p_{2,j} x_{3,i} - 2 p_{1,j} p_{2,j} x_{3,i} + \frac{(1 + T_2) p_{3,j} x_{3,i}}{T_1 (-1 + T_1 T_2)} - \\
\frac{(1 - 2 T_1 T_2 + T_2^2) p_{1,j} p_{3,j} x_{1,i} x_{3,i}}{-1 + T_2} + \frac{(-T_1 + T_2 - 2 T_1 T_2 + 2 T_1^2 T_2 + T_2^2 - T_1 T_2^2) p_{1,j} p_{3,i} x_{1,j} x_{3,i}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} + \\
\frac{(1 + T_2) p_{1,i} p_{3,j} x_{1,j} x_{3,i}}{T_1 (-1 + T_2)} + \frac{(1 + T_2) (1 - 2 T_1 + T_1 T_2) p_{1,j} p_{3,j} x_{1,j} x_{3,i}}{(-1 + T_1) T_1 (-1 + T_2)} + \\
\frac{(1 - T_1 + 2 T_2 - 2 T_1 T_2 + T_1^2 T_2 + T_2^2 - 2 T_1 T_2^2) p_{2,j} p_{3,j} x_{2,i} x_{3,i}}{(-1 + T_1) T_1} + \\
\left. \frac{(-1 + T_1 T_2 + T_1^2 T_2 + T_2^2 - 2 T_1 T_2^2) p_{2,j} p_{3,i} x_{2,j} x_{3,i}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} - \right. \\
\left. \frac{p_{2,i} p_{3,j} x_{2,j} x_{3,i}}{-1 + T_1} - \frac{(1 - 2 T_2 + T_1 T_2) p_{2,j} p_{3,j} x_{2,j} x_{3,i}}{(-1 + T_1) (-1 + T_2)} - \right. \\
\left. \frac{T_2 p_{1,i} p_{3,j} x_{1,i} x_{3,j}}{(-1 + T_1) (-1 + T_1 T_2)} - \frac{(-1 - T_2 + 2 T_1 T_2) p_{1,j} p_{3,j} x_{1,i} x_{3,j}}{(-1 + T_2) (-1 + T_1 T_2)} + \right. \\
\left. \frac{(-1 - T_2 + T_1 T_2) p_{2,i} p_{3,j} x_{2,i} x_{3,j}}{T_1 (-1 + T_2) (-1 + T_1 T_2)} + \frac{T_2 (-1 - T_2 + 2 T_1 T_2) p_{2,j} p_{3,j} x_{2,i} x_{3,j}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} \right] ]$$

In[\*]:= CF[ $\mathcal{L}@X_{i,j}[-1]$ ] /. { $c_{16|19|31|34|46|49|61|64|81|82|84|85} \rightarrow 0$ ,  $b_{3|5} \rightarrow 1$ ,  $a_{2|4} \rightarrow 1$ ,  $B \rightarrow 1$ } // .  
 { $C_{21} \rightarrow -C_6$ ,  $C_{22} \rightarrow -C_7$ ,  $C_{24} \rightarrow -C_9$ ,  $C_7 \rightarrow 0$ ,  $C_{8|10|23|25|83} \rightarrow 0$ }

Out[\*]=

$$\frac{1}{T_1 T_2} \mathbb{E} \left[ \text{Series} \left[ -p_{1,i} x_{1,i} + \frac{p_{1,1+i} x_{1,i}}{T_1} + \frac{(-1 + T_1) p_{1,1+j} x_{1,i}}{T_1} - p_{1,j} x_{1,j} + p_{1,1+j} x_{1,j} - p_{2,i} x_{2,i} + \frac{p_{2,1+i} x_{2,i}}{T_2} \right. \right. \\
\frac{(-1 + T_2) p_{2,1+j} x_{2,i}}{T_2} + \frac{(-1 + T_1 - T_2) p_{3,j} x_{1,i} x_{2,i}}{T_1^2 T_2} + \frac{(1 + T_2) p_{3,j} x_{1,j} x_{2,i}}{T_1 T_2} - p_{2,j} x_{2,j} + p_{2,1+j} x_{2,j} - \\
\frac{p_{3,j} x_{1,i} x_{2,j}}{T_1} - p_{3,i} x_{3,i} + \frac{p_{3,1+i} x_{3,i}}{T_1 T_2} + \frac{(-1 + T_1 T_2) p_{3,1+j} x_{3,i}}{T_1 T_2} - p_{3,j} x_{3,j} + p_{3,1+j} x_{3,j}, \\
\left. \left. \frac{(-1 + T_2) p_{2,1+j} x_{2,i}}{T_2} + \frac{(-1 + T_1 - T_2) p_{3,j} x_{1,i} x_{2,i}}{T_1^2 T_2} + \frac{(1 + T_2) p_{3,j} x_{1,j} x_{2,i}}{T_1 T_2} - p_{2,j} x_{2,j} + p_{2,1+j} x_{2,j} - \right. \right. \\
\left. \left. \frac{p_{3,j} x_{1,i} x_{2,j}}{T_1} - p_{3,i} x_{3,i} + \frac{p_{3,1+i} x_{3,i}}{T_1 T_2} + \frac{(-1 + T_1 T_2) p_{3,1+j} x_{3,i}}{T_1 T_2} - p_{3,j} x_{3,j} + p_{3,1+j} x_{3,j} \right] \right]$$

$$\begin{aligned}
& \frac{(T_1 - T_2) (-1 - T_2 + T_1 T_2) \rho_{1,j} \rho_{2,i} x_{1,i} x_{2,i}}{T_1^2 (-1 + T_2) (-1 + T_1 T_2)} + \frac{(T_1 - T_2) \rho_{1,i} \rho_{2,j} x_{1,i} x_{2,i}}{(-1 + T_1) (-1 + T_1 T_2)} - \\
& \frac{(1 - T_1 + 2 T_2 - T_1 T_2 + 2 T_1^2 T_2 + T_2^2 - 2 T_1 T_2^2) \rho_{1,j} \rho_{2,j} x_{1,i} x_{2,i}}{T_1^2 T_2 (-1 + T_1 T_2)} + \\
& \frac{(T_1 - T_2) (-1 - T_2 + T_1 T_2) \rho_{1,j} \rho_{2,i} x_{1,j} x_{2,i}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} - \\
& \frac{(1 - T_1 + 2 T_2 - 2 T_1 T_2 + T_1^2 T_2 + T_2^2 - 2 T_1 T_2^2) \rho_{1,j} \rho_{2,j} x_{1,j} x_{2,i}}{(-1 + T_1) T_1 T_2 (-1 + T_1 T_2)} + \frac{(T_1 - T_2) T_2 \rho_{1,i} \rho_{2,j} x_{1,i} x_{2,j}}{(-1 + T_1) (-1 + T_2) (-1 + T_1 T_2)} - \\
& \frac{(-1 + 2 T_1 T_2 - T_2^2) \rho_{1,j} \rho_{2,j} x_{1,i} x_{2,j}}{T_1 (-1 + T_2) (-1 + T_1 T_2)} - \frac{\rho_{1,j} \rho_{2,i} x_{3,i}}{T_1} - \frac{\rho_{1,i} \rho_{2,j} x_{3,i}}{T_2} + \frac{(T_1 + T_2) \rho_{1,j} \rho_{2,j} x_{3,i}}{T_1 T_2} + \\
& \frac{(-1 - T_2 + T_1 T_2 - T_2^2) \rho_{3,j} x_{3,i}}{T_1 T_2 (-1 + T_1 T_2)} - \frac{(-T_1 + T_2 - 2 T_1 T_2 + 2 T_1^2 T_2 + T_2^2 - T_1 T_2^2) \rho_{1,j} \rho_{3,i} x_{1,i} x_{3,i}}{T_1^2 (-1 + T_2) (-1 + T_1 T_2)} - \\
& \frac{(1 - T_1 + T_2 - 2 T_1 T_2 + 2 T_1^2 T_2 - T_1 T_2^2) \rho_{1,i} \rho_{3,j} x_{1,i} x_{3,i}}{(-1 + T_1) T_1^2 (-1 + T_2) T_2} + \\
& \frac{(1 - 2 T_1 + T_2 - 3 T_1 T_2 + 4 T_1^2 T_2 - T_1 T_2^2) \rho_{1,j} \rho_{3,j} x_{1,i} x_{3,i}}{T_1^3 (-1 + T_2) T_2} - \\
& \frac{(-T_1 + T_2 - 2 T_1 T_2 + 2 T_1^2 T_2 + T_2^2 - T_1 T_2^2) \rho_{1,j} \rho_{3,i} x_{1,j} x_{3,i}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} - \frac{(1 + T_2) \rho_{1,i} \rho_{3,j} x_{1,j} x_{3,i}}{T_1 (-1 + T_2) T_2} + \\
& \frac{(1 - 2 T_1 + T_1^2 + T_2 - 3 T_1 T_2 + 3 T_1^2 T_2 - T_1 T_2^2) \rho_{1,j} \rho_{3,j} x_{1,j} x_{3,i}}{(-1 + T_1) T_1^2 (-1 + T_2) T_2} - \\
& \frac{(-1 + T_1 T_2 + T_1^2 T_2 + T_2^2 - 2 T_1 T_2^2) \rho_{2,j} \rho_{3,i} x_{2,i} x_{3,i}}{(-1 + T_1) T_1 T_2 (-1 + T_1 T_2)} - \\
& \frac{(-T_1 + T_2 + T_1^2 T_2 + T_2^2 - 2 T_1 T_2^2) \rho_{2,i} \rho_{3,j} x_{2,i} x_{3,i}}{(-1 + T_1) T_1^2 (-1 + T_2) T_2} + \\
& \frac{(-T_1 + 2 T_2 + T_1^2 T_2 + 2 T_2^2 - 4 T_1 T_2^2) \rho_{2,j} \rho_{3,j} x_{2,i} x_{3,i}}{(-1 + T_1) T_1^2 T_2^2} - \\
& \frac{(-1 + T_1 T_2 + T_1^2 T_2 + T_2^2 - 2 T_1 T_2^2) \rho_{2,j} \rho_{3,i} x_{2,j} x_{3,i}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} + \frac{\rho_{2,i} \rho_{3,j} x_{2,j} x_{3,i}}{(-1 + T_1) T_1} + \\
& \frac{(-1 - T_1 + T_1^2 + 2 T_1 T_2 + T_1^2 T_2 + T_2^2 - 3 T_1 T_2^2) \rho_{2,j} \rho_{3,j} x_{2,j} x_{3,i}}{(-1 + T_1) T_1^2 (-1 + T_2) T_2} + \frac{T_2 \rho_{1,i} \rho_{3,j} x_{1,i} x_{3,j}}{(-1 + T_1) (-1 + T_1 T_2)} + \\
& \frac{(-1 + 2 T_1 T_2 - T_2^2) \rho_{1,j} \rho_{3,j} x_{1,i} x_{3,j}}{T_1 (-1 + T_2) (-1 + T_1 T_2)} - \frac{(-1 - T_2 + T_1 T_2) \rho_{2,i} \rho_{3,j} x_{2,i} x_{3,j}}{T_1 (-1 + T_2) (-1 + T_1 T_2)} + \\
& \left. \frac{(1 - T_1 + 2 T_2 - 2 T_1 T_2 + T_1^2 T_2 + T_2^2 - 2 T_1 T_2^2) \rho_{2,j} \rho_{3,j} x_{2,i} x_{3,j}}{(-1 + T_1) T_1 T_2 (-1 + T_1 T_2)} \right] ] ]
\end{aligned}$$

```
In[*]:= MatrixForm@CF[ {
  {r0[1, i, j], r1[1, i, j]},
  {r0[-1, i, j], r1[-1, i, j]},
  {(-a2 + a2 T1 - a4 T1 - a4 T2 + 2 a4 T1 T2), (-b3 + b5 + b3 T1 - b5 T2)}
} /. {b3 -> 0, a4 -> 0, b5 -> T1, a2 -> T1}]
```

```
Out[*]//MatrixForm=

$$\begin{pmatrix} T_1 p_{3,j} x_{1,i} x_{2,i} - p_{3,j} x_{1,j} x_{2,i} & T_1 p_{1,j} p_{2,i} x_{3,i} - T_1 p_{1,j} p_{2,j} x_{3,i} \\ -\frac{p_{3,j} x_{1,i} x_{2,i}}{T_1 T_2} + \frac{p_{3,j} x_{1,j} x_{2,i}}{T_2} & -p_{1,j} p_{2,i} x_{3,i} + p_{1,j} p_{2,j} x_{3,i} \\ (-1 + T_1) T_1 & -T_1 (-1 + T_2) \end{pmatrix}$$

```

```
In[*]:= MatrixForm@CF[ {
  {r0[1, i, j], r1[1, i, j]},
  {r0[-1, i, j], r1[-1, i, j]},
  {(-a2 + a2 T1 - a4 T1 - a4 T2 + 2 a4 T1 T2), (-b3 + b5 + b3 T1 - b5 T2)}
} /. {b3 -> 0, a4 -> 0, b5 -> 1, a2 -> 1}]
```

```
Out[*]//MatrixForm=

$$\begin{pmatrix} p_{3,j} x_{1,i} x_{2,i} - \frac{p_{3,j} x_{1,j} x_{2,i}}{T_1} & p_{1,j} p_{2,i} x_{3,i} - p_{1,j} p_{2,j} x_{3,i} \\ -\frac{p_{3,j} x_{1,i} x_{2,i}}{T_1^2 T_2} + \frac{p_{3,j} x_{1,j} x_{2,i}}{T_1 T_2} & -\frac{p_{1,j} p_{2,i} x_{3,i}}{T_1} + \frac{p_{1,j} p_{2,j} x_{3,i}}{T_1} \\ -1 + T_1 & 1 - T_2 \end{pmatrix}$$

```

```
In[*]:= MatrixForm@CF[ {
  {r0[1, i, j], r1[1, i, j]},
  {r0[-1, i, j], r1[-1, i, j]},
  {(-a2 + a2 T1 - a4 T1 - a4 T2 + 2 a4 T1 T2), (-b3 + b5 + b3 T1 - b5 T2)}
} /. {b3 -> T2^{1/2}, b5 -> -T1^{1/2}, a4 -> T1^{1/2}, a2 -> -\sqrt{T1} T2}]
```

```
Out[*]//MatrixForm=

$$\begin{pmatrix} -\sqrt{T_1} T_2 p_{3,j} x_{1,i} x_{2,i} + \sqrt{T_1} p_{3,j} x_{1,i} x_{2,j} & -\sqrt{T_1} p_{1,j} p_{2,i} x_{3,i} + \sqrt{T_2} p_{1,i} p_{2,j} x_{3,i} + (\sqrt{T_1} - \sqrt{T_2}) p_{1,j} p_{2,j} x_{3,i} \\ \frac{p_{3,j} x_{1,i} x_{2,i}}{\sqrt{T_1} T_2} - \frac{p_{3,j} x_{1,i} x_{2,j}}{\sqrt{T_1}} & \frac{p_{1,j} p_{2,i} x_{3,i}}{\sqrt{T_1}} - \frac{p_{1,i} p_{2,j} x_{3,i}}{\sqrt{T_2}} + \frac{(\sqrt{T_1} - \sqrt{T_2}) p_{1,j} p_{2,j} x_{3,i}}{\sqrt{T_1} \sqrt{T_2}} \\ T_1^{3/2} (-1 + T_2) & (\sqrt{T_1} + \sqrt{T_2}) (-1 + \sqrt{T_1} \sqrt{T_2}) \end{pmatrix}$$

```

```
In[*]:= MatrixForm@CF[ {
  {r0[1, i, j], r1[1, i, j]},
  {r0[-1, i, j], r1[-1, i, j]},
  {(-a2 + a2 T1 - a4 T1 - a4 T2 + 2 a4 T1 T2), (-b3 + b5 + b3 T1 - b5 T2)}
} /. {a2 -> -T2 a4, b5 -> -b3 T1} /. {b3 -> T1 - 1, a4 -> 1}]
```

```
Out[*]//MatrixForm=

$$\begin{pmatrix} -T_2 p_{3,j} x_{1,i} x_{2,i} + p_{3,j} x_{1,i} x_{2,j} & -((-1 + T_1) T_1 p_{1,j} p_{2,i} x_{3,i}) + (-1 + T_1) p_{1,i} p_{2,j} x_{3,i} + (-1 + T_1)^2 p_{1,j} p_{2,j} x_{3,i} \\ \frac{p_{3,j} x_{1,i} x_{2,i}}{T_1 T_2} - \frac{p_{3,j} x_{1,i} x_{2,j}}{T_1} & (-1 + T_1) p_{1,j} p_{2,i} x_{3,i} - \frac{(-1 + T_1) p_{1,i} p_{2,j} x_{3,i}}{T_2} - \frac{(-1 + T_1) (-1 + T_2) p_{1,j} p_{2,j} x_{3,i}}{T_2} \\ T_1 (-1 + T_2) & (-1 + T_1) (-1 + T_1 T_2) \end{pmatrix}$$

```

```
In[*]:= MatrixForm@CF[{
  {r0[1, i, j], r1[1, i, j]},
  {r0[-1, i, j], r1[-1, i, j]},
  {(-a2 + a2 T1 - a4 T1 - a4 T2 + 2 a4 T1 T2), (-b3 + b5 + b3 T1 - b5 T2)}
} /. {a2 -> -T2 a4, b5 -> -b3 T1} /. {b3 -> 1, a4 -> 1 - T1^-1}]
```

Out[\*]//MatrixForm=

$$\begin{pmatrix} -\frac{(-1+T_1) T_2 p_{3,j} x_{1,i} x_{2,i}}{T_1} + \frac{(-1+T_1) p_{3,j} x_{1,i} x_{2,j}}{T_1} & -T_1 p_{1,j} p_{2,i} x_{3,i} + p_{1,i} p_{2,j} x_{3,i} + (-1 + T_1) p_{1,j} p_{2,j} x_{3,i} \\ \frac{(-1+T_1) p_{3,j} x_{1,i} x_{2,i}}{T_1^2 T_2} - \frac{(-1+T_1) p_{3,j} x_{1,i} x_{2,j}}{T_1^2} & p_{1,j} p_{2,i} x_{3,i} - \frac{p_{1,i} p_{2,j} x_{3,i}}{T_2} - \frac{(-1+T_2) p_{1,j} p_{2,j} x_{3,i}}{T_2} \\ (-1 + T_1) (-1 + T_2) & -1 + T_1 T_2 \end{pmatrix}$$

```
In[*]:= L[Ck[1]]
```

Out[\*]=

$$T_1 T_2 \mathbb{E} \left[ \in \text{Series} \left[ (-p_{1,k} + p_{1,1+k}) x_{1,k} + (-p_{2,k} + p_{2,1+k}) x_{2,k} + (-p_{3,k} + p_{3,1+k}) x_{3,k}, \right. \right. \\ \left. \left. \frac{(c_{16} + c_{31} + c_{46} + c_{61}) p_{1,k} x_{1,k} + (c_{19} + c_{34} + c_{49} + c_{64}) p_{2,k} x_{2,k} + (-b_3 + b_5 + b_3 T_1 - b_5 T_2) (-a_2 + a_2 T_1 - a_4 T_1 - a_4 T_2 + 2 a_4 T_1 T_2) p_{3,k} x_{3,k}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} \right] \right]$$

```
In[*]:= Factor \left[ \left( \frac{(-b_3 + b_5 + b_3 T_1 - b_5 T_2) (-a_2 + a_2 T_1 - a_4 T_1 - a_4 T_2 + 2 a_4 T_1 T_2) p_{3,k} x_{3,k}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} \right) /. \right. \\ \left. \{a_2 -> -T_2 a_4, b_5 -> -b_3 T_1\} /. \{b_3 -> T_1 - 1, a_4 -> 1\} \right]
```

Out[\*]=

$$p_{3,k} x_{3,k}$$

```
In[*]:= L[Ck[1]] /. c16|19|31|34|46|49|61|64|81|82|84|85 -> 0 /. T2 -> 1
```

Power: Infinite expression  $\frac{1}{0}$  encountered. ⓘ

Out[\*]=

$$T_1 \mathbb{E} \left[ \in \text{Series} \left[ (-p_{1,k} + p_{1,1+k}) x_{1,k} + (-p_{2,k} + p_{2,1+k}) x_{2,k} + (-p_{3,k} + p_{3,1+k}) x_{3,k}, \text{ComplexInfinity} \right] \right]$$

```
In[*]:= L[Ck[-1]]
```

Out[\*]=

$$\frac{1}{T_1 T_2} \mathbb{E} \left[ \in \text{Series} \left[ (-p_{1,k} + p_{1,1+k}) x_{1,k} + (-p_{2,k} + p_{2,1+k}) x_{2,k} + (-p_{3,k} + p_{3,1+k}) x_{3,k}, \right. \right. \\ \left. \left. \frac{(-c_{16} - c_{31} - c_{46} - c_{61}) p_{1,k} x_{1,k} + (-c_{19} - c_{34} - c_{49} - c_{64}) p_{2,k} x_{2,k} - (-b_3 + b_5 + b_3 T_1 - b_5 T_2) (-a_2 + a_2 T_1 - a_4 T_1 - a_4 T_2 + 2 a_4 T_1 T_2) p_{3,k} x_{3,k}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} \right] \right]$$

## Some Knots

```
In[*]:= K = Knot[3, 1]; K31 = \int L[K] \times d vs[K]
```

Out[\*]=

$$-\frac{1}{(1 - T_1 + T_1^2) (1 - T_2 + T_2^2) (1 - T_1 T_2 + T_1^2 T_2^2)}$$




$$\begin{aligned}
 & i T_1^2 T_2^2 \mathbb{E} [\in \text{Series} [0, - ( (-c_{81} - c_{82} - c_{84} - c_{85} + c_{16} T_1 + c_{31} T_1 + c_{46} T_1 + c_{61} T_1 + 5 c_{81} T_1 + 4 c_{82} T_1 + \\
 & 4 c_{84} T_1 + 4 c_{85} T_1 - 3 c_{16} T_1^2 - 3 c_{31} T_1^2 - 3 c_{46} T_1^2 - 3 c_{61} T_1^2 - 10 c_{81} T_1^2 - 8 c_{82} T_1^2 - 6 c_{84} T_1^2 - \\
 & 8 c_{85} T_1^2 + 5 c_{16} T_1^3 + 5 c_{31} T_1^3 + 5 c_{46} T_1^3 + 5 c_{61} T_1^3 + 11 c_{81} T_1^3 + 10 c_{82} T_1^3 + 5 c_{84} T_1^3 + 10 c_{85} T_1^3 - \\
 & 5 c_{16} T_1^4 - 5 c_{31} T_1^4 - 5 c_{46} T_1^4 - 5 c_{61} T_1^4 - 7 c_{81} T_1^4 - 8 c_{82} T_1^4 - 2 c_{84} T_1^4 - 8 c_{85} T_1^4 + 2 c_{16} T_1^5 + \\
 & 2 c_{31} T_1^5 + 2 c_{46} T_1^5 + 2 c_{61} T_1^5 + 2 c_{81} T_1^5 + 4 c_{82} T_1^5 + 4 c_{85} T_1^5 - c_{82} T_1^6 - c_{85} T_1^6 - a_2 b_3 T_2 + a_2 b_5 T_2 + \\
 & c_{19} T_2 + c_{34} T_2 + c_{49} T_2 + c_{64} T_2 + 4 c_{81} T_2 + 5 c_{82} T_2 + 4 c_{84} T_2 + 4 c_{85} T_2 + 4 a_2 b_3 T_1 T_2 - \\
 & a_4 b_3 T_1 T_2 - 3 a_2 b_5 T_1 T_2 + a_4 b_5 T_1 T_2 - 3 c_{16} T_1 T_2 - 3 c_{19} T_1 T_2 - 3 c_{31} T_1 T_2 - 3 c_{34} T_1 T_2 - \\
 & 3 c_{46} T_1 T_2 - 3 c_{49} T_1 T_2 - 3 c_{61} T_1 T_2 - 3 c_{64} T_1 T_2 - 17 c_{81} T_1 T_2 - 17 c_{82} T_1 T_2 - 13 c_{84} T_1 T_2 - \\
 & 13 c_{85} T_1 T_2 - 8 a_2 b_3 T_1^2 T_2 + 3 a_4 b_3 T_1^2 T_2 + 5 a_2 b_5 T_1^2 T_2 - 2 a_4 b_5 T_1^2 T_2 + 7 c_{16} T_1^2 T_2 + \\
 & 5 c_{19} T_1^2 T_2 + 7 c_{31} T_1^2 T_2 + 5 c_{34} T_1^2 T_2 + 7 c_{46} T_1^2 T_2 + 5 c_{49} T_1^2 T_2 + 7 c_{61} T_1^2 T_2 + 5 c_{64} T_1^2 T_2 + \\
 & 28 c_{81} T_1^2 T_2 + 28 c_{82} T_1^2 T_2 + 15 c_{84} T_1^2 T_2 + 20 c_{85} T_1^2 T_2 + 10 a_2 b_3 T_1^3 T_2 - 5 a_4 b_3 T_1^3 T_2 - \\
 & 5 a_2 b_5 T_1^3 T_2 + 3 a_4 b_5 T_1^3 T_2 - 9 c_{16} T_1^3 T_2 - 5 c_{19} T_1^3 T_2 - 9 c_{31} T_1^3 T_2 - 5 c_{34} T_1^3 T_2 - 9 c_{46} T_1^3 T_2 - \\
 & 5 c_{49} T_1^3 T_2 - 9 c_{61} T_1^3 T_2 - 5 c_{64} T_1^3 T_2 - 23 c_{81} T_1^3 T_2 - 26 c_{82} T_1^3 T_2 - 8 c_{84} T_1^3 T_2 - 16 c_{85} T_1^3 T_2 - \\
 & 8 a_2 b_3 T_1^4 T_2 + 5 a_4 b_3 T_1^4 T_2 + 3 a_2 b_5 T_1^4 T_2 - 2 a_4 b_5 T_1^4 T_2 + 5 c_{16} T_1^4 T_2 + 3 c_{19} T_1^4 T_2 + \\
 & 5 c_{31} T_1^4 T_2 + 3 c_{34} T_1^4 T_2 + 5 c_{46} T_1^4 T_2 + 3 c_{49} T_1^4 T_2 + 5 c_{61} T_1^4 T_2 + 3 c_{64} T_1^4 T_2 + 7 c_{81} T_1^4 T_2 + \\
 & 10 c_{82} T_1^4 T_2 - c_{84} T_1^4 T_2 + 2 c_{85} T_1^4 T_2 + 4 a_2 b_3 T_1^5 T_2 - 3 a_4 b_3 T_1^5 T_2 - a_2 b_5 T_1^5 T_2 + a_4 b_5 T_1^5 T_2 + \\
 & 4 c_{16} T_1^5 T_2 - c_{19} T_1^5 T_2 + 4 c_{31} T_1^5 T_2 - c_{34} T_1^5 T_2 + 4 c_{46} T_1^5 T_2 - c_{49} T_1^5 T_2 + 4 c_{61} T_1^5 T_2 - \\
 & c_{64} T_1^5 T_2 + 4 c_{81} T_1^5 T_2 + 4 c_{82} T_1^5 T_2 + 3 c_{84} T_1^5 T_2 + 8 c_{85} T_1^5 T_2 - a_2 b_3 T_1^6 T_2 + a_4 b_3 T_1^6 T_2 - \\
 & 4 c_{16} T_1^6 T_2 - 4 c_{31} T_1^6 T_2 - 4 c_{46} T_1^6 T_2 - 4 c_{61} T_1^6 T_2 - 3 c_{81} T_1^6 T_2 - 7 c_{82} T_1^6 T_2 - 8 c_{85} T_1^6 T_2 + \\
 & 3 c_{82} T_1^7 T_2 + 3 c_{85} T_1^7 T_2 + 2 a_2 b_3 T_2^2 - a_4 b_3 T_2^2 - 3 a_2 b_5 T_2^2 + a_4 b_5 T_2^2 - 3 c_{19} T_2^2 - 3 c_{34} T_2^2 - \\
 & 3 c_{49} T_2^2 - 3 c_{64} T_2^2 - 8 c_{81} T_2^2 - 10 c_{82} T_2^2 - 8 c_{84} T_2^2 - 6 c_{85} T_2^2 - 7 a_2 b_3 T_1 T_2^2 + 7 a_4 b_3 T_1 T_2^2 + \\
 & 8 a_2 b_5 T_1 T_2^2 - 7 a_4 b_5 T_1 T_2^2 + 5 c_{16} T_1 T_2^2 + 7 c_{19} T_1 T_2^2 + 5 c_{31} T_1 T_2^2 + 7 c_{34} T_1 T_2^2 + 5 c_{46} T_1 T_2^2 + \\
 & 7 c_{49} T_1 T_2^2 + 5 c_{61} T_1 T_2^2 + 7 c_{64} T_1 T_2^2 + 28 c_{81} T_1 T_2^2 + 28 c_{82} T_1 T_2^2 + 20 c_{84} T_1 T_2^2 + 15 c_{85} T_1 T_2^2 + \\
 & 11 a_2 b_3 T_1^2 T_2^2 - 16 a_4 b_3 T_1^2 T_2^2 - 11 a_2 b_5 T_1^2 T_2^2 + 12 a_4 b_5 T_1^2 T_2^2 - 9 c_{16} T_1^2 T_2^2 - 9 c_{19} T_1^2 T_2^2 - \\
 & 9 c_{31} T_1^2 T_2^2 - 9 c_{34} T_1^2 T_2^2 - 9 c_{46} T_1^2 T_2^2 - 9 c_{49} T_1^2 T_2^2 - 9 c_{61} T_1^2 T_2^2 - 9 c_{64} T_1^2 T_2^2 - 34 c_{81} T_1^2 T_2^2 - \\
 & 34 c_{82} T_1^2 T_2^2 - 14 c_{84} T_1^2 T_2^2 - 14 c_{85} T_1^2 T_2^2 - 8 a_2 b_3 T_1^3 T_2^2 + 21 a_4 b_3 T_1^3 T_2^2 + 7 a_2 b_5 T_1^3 T_2^2 - \\
 & 14 a_4 b_5 T_1^3 T_2^2 + 9 c_{16} T_1^3 T_2^2 + 5 c_{19} T_1^3 T_2^2 + 9 c_{31} T_1^3 T_2^2 + 5 c_{34} T_1^3 T_2^2 + 9 c_{46} T_1^3 T_2^2 + 5 c_{49} T_1^3 T_2^2 + \\
 & 9 c_{61} T_1^3 T_2^2 + 5 c_{64} T_1^3 T_2^2 + 11 c_{81} T_1^3 T_2^2 + 12 c_{82} T_1^3 T_2^2 - 3 c_{84} T_1^3 T_2^2 - 4 c_{85} T_1^3 T_2^2 - a_2 b_3 T_1^4 T_2^2 - \\
 & 15 a_4 b_3 T_1^4 T_2^2 + 6 a_2 b_5 T_1^4 T_2^2 - c_{16} T_1^4 T_2^2 + c_{19} T_1^4 T_2^2 - c_{31} T_1^4 T_2^2 + c_{34} T_1^4 T_2^2 - c_{46} T_1^4 T_2^2 + \\
 & c_{49} T_1^4 T_2^2 - c_{61} T_1^4 T_2^2 + c_{64} T_1^4 T_2^2 + 17 c_{81} T_1^4 T_2^2 + 24 c_{82} T_1^4 T_2^2 + 14 c_{84} T_1^4 T_2^2 + 26 c_{85} T_1^4 T_2^2 + \\
 & 7 a_2 b_3 T_1^5 T_2^2 + 4 a_4 b_3 T_1^5 T_2^2 - 3 a_2 b_5 T_1^5 T_2^2 - a_4 b_5 T_1^5 T_2^2 - 10 c_{16} T_1^5 T_2^2 - 3 c_{19} T_1^5 T_2^2 - \\
 & 10 c_{31} T_1^5 T_2^2 - 3 c_{34} T_1^5 T_2^2 - 10 c_{46} T_1^5 T_2^2 - 3 c_{49} T_1^5 T_2^2 - 10 c_{61} T_1^5 T_2^2 - 3 c_{64} T_1^5 T_2^2 - 22 c_{81} T_1^5 T_2^2 - \\
 & 36 c_{82} T_1^5 T_2^2 - 8 c_{84} T_1^5 T_2^2 - 28 c_{85} T_1^5 T_2^2 - 6 a_2 b_3 T_1^6 T_2^2 + 2 a_4 b_3 T_1^6 T_2^2 + 2 a_2 b_5 T_1^6 T_2^2 - \\
 & 2 a_4 b_5 T_1^6 T_2^2 + 2 c_{16} T_1^6 T_2^2 + 2 c_{19} T_1^6 T_2^2 + 2 c_{31} T_1^6 T_2^2 + 2 c_{34} T_1^6 T_2^2 + 2 c_{46} T_1^6 T_2^2 + 2 c_{49} T_1^6 T_2^2 + \\
 & 2 c_{61} T_1^6 T_2^2 + 2 c_{64} T_1^6 T_2^2 + 7 c_{81} T_1^6 T_2^2 + 22 c_{82} T_1^6 T_2^2 - c_{84} T_1^6 T_2^2 + 14 c_{85} T_1^6 T_2^2 + 2 a_2 b_3 T_1^7 T_2^2 - \\
 & 2 a_4 b_3 T_1^7 T_2^2 + 4 c_{16} T_1^7 T_2^2 + 4 c_{31} T_1^7 T_2^2 + 4 c_{46} T_1^7 T_2^2 + 4 c_{61} T_1^7 T_2^2 + c_{81} T_1^7 T_2^2 - 4 c_{82} T_1^7 T_2^2 - \\
 & c_{85} T_1^7 T_2^2 - 2 c_{82} T_1^8 T_2^2 - 2 c_{85} T_1^8 T_2^2 - 3 a_2 b_3 T_2^3 + 2 a_4 b_3 T_2^3 + 5 a_2 b_5 T_2^3 - 3 a_4 b_5 T_2^3 + 5 c_{19} T_2^3 + \\
 & 5 c_{34} T_2^3 + 5 c_{49} T_2^3 + 5 c_{64} T_2^3 + 10 c_{81} T_2^3 + 11 c_{82} T_2^3 + 10 c_{84} T_2^3 + 5 c_{85} T_2^3 + 9 a_2 b_3 T_1 T_2^3 - \\
 & 12 a_4 b_3 T_1 T_2^3 - 11 a_2 b_5 T_1 T_2^3 + 16 a_4 b_5 T_1 T_2^3 - 5 c_{16} T_1 T_2^3 - 9 c_{19} T_1 T_2^3 - 5 c_{31} T_1 T_2^3 - \\
 & 9 c_{34} T_1 T_2^3 - 5 c_{46} T_1 T_2^3 - 9 c_{49} T_1 T_2^3 - 5 c_{61} T_1 T_2^3 - 9 c_{64} T_1 T_2^3 - 26 c_{81} T_1 T_2^3 - 23 c_{82} T_1 T_2^3 - \\
 & 16 c_{84} T_1 T_2^3 - 8 c_{85} T_1 T_2^3 - 11 a_2 b_3 T_1^2 T_2^3 + 22 a_4 b_3 T_1^2 T_2^3 + 11 a_2 b_5 T_1^2 T_2^3 - 22 a_4 b_5 T_1^2 T_2^3 + \\
 & 5 c_{16} T_1^2 T_2^3 + 9 c_{19} T_1^2 T_2^3 + 5 c_{31} T_1^2 T_2^3 + 9 c_{34} T_1^2 T_2^3 + 5 c_{46} T_1^2 T_2^3 + 9 c_{49} T_1^2 T_2^3 + 5 c_{61} T_1^2 T_2^3 + \\
 & 9 c_{64} T_1^2 T_2^3 + 12 c_{81} T_1^2 T_2^3 + 11 c_{82} T_1^2 T_2^3 - 4 c_{84} T_1^2 T_2^3 - 3 c_{85} T_1^2 T_2^3 + 2 a_2 b_3 T_1^3 T_2^3 - 19 a_4 b_3 T_1^3 T_2^3 + \\
 & a_2 b_5 T_1^3 T_2^3 + 18 a_4 b_5 T_1^3 T_2^3 - c_{16} T_1^3 T_2^3 - c_{19} T_1^3 T_2^3 - c_{31} T_1^3 T_2^3 - c_{34} T_1^3 T_2^3 - c_{46} T_1^3 T_2^3 - c_{49} T_1^3 T_2^3 - \\
 & c_{61} T_1^3 T_2^3 - c_{64} T_1^3 T_2^3 + 30 c_{81} T_1^3 T_2^3 + 30 c_{82} T_1^3 T_2^3 + 26 c_{84} T_1^3 T_2^3 + 26 c_{85} T_1^3 T_2^3 + 13 a_2 b_3 T_1^4 T_2^3 - \\
 & 2 a_4 b_3 T_1^4 T_2^3 - 13 a_2 b_5 T_1^4 T_2^3 + 5 a_4 b_5 T_1^4 T_2^3 - 8 c_{16} T_1^4 T_2^3 - 5 c_{19} T_1^4 T_2^3 - 8 c_{31} T_1^4 T_2^3 - 5 c_{34} T_1^4 T_2^3 -
 \end{aligned}$$

$$\begin{aligned}
& 8 c_{46} T_1^4 T_2^3 - 5 c_{49} T_1^4 T_2^3 - 8 c_{61} T_1^4 T_2^3 - 5 c_{64} T_1^4 T_2^3 - 54 c_{81} T_1^4 T_2^3 - 66 c_{82} T_1^4 T_2^3 - 28 c_{84} T_1^4 T_2^3 - \\
& 40 c_{85} T_1^4 T_2^3 - 19 a_2 b_3 T_1^5 T_2^3 + 20 a_4 b_3 T_1^5 T_2^3 + 13 a_2 b_5 T_1^5 T_2^3 - 11 a_4 b_5 T_1^5 T_2^3 + 13 c_{16} T_1^5 T_2^3 + \\
& 3 c_{19} T_1^5 T_2^3 + 13 c_{31} T_1^5 T_2^3 + 3 c_{34} T_1^5 T_2^3 + 13 c_{46} T_1^5 T_2^3 + 3 c_{49} T_1^5 T_2^3 + 13 c_{61} T_1^5 T_2^3 + 3 c_{64} T_1^5 T_2^3 + \\
& 36 c_{81} T_1^5 T_2^3 + 54 c_{82} T_1^5 T_2^3 + 8 c_{84} T_1^5 T_2^3 + 26 c_{85} T_1^5 T_2^3 + 13 a_2 b_3 T_1^6 T_2^3 - 19 a_4 b_3 T_1^6 T_2^3 - \\
& 6 a_2 b_5 T_1^6 T_2^3 + 10 a_4 b_5 T_1^6 T_2^3 + 5 c_{16} T_1^6 T_2^3 + 5 c_{31} T_1^6 T_2^3 + 5 c_{46} T_1^6 T_2^3 + 5 c_{61} T_1^6 T_2^3 - 4 c_{81} T_1^6 T_2^3 - \\
& 17 c_{82} T_1^6 T_2^3 + 4 c_{84} T_1^6 T_2^3 - 3 c_{85} T_1^6 T_2^3 - 4 a_2 b_3 T_1^7 T_2^3 + 8 a_4 b_3 T_1^7 T_2^3 - 7 c_{16} T_1^7 T_2^3 - 2 c_{19} T_1^7 T_2^3 - \\
& 7 c_{31} T_1^7 T_2^3 - 2 c_{34} T_1^7 T_2^3 - 7 c_{46} T_1^7 T_2^3 - 2 c_{49} T_1^7 T_2^3 - 7 c_{61} T_1^7 T_2^3 - 2 c_{64} T_1^7 T_2^3 - 4 c_{81} T_1^7 T_2^3 - \\
& 7 c_{82} T_1^7 T_2^3 - 8 c_{85} T_1^7 T_2^3 - 2 c_{16} T_1^8 T_2^3 - 2 c_{31} T_1^8 T_2^3 - 2 c_{46} T_1^8 T_2^3 - 2 c_{61} T_1^8 T_2^3 + 7 c_{82} T_1^8 T_2^3 + \\
& 5 c_{85} T_1^8 T_2^3 + 2 a_2 b_3 T_1^4 T_2 - 3 a_4 b_3 T_1^4 T_2 - 5 a_2 b_5 T_1^4 T_2 + 5 a_4 b_5 T_1^4 T_2 - 5 c_{19} T_1^4 T_2 - 5 c_{34} T_1^4 T_2 - 5 c_{49} T_1^4 T_2 - \\
& 5 c_{64} T_1^4 T_2 - 8 c_{81} T_1^4 T_2 - 7 c_{82} T_1^4 T_2 - 8 c_{84} T_1^4 T_2 - 2 c_{85} T_1^4 T_2 - 3 a_2 b_3 T_1 T_2^4 + 14 a_4 b_3 T_1 T_2^4 + 7 a_2 b_5 T_1 T_2^4 - \\
& 21 a_4 b_5 T_1 T_2^4 + 3 c_{16} T_1 T_2^4 + 5 c_{19} T_1 T_2^4 + 3 c_{31} T_1 T_2^4 + 5 c_{34} T_1 T_2^4 + 3 c_{46} T_1 T_2^4 + 5 c_{49} T_1 T_2^4 + \\
& 3 c_{61} T_1 T_2^4 + 5 c_{64} T_1 T_2^4 + 10 c_{81} T_1 T_2^4 + 7 c_{82} T_1 T_2^4 + 2 c_{84} T_1 T_2^4 - c_{85} T_1 T_2^4 - 5 a_2 b_3 T_1^2 T_2^4 - \\
& 18 a_4 b_3 T_1^2 T_2^4 + a_2 b_5 T_1^2 T_2^4 + 19 a_4 b_5 T_1^2 T_2^4 + c_{16} T_1^2 T_2^4 - c_{19} T_1^2 T_2^4 + c_{31} T_1^2 T_2^4 - c_{34} T_1^2 T_2^4 + \\
& c_{46} T_1^2 T_2^4 - c_{49} T_1^2 T_2^4 + c_{61} T_1^2 T_2^4 - c_{64} T_1^2 T_2^4 + 24 c_{81} T_1^2 T_2^4 + 17 c_{82} T_1^2 T_2^4 + 26 c_{84} T_1^2 T_2^4 + \\
& 14 c_{85} T_1^2 T_2^4 + 24 a_2 b_3 T_1^3 T_2^4 + a_4 b_3 T_1^3 T_2^4 - 21 a_2 b_5 T_1^3 T_2^4 - a_4 b_5 T_1^3 T_2^4 - 5 c_{16} T_1^3 T_2^4 - \\
& 8 c_{19} T_1^3 T_2^4 - 5 c_{31} T_1^3 T_2^4 - 8 c_{34} T_1^3 T_2^4 - 5 c_{46} T_1^3 T_2^4 - 8 c_{49} T_1^3 T_2^4 - 5 c_{61} T_1^3 T_2^4 - 8 c_{64} T_1^3 T_2^4 - \\
& 66 c_{81} T_1^3 T_2^4 - 54 c_{82} T_1^3 T_2^4 - 40 c_{84} T_1^3 T_2^4 - 28 c_{85} T_1^3 T_2^4 - 41 a_2 b_3 T_1^4 T_2^4 + 34 a_4 b_3 T_1^4 T_2^4 + \\
& 33 a_2 b_5 T_1^4 T_2^4 - 37 a_4 b_5 T_1^4 T_2^4 + 8 c_{16} T_1^4 T_2^4 + 8 c_{19} T_1^4 T_2^4 + 8 c_{31} T_1^4 T_2^4 + 8 c_{34} T_1^4 T_2^4 + \\
& 8 c_{46} T_1^4 T_2^4 + 8 c_{49} T_1^4 T_2^4 + 8 c_{61} T_1^4 T_2^4 + 8 c_{64} T_1^4 T_2^4 + 66 c_{81} T_1^4 T_2^4 + 66 c_{82} T_1^4 T_2^4 + 26 c_{84} T_1^4 T_2^4 + \\
& 26 c_{85} T_1^4 T_2^4 + 39 a_2 b_3 T_1^5 T_2^4 - 52 a_4 b_3 T_1^5 T_2^4 - 25 a_2 b_5 T_1^5 T_2^4 + 35 a_4 b_5 T_1^5 T_2^4 - 3 c_{16} T_1^5 T_2^4 - \\
& 3 c_{31} T_1^5 T_2^4 - 3 c_{46} T_1^5 T_2^4 - 3 c_{61} T_1^5 T_2^4 - 24 c_{81} T_1^5 T_2^4 - 30 c_{82} T_1^5 T_2^4 + 2 c_{84} T_1^5 T_2^4 - 4 c_{85} T_1^5 T_2^4 - \\
& 22 a_2 b_3 T_1^6 T_2^4 + 38 a_4 b_3 T_1^6 T_2^4 + 10 a_2 b_5 T_1^6 T_2^4 - 22 a_4 b_5 T_1^6 T_2^4 - 15 c_{16} T_1^6 T_2^4 - 3 c_{19} T_1^6 T_2^4 - \\
& 15 c_{31} T_1^6 T_2^4 - 3 c_{34} T_1^6 T_2^4 - 15 c_{46} T_1^6 T_2^4 - 3 c_{49} T_1^6 T_2^4 - 15 c_{61} T_1^6 T_2^4 - 3 c_{64} T_1^6 T_2^4 - 10 c_{81} T_1^6 T_2^4 - \\
& 11 c_{82} T_1^6 T_2^4 - 8 c_{84} T_1^6 T_2^4 - 14 c_{85} T_1^6 T_2^4 + 6 a_2 b_3 T_1^7 T_2^4 - 14 a_4 b_3 T_1^7 T_2^4 + 5 c_{16} T_1^7 T_2^4 + \\
& 3 c_{19} T_1^7 T_2^4 + 5 c_{31} T_1^7 T_2^4 + 3 c_{34} T_1^7 T_2^4 + 5 c_{46} T_1^7 T_2^4 + 3 c_{49} T_1^7 T_2^4 + 5 c_{61} T_1^7 T_2^4 + 3 c_{64} T_1^7 T_2^4 + \\
& 8 c_{81} T_1^7 T_2^4 + 23 c_{82} T_1^7 T_2^4 + 15 c_{85} T_1^7 T_2^4 + 6 c_{16} T_1^8 T_2^4 + c_{19} T_1^8 T_2^4 + 6 c_{31} T_1^8 T_2^4 + c_{34} T_1^8 T_2^4 + \\
& 6 c_{46} T_1^8 T_2^4 + c_{49} T_1^8 T_2^4 + 6 c_{61} T_1^8 T_2^4 + c_{64} T_1^8 T_2^4 - 11 c_{82} T_1^8 T_2^4 - 6 c_{85} T_1^8 T_2^4 - a_2 b_3 T_1^5 + \\
& 2 a_4 b_3 T_1^5 + 3 a_2 b_5 T_1^5 - 5 a_4 b_5 T_1^5 + 2 c_{19} T_1^5 + 2 c_{34} T_1^5 + 2 c_{49} T_1^5 + 2 c_{64} T_1^5 + 4 c_{81} T_1^5 + \\
& 2 c_{82} T_1^5 + 4 c_{84} T_1^5 - 6 a_4 b_3 T_1 T_2^5 + 15 a_4 b_5 T_1 T_2^5 - c_{16} T_1 T_2^5 + 4 c_{19} T_1 T_2^5 - c_{31} T_1 T_2^5 + \\
& 4 c_{34} T_1 T_2^5 - c_{46} T_1 T_2^5 + 4 c_{49} T_1 T_2^5 - c_{61} T_1 T_2^5 + 4 c_{64} T_1 T_2^5 + 4 c_{81} T_1 T_2^5 + 4 c_{82} T_1 T_2^5 + \\
& 8 c_{84} T_1 T_2^5 + 3 c_{85} T_1 T_2^5 + 8 a_2 b_3 T_1^2 T_2^5 - 5 a_4 b_3 T_1^2 T_2^5 - 13 a_2 b_5 T_1^2 T_2^5 + 2 a_4 b_5 T_1^2 T_2^5 - \\
& 3 c_{16} T_1^2 T_2^5 - 10 c_{19} T_1^2 T_2^5 - 3 c_{31} T_1^2 T_2^5 - 10 c_{34} T_1^2 T_2^5 - 3 c_{46} T_1^2 T_2^5 - 10 c_{49} T_1^2 T_2^5 - 3 c_{61} T_1^2 T_2^5 - \\
& 10 c_{64} T_1^2 T_2^5 - 36 c_{81} T_1^2 T_2^5 - 22 c_{82} T_1^2 T_2^5 - 28 c_{84} T_1^2 T_2^5 - 8 c_{85} T_1^2 T_2^5 - 22 a_2 b_3 T_1^3 T_2^5 + \\
& 37 a_4 b_3 T_1^3 T_2^5 + 33 a_2 b_5 T_1^3 T_2^5 - 34 a_4 b_5 T_1^3 T_2^5 + 3 c_{16} T_1^3 T_2^5 + 13 c_{19} T_1^3 T_2^5 + 3 c_{31} T_1^3 T_2^5 + \\
& 13 c_{34} T_1^3 T_2^5 + 3 c_{46} T_1^3 T_2^5 + 13 c_{49} T_1^3 T_2^5 + 3 c_{61} T_1^3 T_2^5 + 13 c_{64} T_1^3 T_2^5 + 54 c_{81} T_1^3 T_2^5 + 36 c_{82} T_1^3 T_2^5 + \\
& 26 c_{84} T_1^3 T_2^5 + 8 c_{85} T_1^3 T_2^5 + 32 a_2 b_3 T_1^4 T_2^5 - 74 a_4 b_3 T_1^4 T_2^5 - 40 a_2 b_5 T_1^4 T_2^5 + 74 a_4 b_5 T_1^4 T_2^5 - \\
& 3 c_{19} T_1^4 T_2^5 - 3 c_{34} T_1^4 T_2^5 - 3 c_{49} T_1^4 T_2^5 - 3 c_{64} T_1^4 T_2^5 - 30 c_{81} T_1^4 T_2^5 - 24 c_{82} T_1^4 T_2^5 - 4 c_{84} T_1^4 T_2^5 + \\
& 2 c_{85} T_1^4 T_2^5 - 28 a_2 b_3 T_1^5 T_2^5 + 79 a_4 b_3 T_1^5 T_2^5 + 27 a_2 b_5 T_1^5 T_2^5 - 57 a_4 b_5 T_1^5 T_2^5 - 7 c_{16} T_1^5 T_2^5 - \\
& 7 c_{19} T_1^5 T_2^5 - 7 c_{31} T_1^5 T_2^5 - 7 c_{34} T_1^5 T_2^5 - 7 c_{46} T_1^5 T_2^5 - 7 c_{49} T_1^5 T_2^5 - 7 c_{61} T_1^5 T_2^5 - 7 c_{64} T_1^5 T_2^5 - \\
& 12 c_{81} T_1^5 T_2^5 - 12 c_{82} T_1^5 T_2^5 - 16 c_{84} T_1^5 T_2^5 - 16 c_{85} T_1^5 T_2^5 + 15 a_2 b_3 T_1^6 T_2^5 - 49 a_4 b_3 T_1^6 T_2^5 - \\
& 10 a_2 b_5 T_1^6 T_2^5 + 30 a_4 b_5 T_1^6 T_2^5 + 13 c_{16} T_1^6 T_2^5 + 5 c_{19} T_1^6 T_2^5 + 13 c_{31} T_1^6 T_2^5 + 5 c_{34} T_1^6 T_2^5 + \\
& 13 c_{46} T_1^6 T_2^5 + 5 c_{49} T_1^6 T_2^5 + 13 c_{61} T_1^6 T_2^5 + 5 c_{64} T_1^6 T_2^5 + 26 c_{81} T_1^6 T_2^5 + 34 c_{82} T_1^6 T_2^5 + \\
& 10 c_{84} T_1^6 T_2^5 + 20 c_{85} T_1^6 T_2^5 - 4 a_2 b_3 T_1^7 T_2^5 + 16 a_4 b_3 T_1^7 T_2^5 + 5 c_{16} T_1^7 T_2^5 - c_{19} T_1^7 T_2^5 + 5 c_{31} T_1^7 T_2^5 - \\
& c_{34} T_1^7 T_2^5 + 5 c_{46} T_1^7 T_2^5 - c_{49} T_1^7 T_2^5 + 5 c_{61} T_1^7 T_2^5 - c_{64} T_1^7 T_2^5 - 10 c_{81} T_1^7 T_2^5 - 28 c_{82} T_1^7 T_2^5 - \\
& 13 c_{85} T_1^7 T_2^5 - 10 c_{16} T_1^8 T_2^5 - 3 c_{19} T_1^8 T_2^5 - 10 c_{31} T_1^8 T_2^5 - 3 c_{34} T_1^8 T_2^5 - 10 c_{46} T_1^8 T_2^5 - 3 c_{49} T_1^8 T_2^5 - \\
& 10 c_{61} T_1^8 T_2^5 - 3 c_{64} T_1^8 T_2^5 + 10 c_{82} T_1^8 T_2^5 + 4 c_{85} T_1^8 T_2^5 - a_4 b_3 T_1^6 - a_2 b_5 T_1^6 + 3 a_4 b_5 T_1^6 -
\end{aligned}$$

$$\begin{aligned}
 & c_{81} T_2^6 - c_{84} T_2^6 + 2 a_2 b_3 T_1 T_2^6 + a_4 b_3 T_1 T_2^6 - 3 a_2 b_5 T_1 T_2^6 - 4 a_4 b_5 T_1 T_2^6 - 4 c_{19} T_1 T_2^6 - \\
 & 4 c_{34} T_1 T_2^6 - 4 c_{49} T_1 T_2^6 - 4 c_{64} T_1 T_2^6 - 7 c_{81} T_1 T_2^6 - 3 c_{82} T_1 T_2^6 - 8 c_{84} T_1 T_2^6 - 8 a_2 b_3 T_1^2 T_2^6 + \\
 & 11 a_4 b_3 T_1^2 T_2^6 + 13 a_2 b_5 T_1^2 T_2^6 - 20 a_4 b_5 T_1^2 T_2^6 + 2 c_{16} T_1^2 T_2^6 + 2 c_{19} T_1^2 T_2^6 + 2 c_{31} T_1^2 T_2^6 + \\
 & 2 c_{34} T_1^2 T_2^6 + 2 c_{46} T_1^2 T_2^6 + 2 c_{49} T_1^2 T_2^6 + 2 c_{61} T_1^2 T_2^6 + 2 c_{64} T_1^2 T_2^6 + 22 c_{81} T_1^2 T_2^6 + 7 c_{82} T_1^2 T_2^6 + \\
 & 14 c_{84} T_1^2 T_2^6 - c_{85} T_1^2 T_2^6 + 16 a_2 b_3 T_1^3 T_2^6 - 35 a_4 b_3 T_1^3 T_2^6 - 25 a_2 b_5 T_1^3 T_2^6 + 52 a_4 b_5 T_1^3 T_2^6 + \\
 & 5 c_{19} T_1^3 T_2^6 + 5 c_{34} T_1^3 T_2^6 + 5 c_{49} T_1^3 T_2^6 + 5 c_{64} T_1^3 T_2^6 - 17 c_{81} T_1^3 T_2^6 - 4 c_{82} T_1^3 T_2^6 - 3 c_{84} T_1^3 T_2^6 + \\
 & 4 c_{85} T_1^3 T_2^6 - 20 a_2 b_3 T_1^4 T_2^6 + 57 a_4 b_3 T_1^4 T_2^6 + 27 a_2 b_5 T_1^4 T_2^6 - 79 a_4 b_5 T_1^4 T_2^6 - 3 c_{16} T_1^4 T_2^6 - \\
 & 15 c_{19} T_1^4 T_2^6 - 3 c_{31} T_1^4 T_2^6 - 15 c_{34} T_1^4 T_2^6 - 3 c_{46} T_1^4 T_2^6 - 15 c_{49} T_1^4 T_2^6 - 3 c_{61} T_1^4 T_2^6 - 15 c_{64} T_1^4 T_2^6 - \\
 & 11 c_{81} T_1^4 T_2^6 - 10 c_{82} T_1^4 T_2^6 - 14 c_{84} T_1^4 T_2^6 - 8 c_{85} T_1^4 T_2^6 + 16 a_2 b_3 T_1^5 T_2^6 - 55 a_4 b_3 T_1^5 T_2^6 - \\
 & 17 a_2 b_5 T_1^5 T_2^6 + 55 a_4 b_5 T_1^5 T_2^6 + 5 c_{16} T_1^5 T_2^6 + 13 c_{19} T_1^5 T_2^6 + 5 c_{31} T_1^5 T_2^6 + 13 c_{34} T_1^5 T_2^6 + \\
 & 5 c_{46} T_1^5 T_2^6 + 13 c_{49} T_1^5 T_2^6 + 5 c_{61} T_1^5 T_2^6 + 13 c_{64} T_1^5 T_2^6 + 34 c_{81} T_1^5 T_2^6 + 26 c_{82} T_1^5 T_2^6 + 20 c_{84} T_1^5 T_2^6 + \\
 & 10 c_{85} T_1^5 T_2^6 - 8 a_2 b_3 T_1^6 T_2^6 + 32 a_4 b_3 T_1^6 T_2^6 + 6 a_2 b_5 T_1^6 T_2^6 - 26 a_4 b_5 T_1^6 T_2^6 - c_{16} T_1^6 T_2^6 - \\
 & c_{19} T_1^6 T_2^6 - c_{31} T_1^6 T_2^6 - c_{34} T_1^6 T_2^6 - c_{46} T_1^6 T_2^6 - c_{49} T_1^6 T_2^6 - c_{61} T_1^6 T_2^6 - c_{64} T_1^6 T_2^6 - 28 c_{81} T_1^6 T_2^6 - \\
 & 28 c_{82} T_1^6 T_2^6 - 8 c_{84} T_1^6 T_2^6 - 8 c_{85} T_1^6 T_2^6 + 2 a_2 b_3 T_1^7 T_2^6 - 10 a_4 b_3 T_1^7 T_2^6 - 13 c_{16} T_1^7 T_2^6 - \\
 & 5 c_{19} T_1^7 T_2^6 - 13 c_{31} T_1^7 T_2^6 - 5 c_{34} T_1^7 T_2^6 - 13 c_{46} T_1^7 T_2^6 - 5 c_{49} T_1^7 T_2^6 - 13 c_{61} T_1^7 T_2^6 - 5 c_{64} T_1^7 T_2^6 + \\
 & 8 c_{81} T_1^7 T_2^6 + 17 c_{82} T_1^7 T_2^6 + 4 c_{85} T_1^7 T_2^6 + 10 c_{16} T_1^8 T_2^6 + 5 c_{19} T_1^8 T_2^6 + 10 c_{31} T_1^8 T_2^6 + 5 c_{34} T_1^8 T_2^6 + \\
 & 10 c_{46} T_1^8 T_2^6 + 5 c_{49} T_1^8 T_2^6 + 10 c_{61} T_1^8 T_2^6 + 5 c_{64} T_1^8 T_2^6 - 5 c_{82} T_1^8 T_2^6 - c_{85} T_1^8 T_2^6 - a_4 b_5 T_1^7 + \\
 & 2 a_4 b_3 T_1 T_2^7 + 2 a_2 b_5 T_1 T_2^7 - 2 a_4 b_5 T_1 T_2^7 + 3 c_{81} T_1 T_2^7 + 3 c_{84} T_1 T_2^7 - 10 a_4 b_3 T_1^2 T_2^7 - \\
 & 6 a_2 b_5 T_1^2 T_2^7 + 19 a_4 b_5 T_1^2 T_2^7 + 4 c_{19} T_1^2 T_2^7 + 4 c_{34} T_1^2 T_2^7 + 4 c_{49} T_1^2 T_2^7 + 4 c_{64} T_1^2 T_2^7 - 4 c_{81} T_1^2 T_2^7 + \\
 & c_{82} T_1^2 T_2^7 - c_{84} T_1^2 T_2^7 + 22 a_4 b_3 T_1^3 T_2^7 + 10 a_2 b_5 T_1^3 T_2^7 - 38 a_4 b_5 T_1^3 T_2^7 - 2 c_{16} T_1^3 T_2^7 - \\
 & 7 c_{19} T_1^3 T_2^7 - 2 c_{31} T_1^3 T_2^7 - 7 c_{34} T_1^3 T_2^7 - 2 c_{46} T_1^3 T_2^7 - 7 c_{49} T_1^3 T_2^7 - 2 c_{61} T_1^3 T_2^7 - 7 c_{64} T_1^3 T_2^7 - \\
 & 7 c_{81} T_1^3 T_2^7 - 4 c_{82} T_1^3 T_2^7 - 8 c_{84} T_1^3 T_2^7 - 30 a_4 b_3 T_1^4 T_2^7 - 10 a_2 b_5 T_1^4 T_2^7 + 49 a_4 b_5 T_1^4 T_2^7 + \\
 & 3 c_{16} T_1^4 T_2^7 + 5 c_{19} T_1^4 T_2^7 + 3 c_{31} T_1^4 T_2^7 + 5 c_{34} T_1^4 T_2^7 + 3 c_{46} T_1^4 T_2^7 + 5 c_{49} T_1^4 T_2^7 + 3 c_{61} T_1^4 T_2^7 + \\
 & 5 c_{64} T_1^4 T_2^7 + 23 c_{81} T_1^4 T_2^7 + 8 c_{82} T_1^4 T_2^7 + 15 c_{84} T_1^4 T_2^7 + 26 a_4 b_3 T_1^5 T_2^7 + 6 a_2 b_5 T_1^5 T_2^7 - \\
 & 32 a_4 b_5 T_1^5 T_2^7 - c_{16} T_1^5 T_2^7 + 5 c_{19} T_1^5 T_2^7 - c_{31} T_1^5 T_2^7 + 5 c_{34} T_1^5 T_2^7 - c_{46} T_1^5 T_2^7 + 5 c_{49} T_1^5 T_2^7 - \\
 & c_{61} T_1^5 T_2^7 + 5 c_{64} T_1^5 T_2^7 - 28 c_{81} T_1^5 T_2^7 - 10 c_{82} T_1^5 T_2^7 - 13 c_{84} T_1^5 T_2^7 - 14 a_4 b_3 T_1^6 T_2^7 - \\
 & 2 a_2 b_5 T_1^6 T_2^7 + 14 a_4 b_5 T_1^6 T_2^7 - 5 c_{16} T_1^6 T_2^7 - 13 c_{19} T_1^6 T_2^7 - 5 c_{31} T_1^6 T_2^7 - 13 c_{34} T_1^6 T_2^7 - \\
 & 5 c_{46} T_1^6 T_2^7 - 13 c_{49} T_1^6 T_2^7 - 5 c_{61} T_1^6 T_2^7 - 13 c_{64} T_1^6 T_2^7 + 17 c_{81} T_1^6 T_2^7 + 8 c_{82} T_1^6 T_2^7 + 4 c_{84} T_1^6 T_2^7 + \\
 & 4 a_4 b_3 T_1^7 T_2^7 + 11 c_{16} T_1^7 T_2^7 + 11 c_{19} T_1^7 T_2^7 + 11 c_{31} T_1^7 T_2^7 + 11 c_{34} T_1^7 T_2^7 + 11 c_{46} T_1^7 T_2^7 + \\
 & 11 c_{49} T_1^7 T_2^7 + 11 c_{61} T_1^7 T_2^7 + 11 c_{64} T_1^7 T_2^7 - 4 c_{81} T_1^7 T_2^7 - 4 c_{82} T_1^7 T_2^7 - 6 c_{16} T_1^8 T_2^7 - 5 c_{19} T_1^8 T_2^7 - \\
 & 6 c_{31} T_1^8 T_2^7 - 5 c_{34} T_1^8 T_2^7 - 6 c_{46} T_1^8 T_2^7 - 5 c_{49} T_1^8 T_2^7 - 6 c_{61} T_1^8 T_2^7 - 5 c_{64} T_1^8 T_2^7 + c_{82} T_1^8 T_2^7 + \\
 & 2 a_4 b_5 T_1 T_2^8 - 8 a_4 b_5 T_1^2 T_2^8 - 2 c_{81} T_1^2 T_2^8 - 2 c_{84} T_1^2 T_2^8 + 14 a_4 b_5 T_1^3 T_2^8 - 2 c_{19} T_1^3 T_2^8 - \\
 & 2 c_{34} T_1^3 T_2^8 - 2 c_{49} T_1^3 T_2^8 - 2 c_{64} T_1^3 T_2^8 + 7 c_{81} T_1^3 T_2^8 + 5 c_{84} T_1^3 T_2^8 - 16 a_4 b_5 T_1^4 T_2^8 + c_{16} T_1^4 T_2^8 + \\
 & 6 c_{19} T_1^4 T_2^8 + c_{31} T_1^4 T_2^8 + 6 c_{34} T_1^4 T_2^8 + c_{46} T_1^4 T_2^8 + 6 c_{49} T_1^4 T_2^8 + c_{61} T_1^4 T_2^8 + 6 c_{64} T_1^4 T_2^8 - \\
 & 11 c_{81} T_1^4 T_2^8 - 6 c_{84} T_1^4 T_2^8 + 10 a_4 b_5 T_1^5 T_2^8 - 3 c_{16} T_1^5 T_2^8 - 10 c_{19} T_1^5 T_2^8 - 3 c_{31} T_1^5 T_2^8 - \\
 & 10 c_{34} T_1^5 T_2^8 - 3 c_{46} T_1^5 T_2^8 - 10 c_{49} T_1^5 T_2^8 - 3 c_{61} T_1^5 T_2^8 - 10 c_{64} T_1^5 T_2^8 + 10 c_{81} T_1^5 T_2^8 + 4 c_{84} T_1^5 T_2^8 - \\
 & 4 a_4 b_5 T_1^6 T_2^8 + 5 c_{16} T_1^6 T_2^8 + 10 c_{19} T_1^6 T_2^8 + 5 c_{31} T_1^6 T_2^8 + 10 c_{34} T_1^6 T_2^8 + 5 c_{46} T_1^6 T_2^8 + 10 c_{49} T_1^6 T_2^8 + \\
 & 5 c_{61} T_1^6 T_2^8 + 10 c_{64} T_1^6 T_2^8 - 5 c_{81} T_1^6 T_2^8 - c_{84} T_1^6 T_2^8 - 5 c_{16} T_1^7 T_2^8 - 6 c_{19} T_1^7 T_2^8 - 5 c_{31} T_1^7 T_2^8 - \\
 & 6 c_{34} T_1^7 T_2^8 - 5 c_{46} T_1^7 T_2^8 - 6 c_{49} T_1^7 T_2^8 - 5 c_{61} T_1^7 T_2^8 - 6 c_{64} T_1^7 T_2^8 + c_{81} T_1^7 T_2^8 + 2 c_{16} T_1^8 T_2^8 + \\
 & 2 c_{19} T_1^8 T_2^8 + 2 c_{31} T_1^8 T_2^8 + 2 c_{34} T_1^8 T_2^8 + 2 c_{46} T_1^8 T_2^8 + 2 c_{49} T_1^8 T_2^8 + 2 c_{61} T_1^8 T_2^8 + 2 c_{64} T_1^8 T_2^8) / \\
 & ((-1 + T_1) (1 - T_1 + T_1^2))^2 (-1 + T_2) (-1 + T_1 T_2) (1 - T_2 + T_2^2) (1 - T_1 T_2 + T_1^2 T_2^2)
 \end{aligned}$$

$$\text{In[*]:= } K = \text{Knot}["K11n34"]; \text{ Conway} = \int \mathcal{L}[K] \times \mathfrak{d} \text{ vs } [K]$$

 KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

 **KnotTheory**: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[\*]=

$$\begin{aligned}
 & -i \mathbb{E} \left[ \epsilon \text{Series} \left[ 0, \right. \right. \\
 & \quad \left. \left. \frac{1}{(-1 + T_1) T_1^7 (-1 + T_2) T_2^6 (-1 + T_1 T_2)} \left( a_2 b_3 T_1^2 - a_2 b_5 T_1^2 - 4 a_2 b_3 T_1^3 + a_4 b_3 T_1^3 + 3 a_2 b_5 T_1^3 - \right. \right. \right. \\
 & \quad a_4 b_5 T_1^3 + 6 a_2 b_3 T_1^4 - 3 a_4 b_3 T_1^4 - 3 a_2 b_5 T_1^4 + 2 a_4 b_5 T_1^4 - 4 a_2 b_3 T_1^5 + 3 a_4 b_3 T_1^5 + a_2 b_5 T_1^5 - \\
 & \quad a_4 b_5 T_1^5 + a_2 b_3 T_1^6 - a_4 b_3 T_1^6 - 2 a_2 b_3 T_1 T_2 + 2 a_2 b_5 T_1 T_2 + 6 a_2 b_3 T_1^2 T_2 - a_4 b_3 T_1^2 T_2 - \\
 & \quad 3 a_2 b_5 T_1^2 T_2 + a_4 b_5 T_1^2 T_2 - 6 a_2 b_3 T_1^3 T_2 - a_4 b_3 T_1^3 T_2 - a_2 b_5 T_1^3 T_2 + 3 a_4 b_5 T_1^3 T_2 + 2 a_2 b_3 T_1^4 T_2 + \\
 & \quad 7 a_4 b_3 T_1^4 T_2 + 3 a_2 b_5 T_1^4 T_2 - 7 a_4 b_5 T_1^4 T_2 + 2 a_2 b_3 T_1^5 T_2 - 7 a_4 b_3 T_1^5 T_2 - 3 a_2 b_5 T_1^5 T_2 + \\
 & \quad 3 a_4 b_5 T_1^5 T_2 - 6 a_2 b_3 T_1^6 T_2 + 4 a_4 b_3 T_1^6 T_2 + 4 a_2 b_5 T_1^6 T_2 - 2 a_4 b_5 T_1^6 T_2 + 6 a_2 b_3 T_1^7 T_2 - \\
 & \quad 4 a_4 b_3 T_1^7 T_2 - 2 a_2 b_5 T_1^7 T_2 + 2 a_4 b_5 T_1^7 T_2 - 2 a_2 b_3 T_1^8 T_2 + 2 a_4 b_3 T_1^8 T_2 + a_2 b_5 T_1^8 T_2 - a_4 b_5 T_1^8 T_2 - \\
 & \quad a_4 b_3 T_1 T_2^2 - 3 a_2 b_5 T_1 T_2^2 + a_4 b_5 T_1 T_2^2 - 5 a_2 b_3 T_1^2 T_2^2 + 9 a_4 b_3 T_1^2 T_2^2 + 8 a_2 b_5 T_1^2 T_2^2 - 9 a_4 b_5 T_1^2 T_2^2 + \\
 & \quad 6 a_2 b_3 T_1^3 T_2^2 - 14 a_4 b_3 T_1^3 T_2^2 - 4 a_2 b_5 T_1^3 T_2^2 + 4 a_4 b_5 T_1^3 T_2^2 - 4 a_2 b_3 T_1^4 T_2^2 + 6 a_4 b_3 T_1^4 T_2^2 + \\
 & \quad 2 a_2 b_5 T_1^4 T_2^2 + 5 a_4 b_5 T_1^4 T_2^2 + 4 a_2 b_3 T_1^5 T_2^2 - 2 a_4 b_5 T_1^5 T_2^2 - 4 a_2 b_3 T_1^6 T_2^2 - 6 a_4 b_3 T_1^6 T_2^2 - \\
 & \quad 2 a_2 b_5 T_1^6 T_2^2 + 8 a_4 b_5 T_1^6 T_2^2 + 6 a_2 b_3 T_1^7 T_2^2 + 8 a_4 b_3 T_1^7 T_2^2 - 2 a_2 b_5 T_1^7 T_2^2 - 4 a_4 b_5 T_1^7 T_2^2 - \\
 & \quad 5 a_2 b_3 T_1^8 T_2^2 + a_2 b_5 T_1^8 T_2^2 - 2 a_4 b_5 T_1^8 T_2^2 - a_4 b_3 T_1^9 T_2^2 + a_2 b_5 T_1^9 T_2^2 - a_4 b_5 T_1^9 T_2^2 + a_2 b_3 T_1^{10} T_2^2 - \\
 & \quad a_4 b_3 T_1^{10} T_2^2 - 2 a_2 b_3 T_2^3 + a_4 b_3 T_2^3 + 3 a_2 b_5 T_2^3 - a_4 b_5 T_2^3 + 4 a_2 b_3 T_1 T_2^3 - 3 a_4 b_3 T_1 T_2^3 - a_2 b_5 T_1 T_2^3 + \\
 & \quad a_4 b_5 T_1 T_2^3 - 2 a_2 b_3 T_1^2 T_2^3 - 4 a_4 b_3 T_1^2 T_2^3 - 4 a_2 b_5 T_1^2 T_2^3 + 14 a_4 b_5 T_1^2 T_2^3 + 10 a_4 b_3 T_1^3 T_2^3 + \\
 & \quad 2 a_2 b_5 T_1^3 T_2^3 - 10 a_4 b_5 T_1^3 T_2^3 + a_2 b_3 T_1^4 T_2^3 - 6 a_4 b_3 T_1^4 T_2^3 - 3 a_2 b_5 T_1^4 T_2^3 + 2 a_4 b_5 T_1^4 T_2^3 - \\
 & \quad a_2 b_3 T_1^5 T_2^3 + 7 a_4 b_3 T_1^5 T_2^3 + 2 a_2 b_5 T_1^5 T_2^3 - 5 a_4 b_5 T_1^5 T_2^3 - a_2 b_3 T_1^6 T_2^3 - 6 a_4 b_3 T_1^6 T_2^3 - a_2 b_5 T_1^6 T_2^3 - \\
 & \quad 5 a_4 b_5 T_1^6 T_2^3 + a_2 b_3 T_1^7 T_2^3 + 7 a_4 b_3 T_1^7 T_2^3 + 4 a_2 b_5 T_1^7 T_2^3 - 4 a_4 b_5 T_1^7 T_2^3 + 2 c_{19} T_1^7 T_2^3 + 2 c_{34} T_1^7 T_2^3 + \\
 & \quad 2 c_{49} T_1^7 T_2^3 + 2 c_{64} T_1^7 T_2^3 - 9 a_4 b_3 T_1^8 T_2^3 - a_2 b_5 T_1^8 T_2^3 + 5 a_4 b_5 T_1^8 T_2^3 - 2 c_{19} T_1^8 T_2^3 - 2 c_{34} T_1^8 T_2^3 - \\
 & \quad 2 c_{49} T_1^8 T_2^3 - 2 c_{64} T_1^8 T_2^3 - 2 a_2 b_3 T_1^9 T_2^3 + a_4 b_3 T_1^9 T_2^3 + a_2 b_5 T_1^9 T_2^3 + 3 a_4 b_5 T_1^9 T_2^3 + 4 a_2 b_3 T_1^{10} T_2^3 - \\
 & \quad 2 a_2 b_5 T_1^{10} T_2^3 + 2 a_4 b_5 T_1^{10} T_2^3 - 2 a_2 b_3 T_1^{11} T_2^3 + 2 a_4 b_3 T_1^{11} T_2^3 + a_2 b_5 T_1^{11} T_2^3 - 2 a_4 b_3 T_2^4 - 3 a_2 b_5 T_2^4 + \\
 & \quad 3 a_4 b_5 T_2^4 - 2 a_2 b_3 T_1 T_2^4 + 7 a_4 b_3 T_1 T_2^4 + 3 a_2 b_5 T_1 T_2^4 - 7 a_4 b_5 T_1 T_2^4 - a_2 b_3 T_1^2 T_2^4 - 5 a_4 b_3 T_1^2 T_2^4 + \\
 & \quad 2 a_2 b_5 T_1^2 T_2^4 - 6 a_4 b_5 T_1^2 T_2^4 + 5 a_2 b_3 T_1^3 T_2^4 - 2 a_4 b_3 T_1^3 T_2^4 - 3 a_2 b_5 T_1^3 T_2^4 + 6 a_4 b_5 T_1^3 T_2^4 - \\
 & \quad 2 a_2 b_3 T_1^4 T_2^4 + 4 a_4 b_3 T_1^4 T_2^4 - 4 a_4 b_5 T_1^4 T_2^4 - 3 a_2 b_3 T_1^5 T_2^4 - a_4 b_3 T_1^5 T_2^4 + 2 a_2 b_5 T_1^5 T_2^4 + 4 a_4 b_5 T_1^5 T_2^4 + \\
 & \quad 4 a_2 b_3 T_1^6 T_2^4 - 3 a_4 b_3 T_1^6 T_2^4 - 3 a_2 b_5 T_1^6 T_2^4 + a_4 b_5 T_1^6 T_2^4 - 3 a_2 b_3 T_1^7 T_2^4 + 4 a_4 b_3 T_1^7 T_2^4 + a_2 b_5 T_1^7 T_2^4 + \\
 & \quad 4 a_4 b_5 T_1^7 T_2^4 - 6 c_{19} T_1^7 T_2^4 - 6 c_{34} T_1^7 T_2^4 - 6 c_{49} T_1^7 T_2^4 - 6 c_{64} T_1^7 T_2^4 - 2 a_2 b_3 T_1^8 T_2^4 - a_4 b_3 T_1^8 T_2^4 + \\
 & \quad 3 a_2 b_5 T_1^8 T_2^4 - 4 a_4 b_5 T_1^8 T_2^4 + 4 c_{19} T_1^8 T_2^4 + 4 c_{34} T_1^8 T_2^4 + 4 c_{49} T_1^8 T_2^4 + 4 c_{64} T_1^8 T_2^4 + 5 a_2 b_3 T_1^9 T_2^4 - \\
 & \quad 5 a_4 b_3 T_1^9 T_2^4 - 4 a_2 b_5 T_1^9 T_2^4 + 2 a_4 b_5 T_1^9 T_2^4 + 2 c_{19} T_1^9 T_2^4 + 2 c_{34} T_1^9 T_2^4 + 2 c_{49} T_1^9 T_2^4 + 2 c_{64} T_1^9 T_2^4 - \\
 & \quad a_2 b_3 T_1^{10} T_2^4 + 8 a_4 b_3 T_1^{10} T_2^4 + a_2 b_5 T_1^{10} T_2^4 - 6 a_4 b_5 T_1^{10} T_2^4 - 2 a_2 b_3 T_1^{11} T_2^4 - 3 a_4 b_3 T_1^{11} T_2^4 + \\
 & \quad a_2 b_5 T_1^{11} T_2^4 - a_4 b_5 T_1^{11} T_2^4 + a_2 b_3 T_1^{12} T_2^4 - a_4 b_3 T_1^{12} T_2^4 + a_4 b_3 T_2^5 + a_2 b_5 T_2^5 - 3 a_4 b_5 T_2^5 + \\
 & \quad 2 a_2 b_3 T_1 T_2^5 - 3 a_4 b_3 T_1 T_2^5 - 3 a_2 b_5 T_1 T_2^5 + 7 a_4 b_5 T_1 T_2^5 - 4 a_2 b_3 T_1^2 T_2^5 + 2 a_4 b_3 T_1^2 T_2^5 + \\
 & \quad 3 a_2 b_5 T_1^3 T_2^5 + 5 a_4 b_3 T_1^3 T_2^5 + 2 a_2 b_5 T_1^3 T_2^5 - 7 a_4 b_5 T_1^3 T_2^5 - 2 a_2 b_3 T_1^4 T_2^5 - 4 a_4 b_3 T_1^4 T_2^5 + \\
 & \quad 2 a_2 b_5 T_1^4 T_2^5 + a_4 b_5 T_1^4 T_2^5 - 3 a_2 b_3 T_1^5 T_2^5 - 5 a_4 b_3 T_1^5 T_2^5 + 2 a_2 b_5 T_1^5 T_2^5 + 5 a_4 b_5 T_1^5 T_2^5 + \\
 & \quad 4 a_2 b_3 T_1^6 T_2^5 + 2 a_4 b_3 T_1^6 T_2^5 + 2 a_2 b_5 T_1^6 T_2^5 + 4 a_2 b_3 T_1^7 T_2^5 - 5 a_4 b_3 T_1^7 T_2^5 - 5 a_2 b_5 T_1^7 T_2^5 + \\
 & \quad 9 a_4 b_5 T_1^7 T_2^5 + 6 c_{19} T_1^7 T_2^5 + 6 c_{34} T_1^7 T_2^5 + 6 c_{49} T_1^7 T_2^5 + 6 c_{64} T_1^7 T_2^5 - 3 a_2 b_3 T_1^8 T_2^5 + 3 a_4 b_3 T_1^8 T_2^5 - \\
 & \quad 4 a_2 b_5 T_1^8 T_2^5 + 5 a_4 b_5 T_1^8 T_2^5 - 2 a_2 b_3 T_1^9 T_2^5 + 9 a_4 b_3 T_1^9 T_2^5 + 3 a_2 b_5 T_1^9 T_2^5 - 9 a_4 b_5 T_1^9 T_2^5 - \\
 & \quad 6 c_{19} T_1^9 T_2^5 - 6 c_{34} T_1^9 T_2^5 - 6 c_{49} T_1^9 T_2^5 - 6 c_{64} T_1^9 T_2^5 + 3 a_2 b_3 T_1^{10} T_2^5 - 4 a_4 b_3 T_1^{10} T_2^5 - a_2 b_5 T_1^{10} T_2^5 + \\
 & \quad 3 a_4 b_5 T_1^{10} T_2^5 - 4 a_2 b_3 T_1^{11} T_2^5 - a_4 b_3 T_1^{11} T_2^5 + a_2 b_5 T_1^{11} T_2^5 + a_4 b_5 T_1^{11} T_2^5 + 2 a_2 b_3 T_1^{12} T_2^5 + a_4 b_5 T_2^6 - \\
 & \quad 2 a_2 b_3 T_1 T_2^6 + 2 a_4 b_3 T_1 T_2^6 + 4 a_2 b_5 T_1 T_2^6 - 4 a_4 b_5 T_1 T_2^6 + 2 a_2 b_3 T_1^2 T_2^6 - 8 a_4 b_3 T_1^2 T_2^6 - \\
 & \quad 2 a_2 b_5 T_1^2 T_2^6 + 6 a_4 b_5 T_1^2 T_2^6 + 2 a_2 b_3 T_1^3 T_2^6 + 5 a_4 b_3 T_1^3 T_2^6 - a_2 b_5 T_1^3 T_2^6 + 6 a_4 b_5 T_1^3 T_2^6 - a_4 b_3 T_1^4 T_2^6 - \\
 & \quad 3 a_2 b_5 T_1^4 T_2^6 + 3 a_4 b_5 T_1^4 T_2^6 + 2 c_{16} T_1^4 T_2^6 + 2 c_{31} T_1^4 T_2^6 + 2 c_{46} T_1^4 T_2^6 + 2 c_{61} T_1^4 T_2^6 - 8 a_2 b_3 T_1^5 T_2^6 + \\
 & \quad 2 a_2 b_5 T_1^5 T_2^6 - 2 a_4 b_5 T_1^5 T_2^6 - 6 c_{16} T_1^5 T_2^6 - 6 c_{31} T_1^5 T_2^6 - 6 c_{46} T_1^5 T_2^6 - 6 c_{61} T_1^5 T_2^6 + 22 a_2 b_3 T_1^6 T_2^6 +
 \end{aligned}$$

$$\begin{aligned}
 & 2 a_4 b_3 T_1^6 T_2^6 - 16 a_2 b_5 T_1^6 T_2^6 - 2 a_4 b_5 T_1^6 T_2^6 + 6 c_{16} T_1^6 T_2^6 + 6 c_{31} T_1^6 T_2^6 + 6 c_{46} T_1^6 T_2^6 + 6 c_{61} T_1^6 T_2^6 - \\
 & 32 a_2 b_3 T_1^7 T_2^6 + 20 a_4 b_3 T_1^7 T_2^6 + 20 a_2 b_5 T_1^7 T_2^6 - 27 a_4 b_5 T_1^7 T_2^6 - 2 c_{16} T_1^7 T_2^6 - 2 c_{19} T_1^7 T_2^6 - \\
 & 2 c_{31} T_1^7 T_2^6 - 2 c_{34} T_1^7 T_2^6 - 2 c_{46} T_1^7 T_2^6 - 2 c_{49} T_1^7 T_2^6 - 2 c_{61} T_1^7 T_2^6 - 2 c_{64} T_1^7 T_2^6 + 22 a_2 b_3 T_1^8 T_2^6 - \\
 & 23 a_4 b_3 T_1^8 T_2^6 - 5 a_2 b_5 T_1^8 T_2^6 - a_4 b_5 T_1^8 T_2^6 + 2 c_{16} T_1^8 T_2^6 - 4 c_{19} T_1^8 T_2^6 + 2 c_{31} T_1^8 T_2^6 - 4 c_{34} T_1^8 T_2^6 + \\
 & 2 c_{46} T_1^8 T_2^6 - 4 c_{49} T_1^8 T_2^6 + 2 c_{61} T_1^8 T_2^6 - 4 c_{64} T_1^8 T_2^6 - 8 a_2 b_3 T_1^9 T_2^6 + 3 a_4 b_3 T_1^9 T_2^6 + a_2 b_5 T_1^9 T_2^6 + \\
 & 5 a_4 b_5 T_1^9 T_2^6 - 6 c_{16} T_1^9 T_2^6 + 6 c_{19} T_1^9 T_2^6 - 6 c_{31} T_1^9 T_2^6 + 6 c_{34} T_1^9 T_2^6 - 6 c_{46} T_1^9 T_2^6 + 6 c_{49} T_1^9 T_2^6 - \\
 & 6 c_{61} T_1^9 T_2^6 + 6 c_{64} T_1^9 T_2^6 + 2 a_4 b_3 T_1^{10} T_2^6 + 4 a_2 b_5 T_1^{10} T_2^6 - a_4 b_5 T_1^{10} T_2^6 + 6 c_{16} T_1^{10} T_2^6 + 6 c_{31} T_1^{10} T_2^6 + \\
 & 6 c_{46} T_1^{10} T_2^6 + 6 c_{61} T_1^{10} T_2^6 + 2 a_2 b_3 T_1^{11} T_2^6 - 8 a_4 b_3 T_1^{11} T_2^6 - 2 a_2 b_5 T_1^{11} T_2^6 + 6 a_4 b_5 T_1^{11} T_2^6 - \\
 & 2 c_{16} T_1^{11} T_2^6 - 2 c_{31} T_1^{11} T_2^6 - 2 c_{46} T_1^{11} T_2^6 - 2 c_{61} T_1^{11} T_2^6 + 2 a_2 b_3 T_1^{12} T_2^6 + 4 a_4 b_3 T_1^{12} T_2^6 - 2 a_2 b_5 T_1^{12} T_2^6 + \\
 & 2 a_4 b_5 T_1^{12} T_2^6 - 2 a_2 b_3 T_1^{13} T_2^6 + 2 a_4 b_3 T_1^{13} T_2^6 - 2 a_4 b_3 T_1 T_2^7 - 2 a_2 b_5 T_1 T_2^7 + 4 a_4 b_5 T_1 T_2^7 + \\
 & 2 a_2 b_3 T_1^2 T_2^7 + 4 a_4 b_3 T_1^2 T_2^7 - 2 a_2 b_5 T_1^2 T_2^7 - 8 a_4 b_5 T_1^2 T_2^7 - 4 a_2 b_3 T_1^3 T_2^7 + 4 a_4 b_3 T_1^3 T_2^7 + \\
 & 4 a_2 b_5 T_1^3 T_2^7 - 7 a_4 b_5 T_1^3 T_2^7 + 3 a_2 b_3 T_1^4 T_2^7 - 4 a_4 b_3 T_1^4 T_2^7 + a_2 b_5 T_1^4 T_2^7 - 4 a_4 b_5 T_1^4 T_2^7 - 2 c_{16} T_1^4 T_2^7 - \\
 & 2 c_{31} T_1^4 T_2^7 - 2 c_{46} T_1^4 T_2^7 - 2 c_{61} T_1^4 T_2^7 - 2 a_2 b_3 T_1^5 T_2^7 - 9 a_4 b_3 T_1^5 T_2^7 - 5 a_2 b_5 T_1^5 T_2^7 + 5 a_4 b_5 T_1^5 T_2^7 + \\
 & 4 c_{16} T_1^5 T_2^7 + 4 c_{31} T_1^5 T_2^7 + 4 c_{46} T_1^5 T_2^7 + 4 c_{61} T_1^5 T_2^7 - 3 a_2 b_3 T_1^6 T_2^7 + 27 a_4 b_3 T_1^6 T_2^7 + 20 a_2 b_5 T_1^6 T_2^7 - \\
 & 20 a_4 b_5 T_1^6 T_2^7 + 4 a_2 b_3 T_1^7 T_2^7 - 52 a_4 b_3 T_1^7 T_2^7 - 16 a_2 b_5 T_1^7 T_2^7 + 52 a_4 b_5 T_1^7 T_2^7 - 4 c_{16} T_1^7 T_2^7 + \\
 & 2 c_{19} T_1^7 T_2^7 - 4 c_{31} T_1^7 T_2^7 + 2 c_{34} T_1^7 T_2^7 - 4 c_{46} T_1^7 T_2^7 + 2 c_{49} T_1^7 T_2^7 - 4 c_{61} T_1^7 T_2^7 + 2 c_{64} T_1^7 T_2^7 + \\
 & 4 a_2 b_3 T_1^8 T_2^7 + 38 a_4 b_3 T_1^8 T_2^7 + 2 a_2 b_5 T_1^8 T_2^7 - 9 a_4 b_5 T_1^8 T_2^7 - 3 a_2 b_3 T_1^9 T_2^7 - 10 a_4 b_3 T_1^9 T_2^7 - \\
 & 3 a_2 b_5 T_1^9 T_2^7 + 4 a_4 b_5 T_1^9 T_2^7 + 4 c_{16} T_1^9 T_2^7 - 2 c_{19} T_1^9 T_2^7 + 4 c_{31} T_1^9 T_2^7 - 2 c_{34} T_1^9 T_2^7 + 4 c_{46} T_1^9 T_2^7 - \\
 & 2 c_{49} T_1^9 T_2^7 + 4 c_{61} T_1^9 T_2^7 - 2 c_{64} T_1^9 T_2^7 - 2 a_2 b_3 T_1^{10} T_2^7 + 3 a_4 b_3 T_1^{10} T_2^7 - a_2 b_5 T_1^{10} T_2^7 + 3 a_4 b_5 T_1^{10} T_2^7 + \\
 & 3 a_2 b_3 T_1^{11} T_2^7 + 3 a_4 b_3 T_1^{11} T_2^7 - 2 a_2 b_5 T_1^{11} T_2^7 - 8 a_4 b_5 T_1^{11} T_2^7 - 4 c_{16} T_1^{11} T_2^7 - 4 c_{31} T_1^{11} T_2^7 - \\
 & 4 c_{46} T_1^{11} T_2^7 - 4 c_{61} T_1^{11} T_2^7 - 4 a_2 b_3 T_1^{12} T_2^7 + 4 a_4 b_3 T_1^{12} T_2^7 + 4 a_2 b_5 T_1^{12} T_2^7 - 8 a_4 b_5 T_1^{12} T_2^7 + \\
 & 2 c_{16} T_1^{12} T_2^7 + 2 c_{31} T_1^{12} T_2^7 + 2 c_{46} T_1^{12} T_2^7 + 2 c_{61} T_1^{12} T_2^7 + 2 a_2 b_3 T_1^{13} T_2^7 - 6 a_4 b_3 T_1^{13} T_2^7 - 2 a_4 b_5 T_1 T_2^8 + \\
 & a_2 b_3 T_1^2 T_2^8 + 2 a_4 b_3 T_1^2 T_2^8 + a_2 b_5 T_1^2 T_2^8 - 2 a_2 b_3 T_1^3 T_2^8 - 5 a_4 b_3 T_1^3 T_2^8 - a_2 b_5 T_1^3 T_2^8 + 9 a_4 b_5 T_1^3 T_2^8 - \\
 & a_2 b_3 T_1^4 T_2^8 + 4 a_4 b_3 T_1^4 T_2^8 + 3 a_2 b_5 T_1^4 T_2^8 + a_4 b_5 T_1^4 T_2^8 + 5 a_2 b_3 T_1^5 T_2^8 - 5 a_4 b_3 T_1^5 T_2^8 - 4 a_2 b_5 T_1^5 T_2^8 - \\
 & 3 a_4 b_5 T_1^5 T_2^8 + 2 c_{16} T_1^5 T_2^8 + 2 c_{31} T_1^5 T_2^8 + 2 c_{46} T_1^5 T_2^8 + 2 c_{61} T_1^5 T_2^8 - 2 a_2 b_3 T_1^6 T_2^8 + a_4 b_3 T_1^6 T_2^8 - \\
 & 5 a_2 b_5 T_1^6 T_2^8 + 23 a_4 b_5 T_1^6 T_2^8 - 6 c_{16} T_1^6 T_2^8 - 6 c_{31} T_1^6 T_2^8 - 6 c_{46} T_1^6 T_2^8 - 6 c_{61} T_1^6 T_2^8 - 3 a_2 b_3 T_1^7 T_2^8 + \\
 & 9 a_4 b_3 T_1^7 T_2^8 + 2 a_2 b_5 T_1^7 T_2^8 - 38 a_4 b_5 T_1^7 T_2^8 + 6 c_{16} T_1^7 T_2^8 - 6 c_{19} T_1^7 T_2^8 + 6 c_{31} T_1^7 T_2^8 - 6 c_{34} T_1^7 T_2^8 + \\
 & 6 c_{46} T_1^7 T_2^8 - 6 c_{49} T_1^7 T_2^8 + 6 c_{61} T_1^7 T_2^8 - 6 c_{64} T_1^7 T_2^8 + 4 a_2 b_3 T_1^8 T_2^8 + 2 a_4 b_3 T_1^8 T_2^8 + 2 a_2 b_5 T_1^8 T_2^8 - \\
 & 2 a_4 b_5 T_1^8 T_2^8 - 2 c_{16} T_1^8 T_2^8 + 4 c_{19} T_1^8 T_2^8 - 2 c_{31} T_1^8 T_2^8 + 4 c_{34} T_1^8 T_2^8 - 2 c_{46} T_1^8 T_2^8 + 4 c_{49} T_1^8 T_2^8 - \\
 & 2 c_{61} T_1^8 T_2^8 + 4 c_{64} T_1^8 T_2^8 - 3 a_2 b_3 T_1^9 T_2^8 - 5 a_4 b_3 T_1^9 T_2^8 + 2 a_2 b_5 T_1^9 T_2^8 - 7 a_4 b_5 T_1^9 T_2^8 + 2 c_{16} T_1^9 T_2^8 + \\
 & 2 c_{19} T_1^9 T_2^8 + 2 c_{31} T_1^9 T_2^8 + 2 c_{34} T_1^9 T_2^8 + 2 c_{46} T_1^9 T_2^8 + 2 c_{49} T_1^9 T_2^8 + 2 c_{61} T_1^9 T_2^8 + 2 c_{64} T_1^9 T_2^8 - \\
 & 2 a_2 b_3 T_1^{10} T_2^8 - 4 a_4 b_3 T_1^{10} T_2^8 + 2 a_2 b_5 T_1^{10} T_2^8 - 6 c_{16} T_1^{10} T_2^8 - 6 c_{31} T_1^{10} T_2^8 - 6 c_{46} T_1^{10} T_2^8 - \\
 & 6 c_{61} T_1^{10} T_2^8 + 5 a_2 b_3 T_1^{11} T_2^8 + a_4 b_3 T_1^{11} T_2^8 + 2 a_4 b_5 T_1^{11} T_2^8 + 6 c_{16} T_1^{11} T_2^8 + 6 c_{31} T_1^{11} T_2^8 + 6 c_{46} T_1^{11} T_2^8 + \\
 & 6 c_{61} T_1^{11} T_2^8 - a_2 b_3 T_1^{12} T_2^8 - 4 a_4 b_3 T_1^{12} T_2^8 - 3 a_2 b_5 T_1^{12} T_2^8 + 10 a_4 b_5 T_1^{12} T_2^8 - 2 c_{16} T_1^{12} T_2^8 - \\
 & 2 c_{31} T_1^{12} T_2^8 - 2 c_{46} T_1^{12} T_2^8 - 2 c_{61} T_1^{12} T_2^8 - 2 a_2 b_3 T_1^{13} T_2^8 + 5 a_4 b_3 T_1^{13} T_2^8 + a_2 b_5 T_1^{13} T_2^8 - \\
 & a_4 b_5 T_1^{13} T_2^8 + a_2 b_3 T_1^{14} T_2^8 - a_4 b_3 T_1^{14} T_2^8 + a_4 b_3 T_1^2 T_2^9 + a_2 b_5 T_1^2 T_2^9 + a_4 b_5 T_1^2 T_2^9 - 2 a_2 b_3 T_1^3 T_2^9 - \\
 & 3 a_4 b_3 T_1^3 T_2^9 + a_2 b_5 T_1^3 T_2^9 - a_4 b_5 T_1^3 T_2^9 + 4 a_2 b_3 T_1^4 T_2^9 - 2 a_4 b_3 T_1^4 T_2^9 - 4 a_2 b_5 T_1^4 T_2^9 + 5 a_4 b_5 T_1^4 T_2^9 - \\
 & 2 a_2 b_3 T_1^5 T_2^9 + 9 a_4 b_3 T_1^5 T_2^9 + 3 a_2 b_5 T_1^5 T_2^9 - 9 a_4 b_5 T_1^5 T_2^9 - 5 a_4 b_3 T_1^6 T_2^9 + a_2 b_5 T_1^6 T_2^9 - \\
 & 3 a_4 b_5 T_1^6 T_2^9 + a_2 b_3 T_1^7 T_2^9 - 4 a_4 b_3 T_1^7 T_2^9 - 3 a_2 b_5 T_1^7 T_2^9 + 10 a_4 b_5 T_1^7 T_2^9 + 6 c_{19} T_1^7 T_2^9 + 6 c_{34} T_1^7 T_2^9 + \\
 & 6 c_{49} T_1^7 T_2^9 + 6 c_{64} T_1^7 T_2^9 - a_2 b_3 T_1^8 T_2^9 + 7 a_4 b_3 T_1^8 T_2^9 + 2 a_2 b_5 T_1^8 T_2^9 + 5 a_4 b_5 T_1^8 T_2^9 - a_2 b_3 T_1^9 T_2^9 - \\
 & 5 a_4 b_3 T_1^9 T_2^9 + 5 a_4 b_5 T_1^9 T_2^9 - 6 c_{19} T_1^9 T_2^9 - 6 c_{34} T_1^9 T_2^9 - 6 c_{49} T_1^9 T_2^9 - 6 c_{64} T_1^9 T_2^9 + a_2 b_3 T_1^{10} T_2^9 - \\
 & 2 a_4 b_3 T_1^{10} T_2^9 - 3 a_2 b_5 T_1^{10} T_2^9 + 3 a_4 b_5 T_1^{10} T_2^9 + 8 a_4 b_3 T_1^{11} T_2^9 + 2 a_2 b_5 T_1^{11} T_2^9 - 4 a_4 b_5 T_1^{11} T_2^9 - \\
 & 2 a_2 b_3 T_1^{12} T_2^9 - 3 a_4 b_3 T_1^{12} T_2^9 + 3 a_2 b_5 T_1^{12} T_2^9 - 5 a_4 b_5 T_1^{12} T_2^9 + 4 a_2 b_3 T_1^{13} T_2^9 - 5 a_4 b_3 T_1^{13} T_2^9 - \\
 & 3 a_2 b_5 T_1^{13} T_2^9 + 5 a_4 b_5 T_1^{13} T_2^9 - 2 a_2 b_3 T_1^{14} T_2^9 + 4 a_4 b_3 T_1^{14} T_2^9 + a_4 b_5 T_1^2 T_2^{10} - 2 a_4 b_3 T_1^3 T_2^{10} - \\
 & 2 a_2 b_5 T_1^3 T_2^{10} + a_2 b_3 T_1^4 T_2^{10} + 6 a_4 b_3 T_1^4 T_2^{10} + a_2 b_5 T_1^4 T_2^{10} - 8 a_4 b_5 T_1^4 T_2^{10} - 3 a_4 b_3 T_1^5 T_2^{10} - \\
 & a_2 b_5 T_1^5 T_2^{10} + 4 a_4 b_5 T_1^5 T_2^{10} - 5 a_2 b_3 T_1^6 T_2^{10} + a_4 b_3 T_1^6 T_2^{10} + 4 a_2 b_5 T_1^6 T_2^{10} - 2 a_4 b_5 T_1^6 T_2^{10} +
 \end{aligned}$$

$$\begin{aligned}
 & 6 a_2 b_3 T_1^7 T_2^{10} - 3 a_4 b_3 T_1^7 T_2^{10} - a_2 b_5 T_1^7 T_2^{10} - 3 a_4 b_5 T_1^7 T_2^{10} - 2 c_{19} T_1^7 T_2^{10} - 2 c_{34} T_1^7 T_2^{10} - \\
 & 2 c_{49} T_1^7 T_2^{10} - 2 c_{64} T_1^7 T_2^{10} - 4 a_2 b_3 T_1^8 T_2^{10} + 2 a_2 b_5 T_1^8 T_2^{10} + 4 a_4 b_5 T_1^8 T_2^{10} - 4 c_{19} T_1^8 T_2^{10} - \\
 & 4 c_{34} T_1^8 T_2^{10} - 4 c_{49} T_1^8 T_2^{10} - 4 c_{64} T_1^8 T_2^{10} + 4 a_2 b_3 T_1^9 T_2^{10} - 3 a_4 b_3 T_1^9 T_2^{10} - 3 a_2 b_5 T_1^9 T_2^{10} + \\
 & 2 a_4 b_5 T_1^9 T_2^{10} + 6 c_{19} T_1^9 T_2^{10} + 6 c_{34} T_1^9 T_2^{10} + 6 c_{49} T_1^9 T_2^{10} + 6 c_{64} T_1^9 T_2^{10} - 4 a_2 b_3 T_1^{10} T_2^{10} + \\
 & 4 a_4 b_3 T_1^{10} T_2^{10} + 2 a_2 b_5 T_1^{10} T_2^{10} - 4 a_4 b_5 T_1^{10} T_2^{10} + 6 a_2 b_3 T_1^{11} T_2^{10} - 2 a_4 b_3 T_1^{11} T_2^{10} - 4 a_2 b_5 T_1^{11} T_2^{10} + \\
 & 6 a_4 b_5 T_1^{11} T_2^{10} - 5 a_2 b_3 T_1^{12} T_2^{10} + 2 a_4 b_3 T_1^{12} T_2^{10} - a_2 b_5 T_1^{12} T_2^{10} + a_4 b_5 T_1^{12} T_2^{10} + 5 a_4 b_3 T_1^{13} T_2^{10} + \\
 & 3 a_2 b_5 T_1^{13} T_2^{10} - 9 a_4 b_5 T_1^{13} T_2^{10} + a_2 b_3 T_1^{14} T_2^{10} - 5 a_4 b_3 T_1^{14} T_2^{10} - 2 a_4 b_5 T_1^3 T_2^{11} + a_4 b_3 T_1^4 T_2^{11} + \\
 & a_2 b_5 T_1^4 T_2^{11} + 3 a_4 b_5 T_1^4 T_2^{11} - a_4 b_3 T_1^5 T_2^{11} + a_2 b_5 T_1^5 T_2^{11} + a_4 b_5 T_1^5 T_2^{11} - 2 a_2 b_3 T_1^6 T_2^{11} - \\
 & 6 a_4 b_3 T_1^6 T_2^{11} - 2 a_2 b_5 T_1^6 T_2^{11} + 8 a_4 b_5 T_1^6 T_2^{11} + 6 a_2 b_3 T_1^7 T_2^{11} + 8 a_4 b_3 T_1^7 T_2^{11} - 2 a_2 b_5 T_1^7 T_2^{11} - \\
 & 3 a_4 b_5 T_1^7 T_2^{11} - 6 a_2 b_3 T_1^8 T_2^{11} - 2 a_4 b_3 T_1^8 T_2^{11} - a_4 b_5 T_1^8 T_2^{11} + 2 c_{19} T_1^8 T_2^{11} + 2 c_{34} T_1^8 T_2^{11} + \\
 & 2 c_{49} T_1^8 T_2^{11} + 2 c_{64} T_1^8 T_2^{11} + 2 a_2 b_3 T_1^9 T_2^{11} + 4 a_4 b_3 T_1^9 T_2^{11} + 2 a_2 b_5 T_1^9 T_2^{11} - 8 a_4 b_5 T_1^9 T_2^{11} - \\
 & 2 c_{19} T_1^9 T_2^{11} - 2 c_{34} T_1^9 T_2^{11} - 2 c_{49} T_1^9 T_2^{11} - 2 c_{64} T_1^9 T_2^{11} + 2 a_2 b_3 T_1^{10} T_2^{11} - 6 a_4 b_3 T_1^{10} T_2^{11} - \\
 & 4 a_2 b_5 T_1^{10} T_2^{11} + 2 a_4 b_5 T_1^{10} T_2^{11} - 6 a_2 b_3 T_1^{11} T_2^{11} + 10 a_4 b_3 T_1^{11} T_2^{11} + 8 a_2 b_5 T_1^{11} T_2^{11} - \\
 & 10 a_4 b_5 T_1^{11} T_2^{11} + 6 a_2 b_3 T_1^{12} T_2^{11} - 13 a_4 b_3 T_1^{12} T_2^{11} - 3 a_2 b_5 T_1^{12} T_2^{11} + 5 a_4 b_5 T_1^{12} T_2^{11} - \\
 & 2 a_2 b_3 T_1^{13} T_2^{11} + 3 a_4 b_3 T_1^{13} T_2^{11} - a_2 b_5 T_1^{13} T_2^{11} + 7 a_4 b_5 T_1^{13} T_2^{11} + 2 a_4 b_3 T_1^{14} T_2^{11} + a_4 b_5 T_1^4 T_2^{12} - \\
 & 2 a_4 b_3 T_1^6 T_2^{12} - 2 a_2 b_5 T_1^6 T_2^{12} - 4 a_4 b_5 T_1^6 T_2^{12} + 8 a_4 b_3 T_1^7 T_2^{12} + 4 a_2 b_5 T_1^7 T_2^{12} - 4 a_4 b_5 T_1^7 T_2^{12} + \\
 & a_2 b_3 T_1^8 T_2^{12} - 10 a_4 b_3 T_1^8 T_2^{12} - 3 a_2 b_5 T_1^8 T_2^{12} + 4 a_4 b_5 T_1^8 T_2^{12} - 4 a_2 b_3 T_1^9 T_2^{12} + 5 a_4 b_3 T_1^9 T_2^{12} + \\
 & 3 a_2 b_5 T_1^9 T_2^{12} + 3 a_4 b_5 T_1^9 T_2^{12} + 6 a_2 b_3 T_1^{10} T_2^{12} - a_4 b_3 T_1^{10} T_2^{12} - a_2 b_5 T_1^{10} T_2^{12} - 2 a_4 b_5 T_1^{10} T_2^{12} - \\
 & 4 a_2 b_3 T_1^{11} T_2^{12} - 5 a_4 b_3 T_1^{11} T_2^{12} - 3 a_2 b_5 T_1^{11} T_2^{12} + 13 a_4 b_5 T_1^{11} T_2^{12} + a_2 b_3 T_1^{12} T_2^{12} + 9 a_4 b_3 T_1^{12} T_2^{12} + \\
 & 2 a_2 b_5 T_1^{12} T_2^{12} - 9 a_4 b_5 T_1^{12} T_2^{12} - 4 a_4 b_3 T_1^{13} T_2^{12} - 2 a_4 b_5 T_1^{13} T_2^{12} - 2 a_4 b_5 T_1^6 T_2^{13} + 6 a_4 b_5 T_1^7 T_2^{13} + \\
 & a_4 b_3 T_1^8 T_2^{13} + a_2 b_5 T_1^8 T_2^{13} - 5 a_4 b_5 T_1^8 T_2^{13} - 5 a_4 b_3 T_1^9 T_2^{13} - 3 a_2 b_5 T_1^9 T_2^{13} + 5 a_4 b_5 T_1^9 T_2^{13} + \\
 & 9 a_4 b_3 T_1^{10} T_2^{13} + 3 a_2 b_5 T_1^{10} T_2^{13} - 5 a_4 b_5 T_1^{10} T_2^{13} - 7 a_4 b_3 T_1^{11} T_2^{13} - a_2 b_5 T_1^{11} T_2^{13} - 3 a_4 b_5 T_1^{11} T_2^{13} + \\
 & 2 a_4 b_3 T_1^{12} T_2^{13} + 4 a_4 b_5 T_1^{12} T_2^{13} + a_4 b_5 T_1^8 T_2^{14} - 4 a_4 b_5 T_1^9 T_2^{14} + 5 a_4 b_5 T_1^{10} T_2^{14} - 2 a_4 b_5 T_1^{11} T_2^{14} ) ] ]
 \end{aligned}$$

$$\text{In[*]:= } K = \text{Knot}["K11n42"]; \text{KT} = \int \mathcal{L}[K] \times \text{d vs}[K]$$

Out[\*]=

- i

$$\begin{aligned}
 & E \left[ \epsilon \text{Series} \left[ 0, \frac{1}{(-1 + T_1) T_1^4 (-1 + T_2) T_2^3 (-1 + T_1 T_2)} (a_2 b_3 T_1 - a_2 b_5 T_1 - a_2 b_3 T_1^2 + a_4 b_3 T_1^2 - a_4 b_5 T_1^2 - \right. \right. \\
 & a_2 b_3 T_1^3 + a_2 b_5 T_1^3 - a_4 b_5 T_1^3 + a_2 b_3 T_1^4 - a_4 b_3 T_1^4 - 2 c_{19} T_1^4 - 2 c_{34} T_1^4 - 2 c_{49} T_1^4 - 2 c_{64} T_1^4 + 2 c_{19} T_1^5 + \\
 & 2 c_{34} T_1^5 + 2 c_{49} T_1^5 + 2 c_{64} T_1^5 + a_2 b_3 T_2 - a_2 b_5 T_2 - 4 a_2 b_3 T_1 T_2 + 2 a_4 b_3 T_1 T_2 + 4 a_2 b_5 T_1 T_2 - \\
 & 2 a_4 b_5 T_1 T_2 + 3 a_2 b_3 T_1^2 T_2 - 5 a_4 b_3 T_1^2 T_2 + 4 a_4 b_5 T_1^2 T_2 - a_4 b_3 T_1^3 T_2 - a_2 b_5 T_1^3 T_2 + 5 a_4 b_5 T_1^3 T_2 + \\
 & 3 a_2 b_3 T_1^4 T_2 + 2 a_4 b_3 T_1^4 T_2 - 3 a_2 b_5 T_1^4 T_2 + 2 a_4 b_5 T_1^4 T_2 + 6 c_{19} T_1^4 T_2 + 6 c_{34} T_1^4 T_2 + 6 c_{49} T_1^4 T_2 + \\
 & 6 c_{64} T_1^4 T_2 - 4 a_2 b_3 T_1^5 T_2 + 3 a_4 b_3 T_1^5 T_2 + a_2 b_5 T_1^5 T_2 - a_4 b_5 T_1^5 T_2 - 4 c_{19} T_1^5 T_2 - 4 c_{34} T_1^5 T_2 - \\
 & 4 c_{49} T_1^5 T_2 - 4 c_{64} T_1^5 T_2 + a_2 b_3 T_1^6 T_2 - a_4 b_3 T_1^6 T_2 - 2 c_{19} T_1^6 T_2 - 2 c_{34} T_1^6 T_2 - 2 c_{49} T_1^6 T_2 - \\
 & 2 c_{64} T_1^6 T_2 + a_2 b_3 T_2^2 + a_4 b_3 T_2^2 - a_4 b_5 T_2^2 - 4 a_2 b_3 T_1 T_2^2 - 4 a_4 b_3 T_1 T_2^2 + 5 a_4 b_5 T_1 T_2^2 + 7 a_2 b_3 T_1^2 T_2^2 + \\
 & 3 a_4 b_3 T_1^2 T_2^2 - 4 a_2 b_5 T_1^2 T_2^2 - 3 a_4 b_5 T_1^2 T_2^2 - 4 a_2 b_3 T_1^3 T_2^2 + 4 a_4 b_3 T_1^3 T_2^2 - 8 a_4 b_5 T_1^3 T_2^2 - 4 a_2 b_3 T_1^4 T_2^2 + \\
 & 3 a_4 b_3 T_1^4 T_2^2 + 7 a_2 b_5 T_1^4 T_2^2 - 9 a_4 b_5 T_1^4 T_2^2 - 6 c_{19} T_1^4 T_2^2 - 6 c_{34} T_1^4 T_2^2 - 6 c_{49} T_1^4 T_2^2 - 6 c_{64} T_1^4 T_2^2 + \\
 & 7 a_2 b_3 T_1^5 T_2^2 - 11 a_4 b_3 T_1^5 T_2^2 - 4 a_2 b_5 T_1^5 T_2^2 + 5 a_4 b_5 T_1^5 T_2^2 - 4 a_2 b_3 T_1^6 T_2^2 + 5 a_4 b_3 T_1^6 T_2^2 + a_2 b_5 T_1^6 T_2^2 - \\
 & a_4 b_5 T_1^6 T_2^2 + 6 c_{19} T_1^6 T_2^2 + 6 c_{34} T_1^6 T_2^2 + 6 c_{49} T_1^6 T_2^2 + 6 c_{64} T_1^6 T_2^2 + a_2 b_3 T_1^7 T_2^2 - a_4 b_3 T_1^7 T_2^2 + a_4 b_3 T_2^3 + \\
 & a_2 b_5 T_2^3 - 2 a_2 b_3 T_1 T_2^3 - 5 a_4 b_3 T_1 T_2^3 - a_2 b_5 T_1 T_2^3 + a_4 b_5 T_1 T_2^3 - 2 c_{16} T_1 T_2^3 - 2 c_{31} T_1 T_2^3 - \\
 & 2 c_{46} T_1 T_2^3 - 2 c_{61} T_1 T_2^3 + 6 a_2 b_3 T_1^2 T_2^3 + 8 a_4 b_3 T_1^2 T_2^3 - 4 a_4 b_5 T_1^2 T_2^3 + 6 c_{16} T_1^2 T_2^3 + 6 c_{31} T_1^2 T_2^3 + \\
 & 6 c_{46} T_1^2 T_2^3 + 6 c_{61} T_1^2 T_2^3 - 6 a_2 b_3 T_1^3 T_2^3 - 4 a_4 b_3 T_1^3 T_2^3 + 2 a_2 b_5 T_1^3 T_2^3 + 4 a_4 b_5 T_1^3 T_2^3 - 6 c_{16} T_1^3 T_2^3 - \\
 & 6 c_{31} T_1^3 T_2^3 - 6 c_{46} T_1^3 T_2^3 - 6 c_{61} T_1^3 T_2^3 + 4 a_2 b_3 T_1^4 T_2^3 - 6 a_4 b_3 T_1^4 T_2^3 - 6 a_2 b_5 T_1^4 T_2^3 + 13 a_4 b_5 T_1^4 T_2^3 + \\
 & 2 c_{16} T_1^4 T_2^3 + 2 c_{19} T_1^4 T_2^3 + 2 c_{31} T_1^4 T_2^3 + 2 c_{34} T_1^4 T_2^3 + 2 c_{46} T_1^4 T_2^3 + 2 c_{49} T_1^4 T_2^3 + 2 c_{61} T_1^4 T_2^3 +
 \end{aligned}$$

$$\begin{aligned}
 & 2 c_{64} T_1^4 T_2^3 - 6 a_2 b_3 T_1^5 T_2^3 + 13 a_4 b_3 T_1^5 T_2^3 + 7 a_2 b_5 T_1^5 T_2^3 - 11 a_4 b_5 T_1^5 T_2^3 - 2 c_{16} T_1^5 T_2^3 + 4 c_{19} T_1^5 T_2^3 - \\
 & 2 c_{31} T_1^5 T_2^3 + 4 c_{34} T_1^5 T_2^3 - 2 c_{46} T_1^5 T_2^3 + 4 c_{49} T_1^5 T_2^3 - 2 c_{61} T_1^5 T_2^3 + 4 c_{64} T_1^5 T_2^3 + 6 a_2 b_3 T_1^6 T_2^3 - \\
 & 11 a_4 b_3 T_1^6 T_2^3 - 3 a_2 b_5 T_1^6 T_2^3 + 5 a_4 b_5 T_1^6 T_2^3 + 6 c_{16} T_1^6 T_2^3 - 6 c_{19} T_1^6 T_2^3 + 6 c_{31} T_1^6 T_2^3 - 6 c_{34} T_1^6 T_2^3 + \\
 & 6 c_{46} T_1^6 T_2^3 - 6 c_{49} T_1^6 T_2^3 + 6 c_{61} T_1^6 T_2^3 - 6 c_{64} T_1^6 T_2^3 - 2 a_2 b_3 T_1^7 T_2^3 + 4 a_4 b_3 T_1^7 T_2^3 - 6 c_{16} T_1^7 T_2^3 - \\
 & 6 c_{31} T_1^7 T_2^3 - 6 c_{46} T_1^7 T_2^3 - 6 c_{61} T_1^7 T_2^3 + 2 c_{16} T_1^8 T_2^3 + 2 c_{31} T_1^8 T_2^3 + 2 c_{46} T_1^8 T_2^3 + 2 c_{61} T_1^8 T_2^3 + a_4 b_5 T_2^4 + \\
 & a_2 b_3 T_1 T_2^4 - 2 a_4 b_3 T_1 T_2^4 - 3 a_2 b_5 T_1 T_2^4 - 2 a_4 b_5 T_1 T_2^4 + 2 c_{16} T_1 T_2^4 + 2 c_{31} T_1 T_2^4 + 2 c_{46} T_1 T_2^4 + \\
 & 2 c_{61} T_1 T_2^4 - 4 a_2 b_3 T_1^2 T_2^4 + 9 a_4 b_3 T_1^2 T_2^4 + 7 a_2 b_5 T_1^2 T_2^4 - 3 a_4 b_5 T_1^2 T_2^4 - 4 c_{16} T_1^2 T_2^4 - 4 c_{31} T_1^2 T_2^4 - \\
 & 4 c_{46} T_1^2 T_2^4 - 4 c_{61} T_1^2 T_2^4 + 7 a_2 b_3 T_1^3 T_2^4 - 13 a_4 b_3 T_1^3 T_2^4 - 6 a_2 b_5 T_1^3 T_2^4 + 6 a_4 b_5 T_1^3 T_2^4 - 4 a_2 b_3 T_1^4 T_2^4 + \\
 & 10 a_4 b_3 T_1^4 T_2^4 + 2 a_2 b_5 T_1^4 T_2^4 - 10 a_4 b_5 T_1^4 T_2^4 + 4 c_{16} T_1^4 T_2^4 - 2 c_{19} T_1^4 T_2^4 + 4 c_{31} T_1^4 T_2^4 - 2 c_{34} T_1^4 T_2^4 + \\
 & 4 c_{46} T_1^4 T_2^4 - 2 c_{49} T_1^4 T_2^4 + 4 c_{61} T_1^4 T_2^4 - 2 c_{64} T_1^4 T_2^4 - 4 a_2 b_3 T_1^5 T_2^4 - 8 a_4 b_3 T_1^5 T_2^4 + 11 a_4 b_5 T_1^5 T_2^4 + \\
 & 7 a_2 b_3 T_1^6 T_2^4 + 6 a_4 b_3 T_1^6 T_2^4 - a_2 b_5 T_1^6 T_2^4 - 6 a_4 b_5 T_1^6 T_2^4 - 4 c_{16} T_1^6 T_2^4 + 2 c_{19} T_1^6 T_2^4 - 4 c_{31} T_1^6 T_2^4 + \\
 & 2 c_{34} T_1^6 T_2^4 - 4 c_{46} T_1^6 T_2^4 + 2 c_{49} T_1^6 T_2^4 - 4 c_{61} T_1^6 T_2^4 + 2 c_{64} T_1^6 T_2^4 - 4 a_2 b_3 T_1^7 T_2^4 - a_4 b_3 T_1^7 T_2^4 + \\
 & a_2 b_5 T_1^7 T_2^4 - a_4 b_5 T_1^7 T_2^4 + a_2 b_3 T_1^8 T_2^4 - a_4 b_3 T_1^8 T_2^4 + 4 c_{16} T_1^8 T_2^4 + 4 c_{31} T_1^8 T_2^4 + 4 c_{46} T_1^8 T_2^4 + \\
 & 4 c_{61} T_1^8 T_2^4 - 2 c_{16} T_1^9 T_2^4 - 2 c_{31} T_1^9 T_2^4 - 2 c_{46} T_1^9 T_2^4 - 2 c_{61} T_1^9 T_2^4 + a_4 b_3 T_1 T_2^5 + a_2 b_5 T_1 T_2^5 - \\
 & 3 a_4 b_5 T_1 T_2^5 + a_2 b_3 T_1^2 T_2^5 - 5 a_4 b_3 T_1^2 T_2^5 - 4 a_2 b_5 T_1^2 T_2^5 + 11 a_4 b_5 T_1^2 T_2^5 - 2 c_{16} T_1^2 T_2^5 - 2 c_{31} T_1^2 T_2^5 - \\
 & 2 c_{46} T_1^2 T_2^5 - 2 c_{61} T_1^2 T_2^5 - 4 a_2 b_3 T_1^3 T_2^5 + 11 a_4 b_3 T_1^3 T_2^5 + 7 a_2 b_5 T_1^3 T_2^5 - 13 a_4 b_5 T_1^3 T_2^5 + 6 c_{16} T_1^3 T_2^5 + \\
 & 6 c_{31} T_1^3 T_2^5 + 6 c_{46} T_1^3 T_2^5 + 6 c_{61} T_1^3 T_2^5 + 3 a_2 b_3 T_1^4 T_2^5 - 11 a_4 b_3 T_1^4 T_2^5 + 8 a_4 b_5 T_1^4 T_2^5 - 6 c_{16} T_1^4 T_2^5 + \\
 & 6 c_{19} T_1^4 T_2^5 - 6 c_{31} T_1^4 T_2^5 + 6 c_{34} T_1^4 T_2^5 - 6 c_{46} T_1^4 T_2^5 + 6 c_{49} T_1^4 T_2^5 - 6 c_{61} T_1^4 T_2^5 + 6 c_{64} T_1^4 T_2^5 - \\
 & 4 a_4 b_3 T_1^5 T_2^5 - 4 a_2 b_5 T_1^5 T_2^5 + 4 a_4 b_5 T_1^5 T_2^5 + 2 c_{16} T_1^5 T_2^5 - 4 c_{19} T_1^5 T_2^5 + 2 c_{31} T_1^5 T_2^5 - 4 c_{34} T_1^5 T_2^5 + \\
 & 2 c_{46} T_1^5 T_2^5 - 4 c_{49} T_1^5 T_2^5 + 2 c_{61} T_1^5 T_2^5 - 4 c_{64} T_1^5 T_2^5 + 3 a_2 b_3 T_1^6 T_2^5 + 11 a_4 b_3 T_1^6 T_2^5 - a_4 b_5 T_1^6 T_2^5 - \\
 & 2 c_{16} T_1^6 T_2^5 - 2 c_{19} T_1^6 T_2^5 - 2 c_{31} T_1^6 T_2^5 - 2 c_{34} T_1^6 T_2^5 - 2 c_{46} T_1^6 T_2^5 - 2 c_{49} T_1^6 T_2^5 - 2 c_{61} T_1^6 T_2^5 - \\
 & 2 c_{64} T_1^6 T_2^5 - 4 a_2 b_3 T_1^7 T_2^5 - 4 a_4 b_3 T_1^7 T_2^5 + 2 a_4 b_5 T_1^7 T_2^5 + 6 c_{16} T_1^7 T_2^5 + 6 c_{31} T_1^7 T_2^5 + 6 c_{46} T_1^7 T_2^5 + \\
 & 6 c_{61} T_1^7 T_2^5 + a_2 b_3 T_1^8 T_2^5 + a_4 b_3 T_1^8 T_2^5 - 6 c_{16} T_1^8 T_2^5 - 6 c_{31} T_1^8 T_2^5 - 6 c_{46} T_1^8 T_2^5 - 6 c_{61} T_1^8 T_2^5 + \\
 & 2 c_{16} T_1^9 T_2^5 + 2 c_{31} T_1^9 T_2^5 + 2 c_{46} T_1^9 T_2^5 + 2 c_{61} T_1^9 T_2^5 + a_4 b_5 T_1 T_2^6 + a_4 b_3 T_1^2 T_2^6 + a_2 b_5 T_1^2 T_2^6 - \\
 & 5 a_4 b_5 T_1^2 T_2^6 - 5 a_4 b_3 T_1^3 T_2^6 - 3 a_2 b_5 T_1^3 T_2^6 + 11 a_4 b_5 T_1^3 T_2^6 + a_2 b_3 T_1^4 T_2^6 + 6 a_4 b_3 T_1^4 T_2^6 - a_2 b_5 T_1^4 T_2^6 - \\
 & 6 a_4 b_5 T_1^4 T_2^6 - 6 c_{19} T_1^4 T_2^6 - 6 c_{34} T_1^4 T_2^6 - 6 c_{49} T_1^4 T_2^6 - 6 c_{64} T_1^4 T_2^6 - a_2 b_3 T_1^5 T_2^6 + a_4 b_3 T_1^5 T_2^6 - \\
 & 11 a_4 b_5 T_1^5 T_2^6 - a_2 b_3 T_1^6 T_2^6 + 3 a_4 b_3 T_1^6 T_2^6 + 4 a_2 b_5 T_1^6 T_2^6 - 3 a_4 b_5 T_1^6 T_2^6 + 6 c_{19} T_1^6 T_2^6 + 6 c_{34} T_1^6 T_2^6 + \\
 & 6 c_{49} T_1^6 T_2^6 + 6 c_{64} T_1^6 T_2^6 + a_2 b_3 T_1^7 T_2^6 - 8 a_4 b_3 T_1^7 T_2^6 - a_2 b_5 T_1^7 T_2^6 + a_4 b_5 T_1^7 T_2^6 + 2 a_4 b_3 T_1^8 T_2^6 + \\
 & a_4 b_5 T_1^2 T_2^7 - 4 a_4 b_5 T_1^3 T_2^7 + a_4 b_3 T_1^4 T_2^7 + a_2 b_5 T_1^4 T_2^7 + a_4 b_5 T_1^4 T_2^7 + 2 c_{19} T_1^4 T_2^7 + 2 c_{34} T_1^4 T_2^7 + \\
 & 2 c_{49} T_1^4 T_2^7 + 2 c_{64} T_1^4 T_2^7 - 2 a_4 b_3 T_1^5 T_2^7 + 4 a_4 b_5 T_1^5 T_2^7 + 4 c_{19} T_1^5 T_2^7 + 4 c_{34} T_1^5 T_2^7 + 4 c_{49} T_1^5 T_2^7 + \\
 & 4 c_{64} T_1^5 T_2^7 - a_4 b_3 T_1^6 T_2^7 - a_2 b_5 T_1^6 T_2^7 + 8 a_4 b_5 T_1^6 T_2^7 - 6 c_{19} T_1^6 T_2^7 - 6 c_{34} T_1^6 T_2^7 - 6 c_{49} T_1^6 T_2^7 - \\
 & 6 c_{64} T_1^6 T_2^7 + 2 a_4 b_3 T_1^7 T_2^7 - 2 a_4 b_5 T_1^7 T_2^7 + a_4 b_5 T_1^4 T_2^8 - a_4 b_5 T_1^5 T_2^8 - 2 c_{19} T_1^5 T_2^8 - 2 c_{34} T_1^5 T_2^8 - \\
 & 2 c_{49} T_1^5 T_2^8 - 2 c_{64} T_1^5 T_2^8 - 2 a_4 b_5 T_1^6 T_2^8 + 2 c_{19} T_1^6 T_2^8 + 2 c_{34} T_1^6 T_2^8 + 2 c_{49} T_1^6 T_2^8 + 2 c_{64} T_1^6 T_2^8) ] ]
 \end{aligned}$$

$$\text{In[*]:= } \text{CKT} = \text{Factor} \left[ (-1 + T_1) (-1 + T_2) (-1 + T_1 T_2) \right. \\
 \left. \frac{(\text{Cases} [\{\text{Conway}, \text{KT}\}, \text{eSeries} [0, \mathcal{E}_-] \Rightarrow \mathcal{E}, \infty] / \cdot \text{C}_{16|19|31|34|46|49|61|64|81|82|84|85} \rightarrow \emptyset)}{(-b_3 + b_5 + b_3 T_1 - b_5 T_2) (-a_2 + a_2 T_1 - a_4 T_1 - a_4 T_2 + 2 a_4 T_1 T_2)} \right]$$

Out[\*]=

$$\left\{ -\frac{1}{T_1^7 T_2^6} \left( T_1^2 - 2 T_1^3 + T_1^4 - 2 T_1 T_2 + 2 T_1^2 T_2 + 2 T_1^5 T_2 - 2 T_1^6 T_2 + T_2^2 + 2 T_1 T_2^2 - 2 T_1^2 T_2^2 - 2 T_1^4 T_2^2 - 2 T_1^6 T_2^2 + 2 T_1^7 T_2^2 + T_1^8 T_2^2 - 2 T_2^3 + T_1^4 T_2^3 + T_1^5 T_2^3 - 2 T_1^9 T_2^3 + T_2^4 - 2 T_1^2 T_2^4 + T_1^3 T_2^4 + 2 T_1^4 T_2^4 + 2 T_1^6 T_2^4 + T_1^7 T_2^4 - 2 T_1^8 T_2^4 + T_1^{10} T_2^4 + 2 T_1 T_2^5 + T_1^3 T_2^5 - 4 T_1^5 T_2^5 - 4 T_1^6 T_2^5 + T_1^8 T_2^5 + 2 T_1^{10} T_2^5 - 2 T_1 T_2^6 - 2 T_1^2 T_2^6 + 2 T_1^4 T_2^6 - 4 T_1^5 T_2^6 + 12 T_1^6 T_2^6 - 4 T_1^7 T_2^6 + 2 T_1^8 T_2^6 - 2 T_1^{10} T_2^6 - 2 T_1^{11} T_2^6 + 2 T_1^2 T_2^7 + T_1^4 T_2^7 - 4 T_1^6 T_2^7 - 4 T_1^7 T_2^7 + T_1^9 T_2^7 + 2 T_1^{11} T_2^7 + T_1^2 T_2^8 - 2 T_1^4 T_2^8 + T_1^5 T_2^8 + 2 T_1^6 T_2^8 + 2 T_1^8 T_2^8 + T_1^9 T_2^8 - 2 T_1^{10} T_2^8 + T_1^{12} T_2^8 - 2 T_1^3 T_2^9 + T_1^7 T_2^9 + T_1^8 T_2^9 - 2 T_1^{12} T_2^9 + T_1^4 T_2^{10} + 2 T_1^5 T_2^{10} - 2 T_1^6 T_2^{10} - 2 T_1^8 T_2^{10} - 2 T_1^{10} T_2^{10} + 2 T_1^{11} T_2^{10} + T_1^{12} T_2^{10} - 2 T_1^6 T_2^{11} + 2 T_1^7 T_2^{11} + 2 T_1^{10} T_2^{11} - 2 T_1^{11} T_2^{11} + T_1^8 T_2^{12} - 2 T_1^9 T_2^{12} + T_1^{10} T_2^{12} \right), \right. \\ \left. \frac{1}{T_1^4 T_2^3} \left( T_1 + T_1^2 + T_2 - 2 T_1 T_2 - 2 T_1^2 T_2 - 2 T_1^3 T_2 + T_1^4 T_2 + T_2^2 - 2 T_1 T_2^2 + 2 T_1^2 T_2^2 + 2 T_1^3 T_2^2 - 2 T_1^4 T_2^2 + T_1^5 T_2^2 - 2 T_1 T_2^3 + 2 T_1^2 T_2^3 + 2 T_1^4 T_2^3 - 2 T_1^5 T_2^3 + T_1 T_2^4 - 2 T_1^2 T_2^4 + 2 T_1^3 T_2^4 + 2 T_1^4 T_2^4 - 2 T_1^5 T_2^4 + T_1^6 T_2^4 + T_1^2 T_2^5 - 2 T_1^3 T_2^5 - 2 T_1^4 T_2^5 - 2 T_1^5 T_2^5 + T_1^6 T_2^5 + T_1 T_2^6 + T_1^5 T_2^6 \right) \right\}$$

In[\*]:= Factor[(-1 + T1 T2) (Cases[{Conway, KT}, eSeries[0, ε\_] => ε, ∞] /.

{C16|19|31|34|46|49|61|64|81|82|84|85 -> 0, a2 -> 1, a4 -> 0, b3 -> 0, b5 -> 1})]

Out[\*]=

$$\left\{ \frac{1}{T_1^7 T_2^6} \left( T_1^2 - 2 T_1^3 + T_1^4 - 2 T_1 T_2 + 2 T_1^2 T_2 + 2 T_1^5 T_2 - 2 T_1^6 T_2 + T_2^2 + 2 T_1 T_2^2 - 2 T_1^2 T_2^2 - 2 T_1^4 T_2^2 - 2 T_1^6 T_2^2 + 2 T_1^7 T_2^2 + T_1^8 T_2^2 - 2 T_2^3 + T_1^4 T_2^3 + T_1^5 T_2^3 - 2 T_1^9 T_2^3 + T_2^4 - 2 T_1^2 T_2^4 + T_1^3 T_2^4 + 2 T_1^4 T_2^4 + 2 T_1^6 T_2^4 + T_1^7 T_2^4 - 2 T_1^8 T_2^4 + T_1^{10} T_2^4 + 2 T_1 T_2^5 + T_1^3 T_2^5 - 4 T_1^5 T_2^5 - 4 T_1^6 T_2^5 + T_1^8 T_2^5 + 2 T_1^{10} T_2^5 - 2 T_1 T_2^6 - 2 T_1^2 T_2^6 + 2 T_1^4 T_2^6 - 4 T_1^5 T_2^6 + 12 T_1^6 T_2^6 - 4 T_1^7 T_2^6 + 2 T_1^8 T_2^6 - 2 T_1^{10} T_2^6 - 2 T_1^{11} T_2^6 + 2 T_1^2 T_2^7 + T_1^4 T_2^7 - 4 T_1^6 T_2^7 - 4 T_1^7 T_2^7 + T_1^9 T_2^7 + 2 T_1^{11} T_2^7 + T_1^2 T_2^8 - 2 T_1^4 T_2^8 + T_1^5 T_2^8 + 2 T_1^6 T_2^8 + 2 T_1^8 T_2^8 + T_1^9 T_2^8 - 2 T_1^{10} T_2^8 + T_1^{12} T_2^8 - 2 T_1^3 T_2^9 + T_1^7 T_2^9 + T_1^8 T_2^9 - 2 T_1^{12} T_2^9 + T_1^4 T_2^{10} + 2 T_1^5 T_2^{10} - 2 T_1^6 T_2^{10} - 2 T_1^8 T_2^{10} - 2 T_1^{10} T_2^{10} + 2 T_1^{11} T_2^{10} + T_1^{12} T_2^{10} - 2 T_1^6 T_2^{11} + 2 T_1^7 T_2^{11} + 2 T_1^{10} T_2^{11} - 2 T_1^{11} T_2^{11} + T_1^8 T_2^{12} - 2 T_1^9 T_2^{12} + T_1^{10} T_2^{12} \right), \right. \\ \left. -\frac{1}{T_1^4 T_2^3} \left( T_1 + T_1^2 + T_2 - 2 T_1 T_2 - 2 T_1^2 T_2 - 2 T_1^3 T_2 + T_1^4 T_2 + T_2^2 - 2 T_1 T_2^2 + 2 T_1^2 T_2^2 + 2 T_1^3 T_2^2 - 2 T_1^4 T_2^2 + T_1^5 T_2^2 - 2 T_1 T_2^3 + 2 T_1^2 T_2^3 + 2 T_1^4 T_2^3 - 2 T_1^5 T_2^3 + T_1 T_2^4 - 2 T_1^2 T_2^4 + 2 T_1^3 T_2^4 + 2 T_1^4 T_2^4 - 2 T_1^5 T_2^4 + T_1^6 T_2^4 + T_1^2 T_2^5 - 2 T_1^3 T_2^5 - 2 T_1^4 T_2^5 - 2 T_1^5 T_2^5 + T_1^6 T_2^5 + T_1 T_2^6 + T_1^5 T_2^6 \right) \right\}$$

In[\*]:= Factor[CKT /. T1 -> 1]

Out[\*]=

$$\left\{ \frac{2 (-1 + T_2)^2 (1 + T_2^4)}{T_2^3}, \frac{2 (-1 + T_2)^2 (1 + T_2^4)}{T_2^3} \right\}$$

In[\*]:= Factor[CKT /. T2 -> 1]

Out[\*]=

$$\left\{ \frac{2 (-1 + T_1)^2 (1 + T_1^4)}{T_1^4}, \frac{2 (-1 + T_1)^2 (1 + T_1^4)}{T_1^4} \right\}$$



$$\text{In[*]} := \text{Factor} \left[ \frac{(\text{CKT} /. \{T_1 \rightarrow T_2, T_2 \rightarrow T_1\})}{\text{CKT}} \right]$$

Out[\*]=

$$\left\{ \frac{T_1}{T_2}, \frac{T_1}{T_2} \right\}$$

## Tweaking $\gamma_{42}$

$$\text{In[*]} := \text{CF}[\gamma_{42}[1, k]]$$

Out[\*]=

$$\frac{(c_{16} + c_{31} + c_{46} + c_{61}) p_{1,k} x_{1,k} + (c_{19} + c_{34} + c_{49} + c_{64}) p_{2,k} x_{2,k} + (-b_3 + b_5 + b_3 T_1 - b_5 T_2) (-a_2 + a_2 T_1 - a_4 T_1 - a_4 T_2 + 2 a_4 T_1 T_2) p_{3,k} x_{3,k}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)}$$

$$\text{In[*]} := \mathcal{L}[\mathbf{C}_k[1]] /. \{c_{-} \rightarrow 0, a_2 \rightarrow 1, a_4 \rightarrow 0, b_3 \rightarrow 0, b_5 \rightarrow 1\}$$

Out[\*]=

$$T_1 T_2 \mathbb{E} \left[ \in \text{Series} \left[ (-p_{1,k} + p_{1,1+k}) x_{1,k} + (-p_{2,k} + p_{2,1+k}) x_{2,k} + (-p_{3,k} + p_{3,1+k}) x_{3,k}, \frac{(1 - T_2) p_{3,k} x_{3,k}}{T_1 (-1 + T_2) (-1 + T_1 T_2)} \right] \right]$$

$$\text{In[*]} := \text{CF}[\gamma_{42}[-1, k]]$$

Out[\*]=

$$\frac{(-c_{16} - c_{31} - c_{46} - c_{61}) p_{1,k} x_{1,k} + (-c_{19} - c_{34} - c_{49} - c_{64}) p_{2,k} x_{2,k} - (-b_3 + b_5 + b_3 T_1 - b_5 T_2) (-a_2 + a_2 T_1 - a_4 T_1 - a_4 T_2 + 2 a_4 T_1 T_2) p_{3,k} x_{3,k}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)}$$

$$\text{In[*]} := \mathcal{L}[\mathbf{C}_k[-1]] /. \{c_{-} \rightarrow 0, a_2 \rightarrow 1, a_4 \rightarrow 0, b_3 \rightarrow 0, b_5 \rightarrow 1\}$$

Out[\*]=

$$\frac{\mathbb{E} \left[ \in \text{Series} \left[ (-p_{1,k} + p_{1,1+k}) x_{1,k} + (-p_{2,k} + p_{2,1+k}) x_{2,k} + (-p_{3,k} + p_{3,1+k}) x_{3,k}, -\frac{(1 - T_2) p_{3,k} x_{3,k}}{T_1 (-1 + T_2) (-1 + T_1 T_2)} \right] \right]}{T_1 T_2}$$

$$\text{In[*]} := \gamma_{42}[\varphi, k] := \text{Evaluate}[\text{CF}[\varphi \gamma_{42}[1, k]]];$$

$$\gamma_{42}[\varphi, k]$$

Out[\*]=

$$\frac{\varphi (c_{16} + c_{31} + c_{46} + c_{61}) p_{1,k} x_{1,k} + \varphi (c_{19} + c_{34} + c_{49} + c_{64}) p_{2,k} x_{2,k} + \varphi (-b_3 + b_5 + b_3 T_1 - b_5 T_2) (-a_2 + a_2 T_1 - a_4 T_1 - a_4 T_2 + 2 a_4 T_1 T_2) p_{3,k} x_{3,k}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)}$$

$$In[*]:= \left( \int \mathcal{F}[k] \times \mathcal{L} / @ (C_k[\varphi 1] C_{k+1}[\varphi 2]) \, d\{VS_k, VS_{k'}\} \right) /. \gamma_{0|1}[\_] \rightarrow \theta$$

Out[\*]=

$$T_1^{\varphi 1 + \varphi 2} T_2^{\varphi 1 + \varphi 2}$$

$$\begin{aligned} & E \left[ \in Series \left[ p_{1,2+k} \pi_{1,k} + p_{2,2+k} \pi_{2,k} + p_{3,2+k} \pi_{3,k}, \frac{1}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} \left( \varphi 1 a_2 b_3 + \right. \right. \right. \\ & \quad \varphi 2 a_2 b_3 - \varphi 1 a_2 b_5 - \varphi 2 a_2 b_5 - 2 \varphi 1 a_2 b_3 T_1 - 2 \varphi 2 a_2 b_3 T_1 + \varphi 1 a_4 b_3 T_1 + \varphi 2 a_4 b_3 T_1 + \varphi 1 a_2 b_5 T_1 + \\ & \quad \varphi 2 a_2 b_5 T_1 - \varphi 1 a_4 b_5 T_1 - \varphi 2 a_4 b_5 T_1 - \varphi 1 c_{16} T_1 - \varphi 2 c_{16} T_1 - \varphi 1 c_{19} T_1 - \varphi 2 c_{19} T_1 - \varphi 1 c_{31} T_1 - \\ & \quad \varphi 2 c_{31} T_1 - \varphi 1 c_{34} T_1 - \varphi 2 c_{34} T_1 - \varphi 1 c_{46} T_1 - \varphi 2 c_{46} T_1 - \varphi 1 c_{49} T_1 - \varphi 2 c_{49} T_1 - \varphi 1 c_{61} T_1 - \\ & \quad \varphi 2 c_{61} T_1 - \varphi 1 c_{64} T_1 - \varphi 2 c_{64} T_1 + \varphi 1 a_2 b_3 T_1^2 + \varphi 2 a_2 b_3 T_1^2 - \varphi 1 a_4 b_3 T_1^2 - \varphi 2 a_4 b_3 T_1^2 + \varphi 1 c_{16} T_1^2 + \\ & \quad \varphi 2 c_{16} T_1^2 + \varphi 1 c_{19} T_1^2 + \varphi 2 c_{19} T_1^2 + \varphi 1 c_{31} T_1^2 + \varphi 2 c_{31} T_1^2 + \varphi 1 c_{34} T_1^2 + \varphi 2 c_{34} T_1^2 + \varphi 1 c_{46} T_1^2 + \\ & \quad \varphi 2 c_{46} T_1^2 + \varphi 1 c_{49} T_1^2 + \varphi 2 c_{49} T_1^2 + \varphi 1 c_{61} T_1^2 + \varphi 2 c_{61} T_1^2 + \varphi 1 c_{64} T_1^2 + \varphi 2 c_{64} T_1^2 + \varphi 1 a_4 b_3 T_2 + \\ & \quad \varphi 2 a_4 b_3 T_2 + \varphi 1 a_2 b_5 T_2 + \varphi 2 a_2 b_5 T_2 - \varphi 1 a_4 b_5 T_2 - \varphi 2 a_4 b_5 T_2 - 3 \varphi 1 a_4 b_3 T_1 T_2 - 3 \varphi 2 a_4 b_3 T_1 T_2 - \\ & \quad \varphi 1 a_2 b_5 T_1 T_2 - \varphi 2 a_2 b_5 T_1 T_2 + 3 \varphi 1 a_4 b_5 T_1 T_2 + 3 \varphi 2 a_4 b_5 T_1 T_2 + \varphi 1 c_{16} T_1 T_2 + \varphi 2 c_{16} T_1 T_2 + \\ & \quad \varphi 1 c_{19} T_1 T_2 + \varphi 2 c_{19} T_1 T_2 + \varphi 1 c_{31} T_1 T_2 + \varphi 2 c_{31} T_1 T_2 + \varphi 1 c_{34} T_1 T_2 + \varphi 2 c_{34} T_1 T_2 + \varphi 1 c_{46} T_1 T_2 + \\ & \quad \varphi 2 c_{46} T_1 T_2 + \varphi 1 c_{49} T_1 T_2 + \varphi 2 c_{49} T_1 T_2 + \varphi 1 c_{61} T_1 T_2 + \varphi 2 c_{61} T_1 T_2 + \varphi 1 c_{64} T_1 T_2 + \varphi 2 c_{64} T_1 T_2 + \\ & \quad 2 \varphi 1 a_4 b_3 T_1^3 T_2 + 2 \varphi 2 a_4 b_3 T_1^3 T_2 - \varphi 1 c_{16} T_1^3 T_2 - \varphi 2 c_{16} T_1^3 T_2 - \varphi 1 c_{19} T_1^3 T_2 - \varphi 2 c_{19} T_1^3 T_2 - \\ & \quad \varphi 1 c_{31} T_1^3 T_2 - \varphi 2 c_{31} T_1^3 T_2 - \varphi 1 c_{34} T_1^3 T_2 - \varphi 2 c_{34} T_1^3 T_2 - \varphi 1 c_{46} T_1^3 T_2 - \varphi 2 c_{46} T_1^3 T_2 - \varphi 1 c_{49} T_1^3 T_2 - \\ & \quad \varphi 2 c_{49} T_1^3 T_2 - \varphi 1 c_{61} T_1^3 T_2 - \varphi 2 c_{61} T_1^3 T_2 - \varphi 1 c_{64} T_1^3 T_2 - \varphi 2 c_{64} T_1^3 T_2 + \varphi 1 a_4 b_5 T_2^2 + \varphi 2 a_4 b_5 T_2^2 - \\ & \quad 2 \varphi 1 a_4 b_5 T_1 T_2^2 - 2 \varphi 2 a_4 b_5 T_1 T_2^2 - \varphi 1 c_{16} T_1^2 T_2^2 - \varphi 2 c_{16} T_1^2 T_2^2 - \varphi 1 c_{19} T_1^2 T_2^2 - \varphi 2 c_{19} T_1^2 T_2^2 - \\ & \quad \varphi 1 c_{31} T_1^2 T_2^2 - \varphi 2 c_{31} T_1^2 T_2^2 - \varphi 1 c_{34} T_1^2 T_2^2 - \varphi 2 c_{34} T_1^2 T_2^2 - \varphi 1 c_{46} T_1^2 T_2^2 - \varphi 2 c_{46} T_1^2 T_2^2 - \varphi 1 c_{49} T_1^2 T_2^2 - \\ & \quad \varphi 2 c_{49} T_1^2 T_2^2 - \varphi 1 c_{61} T_1^2 T_2^2 - \varphi 2 c_{61} T_1^2 T_2^2 - \varphi 1 c_{64} T_1^2 T_2^2 - \varphi 2 c_{64} T_1^2 T_2^2 + \varphi 1 c_{16} T_1^3 T_2^2 + \varphi 2 c_{16} T_1^3 T_2^2 + \\ & \quad \varphi 1 c_{19} T_1^3 T_2^2 + \varphi 2 c_{19} T_1^3 T_2^2 + \varphi 1 c_{31} T_1^3 T_2^2 + \varphi 2 c_{31} T_1^3 T_2^2 + \varphi 1 c_{34} T_1^3 T_2^2 + \varphi 2 c_{34} T_1^3 T_2^2 + \varphi 1 c_{46} T_1^3 T_2^2 + \\ & \quad \varphi 2 c_{46} T_1^3 T_2^2 + \varphi 1 c_{49} T_1^3 T_2^2 + \varphi 2 c_{49} T_1^3 T_2^2 + \varphi 1 c_{61} T_1^3 T_2^2 + \varphi 2 c_{61} T_1^3 T_2^2 + \varphi 1 c_{64} T_1^3 T_2^2 + \varphi 2 c_{64} T_1^3 T_2^2 \left. \right) + \\ & \quad \left. \frac{(\varphi 1 + \varphi 2) (c_{16} + c_{31} + c_{46} + c_{61}) p_{1,2+k} \pi_{1,k} + (\varphi 1 + \varphi 2) (c_{19} + c_{34} + c_{49} + c_{64}) p_{2,2+k} \pi_{2,k} + (\varphi 1 + \varphi 2) (-b_3 + b_5 + b_3 T_1 - b_5 T_2) (-a_2 + a_2 T_1 - a_4 T_1 - a_4 T_2 + 2 a_4 T_1 T_2) p_{3,2+k} \pi_{3,k}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} \right] \end{aligned}$$

$$\begin{aligned} In[*]:= & \left( \int \mathcal{F}[k] \times \mathcal{L} / @ (C_k[\varphi 1] C_{k+1}[\varphi 2]) \, d\{VS_k, VS_{k'}\} == \right. \\ & \left. \int \mathcal{F}[k] \times \mathcal{L} / @ (C_k[\varphi 1 + \varphi 2] C_{k+1}[\theta]) \, d\{VS_k, VS_{k'}\} \right) /. \gamma_{0|1}[\_] \rightarrow \theta \end{aligned}$$

Out[\*]=

True

## Saving the Results

```
In[*]:= data = CF[{
  {r0[1, i, j], r0[-1, i, j]},
  {r1[1, i, j], r1[-1, i, j]},
  {r42[1, i, j], r42[-1, i, j]},
  γ42[φ, k]
} /. {c_ → 0, a2 → 1, a4 → 0, b3 → 0, b5 → 1}
]
```

Out[\*]=

$$\left\{ \left\{ p_{3,j} x_{1,i} x_{2,i} - \frac{p_{3,j} x_{1,j} x_{2,i}}{T_1}, -\frac{p_{3,j} x_{1,i} x_{2,i}}{T_1^2 T_2} + \frac{p_{3,j} x_{1,j} x_{2,i}}{T_1 T_2} \right\}, \right. \\ \left. \left\{ p_{1,j} p_{2,i} x_{3,i} - p_{1,j} p_{2,j} x_{3,i}, -\frac{p_{1,j} p_{2,i} x_{3,i}}{T_1} + \frac{p_{1,j} p_{2,j} x_{3,i}}{T_1} \right\}, \right. \\ \left. \left\{ \frac{T_2 p_{1,j} p_{2,j} x_{1,i} x_{2,i}}{-1 + T_1 T_2} - \frac{p_{1,j} p_{2,i} x_{1,j} x_{2,i}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} - \frac{p_{1,j} p_{2,j} x_{1,j} x_{2,i}}{(-1 + T_1) T_1} + \frac{p_{1,i} p_{2,j} x_{1,i} x_{2,j}}{(-1 + T_1) (-1 + T_1 T_2)} + \right. \right. \\ \frac{p_{3,j} x_{3,i}}{T_1 (-1 + T_1 T_2)} - p_{1,j} p_{3,j} x_{1,i} x_{3,i} + \frac{p_{1,j} p_{3,i} x_{1,j} x_{3,i}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} + \frac{p_{1,j} p_{3,j} x_{1,j} x_{3,i}}{-1 + T_1} - \\ \left. \frac{T_2 p_{2,j} p_{3,j} x_{2,i} x_{3,i}}{T_1} - \frac{p_{2,j} p_{3,i} x_{2,j} x_{3,i}}{T_1 (-1 + T_1 T_2)} - \frac{p_{1,i} p_{3,j} x_{1,i} x_{3,j}}{(-1 + T_1) (-1 + T_1 T_2)} + \frac{T_2 p_{2,j} p_{3,j} x_{2,i} x_{3,j}}{T_1 (-1 + T_1 T_2)}, \right. \\ \left. \frac{p_{1,j} p_{2,i} x_{1,i} x_{2,i}}{T_1^2 (-1 + T_1 T_2)} - \frac{(-1 + T_2) p_{1,i} p_{2,j} x_{1,i} x_{2,i}}{(-1 + T_1) T_2 (-1 + T_1 T_2)} + \frac{(-T_1 - T_2 + T_1 T_2) p_{1,j} p_{2,j} x_{1,i} x_{2,i}}{T_1^2 T_2 (-1 + T_1 T_2)} + \right. \\ \frac{p_{1,j} p_{2,i} x_{1,j} x_{2,i}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} + \frac{p_{1,j} p_{2,j} x_{1,j} x_{2,i}}{T_1 (-1 + T_1 T_2)} - \frac{p_{1,i} p_{2,j} x_{1,i} x_{2,j}}{(-1 + T_1) (-1 + T_1 T_2)} + \\ \left. \frac{p_{1,j} p_{2,j} x_{1,i} x_{2,j}}{T_1 (-1 + T_1 T_2)} - \frac{p_{3,j} x_{3,i}}{T_1 (-1 + T_1 T_2)} - \frac{p_{1,j} p_{3,i} x_{1,i} x_{3,i}}{T_1^2 (-1 + T_1 T_2)} + \frac{p_{1,i} p_{3,j} x_{1,i} x_{3,i}}{(-1 + T_1) T_1 T_2} - \right. \\ \left. \frac{p_{1,j} p_{3,j} x_{1,i} x_{3,i}}{T_1^2 T_2} - \frac{p_{1,j} p_{3,i} x_{1,j} x_{3,i}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} + \frac{(-1 + T_2) p_{2,j} p_{3,i} x_{2,i} x_{3,i}}{T_1 T_2 (-1 + T_1 T_2)} + \right. \\ \left. \frac{p_{2,i} p_{3,j} x_{2,i} x_{3,i}}{T_1^2 T_2} - \frac{(-1 + 2 T_2) p_{2,j} p_{3,j} x_{2,i} x_{3,i}}{T_1^2 T_2^2} + \frac{p_{2,j} p_{3,i} x_{2,j} x_{3,i}}{T_1 (-1 + T_1 T_2)} - \frac{p_{2,j} p_{3,j} x_{2,j} x_{3,i}}{T_1^2 T_2} + \right. \\ \left. \frac{p_{1,i} p_{3,j} x_{1,i} x_{3,j}}{(-1 + T_1) (-1 + T_1 T_2)} - \frac{p_{1,j} p_{3,j} x_{1,i} x_{3,j}}{T_1 (-1 + T_1 T_2)} - \frac{p_{2,j} p_{3,j} x_{2,i} x_{3,j}}{T_1 (-1 + T_1 T_2)} \right\}, -\frac{\varphi p_{3,k} x_{3,k}}{T_1 (-1 + T_1 T_2)} \left. \right\}$$

```
If[False, Put[data, "px-data.m"]]
```