Pensieve header: Finding the A2 $d=1$ invariant using undetermined coefficients.

Searching for Q + xxx + ϵ(ppp + 1 + px + ppxx) solutions.

Initialization

(Alt) In[ ]:=

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank"];
Once[<< KnotTheory`; << Rot.m];
<< FormalGaussianIntegration.m;
i_⁺ := i + 1;
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
Loading Rot.m from http://drorbn.net/AP/Projects/HigherRank to compute rotation numbers.
```

(Alt) In[ ]:=

```
Features[Knot[8, 17]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

(Alt) Out[ ]=

Features[18, $C_6[-1]\, C_{14}[-1]\, X_{1,7}[1]\, X_{3,9}[-1]\, X_{5,13}[-1]\, X_{8,16}[1]\, X_{10,4}[-1]\, X_{12,18}[1]\, X_{15,2}[-1]\, X_{17,11}[1]$]

(Alt) In[ ]:=

```
T3 = T1^-1 T2^-1;
𝒮 = {x__, p__};
q[s_, i_, j_] := Sum[
    x_{ν,i} (p_{ν,i⁺} - p_{ν,i}) + x_{ν,j} (p_{ν,j⁺} - p_{ν,j}) + (T_ν^s - 1) x_{ν,i} (p_{ν,i⁺} - p_{ν,j⁺}),
    {ν, 3}];
ℒ[X_{i_,j_}[s_]] :=
   T3^s 𝔼[q[s, i, j] + B^-1 r0[s, i, j] + ϵ B r1[s, i, j] + ϵ r42[s, i, j] + O[ϵ]^2];
(*γ1[φ_,k_]:=φ(3/2-x_{1,k}p_{1,k}-x_{2,k}p_{2,k}-x_{3,k}p_{3,k});*)
ℒ[C_{k_}[0]] := 𝔼[Sum[x_{ν,k} (p_{ν,k⁺} - p_{ν,k}), {ν, 3}] + O[ϵ]^2];
ℒ[C_{k_}[φ_]] :=
   T3^φ 𝔼[Sum[x_{ν,k} (p_{ν,k⁺} - p_{ν,k}), {ν, 3}] + B^-1 γ0[φ, k] + ϵ B γ1[φ, k] + ϵ γ42[φ, k] + O[ϵ]^2];
ps_{i_} := Sequence[p_{1,i}, p_{2,i}, p_{3,i}];
xs_{i_} := Sequence[x_{1,i}, x_{2,i}, x_{3,i}];
vs_{i_} := Sequence[ps_i, xs_i];
ℱ[is___] := 𝔼[Sum[π_{ν,i} p_{ν,i}, {i, {is}}, {ν, 3}]];
ℒ[K_] := CF[ℒ /@ Features[K][[2]]];
vs[K_] := Union @@ Table[{vs_i}, {i, Features[K][[1]]}]
```

(Alt) In[ ]:=

```
vs_i
```

(Alt) Out[ ]=

Sequence[$p_{1,i}, p_{2,i}, p_{3,i}, x_{1,i}, x_{2,i}, x_{3,i}$]

The Various Terms

The xxx Terms (r_0)

(Alt) In[ ]:=

```
κ = 0;
r0[1, i_, j_] := Evaluate[Sum[
    a++κ x3,k3 x1,k1 x2,k2,
    {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
  ]];
r0[1, i, j]
```

(Alt) Out[ ]=

$a_1 x_{1,i} x_{2,i} x_{3,i} + a_5 x_{1,j} x_{2,i} x_{3,i} + a_3 x_{1,i} x_{2,j} x_{3,i} + a_7 x_{1,j} x_{2,j} x_{3,i} + a_2 x_{1,i} x_{2,i} x_{3,j} + a_6 x_{1,j} x_{2,i} x_{3,j} + a_4 x_{1,i} x_{2,j} x_{3,j} + a_8 x_{1,j} x_{2,j} x_{3,j}$

(Alt) In[ ]:=

```
κ = 0;
r0[-1, i_, j_] := Evaluate[Sum[
    d++κ x3,k3 x1,k1 x2,k2,
    {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
  ]];
r0[-1, i, j]
```

(Alt) Out[ ]=

$d_1 x_{1,i} x_{2,i} x_{3,i} + d_5 x_{1,j} x_{2,i} x_{3,i} + d_3 x_{1,i} x_{2,j} x_{3,i} + d_7 x_{1,j} x_{2,j} x_{3,i} + d_2 x_{1,i} x_{2,i} x_{3,j} + d_6 x_{1,j} x_{2,i} x_{3,j} + d_4 x_{1,i} x_{2,j} x_{3,j} + d_8 x_{1,j} x_{2,j} x_{3,j}$

The ppp Terms (r_1)

(Alt) In[ ]:=

```
κ = 0;
r1[1, i_, j_] := Evaluate[Sum[
    b++κ p3,k3 p1,k1 p2,k2,
    {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
  ]];
r1[1, i, j]
```

(Alt) Out[ ]=

$b_1 p_{1,i} p_{2,i} p_{3,i} + b_5 p_{1,j} p_{2,i} p_{3,i} + b_3 p_{1,i} p_{2,j} p_{3,i} + b_7 p_{1,j} p_{2,j} p_{3,i} + b_2 p_{1,i} p_{2,i} p_{3,j} + b_6 p_{1,j} p_{2,i} p_{3,j} + b_4 p_{1,i} p_{2,j} p_{3,j} + b_8 p_{1,j} p_{2,j} p_{3,j}$

(Alt) In[ ]:=

```
κ = 0;
r1[-1, i_, j_] := Evaluate[Sum[
    e++κ p3,k3 p1,k1 p2,k2,
    {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
   ]];
r1[-1, i, j]
```

(Alt) Out[ ]=

$e_1 p_{1,i} p_{2,i} p_{3,i} + e_5 p_{1,j} p_{2,i} p_{3,i} + e_3 p_{1,i} p_{2,j} p_{3,i} + e_7 p_{1,j} p_{2,j} p_{3,i} + e_2 p_{1,i} p_{2,i} p_{3,j} + e_6 p_{1,j} p_{2,i} p_{3,j} + e_4 p_{1,i} p_{2,j} p_{3,j} + e_8 p_{1,j} p_{2,j} p_{3,j}$

The ppxx Terms (r_{42})

(Alt) In[ ]:=

```
κ = 0;
Short[r42[1, i_, j_] = Evaluate[Plus[
    Sum[
     c++κ xν1,k1 pν1,k2 xν2,k3 pν2,k4,
     {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}, {k4, {i, j}}, {ν1, 2}, {ν2, ν1, 3}
    ],
    Sum[
     c++κ xν,k1 pν,k2,
     {k1, {i, j}}, {k2, {i, j}}, {ν, 3}
    ],
    c++κ
   ]]]
```

(Alt) Out[ ]//Short=

$c_{93} + c_{81} p_{1,i} x_{1,i} + \ll 89 \gg + c_{60} p_{2,i} p_{3,j} x_{2,j} x_{3,j} + c_{80} p_{2,j} p_{3,j} x_{2,j} x_{3,j}$

(Alt) In[ ]:=

```
κ = 0;
Short[r42[-1, i_, j_] = Evaluate[Plus[
    Sum[
     f++κ xν1,k1 pν1,k2 xν2,k3 pν2,k4,
     {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}, {k4, {i, j}}, {ν1, 2}, {ν2, ν1, 3}
    ],
    Sum[
     f++κ xν,k1 pν,k2,
     {k1, {i, j}}, {k2, {i, j}}, {ν, 3}
    ],
    f++κ
   ]]]
```

(Alt) Out[ ]//Short=

$f_{93} + f_{81} p_{1,i} x_{1,i} + \ll 89 \gg + f_{60} p_{2,i} p_{3,j} x_{2,j} x_{3,j} + f_{80} p_{2,j} p_{3,j} x_{2,j} x_{3,j}$

The γ Terms (γ_0, γ_1, γ_{42})

(Alt) In[ ]:=

```
κ = 0;
γ0[1, k_] := Evaluate[g++κ p3,k x1,k x2,k];
γ1[1, k_] := Evaluate[g++κ x3,k p1,k p2,k];
γ42[1, k_] := Evaluate[Plus[
    Sum[g++κ xν,k pν,k, {ν, 3}],
    Sum[g++κ xν1,k pν1,k xν2,k pν2,k, {ν1, 2}, {ν2, ν1, 3}]
   ]];
{γ0[1, k], γ0[1, k], γ42[1, k]}
```

(Alt) Out[ ]=

$$\{g_1 p_{3,k} x_{1,k} x_{2,k},\ g_1 p_{3,k} x_{1,k} x_{2,k},\ g_3 p_{1,k} x_{1,k} + g_6 p_{1,k}^2 x_{1,k}^2 + g_4 p_{2,k} x_{2,k} + g_7 p_{1,k} p_{2,k} x_{1,k} x_{2,k} + g_9 p_{2,k}^2 x_{2,k}^2 + g_5 p_{3,k} x_{3,k} + g_8 p_{1,k} p_{3,k} x_{1,k} x_{3,k} + g_{10} p_{2,k} p_{3,k} x_{2,k} x_{3,k}\}$$

(Alt) In[ ]:=

```
κ = 0;
γ0[-1, k_] := Evaluate[h++κ p3,k x1,k x2,k];
γ1[-1, k_] := Evaluate[h++κ x3,k p1,k p2,k];
γ42[-1, k_] := Evaluate[Plus[
    Sum[h++κ xν,k pν,k, {ν, 3}],
    Sum[h++κ xν1,k pν1,k xν2,k pν2,k, {ν1, 2}, {ν2, ν1, 3}]
   ]];
{γ0[-1, k], γ0[-1, k], γ42[-1, k]}
```

(Alt) Out[ ]=

$$\{h_1 p_{3,k} x_{1,k} x_{2,k},\ h_1 p_{3,k} x_{1,k} x_{2,k},\ h_3 p_{1,k} x_{1,k} + h_6 p_{1,k}^2 x_{1,k}^2 + h_4 p_{2,k} x_{2,k} + h_7 p_{1,k} p_{2,k} x_{1,k} x_{2,k} + h_9 p_{2,k}^2 x_{2,k}^2 + h_5 p_{3,k} x_{3,k} + h_8 p_{1,k} p_{3,k} x_{1,k} x_{3,k} + h_{10} p_{2,k} p_{3,k} x_{2,k} x_{3,k}\}$$

Reidemeister 3b

(Alt) In[ ]:=

```
Timing[{LeftR3b} =
  Cases[∫ℱ[i, j, k] × ℒ /@ (Xi,j[1] Xi+,k[1] Xj+,k+[1]) d{vsi, vsj, vsk, vsi+, vsj+, vsk+},
   𝔼[ℰ_] :→ ℰ, ∞]]
```

(Alt) Out[ ]=

$$\{0.609375,\ \{\epsilon\text{Series}[T_1^2 p_{1,2+i} \pi_{1,i} - (-1+T_1) T_1 p_{1,2+j} \pi_{1,i} + (1-T_1) p_{1,2+k} \pi_{1,i} + T_1 p_{1,2+j} \pi_{1,j} + (1-T_1) p_{1,2+k} \pi_{1,j} + p_{1,2+k} \pi_{1,k} + T_2^2 p_{2,2+i} \pi_{2,i} - (-1+T_2) T_2 p_{2,2+j} \pi_{2,i} + (1-T_2) p_{2,2+k} \pi_{2,i} + \langle\langle 38 \rangle\rangle + \frac{(a_6 + a_8 T_2 - a_8 T_2^2) \pi_{1,k} \pi_{2,i} \pi_{3,k}}{B} - \frac{(-1+T_1)(a_2 + a_6 T_1) \pi_{1,i} \pi_{2,j} \pi_{3,k}}{B} + \frac{a_2 \pi_{1,j} \pi_{2,j} \pi_{3,k}}{B} + \frac{a_6 \pi_{1,k} \pi_{2,j} \pi_{3,k}}{B} + \frac{(a_4 + a_8 T_1 - a_8 T_1^2) \pi_{1,i} \pi_{2,k} \pi_{3,k}}{B} + \frac{a_4 \pi_{1,j} \pi_{2,k} \pi_{3,k}}{B} + \frac{2 a_8 \pi_{1,k} \pi_{2,k} \pi_{3,k}}{B},\ \frac{\langle\langle 226 \rangle\rangle + a_7 b_1 T_1^4 T_2^4 + a_7 b_2 T_1^4 T_2^4}{T_1^2 T_2^2} + \langle\langle 520 \rangle\rangle]\}\}$$

Full expression not available (original memory size: 13.9 MB)

(Alt) In[]:=

```
Timing[{RightR3b} =
   Cases[∫𝓕[i, j, k] × 𝓛 /@ (X_{j,k}[1] X_{i⁺,k⁺}[1] X_{i⁺,j⁺}[1]) d{vs_i, vs_j, vs_k, vs_{i⁺}, vs_{j⁺}, vs_{k⁺}},
    𝔼[ℰ_] ⧴ ℰ, ∞];]
```

(Alt) Out[]=

```
{0.296875, Null}
```

(Alt) In[]:=

```
Short[eqn = CF[LeftR3b⟦1⟧ - RightR3b⟦1⟧]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r_0[1, i, j], a_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) ⧴ (c == 0), 3]
{sol} = Solve[eqns, vars]
```

(Alt) Out[]//Short=

$$\ll 40 \gg + \frac{a_8 (-1 + T_1) \ll 1 \gg \pi_{\ll 1 \gg} \pi_{3,k}}{B}$$

(Alt) Out[]=

$$\{\pi_{1,i}, \pi_{1,j}, \pi_{1,k}, \pi_{2,i}, \pi_{2,j}, \pi_{2,k}, \pi_{3,i}, \pi_{3,j}, \pi_{3,k}\}$$

(Alt) Out[]=

$$\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$$

(Alt) Out[]//Short=

$$\Big\{\frac{a_2}{B} + \frac{a_3}{B} - \frac{a_4}{B} + \frac{a_5}{B} - \frac{a_6}{B} - \frac{a_7}{B} + \frac{a_4}{B T_1^2} + \frac{a_1}{B T_1} - \frac{a_2}{B T_1} - \frac{a_3}{B T_1} + \frac{a_8}{B T_1} - \frac{a_1 T_1}{B} + \ll 39 \gg + \frac{a_1 T_1 T_2}{B} - \frac{a_3 T_1 T_2}{B} - \frac{a_5 T_1 T_2}{B} + \frac{a_8 T_1 T_2}{B} + \frac{a_5 T_1^2 T_2}{B} - \frac{a_7 T_1^2 T_2}{B} - \frac{a_3 T_2^2}{B} + \frac{a_3 T_1 T_2^2}{B} - \frac{a_7 T_1 T_2^2}{B} + \frac{a_7 T_1^2 T_2^2}{B} == 0,$$
$$\ll 1 \gg == 0, \ll 22 \gg, \ll 1 \gg == 0, -\frac{a_8}{B} + \frac{a_8}{B \ll 1 \gg \ll 1 \gg} == 0\Big\}$$

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[]=

$$\Big\{\Big\{a_1 \to -\frac{a_4 (1 - T_2 - T_1 T_2 + T_1 T_2^2)}{T_1 T_2}, a_2 \to -a_4 (-1 + T_2), a_3 \to -\frac{a_4 (1 - T_1 T_2)}{T_1 T_2},$$
$$a_5 \to \frac{a_4 (-1 + T_2)(-1 + T_1 T_2)}{(-1 + T_1) T_1 T_2}, a_6 \to -\frac{a_4 (1 - T_2)}{-1 + T_1}, a_7 \to -\frac{a_4 (-1 + T_1 T_2)}{(-1 + T_1) T_1 T_2}, a_8 \to 0\Big\}\Big\}$$

(Alt) In[ ]:=

```
sol /. (v_ → val_) :→ (v = CF[val]);
r0[1, i, j]
```

(Alt) Out[ ]=

$$-\frac{a_4 (-1+T_2)(-1+T_1 T_2) x_{1,i} x_{2,i} x_{3,i}}{T_1 T_2}+\frac{a_4 (-1+T_2)(-1+T_1 T_2) x_{1,j} x_{2,i} x_{3,i}}{(-1+T_1) T_1 T_2}+\frac{a_4 (-1+T_1 T_2) x_{1,i} x_{2,j} x_{3,i}}{T_1 T_2}-\frac{a_4 (-1+T_1 T_2) x_{1,j} x_{2,j} x_{3,i}}{(-1+T_1) T_1 T_2}-a_4 (-1+T_2) x_{1,i} x_{2,i} x_{3,j}+\frac{a_4 (-1+T_2) x_{1,j} x_{2,i} x_{3,j}}{-1+T_1}+a_4 x_{1,i} x_{2,j} x_{3,j}$$

(Alt) In[ ]:=

```
Short[eqn = CF[Coefficient[
    LeftR3b[[2]] - RightR3b[[2]] /. v : (π | p)__ :→ μ v,
    μ^3
   ]], 5]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r1[1, i, j], b_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :→ (c == 0), 3]
{sol} = Solve[eqns, vars]
```

(Alt) Out[ ]//Short=

$$\frac{B b_1 (-1+T_1) p_{1,2+j} p_{2,2+i} p_{3,2+i}}{T_1}-\frac{B b_1 (-1+T_1) p_{1,2+k} p_{2,2+i} p_{3,2+i}}{T_1}+\ll 35\gg+\frac{B (b_1+b_2+b_3+b_4+b_5+b_6+b_7+b_8)(-1+T_1)(-1+T_2)(-1+T_1 T_2) p_{1,2+k} p_{2,2+k} p_{3,2+k}}{T_1 T_2}$$

(Alt) Out[ ]=

$\{p_{1,2+i}, p_{1,2+j}, p_{1,2+k}, p_{2,2+i}, p_{2,2+j}, p_{2,2+k}, p_{3,2+i}, p_{3,2+j}, p_{3,2+k}\}$

(Alt) Out[ ]=

$\{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8\}$

(Alt) Out[ ]//Short=

$$\Big\{B b_1 - B b_1 T_1 T_2 == 0,\ -B b_1 + B b_1 T_1 T_2 == 0,\ \ll 22\gg,\ B b_2 + \ll 43\gg + \frac{B \ll 1\gg}{\ll 1\gg \ll 1\gg} == 0,\ \frac{B b_1}{T_1}+\frac{B b_2}{T_1}+\frac{B b_3}{T_1}+\frac{B b_4}{T_1}+\frac{B b_5}{T_1}+\frac{B b_6}{T_1}+\frac{B b_7}{T_1}+\ll 57\gg+B b_1 T_1 T_2+B b_2 T_1 T_2+B b_3 T_1 T_2+B b_4 T_1 T_2+B b_5 T_1 T_2+B b_6 T_1 T_2+B b_7 T_1 T_2+B b_8 T_1 T_2 == 0\Big\}$$

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]=

$$\Big\{\Big\{b_1 \to 0,\ b_3 \to -\frac{b_2 T_1 (-1+T_2) T_2}{-1+T_1 T_2},\ b_4 \to -b_2 (1-T_2),\ b_5 \to -\frac{b_2 (-1+T_1) T_1 T_2}{-1+T_1 T_2},\ b_6 \to -b_2 (1-T_1),\ b_7 \to -\frac{b_2 \left(T_1 T_2 - T_1^2 T_2 - T_1 T_2^2 + T_1^2 T_2^2\right)}{-1+T_1 T_2},\ b_8 \to -b_2 (-1+T_1+T_2-T_1 T_2)\Big\}\Big\}$$

(Alt) In[#]:=

```
sol /. (v_ → val_) :> (v = CF[val]);
r1[1, i, j]
```

(Alt) Out[#]=

$$-\frac{b_2 (-1+T_1) T_1 T_2 p_{1,j} p_{2,i} p_{3,i}}{-1+T_1 T_2} - \frac{b_2 T_1 (-1+T_2) T_2 p_{1,i} p_{2,j} p_{3,i}}{-1+T_1 T_2} - \frac{b_2 (-1+T_1) T_1 (-1+T_2) T_2 p_{1,j} p_{2,j} p_{3,i}}{-1+T_1 T_2} + b_2 p_{1,i} p_{2,i} p_{3,j} + b_2 (-1+T_1) p_{1,j} p_{2,i} p_{3,j} + b_2 (-1+T_2) p_{1,i} p_{2,j} p_{3,j} + b_2 (-1+T_1)(-1+T_2) p_{1,j} p_{2,j} p_{3,j}$$

(Alt) In[#]:=

```
Short[eqn = CF[LeftR3b[[2]] - RightR3b[[2]]], 5]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r42[1, i, j], c_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0), 3]
Short[{sol} = Solve[eqns, vars]]
```

(Alt) Out[#]//Short=

$$-T_1^2 (-a_4 b_2 - 2 c_{11} - 2 c_{41} - c_{42} - c_{43} - 2 c_{56} - c_{57} - c_{58} - 2 c_{71} - c_{87} + 2 c_{11} T_1 + 2 c_{41} T_1 + c_{42} T_1 + c_{43} T_1 + 2 c_{56} T_1 + c_{57} T_1 + c_{58} T_1 + 2 c_{71} T_1 + c_{87} T_1 + a_4 b_2 T_2) p_{1,2+j} \pi_{1,i} - \frac{(-1+T_1)(\ll 1 \gg) p_{1,2+k} \pi_{1,i}}{-1+T_1 T_2} + 2 c_1 (-1+T_1) T_1^3 p_{1,2+i} p_{1,2+j} \pi_{1,i}^2 + \ll 476 \gg + \frac{\ll 1 \gg}{\ll 1 \gg \ll 1 \gg} - \frac{(-1+T_2)(-c_{55} + c_{55} T_1 T_2 + c_{60} T_1 T_2) \ll 1 \gg \ll 1 \gg \pi_{\ll 1 \gg} \pi_{3,k}}{T_1} + \frac{(-1+T_2)(-c_{55} + c_{55} T_1 T_2 + c_{60} T_1 T_2) p_{2,2+j} p_{3,2+k} \pi_{2,k} \pi_{3,k}}{T_1}$$

(Alt) Out[#]=

$$\{p_{1,2+i}, p_{1,2+j}, p_{1,2+k}, p_{2,2+i}, p_{2,2+j}, p_{2,2+k}, p_{3,2+i}, p_{3,2+j}, p_{3,2+k}, \pi_{1,i}, \pi_{1,j}, \pi_{1,k}, \pi_{2,i}, \pi_{2,j}, \pi_{2,k}, \pi_{3,i}, \pi_{3,j}, \pi_{3,k}\}$$

(Alt) Out[#]=

$$\{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12}, c_{13}, c_{14}, c_{15}, c_{16}, c_{17}, c_{18}, c_{19}, c_{20}, c_{21}, c_{22}, c_{23}, c_{24}, c_{25}, c_{26}, c_{27}, c_{28}, c_{29}, c_{30}, c_{31}, c_{32}, c_{33}, c_{34}, c_{35}, c_{36}, c_{37}, c_{38}, c_{39}, c_{40}, c_{41}, c_{42}, c_{43}, c_{44}, c_{45}, c_{46}, c_{47}, c_{48}, c_{49}, c_{50}, c_{51}, c_{52}, c_{53}, c_{54}, c_{55}, c_{56}, c_{57}, c_{58}, c_{59}, c_{60}, c_{61}, c_{62}, c_{63}, c_{64}, c_{65}, c_{66}, c_{67}, c_{68}, c_{69}, c_{70}, c_{71}, c_{72}, c_{73}, c_{74}, c_{75}, c_{76}, c_{77}, c_{78}, c_{79}, c_{80}, c_{81}, c_{82}, c_{83}, c_{84}, c_{85}, c_{86}, c_{87}, c_{88}, c_{89}, c_{90}, c_{91}, c_{92}, c_{93}\}$$

(Alt) Out[#]//Short=

$$\left\{-c_{11} T_1^4 - c_{41} T_1^4 + c_{11} T_1^5 + c_{41} T_1^5 == 0,\ c_{11} T_1^3 + c_{41} T_1^3 - c_{11} T_1^4 - c_{41} T_1^4 == 0,\ \ll 315 \gg,\ -\frac{a_4 b_2}{T_1} - \frac{c_{13}}{T_1^2 T_2^2} - \frac{c_{15}}{T_1^2 T_2^2} - \frac{c_{73}}{T_1^2 T_2^2} - \frac{c_{75}}{T_1^2 T_2^2} - \frac{c_{89}}{T_1^2 T_2^2} + \frac{a_4 b_2}{T_1 T_2} + \frac{c_{13}}{T_1 T_2} + \frac{c_{15}}{T_1 T_2} + \frac{c_{73}}{T_1 T_2} + \frac{c_{75}}{T_1 T_2} + \frac{c_{89}}{T_1 T_2} == 0\right\}$$

••• Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[#]//Short=

$$\left\{\left\{c_1 \to 0,\ c_2 \to 0,\ \ll 58 \gg,\ c_{91} \to \ll 1 \gg,\ c_{92} \to -c_{83} T_1 T_2 - c_{86} T_1 T_2 - \frac{-a_4 b_2 + \ll 1 \gg + \ll 1 \gg - 2 a_4 b_2 T_1 T_2}{-1+T_1}\right\}\right\}$$

(Alt) In[ ]:=

```
sol /. (v_ → val_) :→ (v = CF[val]);
```

(Alt) In[ ]:=

```
Short[CF[r42[1, i, j]], 20]
```

(Alt) Out[ ]//Short=

$$
\begin{aligned}
&c_{93} + c_{81}\, p_{1,i}\, x_{1,i} + c_{84}\, p_{1,j}\, x_{1,i} + (c_6 + c_{21})\, p_{1,i}\, p_{1,j}\, x_{1,i}^2 + \\
&\frac{1}{2}(-1+T_1)(2 c_6 + 2 c_{21} + c_{16} T_1 + c_{31} T_1 + c_{46} T_1 + c_{61} T_1)\, p_{1,j}^2\, x_{1,i}^2 + \frac{a_4 b_2 (-1+T_2)\, p_{1,i}\, x_{1,j}}{-1+T_1} + \\
&\frac{\left(c_{81} + c_{84} + a_4 b_2 T_1 - 2 a_4 b_2 T_1 T_2 - c_{81} T_1 T_2 - c_{84} T_1 T_2 - a_4 b_2 T_1^2 T_2 + a_4 b_2 T_1^2 T_2^2\right) p_{1,j}\, x_{1,j}}{T_1 (-1 + T_1 T_2)} + \\
&(c_{16} + c_{31} + c_{46} + c_{61})\, p_{1,i}\, p_{1,j}\, x_{1,i}\, x_{1,j} + \\
&\frac{1}{2}(-2 c_6 - c_{16} - 2 c_{21} - c_{31} - c_{46} - c_{61} - c_{16} T_1 - c_{31} T_1 - c_{46} T_1 - c_{61} T_1)\, p_{1,j}^2\, x_{1,i}\, x_{1,j} + c_{82}\, p_{2,i}\, x_{2,i} + \\
&c_{85}\, p_{2,j}\, x_{2,i} + \frac{(-1+T_1)(c_7 - c_{17} + c_{17} T_2)\, p_{1,j}\, p_{2,i}\, x_{1,i}\, x_{2,i}}{-1+T_2} + c_7\, p_{1,i}\, p_{2,j}\, x_{1,i}\, x_{2,i} + \ll 40 \gg + \\
&\frac{\left(a_4 b_2 - a_4 b_2 T_1 - a_4 b_2 T_2 + 2 a_4 b_2 T_1 T_2 + c_{83} T_1 T_2 + c_{86} T_1 T_2 - c_{83} T_1^2 T_2 - c_{86} T_1^2 T_2\right) p_{3,j}\, x_{3,j}}{-1+T_1} + \\
&c_{18}\, p_{1,i}\, p_{3,j}\, x_{1,i}\, x_{3,j} + \frac{T_1 (a_4 b_2 - c_8 + c_8 T_1)\, T_2\, p_{1,j}\, p_{3,j}\, x_{1,i}\, x_{3,j}}{-1 + T_1 T_2} + \\
&\frac{a_4 b_2 T_1 (-1+T_2) T_2\, p_{1,i}\, p_{3,i}\, x_{1,j}\, x_{3,j}}{(-1+T_1)(-1+T_1 T_2)} + \frac{a_4 b_2 T_1 (-1+T_2) T_2\, p_{1,j}\, p_{3,i}\, x_{1,j}\, x_{3,j}}{-1+T_1 T_2} - \\
&\frac{a_4 b_2 (-1+T_2)\, p_{1,i}\, p_{3,j}\, x_{1,j}\, x_{3,j}}{-1+T_1} - a_4 b_2 (-1+T_2)\, p_{1,j}\, p_{3,j}\, x_{1,j}\, x_{3,j} + c_{20}\, p_{2,i}\, p_{3,j}\, x_{2,i}\, x_{3,j} + \\
&\frac{1}{(-1+T_1)(-1+T_1 T_2)} \Big(-c_{20} - c_{25} + c_{20} T_1 + c_{25} T_1 + c_{20} T_2 - a_4 b_2 T_1 T_2 + c_{25} T_1 T_2 - c_{20} T_1^2 T_2 - \\
&\quad c_{25} T_1^2 T_2 + a_4 b_2 T_1 T_2^2 - c_{20} T_1 T_2^2 + c_{20} T_1^2 T_2^2\Big)\, p_{2,j}\, p_{3,j}\, x_{2,i}\, x_{3,j} + \frac{a_4 b_2 T_1 T_2\, p_{2,i}\, p_{3,i}\, x_{2,j}\, x_{3,j}}{-1+T_1 T_2} + \\
&\frac{a_4 b_2 T_1 (-1+T_2) T_2\, p_{2,j}\, p_{3,i}\, x_{2,j}\, x_{3,j}}{-1+T_1 T_2} - a_4 b_2\, p_{2,i}\, p_{3,j}\, x_{2,j}\, x_{3,j} - a_4 b_2 (-1+T_2)\, p_{2,j}\, p_{3,j}\, x_{2,j}\, x_{3,j}
\end{aligned}
$$

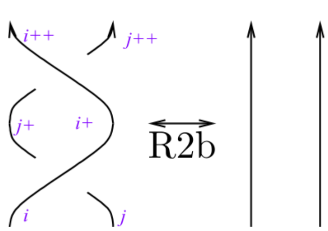
(Alt) In[ ]:=

```
CF[LeftR3b - RightR3b]
```

(Alt) Out[ ]=

```
∈Series[0, 0]
```

Reidemeister 2b



Territory: $(x \mid p)_{(i|j)^+},\ x_{i|j}$.

(Alt) In[]:=

```
Timing[Short[LeftR2b = (∫𝓕[i, j] × ℒ /@ (X_{i,j}[1] X_{i⁺,j⁺}[-1]) d{vs_i, vs_j, vs_{i⁺}, vs_{j⁺}})[[1]]
]]
```

(Alt) Out[]=

$$\left\{0.125,\ \epsilon\text{Series}\left[p_{1,2+i}\,\pi_{1,i}+p_{1,2+j}\,\pi_{1,j}+\ll 14\gg+\frac{d_8\,\pi_{1,j}\,\pi_{2,j}\,\pi_{3,j}}{B},\ \frac{\ll 1\gg}{\ll 1\gg}+\ll 111\gg\right]\right\}$$

(Alt) In[]:=

```
RightR2b = ϵSeries[p_{1,2+i} π_{1,i} + p_{1,2+j} π_{1,j} + p_{2,2+i} π_{2,i} + p_{2,2+j} π_{2,j} + p_{3,2+i} π_{3,i} + p_{3,2+j} π_{3,j}, 0]
```

(Alt) Out[]=

$$\epsilon\text{Series}\left[p_{1,2+i}\,\pi_{1,i}+p_{1,2+j}\,\pi_{1,j}+p_{2,2+i}\,\pi_{2,i}+p_{2,2+j}\,\pi_{2,j}+p_{3,2+i}\,\pi_{3,i}+p_{3,2+j}\,\pi_{3,j},\ 0\right]$$

(Alt) In[]:=

```
Short[eqn = CF[LeftR2b[[1]] - RightR2b[[1]]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r_0[-1, i, j], d_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) ↦ (c == 0), 3]
{sol} = Solve[eqns, vars]
```

(Alt) Out[]//Short=

$$-\frac{\left(a_4-d_7+\ll 38\gg+d_6\,T_1^2\,T_2^2-d_8\,T_1^2\,T_2^2\right)\ll 2\gg\,\pi_{\ll 1\gg}}{B\,T_1\,T_2}+\frac{\ll 1\gg}{\ll 1\gg}+\ll 6\gg+\frac{\ll 1\gg}{B}+\frac{d_8\ll 1\gg\ll 1\gg\ll 1\gg\,\pi_{3\,\ll 1\gg\,\ll 1\gg}}{B}$$

(Alt) Out[]=

$$\{\pi_{1,i},\ \pi_{1,j},\ \pi_{2,i},\ \pi_{2,j},\ \pi_{3,i},\ \pi_{3,j}\}$$

(Alt) Out[]=

$$\{d_1,\ d_2,\ d_3,\ d_4,\ d_5,\ d_6,\ d_7,\ d_8\}$$

(Alt) Out[]//Short=

$$\left\{\frac{a_4}{B}+\frac{d_1}{B}-\frac{d_2}{B}-\frac{d_3}{B}+\frac{d_4}{B}-\frac{d_5}{B}+\frac{d_6}{B}+\frac{d_7}{B}+\frac{a_4}{B\,T_1}+\frac{d_5}{B\,T_1}-\frac{d_6}{B\,T_1}-\frac{d_7}{B\,T_1}+\frac{d_8}{B\,T_1}+\frac{d_4\ll 1\gg}{B}-\frac{\ll 1\gg}{B}+\frac{d_3}{B\ll 1\gg}-\frac{d_4}{B\,T_2}-\frac{d_7}{B\,T_2}+\frac{d_8}{B\,T_2}-\frac{a_4}{B\,T_1\,T_2}+\frac{d_7}{B\,T_1\,T_2}-\frac{d_8}{B\,T_1\,T_2}-\frac{a_4\,T_2}{B}+\frac{d_6\,T_2}{B}-\frac{d_8\,T_2}{B}+\frac{d_2\,T_1\,T_2}{B}-\frac{d_4\,T_1\,T_2}{B}-\frac{d_6\,T_1\,T_2}{B}+\frac{d_8\,T_1\,T_2}{B}==0,\ \ll 6\gg,\ \frac{\ll 1\gg}{B}==0\right\}$$

(Alt) Out[]=

$$\left\{\left\{d_1\to\frac{a_4\,(-1+T_2)\,(-1+T_1\,T_2)}{T_1\,T_2},\ d_2\to-\frac{a_4\,(-1+T_2)}{T_1\,T_2},\ d_3\to-\frac{a_4-a_4\,T_1\,T_2}{T_1},\ d_4\to-\frac{a_4}{T_1},\ d_5\to\frac{a_4\,(-1+T_2)\,(-1+T_1\,T_2)}{(-1+T_1)\,T_2},\ d_6\to-\frac{a_4\,(-1+T_2)}{(-1+T_1)\,T_2},\ d_7\to\frac{a_4\,(-1+T_1\,T_2)}{-1+T_1},\ d_8\to 0\right\}\right\}$$

(Alt) In[]:=

```
sol /. (v_ → val_) :> (v = CF[val]);
r0[-1, i, j]
```

(Alt) Out[]=

$$\frac{a_4\,(-1+T_2)\,(-1+T_1 T_2)\,x_{1,i}\,x_{2,i}\,x_{3,i}}{T_1 T_2}+\frac{a_4\,(-1+T_2)\,(-1+T_1 T_2)\,x_{1,j}\,x_{2,i}\,x_{3,i}}{(-1+T_1)\,T_2}+\frac{a_4\,(-1+T_1 T_2)\,x_{1,i}\,x_{2,j}\,x_{3,i}}{T_1}+\frac{a_4\,(-1+T_1 T_2)\,x_{1,j}\,x_{2,j}\,x_{3,i}}{-1+T_1}-\frac{a_4\,(-1+T_2)\,x_{1,i}\,x_{2,i}\,x_{3,j}}{T_1 T_2}-\frac{a_4\,(-1+T_2)\,x_{1,j}\,x_{2,i}\,x_{3,j}}{(-1+T_1)\,T_2}-\frac{a_4\,x_{1,i}\,x_{2,j}\,x_{3,j}}{T_1}$$

(Alt) In[]:=

```
Short[eqn = CF[LeftR2b[[2]] - RightR2b[[2]]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r1[-1, i, j] + r42[-1, i, j], e_ | f_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0), 3]
Short[{sol} = Solve[eqns, vars]]
```

(Alt) Out[]//Short=

$$\frac{a_4 e_1 + \ll 348 \gg}{(-1+T_1)\,T_1 T_2\,(-1+T_1 T_2)} + \ll 123 \gg$$

(Alt) Out[]=

$\{p_{1,2+i}, p_{1,2+j}, p_{2,2+i}, p_{2,2+j}, p_{3,2+i}, p_{3,2+j}, \pi_{1,i}, \pi_{1,j}, \pi_{2,i}, \pi_{2,j}, \pi_{3,i}, \pi_{3,j}\}$

(Alt) Out[]=

$\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_{12}, f_{13}, f_{14}, f_{15}, f_{16}, f_{17}, f_{18}, f_{19}, f_{20}, f_{21}, f_{22}, f_{23}, f_{24}, f_{25}, f_{26}, f_{27}, f_{28}, f_{29}, f_{30}, f_{31}, f_{32}, f_{33}, f_{34}, f_{35}, f_{36}, f_{37}, f_{38}, f_{39}, f_{40}, f_{41}, f_{42}, f_{43}, f_{44}, f_{45}, f_{46}, f_{47}, f_{48}, f_{49}, f_{50}, f_{51}, f_{52}, f_{53}, f_{54}, f_{55}, f_{56}, f_{57}, f_{58}, f_{59}, f_{60}, f_{61}, f_{62}, f_{63}, f_{64}, f_{65}, f_{66}, f_{67}, f_{68}, f_{69}, f_{70}, f_{71}, f_{72}, f_{73}, f_{74}, f_{75}, f_{76}, f_{77}, f_{78}, f_{79}, f_{80}, f_{81}, f_{82}, f_{83}, f_{84}, f_{85}, f_{86}, f_{87}, f_{88}, f_{89}, f_{90}, f_{91}, f_{92}, f_{93}\}$

(Alt) Out[]//Short=

$\{\ll 1 \gg\}$

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[]//Short=

$\{\{e_1 \to 0, e_2 \to -b_2 T_1 T_2, \ll 84 \gg, f_{93} \to -c_{93}\}\}$

(Alt) In[]:=

```
sol /. (v_ → val_) :> (v = CF[val]);
```

(Alt) In[ ]:=

```
r₁[-1, i, j]
Short[CF[r₄₂[-1, i, j]], 5]
```

(Alt) Out[ ]=

$$\frac{b_2 (-1+T_1) T_2 p_{1,j} p_{2,i} p_{3,i}}{-1+T_1 T_2} + \frac{b_2 T_1 (-1+T_2) p_{1,i} p_{2,j} p_{3,i}}{-1+T_1 T_2} - \frac{b_2 (-1+T_1)(-1+T_2) p_{1,j} p_{2,j} p_{3,i}}{-1+T_1 T_2} - b_2 T_1 T_2 p_{1,i} p_{2,i} p_{3,j} + b_2 (-1+T_1) T_2 p_{1,j} p_{2,i} p_{3,j} + b_2 T_1 (-1+T_2) p_{1,i} p_{2,j} p_{3,j} - b_2 (-1+T_1)(-1+T_2) p_{1,j} p_{2,j} p_{3,j}$$

(Alt) Out[ ]//Short=

$$-c_{93} - c_{81} p_{1,i} x_{1,i} + \ll 72 \gg + \frac{a_4 b_2 (-1+T_2) p_{2,j} p_{3,i} x_{2,j} x_{3,j}}{-1+T_1 T_2} - a_4 b_2 T_2 p_{2,i} p_{3,j} x_{2,j} x_{3,j} + a_4 b_2 (-1+T_2) p_{2,j} p_{3,j} x_{2,j} x_{3,j}$$

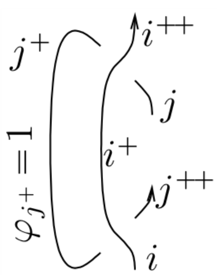
(Alt) In[ ]:=

```
CF[LeftR2b - RightR2b]
```

(Alt) Out[ ]=

```
∈Series[0, 0]
```

Reidemeister 2c



(Alt) In[ ]:=

```
Timing[Short[{LeftR2c} = Cases[
    ∫𝓕[i, j] × ℒ /@ (X_{i+1,j}[1] X_{i,j+2}[-1] C_{j+1}[1]) ⅆ{vs_i, vs_j, vs_{i⁺}, vs_{j⁺}, vs_{j+2}}, 𝔼[ℰ_] ⧴ ℰ]
 ]]
```

(Alt) Out[ ]=

$\{0.234375,$
$\{\in\text{Series}[p_{1,2+i}\,\pi_{1,i} + p_{1,3+j}\,\pi_{1,j} + p_{2,2+i}\,\pi_{2,i} + \ll 19 \gg + p_{3,2+i}\,\pi_{3,i} + p_{3,3+j}\,\pi_{3,j},\ g_1 g_2 + \ll 67 \gg]\}\}$

(Alt) In[ ]:=
```
Timing[Short[{RightR2c} =
  Cases[∫ℱ[i, j] × ℒ /@ (C_i[0] C_{i+1}[0] C_j[0] C_{j+1}[1] C_{j+2}[0]) ⅆ{vs_i, vs_j, vs_{i⁺}, vs_{j⁺}, vs_{j+2}},
   𝔼[ℰ_] :→ ℰ]
]]
```
(Alt) Out[ ]=

$\{0., \{\epsilon\text{Series}[p_{1,2+i}\,\pi_{1,i} + p_{1,3+j}\,\pi_{1,j} + p_{2,2+i}\,\pi_{2,i} + p_{2,\ll 1\gg}\,\pi_{\ll 1\gg} + \frac{\ll 1\gg}{B} + p_{3,2+i}\,\pi_{3,i} + p_{3,3+j}\,\pi_{3,j},\ \ll 1\gg]\}\}$

(Alt) In[ ]:=
```
Short[eqn = CF[LeftR2c⟦1⟧ - RightR2c⟦1⟧]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[γ0[1, k], g_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :→ (c == 0), 3]
{sol} = Solve[eqns, vars]
```
(Alt) Out[ ]//Short=

$\frac{g_1\ (-1+T_1)\ (-1+T_2)\ p_{3\ \ll 1\gg\ \ll 1\gg}\ \pi_{1,i}\ \pi_{2,i}}{B\ T_1\ T_2} - \frac{\ll 1\gg}{B \ll 1\gg} + \ll 11\gg + \frac{\ll 1\gg}{\ll 1\gg} - \frac{a_4\ g_1 \ll 1\gg \pi_{\ll 1\gg}\ \pi^2_{\ll 1\gg}}{B^2\ T_1}$

(Alt) Out[ ]=

$\{p_{3,3+j},\ \pi_{1,i},\ \pi_{1,j},\ \pi_{2,i},\ \pi_{2,j}\}$

(Alt) Out[ ]=

$\{g_1\}$

(Alt) Out[ ]//Short=

$\left\{\frac{g_1}{B} - \frac{g_1}{B\ T_1} - \frac{g_1}{B\ T_2} + \frac{g_1}{B\ T_1\ T_2} == 0,\ -\frac{g_1}{B} + \frac{g_1}{B\ T_1} == 0,\ -\frac{g_1}{B} + \frac{g_1}{B\ T_2} == 0,\ \ll 5\gg,\ -\frac{a_4\ g_1}{B^2\ T_1} == 0,\right.$
$\left.-\frac{a_4\ g_1}{B^2\ (1-T_1)} - \frac{a_4\ g_1}{B^2\ (1-T_1)\ T_2^2} + \frac{2\ a_4\ g_1}{B^2\ (1-T_1)\ T_2} == 0,\ \frac{a_4\ g_1}{B^2\ (1-T_1)} - \frac{a_4\ g_1}{B^2\ (1-T_1)\ T_2} == 0\right\}$

(Alt) Out[ ]=

$\{\{g_1 \to 0\}\}$

(Alt) In[ ]:=
```
sol /. (v_ → val_) :→ (v = CF[val]);
γ0[1, k]
```
(Alt) Out[ ]=

0

(Alt) In[ ]:=

```
Short[eqn = CF[LeftR2c⟦2⟧ - RightR2c⟦2⟧]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[γ1[1, k] + γ42[1, k], g_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0), 3]
Short[{sol} = Solve[eqns, vars]]
```

(Alt) Out[ ]//Short=

$$\frac{(c_{16} + c_{31} + c_{46} + c_{61} - g_3 - 4\, g_6 - g_7 - g_8)\ \ll 2 \gg\ \pi_{\ll 1 \gg}}{T_1} + \ll 42 \gg$$

(Alt) Out[ ]=

$\{p_{1,3+j}, p_{2,3+j}, p_{3,3+j}, \pi_{1,i}, \pi_{1,j}, \pi_{2,i}, \pi_{2,j}, \pi_{3,i}, \pi_{3,j}\}$

(Alt) Out[ ]=

$\{g_2, g_3, g_4, g_5, g_6, g_7, g_8, g_9, g_{10}\}$

(Alt) Out[ ]//Short=

$$\Big\{g_6 + \frac{g_6}{T_1^2} - \frac{2\, g_6}{T_1} == 0,\ -2\, g_6 + \frac{2\, g_6}{T_1} == 0,\ g_7 - \frac{g_7}{T_1} - \frac{g_7}{T_2} + \frac{g_7}{T_1 T_2} == 0,$$
$$-g_7 + \frac{g_7}{T_1} == 0,\ \ll 22 \gg,\ -g_5 - g_8 - g_{10} + g_5 T_1 T_2 + g_8 T_1 T_2 + g_{10} T_1 T_2 == 0,$$
$$-\frac{a_4 g_2}{1 - T_1} + \frac{2\, a_4 g_2 T_1 T_2}{1 - T_1} - \frac{a_4 g_2 T_1^2 T_2^2}{1 - T_1} == 0,\ \frac{a_4 g_2}{1 - T_1} - \frac{a_4 g_2 T_1 T_2}{1 - T_1} == 0\Big\}$$

(Alt) Out[ ]//Short=

$$\{\{g_2 \to 0,\ g_3 \to c_{16} + c_{31} + c_{46} + c_{61},$$
$$g_4 \to c_{19} + c_{34} + c_{49} + c_{64},\ g_5 \to 0,\ g_6 \to 0,\ g_7 \to 0,\ g_8 \to 0,\ g_9 \to 0,\ g_{10} \to 0\}\}$$

(Alt) In[ ]:=

```
sol /. (v_ → val_) :> (v = CF[val]);
```

(Alt) In[ ]:=

```
γ1[1, k]
Short[CF[γ42[1, k]], 5]
```

(Alt) Out[ ]=

0

(Alt) Out[ ]//Short=

$(c_{16} + c_{31} + c_{46} + c_{61})\ p_{1,k}\ x_{1,k} + (c_{19} + c_{34} + c_{49} + c_{64})\ p_{2,k}\ x_{2,k}$

(Alt) In[ ]:=

```
CF[LeftR2c - RightR2c]
```

(Alt) Out[ ]=

∈Series[0, 0]

C_k[1] and C_k[−1] are inverses

(Alt) In[ ]:=

```
Timing[Short[{LeftCC} = Cases[{∫𝓕[k] × ℒ /@ (C_k[1] C_{k+1}[-1]) d{vs_k, vs_{k+}}}, 𝔼[ℰ_] :→ ℰ]
  ]]
```

(Alt) Out[ ]=

$$\left\{0.,\left\{\in\text{Series}\left[p_{1,2+k}\,\pi_{1,k}+p_{2,2+k}\,\pi_{2,k}+\frac{h_1\,p_{\ll 1\gg}\ \ll 1\gg\ \pi_{2,k}}{B}+p_{3,2+k}\,\pi_{3,k},\ c_{16}+\ll 24\gg+\left(h_1\,h_2+h_{10}\right)\ \ll 3\gg\ \pi_{\ll 1\gg}\right]\right\}\right\}$$

(Alt) In[ ]:=

```
Timing[Short[{RightCC} = Cases[{∫𝓕[k] × ℒ /@ (C_k[0] C_{k+1}[0]) d{vs_k, vs_{k+}}}, 𝔼[ℰ_] :→ ℰ]
  ]]
```

(Alt) Out[ ]=

$$\{0.,\ \{\in\text{Series}[p_{1,2+k}\,\pi_{1,k}+p_{2,2+k}\,\pi_{2,k}+p_{3,2+k}\,\pi_{3,k},\ 0]\}\}$$

(Alt) In[ ]:=

```
Short[eqn = CF[LeftCC[[1]] - RightCC[[1]]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[γ0[-1, k], h_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :→ (c == 0), 3]
{sol} = Solve[eqns, vars]
```

(Alt) Out[ ]//Short=

$$\frac{h_1\,p_{3,2+k}\,\pi_{1,k}\,\pi_{2,k}}{B}$$

(Alt) Out[ ]=

$$\{p_{3,2+k},\ \pi_{1,k},\ \pi_{2,k}\}$$

(Alt) Out[ ]=

$$\{h_1\}$$

(Alt) Out[ ]//Short=

$$\left\{\frac{h_1}{B}==0\right\}$$

(Alt) Out[ ]=

$$\{\{h_1\to 0\}\}$$

(Alt) In[ ]:=

```
sol /. (v_ → val_) :→ (v = CF[val]);
γ0[-1, k]
```

(Alt) Out[ ]=

0

(Alt) In[ ]:=

```
Short[eqn = CF[LeftCC⟦2⟧ - RightCC⟦2⟧]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[γ1[-1, k] + γ42[-1, k], h_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) ⧴ (c == 0), 3]
Short[{sol} = Solve[eqns, vars]]
```

(Alt) Out[ ]//Short=

$$c_{16} + c_{19} + c_{31} + \ll 20 \gg + h_8\, p_{1,2+k}\, p_{3,2+k}\, \pi_{1,k}\, \pi_{3,k} + h_{10}\, p_{2,2+k}\, p_{3,2+k}\, \pi_{2,k}\, \pi_{3,k}$$

(Alt) Out[ ]=

$$\{p_{1,2+k},\ p_{2,2+k},\ p_{3,2+k},\ \pi_{1,k},\ \pi_{2,k},\ \pi_{3,k}\}$$

(Alt) Out[ ]=

$$\{h_2,\ h_3,\ h_4,\ h_5,\ h_6,\ h_7,\ h_8,\ h_9,\ h_{10}\}$$

(Alt) Out[ ]//Short=

$$\{h_6 == 0,\ h_7 == 0,\ B\, h_2 == 0,\ h_8 == 0,\ c_{16} + c_{31} + c_{46} + c_{61} + h_3 + 4\, h_6 + h_7 + h_8 == 0,$$
$$h_9 == 0,\ h_{10} == 0,\ c_{19} + c_{34} + c_{49} + c_{64} + h_4 + h_7 + 4\, h_9 + h_{10} == 0,\ h_5 + h_8 + h_{10} == 0,$$
$$c_{16} + c_{19} + c_{31} + c_{34} + c_{46} + c_{49} + c_{61} + c_{64} + h_3 + h_4 + h_5 + 2\, h_6 + h_7 + h_8 + 2\, h_9 + h_{10} == 0\}$$

(Alt) Out[ ]//Short=

$$\{\{h_2 \to 0,\ h_3 \to -c_{16} - c_{31} - c_{46} - c_{61},$$
$$h_4 \to -c_{19} - c_{34} - c_{49} - c_{64},\ h_5 \to 0,\ h_6 \to 0,\ h_7 \to 0,\ h_8 \to 0,\ h_9 \to 0,\ h_{10} \to 0\}\}$$

(Alt) In[ ]:=

```
sol /. (v_ → val_) ⧴ (v = CF[val]);
```

(Alt) In[ ]:=

```
γ1[-1, k]
Short[CF[γ42[-1, k]], 5]
```

(Alt) Out[ ]=

$$0$$

(Alt) Out[ ]//Short=

$$(-c_{16} - c_{31} - c_{46} - c_{61})\, p_{1,k}\, x_{1,k} + (-c_{19} - c_{34} - c_{49} - c_{64})\, p_{2,k}\, x_{2,k}$$

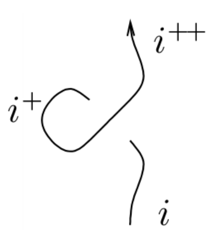
(Alt) In[ ]:=

```
CF[LeftCC - RightCC]
```

(Alt) Out[ ]=

$$\in\text{Series}[0, 0]$$

Invariance Under R1l



(Alt) In[ ]:=

```
{LeftR1l} = Cases[{∫𝓕[i] × ℒ /@ (X_{i+2,i}[1] C_{i+1}[1]) d{vs_i, vs_{i⁺}, vs_{i+2}}}, 𝔼[ℰ_] ⧴ ℰ, ∞]
```

(Alt) Out[ ]=

{∈Series[$p_{1,3+i}\,\pi_{1,i} + p_{2,3+i}\,\pi_{2,i} + p_{3,3+i}\,\pi_{3,i}$, $-a_4\,b_2 + c_{93} + a_4\,b_2\,T_2$]}

(Alt) In[ ]:=

```
{RightR1l} = Cases[{∫𝓕[i] × ℒ /@ (C_i[0] C_{i+1}[0] C_{i+2}[0]) d{vs_i, vs_{i⁺}, vs_{i+2}}}, 𝔼[ℰ_] ⧴ ℰ, ∞]
```

(Alt) Out[ ]=

{∈Series[$p_{1,3+i}\,\pi_{1,i} + p_{2,3+i}\,\pi_{2,i} + p_{3,3+i}\,\pi_{3,i}$, 0]}

(Alt) In[ ]:=

```
LeftR1l⟦1⟧ == RightR1l⟦1⟧
```

(Alt) Out[ ]=

True

(Alt) In[ ]:=

```
Short[eqn = CF[LeftR1l⟦2⟧ - RightR1l⟦2⟧]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[eqn, (c | d | e | f | g | h)_, ∞]
Short[eqns = If[cvs === {},
   {eqn == 0},
   CoefficientRules[eqn, cvs] /. (_ → c_) ⧴ (c == 0)
  ], 3]
{sol} = Solve[eqns, vars]
```

(Alt) Out[ ]//Short=

$-a_4\,b_2 + c_{93} + a_4\,b_2\,T_2$

(Alt) Out[ ]=

{}

(Alt) Out[ ]=

{c_{93}}

(Alt) Out[ ]//Short=

{$-a_4\,b_2 + c_{93} + a_4\,b_2\,T_2 == 0$}

(Alt) Out[ ]=

{{$c_{93} \to a_4\,b_2 - a_4\,b_2\,T_2$}}

(Alt) In[ ]:=

```
sol /. (v_ → val_) ⧴ (v = CF[val]);
```

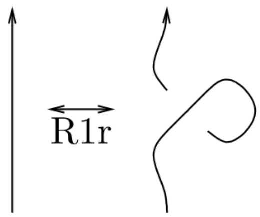
(Alt) In[ ]:=

```
CF[LeftR1l - RightR1l]
```

(Alt) Out[ ]=

∈Series[0, 0]

Invariance Under R1r



(Alt) In[ ]:=

```
{LeftR1r} = Cases[{∫𝓕[i] × 𝓛 /@ (X_{i,i+2}[1] C_{i+1}[-1]) d{vs_i, vs_{i+}, vs_{i+2}}}, 𝔼[ℰ_] ⧴ ℰ, ∞]
```

(Alt) Out[ ]=

$$\Big\{\epsilon\text{Series}\Big[p_{1,3+i}\,\pi_{1,i} + p_{2,3+i}\,\pi_{2,i} + p_{3,3+i}\,\pi_{3,i},\ \frac{1}{(-1+T_1)\,T_1\,T_2\,(-1+T_1\,T_2)} \big(-c_{82}\,T_1 - c_{85}\,T_1 + c_{82}\,T_1^2 + c_{85}\,T_1^2 - c_{81}\,T_2 - c_{84}\,T_2 + 2\,c_{81}\,T_1\,T_2 + c_{82}\,T_1\,T_2 + c_{83}\,T_1\,T_2 + c_{84}\,T_1\,T_2 + a_4\,b_2\,T_1^2\,T_2 - c_{81}\,T_1^2\,T_2 - c_{83}\,T_1^2\,T_2 + c_{85}\,T_1^2\,T_2 - c_{82}\,T_1^3\,T_2 - c_{85}\,T_1^3\,T_2 + a_4\,b_2\,T_1\,T_2^2 + c_{81}\,T_1\,T_2^2 + c_{84}\,T_1\,T_2^2 - 3\,a_4\,b_2\,T_1^2\,T_2^2 - 2\,c_{81}\,T_1^2\,T_2^2 - c_{82}\,T_1^2\,T_2^2 - 2\,c_{83}\,T_1^2\,T_2^2 - c_{84}\,T_1^2\,T_2^2 - c_{86}\,T_1^2\,T_2^2 + c_{81}\,T_1^3\,T_2^2 + c_{82}\,T_1^3\,T_2^2 + 2\,c_{83}\,T_1^3\,T_2^2 + c_{86}\,T_1^3\,T_2^2 + a_4\,b_2\,T_1^3\,T_2^3 + c_{83}\,T_1^3\,T_2^3 + c_{86}\,T_1^3\,T_2^3 - c_{83}\,T_1^4\,T_2^3 - c_{86}\,T_1^4\,T_2^3\big)\Big]\Big\}$$

(Alt) In[ ]:=

```
{RightR1r} = Cases[{∫𝓕[i] × 𝓛 /@ (C_i[0] C_{i+1}[0] C_{i+2}[0]) d{vs_i, vs_{i+}, vs_{i+2}}}, 𝔼[ℰ_] ⧴ ℰ, ∞]
```

(Alt) Out[ ]=

$$\{\epsilon\text{Series}[p_{1,3+i}\,\pi_{1,i} + p_{2,3+i}\,\pi_{2,i} + p_{3,3+i}\,\pi_{3,i},\ 0]\}$$

(Alt) In[ ]:=

```
LeftR1r[[1]] == RightR1r[[1]]
```

(Alt) Out[ ]=

True

(Alt) In[ ]:=

```
Short[eqn = CF[LeftR1r[[2]] - RightR1r[[2]]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[eqn, (c | d | e | f | g | h)_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) ⧴ (c == 0), 3]
{sol} = Solve[eqns, vars]
```

(Alt) Out[ ]//Short=

$$\frac{-c_{82}\,T_1 - c_{85}\,T_1 + \ll 46 \gg}{(-1+T_1)\,T_1\,T_2\,(-1+T_1\,T_2)}$$

(Alt) Out[ ]=

{ }

(Alt) Out[ ]=

$\{c_{81}, c_{82}, c_{83}, c_{84}, c_{85}, c_{86}\}$

(Alt) Out[ ]//Short=

$$\Big\{\frac{-c_{82}\,T_1 - c_{85}\,T_1 + c_{82}\,T_1^2 + \ll 39 \gg + c_{83}\,T_1^3\,T_2^3 + c_{86}\,T_1^3\,T_2^3 - c_{83}\,T_1^4\,T_2^3 - c_{86}\,T_1^4\,T_2^3}{(-1+T_1)\,T_1\,T_2\,(-1+T_1\,T_2)} == 0\Big\}$$

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[ ]=

$$\Bigg\{\Bigg\{c_{86} \to -\frac{c_{85}\left(\frac{T_1}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{T_1^2}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{1}{(-1+T_1)\,T_2\,(-1+T_1\,T_2)}+\frac{T_1}{(-1+T_1)\,T_2\,(-1+T_1\,T_2)}\right)}{-\frac{T_1\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_1^2\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_1^2\,T_2^2}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{T_1^3\,T_2^2}{(-1+T_1)\,(-1+T_1\,T_2)}} -$$

$$\frac{c_{84}\left(\frac{1}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{1}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)}+\frac{T_2}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{T_1\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}\right)}{-\frac{T_1\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_1^2\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_1^2\,T_2^2}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{T_1^3\,T_2^2}{(-1+T_1)\,(-1+T_1\,T_2)}} -$$

$$\left(c_{81}\left(\frac{2}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{1}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)}-\frac{T_1}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_2}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{2\,T_1\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_1^2\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}\right)\right)\Big/$$

$$\left(-\frac{T_1\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_1^2\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_1^2\,T_2^2}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{T_1^3\,T_2^2}{(-1+T_1)\,(-1+T_1\,T_2)}\right) -$$

$$\left(c_{82}\left(\frac{1}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{T_1^2}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{1}{(-1+T_1)\,T_2\,(-1+T_1\,T_2)}+\frac{T_1}{(-1+T_1)\,T_2\,(-1+T_1\,T_2)}-\frac{T_1\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_1^2\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}\right)\right)\Big/$$

$$\left(-\frac{T_1\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_1^2\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_1^2\,T_2^2}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{T_1^3\,T_2^2}{(-1+T_1)\,(-1+T_1\,T_2)}\right) -$$

$$\frac{\frac{a_4\,b_2\,T_1}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{a_4\,b_2\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{3\,a_4\,b_2\,T_1\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{a_4\,b_2\,T_1^2\,T_2^2}{(-1+T_1)\,(-1+T_1\,T_2)}}{-\frac{T_1\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_1^2\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_1^2\,T_2^2}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{T_1^3\,T_2^2}{(-1+T_1)\,(-1+T_1\,T_2)}} -$$

$$\left(c_{83}\left(\frac{1}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{T_1}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{2\,T_1\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{2\,T_1^2\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_1^2\,T_2^2}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{T_1^3\,T_2^2}{(-1+T_1)\,(-1+T_1\,T_2)}\right)\right)\Big/$$

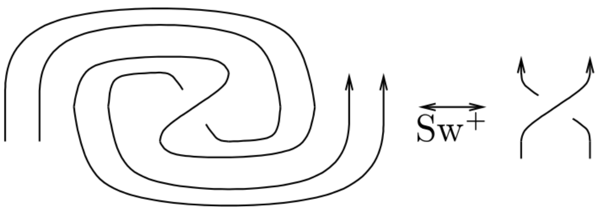
$$\left(-\frac{T_1\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_1^2\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_1^2\,T_2^2}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{T_1^3\,T_2^2}{(-1+T_1)\,(-1+T_1\,T_2)}\right)\Bigg\}\Bigg\}$$

(Alt) In[ ]:=

```
sol /. (v_ → val_) :→ (v = CF[val]);
```

(Alt) In[ ]:=
```
CF[LeftR1r - RightR1r]
```
(Alt) Out[ ]=
```
∈Series[0, 0]
```

Invariance Under Sw



(Alt) In[ ]:=
```
Timing[ Short[{LeftSw} = Cases[{∫ℱ[i, j] × ℒ /@ (X_{i+1,j+1}[1] C_i[-1] C_j[-1] C_{i+2}[1] C_{j+2}[1])
        ⅆ{vs_i, vs_j, vs_{i⁺}, vs_{j⁺}, vs_{i+2}, vs_{j+2}}}, 𝔼[ℰ_] :→ ℰ, ∞]
  ]]
```
(Alt) Out[ ]=

$\{0.046875, \{\in\text{Series}[T_1\, p_{1,3+i}\, \pi_{1,i} + (1 - T_1)\, p_{1,3+j}\, \pi_{1,i} + \ll 16 \gg + \frac{a_4\, \pi_{1,i}\, \pi_{2,j}\, \pi_{3,j}}{B}, a_4\, b_2 + \ll 121 \gg]\}\}$

(Alt) In[ ]:=
```
Timing[ Short[{RightSw} = Cases[{∫ℱ[i, j] × ℒ /@ (X_{i+1,j+1}[1] C_i[0] C_j[0] C_{i+2}[0] C_{j+2}[0])
        ⅆ{vs_i, vs_j, vs_{i⁺}, vs_{j⁺}, vs_{i+2}, vs_{j+2}}}, 𝔼[ℰ_] :→ ℰ, ∞]
  ]]
```
(Alt) Out[ ]=

$\{0.03125, \{\in\text{Series}[T_1\, p_{1,3+i}\, \pi_{1,i} + (1 - T_1)\, p_{1,3+j}\, \pi_{1,i} + \ll 16 \gg + \frac{a_4\, \pi_{1,i}\, \pi_{2,j}\, \pi_{3,j}}{B}, a_4\, b_2 + \ll 121 \gg]\}\}$

(Alt) In[ ]:=
```
LeftSw == RightSw
```
(Alt) Out[ ]=
```
True
```

The Solution

(Alt) In[ ]:=
```
Union@Cases[ℒ@X_{i,j}[1], (a | b | c | d | e | f | g | h)_, ∞]
```
(Alt) Out[ ]=

$\{a_4, b_2, c_6, c_7, c_8, c_9, c_{16}, c_{17}, c_{18}, c_{19}, c_{20}, c_{21}, c_{24}, c_{25}, c_{31}, c_{34}, c_{36}, c_{39}, c_{46}, c_{49}, c_{61}, c_{64}, c_{81}, c_{82}, c_{83}, c_{84}, c_{85}\}$

Some Knots

(Alt) In[ ]:=

```
tab1 = Last /@ Table[K = Knot[n, 1];
   Echo@Timing[K → ∫ℒ[K] ⅆ vs[K]],
   {n, 3, 10}];
```

» {1.75,

$$\text{Knot}[3,1] \to -\Big(\Big(\mathbb{i}\, T_1^6 T_2^6\, \mathbb{E}\Big[\epsilon\text{Series}\Big[0,\ \Big(-2 c_{82} T_1 - 2 c_{85} T_1 + 6 c_{82} T_1^2 + 6 c_{85} T_1^2 - 10 c_{82} T_1^3 - 10 c_{85} T_1^3 + 10 c_{82} T_1^4 +$$
$$10 c_{85} T_1^4 - 6 c_{82} T_1^5 - 6 c_{85} T_1^5 + 2 c_{82} T_1^6 + 2 c_{85} T_1^6 - 2 c_{81} T_2 - 2 c_{84} T_2 + 7 c_{81} T_1 T_2 + 5 c_{82} T_1 T_2 +$$
$$5 c_{84} T_1 T_2 + 3 c_{85} T_1 T_2 + 2 a_4 b_2 T_1^2 T_2 + c_{16} T_1^2 T_2 + c_{31} T_1^2 T_2 + c_{46} T_1^2 T_2 + c_{61} T_1^2 T_2 - 11 c_{81} T_1^2 T_2 -$$
$$12 c_{82} T_1^2 T_2 - 6 c_{84} T_1^2 T_2 - 6 c_{85} T_1^2 T_2 - 4 a_4 b_2 T_1^3 T_2 - 3 c_{16} T_1^3 T_2 - 3 c_{31} T_1^3 T_2 - 3 c_{46} T_1^3 T_2 -$$
$$3 c_{61} T_1^3 T_2 + 10 c_{81} T_1^3 T_2 + 16 c_{82} T_1^3 T_2 + 4 c_{84} T_1^3 T_2 + 6 c_{85} T_1^3 T_2 + 6 a_4 b_2 T_1^4 T_2 + 5 c_{16} T_1^4 T_2 +$$
$$5 c_{31} T_1^4 T_2 + 5 c_{46} T_1^4 T_2 + 5 c_{61} T_1^4 T_2 - 5 c_{81} T_1^4 T_2 - 10 c_{82} T_1^4 T_2 - c_{84} T_1^4 T_2 - 4 a_4 b_2 T_1^5 T_2 - 5 c_{16} T_1^5 T_2 -$$
$$5 c_{31} T_1^5 T_2 - 5 c_{46} T_1^5 T_2 - 5 c_{61} T_1^5 T_2 + c_{81} T_1^5 T_2 - 6 c_{85} T_1^5 T_2 + 2 a_4 b_2 T_1^6 T_2 + 2 c_{16} T_1^6 T_2 + 2 c_{31} T_1^6 T_2 +$$
$$2 c_{46} T_1^6 T_2 + 2 c_{61} T_1^6 T_2 + 4 c_{82} T_1^6 T_2 + 6 c_{85} T_1^6 T_2 - 3 c_{82} T_1^7 T_2 - 3 c_{85} T_1^7 T_2 + 4 c_{81} T_2^2 + 4 c_{84} T_2^2 +$$
$$2 a_4 b_2 T_1 T_2^2 + c_{19} T_1 T_2^2 + c_{34} T_1 T_2^2 + c_{49} T_1 T_2^2 + c_{64} T_1 T_2^2 - 11 c_{81} T_1 T_2^2 - 6 c_{82} T_1 T_2^2 - 7 c_{84} T_1 T_2^2 -$$
$$3 c_{85} T_1 T_2^2 - 10 a_4 b_2 T_1^2 T_2^2 - 2 c_{16} T_1^2 T_2^2 - 3 c_{19} T_1^2 T_2^2 - 2 c_{31} T_1^2 T_2^2 - 3 c_{34} T_1^2 T_2^2 - 2 c_{46} T_1^2 T_2^2 -$$
$$3 c_{49} T_1^2 T_2^2 - 2 c_{61} T_1^2 T_2^2 - 3 c_{64} T_1^2 T_2^2 + 12 c_{81} T_1^2 T_2^2 + 11 c_{82} T_1^2 T_2^2 + 5 c_{84} T_1^2 T_2^2 + 5 c_{85} T_1^2 T_2^2 +$$
$$15 a_4 b_2 T_1^3 T_2^2 + 4 c_{16} T_1^3 T_2^2 + 5 c_{19} T_1^3 T_2^2 + 4 c_{31} T_1^3 T_2^2 + 5 c_{34} T_1^3 T_2^2 + 4 c_{46} T_1^3 T_2^2 + 5 c_{49} T_1^3 T_2^2 +$$
$$4 c_{61} T_1^3 T_2^2 + 5 c_{64} T_1^3 T_2^2 - 6 c_{81} T_1^3 T_2^2 - 10 c_{82} T_1^3 T_2^2 - c_{84} T_1^3 T_2^2 - 4 c_{85} T_1^3 T_2^2 - 17 a_4 b_2 T_1^4 T_2^2 -$$
$$4 c_{16} T_1^4 T_2^2 - 5 c_{19} T_1^4 T_2^2 - 4 c_{31} T_1^4 T_2^2 - 5 c_{34} T_1^4 T_2^2 - 4 c_{46} T_1^4 T_2^2 - 5 c_{49} T_1^4 T_2^2 - 4 c_{61} T_1^4 T_2^2 - 5 c_{64} T_1^4 T_2^2 -$$
$$2 c_{82} T_1^4 T_2^2 - c_{84} T_1^4 T_2^2 - 2 c_{85} T_1^4 T_2^2 + 8 a_4 b_2 T_1^5 T_2^2 + 3 c_{19} T_1^5 T_2^2 + 3 c_{34} T_1^5 T_2^2 + 3 c_{49} T_1^5 T_2^2 + 3 c_{64} T_1^5 T_2^2 +$$
$$12 c_{82} T_1^5 T_2^2 - c_{84} T_1^5 T_2^2 + 6 c_{85} T_1^5 T_2^2 - 3 a_4 b_2 T_1^6 T_2^2 + 6 c_{16} T_1^6 T_2^2 - c_{19} T_1^6 T_2^2 + 6 c_{31} T_1^6 T_2^2 - c_{34} T_1^6 T_2^2 +$$
$$6 c_{46} T_1^6 T_2^2 - c_{49} T_1^6 T_2^2 + 6 c_{61} T_1^6 T_2^2 - c_{64} T_1^6 T_2^2 + 2 c_{81} T_1^6 T_2^2 - 10 c_{82} T_1^6 T_2^2 + c_{84} T_1^6 T_2^2 - 4 c_{85} T_1^6 T_2^2 -$$
$$a_4 b_2 T_1^7 T_2^2 - 4 c_{16} T_1^7 T_2^2 - 4 c_{31} T_1^7 T_2^2 - 4 c_{46} T_1^7 T_2^2 - 4 c_{61} T_1^7 T_2^2 - c_{81} T_1^7 T_2^2 + 4 c_{82} T_1^7 T_2^2 + c_{85} T_1^7 T_2^2 +$$
$$c_{82} T_1^8 T_2^2 + c_{85} T_1^8 T_2^2 - 6 c_{81} T_2^3 - 6 c_{84} T_2^3 - 4 a_4 b_2 T_1 T_2^3 - 2 c_{19} T_1 T_2^3 - 2 c_{34} T_1 T_2^3 - 2 c_{49} T_1 T_2^3 -$$
$$2 c_{64} T_1 T_2^3 + 15 c_{81} T_1 T_2^3 + 4 c_{82} T_1 T_2^3 + 9 c_{84} T_1 T_2^3 + c_{85} T_1 T_2^3 + 15 a_4 b_2 T_1^2 T_2^3 + 3 c_{16} T_1^2 T_2^3 +$$
$$4 c_{19} T_1^2 T_2^3 + 3 c_{31} T_1^2 T_2^3 + 4 c_{34} T_1^2 T_2^3 + 3 c_{46} T_1^2 T_2^3 + 4 c_{49} T_1^2 T_2^3 + 3 c_{61} T_1^2 T_2^3 + 4 c_{64} T_1^2 T_2^3 -$$
$$14 c_{81} T_1^2 T_2^3 - 5 c_{82} T_1^2 T_2^3 - 5 c_{84} T_1^2 T_2^3 - 20 a_4 b_2 T_1^3 T_2^3 - 5 c_{16} T_1^3 T_2^3 - 4 c_{19} T_1^3 T_2^3 - 5 c_{31} T_1^3 T_2^3 -$$
$$4 c_{34} T_1^3 T_2^3 - 5 c_{46} T_1^3 T_2^3 - 4 c_{49} T_1^3 T_2^3 - 5 c_{61} T_1^3 T_2^3 - 4 c_{64} T_1^3 T_2^3 + 4 c_{81} T_1^3 T_2^3 - c_{84} T_1^3 T_2^3 - 4 c_{85} T_1^3 T_2^3 +$$
$$21 a_4 b_2 T_1^4 T_2^3 + 5 c_{16} T_1^4 T_2^3 + 5 c_{31} T_1^4 T_2^3 + 5 c_{46} T_1^4 T_2^3 + 5 c_{61} T_1^4 T_2^3 + 7 c_{81} T_1^4 T_2^3 + 12 c_{82} T_1^4 T_2^3 +$$
$$6 c_{84} T_1^4 T_2^3 + 10 c_{85} T_1^4 T_2^3 - 9 a_4 b_2 T_1^5 T_2^3 - c_{16} T_1^5 T_2^3 + 4 c_{19} T_1^5 T_2^3 - c_{31} T_1^5 T_2^3 + 4 c_{34} T_1^5 T_2^3 - c_{46} T_1^5 T_2^3 +$$
$$4 c_{49} T_1^5 T_2^3 - c_{61} T_1^5 T_2^3 + 4 c_{64} T_1^5 T_2^3 - 11 c_{81} T_1^5 T_2^3 - 18 c_{82} T_1^5 T_2^3 - 5 c_{84} T_1^5 T_2^3 - 12 c_{85} T_1^5 T_2^3 +$$
$$4 a_4 b_2 T_1^6 T_2^3 - 4 c_{16} T_1^6 T_2^3 - 4 c_{19} T_1^6 T_2^3 - 4 c_{31} T_1^6 T_2^3 - 4 c_{34} T_1^6 T_2^3 - 4 c_{46} T_1^6 T_2^3 - 4 c_{49} T_1^6 T_2^3 - 4 c_{61} T_1^6 T_2^3 -$$
$$4 c_{64} T_1^6 T_2^3 + 10 c_{81} T_1^6 T_2^3 + 12 c_{82} T_1^6 T_2^3 + 5 c_{84} T_1^6 T_2^3 + 8 c_{85} T_1^6 T_2^3 + a_4 b_2 T_1^7 T_2^3 - 2 c_{16} T_1^7 T_2^3 +$$
$$2 c_{19} T_1^7 T_2^3 - 2 c_{31} T_1^7 T_2^3 + 2 c_{34} T_1^7 T_2^3 - 2 c_{46} T_1^7 T_2^3 + 2 c_{49} T_1^7 T_2^3 - 2 c_{61} T_1^7 T_2^3 + 2 c_{64} T_1^7 T_2^3 - 8 c_{81} T_1^7 T_2^3 -$$
$$4 c_{82} T_1^7 T_2^3 - 3 c_{84} T_1^7 T_2^3 - 3 c_{85} T_1^7 T_2^3 + 4 c_{16} T_1^8 T_2^3 + 4 c_{31} T_1^8 T_2^3 + 4 c_{46} T_1^8 T_2^3 + 4 c_{61} T_1^8 T_2^3 + 3 c_{81} T_1^8 T_2^3 -$$
$$c_{82} T_1^8 T_2^3 + 4 c_{81} T_2^4 + 4 c_{84} T_2^4 + 6 a_4 b_2 T_1 T_2^4 + 3 c_{19} T_1 T_2^4 + 3 c_{34} T_1 T_2^4 + 3 c_{49} T_1 T_2^4 + 3 c_{64} T_1 T_2^4 -$$
$$5 c_{81} T_1 T_2^4 - c_{82} T_1 T_2^4 - c_{84} T_1 T_2^4 - 17 a_4 b_2 T_1^2 T_2^4 - 2 c_{16} T_1^2 T_2^4 - 5 c_{19} T_1^2 T_2^4 - 2 c_{31} T_1^2 T_2^4 - 5 c_{34} T_1^2 T_2^4 -$$
$$2 c_{46} T_1^2 T_2^4 - 5 c_{49} T_1^2 T_2^4 - 2 c_{61} T_1^2 T_2^4 - 5 c_{64} T_1^2 T_2^4 - 6 c_{81} T_1^2 T_2^4 - 7 c_{84} T_1^2 T_2^4 + 21 a_4 b_2 T_1^3 T_2^4 +$$
$$5 c_{19} T_1^3 T_2^4 + 5 c_{34} T_1^3 T_2^4 + 5 c_{49} T_1^3 T_2^4 + 5 c_{64} T_1^3 T_2^4 + 18 c_{81} T_1^3 T_2^4 + 7 c_{82} T_1^3 T_2^4 + 11 c_{84} T_1^3 T_2^4 +$$
$$3 c_{85} T_1^3 T_2^4 - 18 a_4 b_2 T_1^4 T_2^4 + 4 c_{16} T_1^4 T_2^4 - c_{19} T_1^4 T_2^4 + 4 c_{31} T_1^4 T_2^4 - c_{34} T_1^4 T_2^4 + 4 c_{46} T_1^4 T_2^4 - c_{49} T_1^4 T_2^4 +$$
$$4 c_{61} T_1^4 T_2^4 - c_{64} T_1^4 T_2^4 - 23 c_{81} T_1^4 T_2^4 - 18 c_{82} T_1^4 T_2^4 - 12 c_{84} T_1^4 T_2^4 - 8 c_{85} T_1^4 T_2^4 + 4 a_4 b_2 T_1^5 T_2^4 -$$
$$9 c_{16} T_1^5 T_2^4 - c_{19} T_1^5 T_2^4 - 9 c_{31} T_1^5 T_2^4 - c_{34} T_1^5 T_2^4 - 9 c_{46} T_1^5 T_2^4 - c_{49} T_1^5 T_2^4 - 9 c_{61} T_1^5 T_2^4 - c_{64} T_1^5 T_2^4 +$$
$$19 c_{81} T_1^5 T_2^4 + 24 c_{82} T_1^5 T_2^4 + 7 c_{84} T_1^5 T_2^4 + 12 c_{85} T_1^5 T_2^4 + 3 a_4 b_2 T_1^6 T_2^4 + 9 c_{16} T_1^6 T_2^4 - c_{19} T_1^6 T_2^4 +$$
$$9 c_{31} T_1^6 T_2^4 - c_{34} T_1^6 T_2^4 + 9 c_{46} T_1^6 T_2^4 - c_{49} T_1^6 T_2^4 + 9 c_{61} T_1^6 T_2^4 - c_{64} T_1^6 T_2^4 - 12 c_{81} T_1^6 T_2^4 - 18 c_{82} T_1^6 T_2^4 -$$
$$5 c_{84} T_1^6 T_2^4 - 10 c_{85} T_1^6 T_2^4 - 6 a_4 b_2 T_1^7 T_2^4 + 3 c_{16} T_1^7 T_2^4 + 2 c_{19} T_1^7 T_2^4 + 3 c_{31} T_1^7 T_2^4 + 2 c_{34} T_1^7 T_2^4 +$$

$3 c_{46} T_1^7 T_2^4 + 2 c_{49} T_1^7 T_2^4 + 3 c_{61} T_1^7 T_2^4 + 2 c_{64} T_1^7 T_2^4 + 6 c_{81} T_1^7 T_2^4 + 7 c_{82} T_1^7 T_2^4 + c_{84} T_1^7 T_2^4 + 4 c_{85} T_1^7 T_2^4 +$
$3 a_4 b_2 T_1^8 T_2^4 - 3 c_{16} T_1^8 T_2^4 - 2 c_{19} T_1^8 T_2^4 - 3 c_{31} T_1^8 T_2^4 - 2 c_{34} T_1^8 T_2^4 - 3 c_{46} T_1^8 T_2^4 - 2 c_{49} T_1^8 T_2^4 -$
$3 c_{61} T_1^8 T_2^4 - 2 c_{64} T_1^8 T_2^4 + c_{81} T_1^8 T_2^4 + 2 c_{84} T_1^8 T_2^4 - 2 c_{16} T_1^9 T_2^4 - 2 c_{31} T_1^9 T_2^4 - 2 c_{46} T_1^9 T_2^4 - 2 c_{61} T_1^9 T_2^4 -$
$2 c_{81} T_1^9 T_2^4 - c_{82} T_1^9 T_2^4 - c_{85} T_1^9 T_2^4 - 2 c_{81} T_2^5 - 2 c_{84} T_2^5 - 4 a_4 b_2 T_1 T_2^5 - 2 c_{19} T_1 T_2^5 - 2 c_{34} T_1 T_2^5 -$
$2 c_{49} T_1 T_2^5 - 2 c_{64} T_1 T_2^5 + c_{81} T_1 T_2^5 - c_{84} T_1 T_2^5 + 8 a_4 b_2 T_1^2 T_2^5 + c_{16} T_1^2 T_2^5 + c_{31} T_1^2 T_2^5 + c_{46} T_1^2 T_2^5 +$
$c_{61} T_1^2 T_2^5 + 6 c_{81} T_1^2 T_2^5 - c_{82} T_1^2 T_2^5 + 5 c_{84} T_1^2 T_2^5 - c_{85} T_1^2 T_2^5 - 9 a_4 b_2 T_1^3 T_2^5 + c_{16} T_1^3 T_2^5 + 4 c_{19} T_1^3 T_2^5 +$
$c_{31} T_1^3 T_2^5 + 4 c_{34} T_1^3 T_2^5 + c_{46} T_1^3 T_2^5 + 4 c_{49} T_1^3 T_2^5 + c_{61} T_1^3 T_2^5 + 4 c_{64} T_1^3 T_2^5 - 12 c_{81} T_1^3 T_2^5 - 4 c_{82} T_1^3 T_2^5 -$
$7 c_{84} T_1^3 T_2^5 - c_{85} T_1^3 T_2^5 + 4 a_4 b_2 T_1^4 T_2^5 - c_{16} T_1^4 T_2^5 - 9 c_{19} T_1^4 T_2^5 - c_{31} T_1^4 T_2^5 - 9 c_{34} T_1^4 T_2^5 - c_{46} T_1^4 T_2^5 -$
$9 c_{49} T_1^4 T_2^5 - c_{61} T_1^4 T_2^5 - 9 c_{64} T_1^4 T_2^5 + 19 c_{81} T_1^4 T_2^5 + 12 c_{82} T_1^4 T_2^5 + 12 c_{84} T_1^4 T_2^5 + 4 c_{85} T_1^4 T_2^5 +$
» $6 a_4 b_2 T_1^5 T_2^5 - c_{16} T_1^5 T_2^5 + 7 c_{19} T_1^5 T_2^5 - c_{31} T_1^5 T_2^5 + 7 c_{34} T_1^5 T_2^5 - c_{46} T_1^5 T_2^5 + 7 c_{49} T_1^5 T_2^5 - c_{61} T_1^5 T_2^5 +$
$7 c_{64} T_1^5 T_2^5 - 23 c_{81} T_1^5 T_2^5 - 18 c_{82} T_1^5 T_2^5 - 11 c_{84} T_1^5 T_2^5 - 6 c_{85} T_1^5 T_2^5 - 15 a_4 b_2 T_1^6 T_2^5 + 6 c_{16} T_1^6 T_2^5 -$
$c_{19} T_1^6 T_2^5 + 6 c_{31} T_1^6 T_2^5 - c_{34} T_1^6 T_2^5 + 6 c_{46} T_1^6 T_2^5 - c_{49} T_1^6 T_2^5 + 6 c_{61} T_1^6 T_2^5 - c_{64} T_1^6 T_2^5 + 18 c_{81} T_1^6 T_2^5 +$
$12 c_{82} T_1^6 T_2^5 + 7 c_{84} T_1^6 T_2^5 + 2 c_{85} T_1^6 T_2^5 + 14 a_4 b_2 T_1^7 T_2^5 - 12 c_{16} T_1^7 T_2^5 - c_{19} T_1^7 T_2^5 - 12 c_{31} T_1^7 T_2^5 -$
$c_{34} T_1^7 T_2^5 - 12 c_{46} T_1^7 T_2^5 - c_{49} T_1^7 T_2^5 - 12 c_{61} T_1^7 T_2^5 - c_{64} T_1^7 T_2^5 - 6 c_{81} T_1^7 T_2^5 + c_{84} T_1^7 T_2^5 + 4 c_{85} T_1^7 T_2^5 -$
$8 a_4 b_2 T_1^8 T_2^5 + 2 c_{16} T_1^8 T_2^5 + c_{19} T_1^8 T_2^5 + 2 c_{31} T_1^8 T_2^5 + c_{34} T_1^8 T_2^5 + 2 c_{46} T_1^8 T_2^5 + c_{49} T_1^8 T_2^5 + 2 c_{61} T_1^8 T_2^5 +$
$c_{64} T_1^8 T_2^5 - 5 c_{81} T_1^8 T_2^5 - 5 c_{82} T_1^8 T_2^5 - 4 c_{84} T_1^8 T_2^5 - 5 c_{85} T_1^8 T_2^5 + 4 c_{16} T_1^9 T_2^5 + c_{19} T_1^9 T_2^5 + 4 c_{31} T_1^9 T_2^5 +$
$c_{34} T_1^9 T_2^5 + 4 c_{46} T_1^9 T_2^5 + c_{49} T_1^9 T_2^5 + 4 c_{61} T_1^9 T_2^5 + c_{64} T_1^9 T_2^5 + 4 c_{81} T_1^9 T_2^5 + 4 c_{82} T_1^9 T_2^5 + 3 c_{85} T_1^9 T_2^5 +$
$2 a_4 b_2 T_1 T_2^6 + 3 c_{81} T_1 T_2^6 + 3 c_{84} T_1 T_2^6 - 3 a_4 b_2 T_1^2 T_2^6 + 4 c_{19} T_1^2 T_2^6 + 4 c_{34} T_1^2 T_2^6 + 4 c_{49} T_1^2 T_2^6 +$
$4 c_{64} T_1^2 T_2^6 - 8 c_{81} T_1^2 T_2^6 + c_{82} T_1^2 T_2^6 - 5 c_{84} T_1^2 T_2^6 + 4 a_4 b_2 T_1^3 T_2^6 - 2 c_{16} T_1^3 T_2^6 - 6 c_{19} T_1^3 T_2^6 - 2 c_{31} T_1^3 T_2^6 -$
$6 c_{34} T_1^3 T_2^6 - 2 c_{46} T_1^3 T_2^6 - 6 c_{49} T_1^3 T_2^6 - 2 c_{61} T_1^3 T_2^6 - 6 c_{64} T_1^3 T_2^6 + 10 c_{81} T_1^3 T_2^6 + 4 c_{82} T_1^3 T_2^6 +$
$5 c_{84} T_1^3 T_2^6 + 3 c_{85} T_1^3 T_2^6 + 3 a_4 b_2 T_1^4 T_2^6 + 2 c_{16} T_1^4 T_2^6 + 4 c_{19} T_1^4 T_2^6 + 2 c_{31} T_1^4 T_2^6 + 4 c_{34} T_1^4 T_2^6 +$
$2 c_{46} T_1^4 T_2^6 + 4 c_{49} T_1^4 T_2^6 + 2 c_{61} T_1^4 T_2^6 + 4 c_{64} T_1^4 T_2^6 - 11 c_{81} T_1^4 T_2^6 - 10 c_{82} T_1^4 T_2^6 - 6 c_{84} T_1^4 T_2^6 -$
$6 c_{85} T_1^4 T_2^6 - 15 a_4 b_2 T_1^5 T_2^6 - c_{16} T_1^5 T_2^6 + 4 c_{19} T_1^5 T_2^6 - c_{31} T_1^5 T_2^6 + 4 c_{34} T_1^5 T_2^6 - c_{46} T_1^5 T_2^6 + 4 c_{49} T_1^5 T_2^6 -$
$c_{61} T_1^5 T_2^6 + 4 c_{64} T_1^5 T_2^6 + 7 c_{81} T_1^5 T_2^6 + 12 c_{82} T_1^5 T_2^6 + c_{84} T_1^5 T_2^6 + 6 c_{85} T_1^5 T_2^6 + 28 a_4 b_2 T_1^6 T_2^6 - c_{16} T_1^6 T_2^6 -$
$8 c_{19} T_1^6 T_2^6 - c_{31} T_1^6 T_2^6 - 8 c_{34} T_1^6 T_2^6 - c_{46} T_1^6 T_2^6 - 8 c_{49} T_1^6 T_2^6 - c_{61} T_1^6 T_2^6 - 8 c_{64} T_1^6 T_2^6 + 4 c_{81} T_1^6 T_2^6 -$
$2 c_{82} T_1^6 T_2^6 + 5 c_{84} T_1^6 T_2^6 - 23 a_4 b_2 T_1^7 T_2^6 + c_{16} T_1^7 T_2^6 + 4 c_{19} T_1^7 T_2^6 + c_{31} T_1^7 T_2^6 + 4 c_{34} T_1^7 T_2^6 + c_{46} T_1^7 T_2^6 +$
$4 c_{49} T_1^7 T_2^6 + c_{61} T_1^7 T_2^6 + 4 c_{64} T_1^7 T_2^6 - 14 c_{81} T_1^7 T_2^6 - 10 c_{82} T_1^7 T_2^6 - 9 c_{84} T_1^7 T_2^6 - 6 c_{85} T_1^7 T_2^6 +$
$12 a_4 b_2 T_1^8 T_2^6 + 7 c_{16} T_1^8 T_2^6 + 7 c_{31} T_1^8 T_2^6 + 7 c_{46} T_1^8 T_2^6 + 7 c_{61} T_1^8 T_2^6 + 15 c_{81} T_1^8 T_2^6 + 11 c_{82} T_1^8 T_2^6 +$
$6 c_{84} T_1^8 T_2^6 + 6 c_{85} T_1^8 T_2^6 - 6 c_{16} T_1^9 T_2^6 - 2 c_{19} T_1^9 T_2^6 - 6 c_{31} T_1^9 T_2^6 - 2 c_{34} T_1^9 T_2^6 - 6 c_{46} T_1^9 T_2^6 - 2 c_{49} T_1^9 T_2^6 -$
$6 c_{61} T_1^9 T_2^6 - 2 c_{64} T_1^9 T_2^6 - 6 c_{81} T_1^9 T_2^6 - 6 c_{82} T_1^9 T_2^6 - 3 c_{85} T_1^9 T_2^6 - a_4 b_2 T_1^2 T_2^7 - c_{81} T_1^2 T_2^7 - c_{84} T_1^2 T_2^7 +$
$a_4 b_2 T_1^3 T_2^7 - 4 c_{19} T_1^3 T_2^7 - 4 c_{34} T_1^3 T_2^7 - 4 c_{49} T_1^3 T_2^7 - 4 c_{64} T_1^3 T_2^7 + 2 c_{81} T_1^3 T_2^7 - 3 c_{82} T_1^3 T_2^7 + c_{84} T_1^3 T_2^7 -$
$6 a_4 b_2 T_1^4 T_2^7 + 2 c_{16} T_1^4 T_2^7 + 9 c_{19} T_1^4 T_2^7 + 2 c_{31} T_1^4 T_2^7 + 9 c_{34} T_1^4 T_2^7 + 2 c_{46} T_1^4 T_2^7 + 9 c_{49} T_1^4 T_2^7 +$
$2 c_{61} T_1^4 T_2^7 + 9 c_{64} T_1^4 T_2^7 + 4 c_{82} T_1^4 T_2^7 + c_{84} T_1^4 T_2^7 - 2 c_{85} T_1^4 T_2^7 + 14 a_4 b_2 T_1^5 T_2^7 - 4 c_{16} T_1^5 T_2^7 -$
$11 c_{19} T_1^5 T_2^7 - 4 c_{31} T_1^5 T_2^7 - 11 c_{34} T_1^5 T_2^7 - 4 c_{46} T_1^5 T_2^7 - 11 c_{49} T_1^5 T_2^7 - 4 c_{61} T_1^5 T_2^7 - 11 c_{64} T_1^5 T_2^7 +$
$c_{84} T_1^5 T_2^7 + 6 c_{85} T_1^5 T_2^7 - 23 a_4 b_2 T_1^6 T_2^7 + 4 c_{16} T_1^6 T_2^7 + 5 c_{19} T_1^6 T_2^7 + 4 c_{31} T_1^6 T_2^7 + 5 c_{34} T_1^6 T_2^7 +$
$4 c_{46} T_1^6 T_2^7 + 5 c_{49} T_1^6 T_2^7 + 4 c_{61} T_1^6 T_2^7 + 5 c_{64} T_1^6 T_2^7 - 6 c_{81} T_1^6 T_2^7 - 10 c_{82} T_1^6 T_2^7 - 5 c_{84} T_1^6 T_2^7 -$
$10 c_{85} T_1^6 T_2^7 + 18 a_4 b_2 T_1^7 T_2^7 + 3 c_{19} T_1^7 T_2^7 + 3 c_{34} T_1^7 T_2^7 + 3 c_{49} T_1^7 T_2^7 + 3 c_{64} T_1^7 T_2^7 + 12 c_{81} T_1^7 T_2^7 +$
$16 c_{82} T_1^7 T_2^7 + 7 c_{84} T_1^7 T_2^7 + 10 c_{85} T_1^7 T_2^7 - 9 a_4 b_2 T_1^8 T_2^7 - 6 c_{16} T_1^8 T_2^7 - 5 c_{19} T_1^8 T_2^7 - 6 c_{31} T_1^8 T_2^7 -$
$5 c_{34} T_1^8 T_2^7 - 6 c_{46} T_1^8 T_2^7 - 5 c_{49} T_1^8 T_2^7 - 6 c_{61} T_1^8 T_2^7 - 5 c_{64} T_1^8 T_2^7 - 11 c_{81} T_1^8 T_2^7 - 12 c_{82} T_1^8 T_2^7 -$
$4 c_{84} T_1^8 T_2^7 - 6 c_{85} T_1^8 T_2^7 + 4 c_{16} T_1^9 T_2^7 + 3 c_{19} T_1^9 T_2^7 + 4 c_{31} T_1^9 T_2^7 + 3 c_{34} T_1^9 T_2^7 + 4 c_{46} T_1^9 T_2^7 +$
$3 c_{49} T_1^9 T_2^7 + 4 c_{61} T_1^9 T_2^7 + 3 c_{64} T_1^9 T_2^7 + 4 c_{81} T_1^9 T_2^7 + 5 c_{82} T_1^9 T_2^7 + 2 c_{85} T_1^9 T_2^7 + 3 a_4 b_2 T_1^4 T_2^8 +$
$2 c_{19} T_1^4 T_2^8 + 2 c_{34} T_1^4 T_2^8 + 2 c_{49} T_1^4 T_2^8 + 2 c_{64} T_1^4 T_2^8 + c_{81} T_1^4 T_2^8 + 2 c_{82} T_1^4 T_2^8 + c_{84} T_1^4 T_2^8 - 8 a_4 b_2 T_1^5 T_2^8 -$
$c_{16} T_1^5 T_2^8 - 6 c_{19} T_1^5 T_2^8 - c_{31} T_1^5 T_2^8 - 6 c_{34} T_1^5 T_2^8 - c_{46} T_1^5 T_2^8 - 6 c_{49} T_1^5 T_2^8 - c_{61} T_1^5 T_2^8 - 6 c_{64} T_1^5 T_2^8 -$
$5 c_{81} T_1^5 T_2^8 - 6 c_{82} T_1^5 T_2^8 - 4 c_{84} T_1^5 T_2^8 + 12 a_4 b_2 T_1^6 T_2^8 + 3 c_{16} T_1^6 T_2^8 + 10 c_{19} T_1^6 T_2^8 + 3 c_{31} T_1^6 T_2^8 +$
$10 c_{34} T_1^6 T_2^8 + 3 c_{46} T_1^6 T_2^8 + 10 c_{49} T_1^6 T_2^8 + 3 c_{61} T_1^6 T_2^8 + 10 c_{64} T_1^6 T_2^8 + 10 c_{81} T_1^6 T_2^8 + 10 c_{82} T_1^6 T_2^8 +$
$6 c_{84} T_1^6 T_2^8 - 9 a_4 b_2 T_1^7 T_2^8 - 5 c_{16} T_1^7 T_2^8 - 10 c_{19} T_1^7 T_2^8 - 5 c_{31} T_1^7 T_2^8 - 10 c_{34} T_1^7 T_2^8 - 5 c_{46} T_1^7 T_2^8 -$
$10 c_{49} T_1^7 T_2^8 - 5 c_{61} T_1^7 T_2^8 - 10 c_{64} T_1^7 T_2^8 - 11 c_{81} T_1^7 T_2^8 - 10 c_{82} T_1^7 T_2^8 - 5 c_{84} T_1^7 T_2^8 + 4 a_4 b_2 T_1^8 T_2^8 +$
$5 c_{16} T_1^8 T_2^8 + 6 c_{19} T_1^8 T_2^8 + 5 c_{31} T_1^8 T_2^8 + 6 c_{34} T_1^8 T_2^8 + 5 c_{46} T_1^8 T_2^8 + 6 c_{49} T_1^8 T_2^8 + 5 c_{61} T_1^8 T_2^8 +$
$6 c_{64} T_1^8 T_2^8 + 7 c_{81} T_1^8 T_2^8 + 6 c_{82} T_1^8 T_2^8 + 2 c_{84} T_1^8 T_2^8 - 2 c_{16} T_1^9 T_2^8 - 2 c_{19} T_1^9 T_2^8 - 2 c_{31} T_1^9 T_2^8 -$
$2 c_{34} T_1^9 T_2^8 - 2 c_{46} T_1^9 T_2^8 - 2 c_{49} T_1^9 T_2^8 - 2 c_{61} T_1^9 T_2^8 - 2 c_{64} T_1^9 T_2^8 - 2 c_{81} T_1^9 T_2^8 - 2 c_{82} T_1^9 T_2^8\big) \Big/$

$((-1 + T_1)\, T_1\, (1 - T_1 + T_1^2)^2\, T_2\, (-1 + T_1 T_2)\, (1 - T_2 + T_2^2)^2\, (1 - T_1 T_2 + T_1^2 T_2^2))\,]\,]\,)\,/$
$((1 - T_1 + T_1^2)\, (1 - T_2 + T_2^2)\, (1 - T_1 T_2 + T_1^2 T_2^2))))\}$

» $\{2.70313,$

$\text{Knot}[4, 1] \to (T_1^4 T_2^4\, \mathbb{E}[\in\text{Series}[0, -((2 c_{82} T_1 + 2 c_{85} T_1 - 8 c_{82} T_1^2 - 8 c_{85} T_1^2 + 8 c_{82} T_1^3 + 8 c_{85} T_1^3 - 2 c_{82} T_1^4 -$
$2 c_{85} T_1^4 + 2 c_{81} T_2 + 2 c_{84} T_2 - c_{16} T_1 T_2 - c_{19} T_1 T_2 - c_{31} T_1 T_2 - c_{34} T_1 T_2 - c_{46} T_1 T_2 - c_{49} T_1 T_2 -$
$c_{61} T_1 T_2 - c_{64} T_1 T_2 - 7 c_{81} T_1 T_2 - 5 c_{82} T_1 T_2 - 5 c_{84} T_1 T_2 - 3 c_{85} T_1 T_2 - 2 a_4 b_2 T_1^2 T_2 + c_{16} T_1^2 T_2 +$
$4 c_{19} T_1^2 T_2 + c_{31} T_1^2 T_2 + 4 c_{34} T_1^2 T_2 + c_{46} T_1^2 T_2 + 4 c_{49} T_1^2 T_2 + c_{61} T_1^2 T_2 + 4 c_{64} T_1^2 T_2 + 8 c_{81} T_1^2 T_2 +$
$15 c_{82} T_1^2 T_2 + 3 c_{84} T_1^2 T_2 + 7 c_{85} T_1^2 T_2 + 6 a_4 b_2 T_1^3 T_2 + c_{16} T_1^3 T_2 - 4 c_{19} T_1^3 T_2 + c_{31} T_1^3 T_2 - 4 c_{34} T_1^3 T_2 +$
$c_{46} T_1^3 T_2 - 4 c_{49} T_1^3 T_2 + c_{61} T_1^3 T_2 - 4 c_{64} T_1^3 T_2 - 3 c_{81} T_1^3 T_2 + 8 c_{85} T_1^3 T_2 - 2 a_4 b_2 T_1^4 T_2 - c_{16} T_1^4 T_2 +$
$c_{19} T_1^4 T_2 - c_{31} T_1^4 T_2 + c_{34} T_1^4 T_2 - c_{46} T_1^4 T_2 + c_{49} T_1^4 T_2 - c_{61} T_1^4 T_2 + c_{64} T_1^4 T_2 - 15 c_{82} T_1^4 T_2 -$
$17 c_{85} T_1^4 T_2 + 5 c_{82} T_1^5 T_2 + 5 c_{85} T_1^5 T_2 - 6 c_{81} T_2^2 - 6 c_{84} T_2^2 - 2 a_4 b_2 T_1 T_2^2 + 3 c_{16} T_1 T_2^2 + 3 c_{31} T_1 T_2^2 +$
$3 c_{46} T_1 T_2^2 + 3 c_{61} T_1 T_2^2 + 16 c_{81} T_1 T_2^2 + 3 c_{82} T_1 T_2^2 + 10 c_{84} T_1 T_2^2 + 10 a_4 b_2 T_1^2 T_2^2 + c_{16} T_1^2 T_2^2 +$
$4 c_{19} T_1^2 T_2^2 + c_{31} T_1^2 T_2^2 + 4 c_{34} T_1^2 T_2^2 + c_{46} T_1^2 T_2^2 + 4 c_{49} T_1^2 T_2^2 + c_{61} T_1^2 T_2^2 + 4 c_{64} T_1^2 T_2^2 - 11 c_{81} T_1^2 T_2^2 -$
$4 c_{82} T_1^2 T_2^2 - c_{84} T_1^2 T_2^2 + 3 c_{85} T_1^2 T_2^2 - 15 a_4 b_2 T_1^3 T_2^2 - 7 c_{16} T_1^3 T_2^2 - 16 c_{19} T_1^3 T_2^2 - 7 c_{31} T_1^3 T_2^2 -$
$16 c_{34} T_1^3 T_2^2 - 7 c_{46} T_1^3 T_2^2 - 16 c_{49} T_1^3 T_2^2 - 7 c_{61} T_1^3 T_2^2 - 16 c_{64} T_1^3 T_2^2 + c_{81} T_1^3 T_2^2 - 17 c_{82} T_1^3 T_2^2 -$
$9 c_{85} T_1^3 T_2^2 - 4 a_4 b_2 T_1^4 T_2^2 - c_{16} T_1^4 T_2^2 + 16 c_{19} T_1^4 T_2^2 - c_{31} T_1^4 T_2^2 + 16 c_{34} T_1^4 T_2^2 - c_{46} T_1^4 T_2^2 + 16 c_{49} T_1^4 T_2^2 -$
$c_{61} T_1^4 T_2^2 + 16 c_{64} T_1^4 T_2^2 - 3 c_{81} T_1^4 T_2^2 + 17 c_{82} T_1^4 T_2^2 - 3 c_{84} T_1^4 T_2^2 + 3 a_4 b_2 T_1^5 T_2^2 + 4 c_{16} T_1^5 T_2^2 -$
$4 c_{19} T_1^5 T_2^2 + 4 c_{31} T_1^5 T_2^2 - 4 c_{34} T_1^5 T_2^2 + 4 c_{46} T_1^5 T_2^2 - 4 c_{49} T_1^5 T_2^2 + 4 c_{61} T_1^5 T_2^2 - 4 c_{64} T_1^5 T_2^2 + 3 c_{81} T_1^5 T_2^2 +$
$4 c_{82} T_1^5 T_2^2 + 9 c_{85} T_1^5 T_2^2 - 3 c_{82} T_1^6 T_2^2 - 3 c_{85} T_1^6 T_2^2 + 2 c_{81} T_2^3 + 2 c_{84} T_2^3 + 6 a_4 b_2 T_1 T_2^3 - c_{16} T_1 T_2^3 +$
$c_{19} T_1 T_2^3 - c_{31} T_1 T_2^3 + c_{34} T_1 T_2^3 - c_{46} T_1 T_2^3 + c_{49} T_1 T_2^3 - c_{61} T_1 T_2^3 + c_{64} T_1 T_2^3 + 8 c_{81} T_1 T_2^3 + 10 c_{84} T_1 T_2^3 -$
$15 a_4 b_2 T_1^2 T_2^3 - 11 c_{16} T_1^2 T_2^3 - 4 c_{19} T_1^2 T_2^3 - 11 c_{31} T_1^2 T_2^3 - 4 c_{34} T_1^2 T_2^3 - 11 c_{46} T_1^2 T_2^3 - 4 c_{49} T_1^2 T_2^3 -$
$11 c_{61} T_1^2 T_2^3 - 4 c_{64} T_1^2 T_2^3 - 28 c_{81} T_1^2 T_2^3 - 18 c_{84} T_1^2 T_2^3 + 3 c_{85} T_1^2 T_2^3 + 12 a_4 b_2 T_1^3 T_2^3 + 9 c_{16} T_1^3 T_2^3 +$
$9 c_{31} T_1^3 T_2^3 + 9 c_{46} T_1^3 T_2^3 + 9 c_{61} T_1^3 T_2^3 + 18 c_{81} T_1^3 T_2^3 - 9 c_{85} T_1^3 T_2^3 + 3 a_4 b_2 T_1^4 T_2^3 + 15 c_{16} T_1^4 T_2^3 +$
$15 c_{19} T_1^4 T_2^3 + 15 c_{31} T_1^4 T_2^3 + 15 c_{34} T_1^4 T_2^3 + 15 c_{46} T_1^4 T_2^3 + 15 c_{49} T_1^4 T_2^3 + 15 c_{61} T_1^4 T_2^3 + 15 c_{64} T_1^4 T_2^3 +$
$c_{81} T_1^4 T_2^3 + c_{84} T_1^4 T_2^3 - 8 c_{16} T_1^5 T_2^3 - 16 c_{19} T_1^5 T_2^3 - 8 c_{31} T_1^5 T_2^3 - 16 c_{34} T_1^5 T_2^3 - 8 c_{46} T_1^5 T_2^3 - 16 c_{49} T_1^5 T_2^3 -$
$8 c_{61} T_1^5 T_2^3 - 16 c_{64} T_1^5 T_2^3 + 4 c_{81} T_1^5 T_2^3 + 5 c_{84} T_1^5 T_2^3 + 9 c_{85} T_1^5 T_2^3 - 4 c_{16} T_1^6 T_2^3 + 4 c_{19} T_1^6 T_2^3 - 4 c_{31} T_1^6 T_2^3 +$
$4 c_{34} T_1^6 T_2^3 - 4 c_{46} T_1^6 T_2^3 + 4 c_{49} T_1^6 T_2^3 - 4 c_{61} T_1^6 T_2^3 + 4 c_{64} T_1^6 T_2^3 - 5 c_{81} T_1^6 T_2^3 - 3 c_{85} T_1^6 T_2^3 - 2 a_4 b_2 T_1 T_2^4 -$
$5 c_{81} T_1 T_2^4 - 5 c_{84} T_1 T_2^4 - 4 a_4 b_2 T_1^2 T_2^4 + 4 c_{16} T_1^2 T_2^4 - 4 c_{19} T_1^2 T_2^4 + 4 c_{31} T_1^2 T_2^4 - 4 c_{34} T_1^2 T_2^4 + 4 c_{46} T_1^2 T_2^4 -$
$4 c_{49} T_1^2 T_2^4 + 4 c_{61} T_1^2 T_2^4 - 4 c_{64} T_1^2 T_2^4 + 4 c_{81} T_1^2 T_2^4 - 3 c_{82} T_1^2 T_2^4 - c_{84} T_1^2 T_2^4 + 3 a_4 b_2 T_1^3 T_2^4 + 8 c_{16} T_1^3 T_2^4 +$
$16 c_{19} T_1^3 T_2^4 + 8 c_{31} T_1^3 T_2^4 + 16 c_{34} T_1^3 T_2^4 + 8 c_{46} T_1^3 T_2^4 + 16 c_{49} T_1^3 T_2^4 + 8 c_{61} T_1^3 T_2^4 + 16 c_{64} T_1^3 T_2^4 +$
$c_{81} T_1^3 T_2^4 + 4 c_{82} T_1^3 T_2^4 - 5 c_{85} T_1^3 T_2^4 + 12 a_4 b_2 T_1^4 T_2^4 - 15 c_{16} T_1^4 T_2^4 - 15 c_{19} T_1^4 T_2^4 - 15 c_{31} T_1^4 T_2^4 -$
$15 c_{34} T_1^4 T_2^4 - 15 c_{46} T_1^4 T_2^4 - 15 c_{49} T_1^4 T_2^4 - 15 c_{61} T_1^4 T_2^4 - 15 c_{64} T_1^4 T_2^4 + 18 c_{81} T_1^4 T_2^4 + 17 c_{82} T_1^4 T_2^4 +$
$18 c_{84} T_1^4 T_2^4 + 17 c_{85} T_1^4 T_2^4 + 2 a_4 b_2 T_1^5 T_2^4 - 9 c_{16} T_1^5 T_2^4 - 9 c_{31} T_1^5 T_2^4 - 9 c_{46} T_1^5 T_2^4 - 9 c_{61} T_1^5 T_2^4 -$
$28 c_{81} T_1^5 T_2^4 - 17 c_{82} T_1^5 T_2^4 - 10 c_{84} T_1^5 T_2^4 - 8 c_{85} T_1^5 T_2^4 - 5 a_4 b_2 T_1^6 T_2^4 + 11 c_{16} T_1^6 T_2^4 + 4 c_{19} T_1^6 T_2^4 +$
$11 c_{31} T_1^6 T_2^4 + 4 c_{34} T_1^6 T_2^4 + 11 c_{46} T_1^6 T_2^4 + 4 c_{49} T_1^6 T_2^4 + 11 c_{61} T_1^6 T_2^4 + 4 c_{64} T_1^6 T_2^4 + 8 c_{81} T_1^6 T_2^4 -$
$4 c_{82} T_1^6 T_2^4 - 2 c_{84} T_1^6 T_2^4 - 7 c_{85} T_1^6 T_2^4 + c_{16} T_1^7 T_2^4 - c_{19} T_1^7 T_2^4 + c_{31} T_1^7 T_2^4 - c_{34} T_1^7 T_2^4 + c_{46} T_1^7 T_2^4 -$
$c_{49} T_1^7 T_2^4 + c_{61} T_1^7 T_2^4 - c_{64} T_1^7 T_2^4 + 2 c_{81} T_1^7 T_2^4 + 3 c_{82} T_1^7 T_2^4 + 3 c_{85} T_1^7 T_2^4 + 3 a_4 b_2 T_1^2 T_2^5 + 3 c_{81} T_1^2 T_2^5 +$
$3 c_{84} T_1^2 T_2^5 - 4 c_{16} T_1^3 T_2^5 + 4 c_{19} T_1^3 T_2^5 - 4 c_{31} T_1^3 T_2^5 + 4 c_{34} T_1^3 T_2^5 - 4 c_{46} T_1^3 T_2^5 + 4 c_{49} T_1^3 T_2^5 - 4 c_{61} T_1^3 T_2^5 +$
$4 c_{64} T_1^3 T_2^5 - 3 c_{81} T_1^3 T_2^5 + 5 c_{82} T_1^3 T_2^5 + 2 a_4 b_2 T_1^4 T_2^5 + c_{16} T_1^4 T_2^5 - 16 c_{19} T_1^4 T_2^5 + c_{31} T_1^4 T_2^5 - 16 c_{34} T_1^4 T_2^5 +$
$c_{46} T_1^4 T_2^5 - 16 c_{49} T_1^4 T_2^5 + c_{61} T_1^4 T_2^5 - 16 c_{64} T_1^4 T_2^5 + c_{81} T_1^4 T_2^5 - 15 c_{82} T_1^4 T_2^5 + c_{84} T_1^4 T_2^5 + 2 c_{85} T_1^4 T_2^5 -$
$24 a_4 b_2 T_1^5 T_2^5 + 7 c_{16} T_1^5 T_2^5 + 16 c_{19} T_1^5 T_2^5 + 7 c_{31} T_1^5 T_2^5 + 16 c_{34} T_1^5 T_2^5 + 7 c_{46} T_1^5 T_2^5 + 16 c_{49} T_1^5 T_2^5 +$
$7 c_{61} T_1^5 T_2^5 + 16 c_{64} T_1^5 T_2^5 - 11 c_{81} T_1^5 T_2^5 - 10 c_{84} T_1^5 T_2^5 - 8 c_{85} T_1^5 T_2^5 + 11 a_4 b_2 T_1^6 T_2^5 - c_{16} T_1^6 T_2^5 -$
$4 c_{19} T_1^6 T_2^5 - c_{31} T_1^6 T_2^5 - 4 c_{34} T_1^6 T_2^5 - c_{46} T_1^6 T_2^5 - 4 c_{49} T_1^6 T_2^5 - c_{61} T_1^6 T_2^5 - 4 c_{64} T_1^6 T_2^5 + 16 c_{81} T_1^6 T_2^5 +$
$15 c_{82} T_1^6 T_2^5 + 6 c_{84} T_1^6 T_2^5 + 8 c_{85} T_1^6 T_2^5 - 3 c_{16} T_1^7 T_2^5 - 3 c_{31} T_1^7 T_2^5 - 3 c_{46} T_1^7 T_2^5 - 3 c_{61} T_1^7 T_2^5 - 6 c_{81} T_1^7 T_2^5 -$
$5 c_{82} T_1^7 T_2^5 - 2 c_{85} T_1^7 T_2^5 - 5 a_4 b_2 T_1^4 T_2^6 + c_{16} T_1^4 T_2^6 - c_{19} T_1^4 T_2^6 + c_{31} T_1^4 T_2^6 - c_{34} T_1^4 T_2^6 + c_{46} T_1^4 T_2^6 -$
$c_{49} T_1^4 T_2^6 + c_{61} T_1^4 T_2^6 - c_{64} T_1^4 T_2^6 - 3 c_{81} T_1^4 T_2^6 - 2 c_{82} T_1^4 T_2^6 - 3 c_{84} T_1^4 T_2^6 + 11 a_4 b_2 T_1^5 T_2^6 - c_{16} T_1^5 T_2^6 +$
$4 c_{19} T_1^5 T_2^6 - c_{31} T_1^5 T_2^6 + 4 c_{34} T_1^5 T_2^6 - c_{46} T_1^5 T_2^6 + 4 c_{49} T_1^5 T_2^6 - c_{61} T_1^5 T_2^6 + 4 c_{64} T_1^5 T_2^6 + 8 c_{81} T_1^5 T_2^6 +$
$8 c_{82} T_1^5 T_2^6 + 5 c_{84} T_1^5 T_2^6 - 4 a_4 b_2 T_1^6 T_2^6 - c_{16} T_1^6 T_2^6 - 4 c_{19} T_1^6 T_2^6 - c_{31} T_1^6 T_2^6 - 4 c_{34} T_1^6 T_2^6 - c_{46} T_1^6 T_2^6 -$
$4 c_{49} T_1^6 T_2^6 - c_{61} T_1^6 T_2^6 - 4 c_{64} T_1^6 T_2^6 - 7 c_{81} T_1^6 T_2^6 - 8 c_{82} T_1^6 T_2^6 - 2 c_{84} T_1^6 T_2^6 + c_{16} T_1^7 T_2^6 + c_{19} T_1^7 T_2^6 +$

$c_{31} T_1^7 T_2^6 + c_{34} T_1^7 T_2^6 + c_{46} T_1^7 T_2^6 + c_{49} T_1^7 T_2^6 + c_{61} T_1^7 T_2^6 + c_{64} T_1^7 T_2^6 + 2 c_{81} T_1^7 T_2^6 + 2 c_{82} T_1^7 T_2^6) /$
$((-1 + T_1) T_1 (1 - 3 T_1 + T_1^2) T_2 (-1 + T_1 T_2) (1 - 3 T_2 + T_2^2) (1 - 3 T_1 T_2 + T_1^2 T_2^2)))]]) /$
» $((1 - 3 T_1 + T_1^2) (1 - 3 T_2 + T_2^2) (1 - 3 T_1 T_2 + T_1^2 T_2^2))\}$

(Alt) Out[ ]=

$Aborted

In[ ]:= **tab1**[[1]]

(Alt) In[ ]:=

```
K = Knot["K11n34"]; Conway = ∫ℒ[K] ⅆ vs[K]
```

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

(Alt) Out[ ]=

$$-i \, \mathbb{E}\Big[\epsilon\text{Series}\Big[0, \frac{1}{T_1^3 T_2^3} 2 (c_{19} T_1^3 + c_{34} T_1^3 + c_{49} T_1^3 + c_{64} T_1^3 - 2 c_{19} T_1^3 T_2 - 2 c_{34} T_1^3 T_2 - 2 c_{49} T_1^3 T_2 - 2 c_{64} T_1^3 T_2 + c_{19} T_1^3 T_2^2 + c_{34} T_1^3 T_2^2 + c_{49} T_1^3 T_2^2 + c_{64} T_1^3 T_2^2 + c_{16} T_2^3 + c_{31} T_2^3 + c_{46} T_2^3 + c_{61} T_2^3 - 2 c_{16} T_1 T_2^3 - 2 c_{31} T_1 T_2^3 - 2 c_{46} T_1 T_2^3 - 2 c_{61} T_1 T_2^3 + c_{16} T_1^2 T_2^3 + c_{31} T_1^2 T_2^3 + c_{46} T_1^2 T_2^3 + c_{61} T_1^2 T_2^3 + c_{16} T_1^4 T_2^3 + c_{31} T_1^4 T_2^3 + c_{46} T_1^4 T_2^3 + c_{61} T_1^4 T_2^3 - 2 c_{16} T_1^5 T_2^3 - 2 c_{31} T_1^5 T_2^3 - 2 c_{46} T_1^5 T_2^3 - 2 c_{61} T_1^5 T_2^3 + c_{16} T_1^6 T_2^3 + c_{31} T_1^6 T_2^3 + c_{46} T_1^6 T_2^3 + c_{61} T_1^6 T_2^3 + c_{19} T_1^3 T_2^4 + c_{34} T_1^3 T_2^4 + c_{49} T_1^3 T_2^4 + c_{64} T_1^3 T_2^4 - 2 c_{19} T_1^3 T_2^5 - 2 c_{34} T_1^3 T_2^5 - 2 c_{49} T_1^3 T_2^5 - 2 c_{64} T_1^3 T_2^5 + c_{19} T_1^3 T_2^6 + c_{34} T_1^3 T_2^6 + c_{49} T_1^3 T_2^6 + c_{64} T_1^3 T_2^6)\Big]\Big]$$

(Alt) In[ ]:=

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K = Knot["K11n42"]; KT = ∫ℒ[K] ⅆ vs[K]
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(Alt) Out[ ]=

$$-i \, T_1^2 T_2^2 \, \mathbb{E}\Big[\epsilon\text{Series}\Big[0, \frac{1}{T_1^3 T_2^3} 2 (c_{19} T_1^3 + c_{34} T_1^3 + c_{49} T_1^3 + c_{64} T_1^3 - 2 c_{19} T_1^3 T_2 - 2 c_{34} T_1^3 T_2 - 2 c_{49} T_1^3 T_2 - 2 c_{64} T_1^3 T_2 + c_{19} T_1^3 T_2^2 + c_{34} T_1^3 T_2^2 + c_{49} T_1^3 T_2^2 + c_{64} T_1^3 T_2^2 + c_{16} T_2^3 + c_{31} T_2^3 + c_{46} T_2^3 + c_{61} T_2^3 - 2 c_{16} T_1 T_2^3 - 2 c_{31} T_1 T_2^3 - 2 c_{46} T_1 T_2^3 - 2 c_{61} T_1 T_2^3 + c_{16} T_1^2 T_2^3 + c_{31} T_1^2 T_2^3 + c_{46} T_1^2 T_2^3 + c_{61} T_1^2 T_2^3 + c_{16} T_1^4 T_2^3 + c_{31} T_1^4 T_2^3 + c_{46} T_1^4 T_2^3 + c_{61} T_1^4 T_2^3 - 2 c_{16} T_1^5 T_2^3 - 2 c_{31} T_1^5 T_2^3 - 2 c_{46} T_1^5 T_2^3 - 2 c_{61} T_1^5 T_2^3 + c_{16} T_1^6 T_2^3 + c_{31} T_1^6 T_2^3 + c_{46} T_1^6 T_2^3 + c_{61} T_1^6 T_2^3 + c_{19} T_1^3 T_2^4 + c_{34} T_1^3 T_2^4 + c_{49} T_1^3 T_2^4 + c_{64} T_1^3 T_2^4 - 2 c_{19} T_1^3 T_2^5 - 2 c_{34} T_1^3 T_2^5 - 2 c_{49} T_1^3 T_2^5 - 2 c_{64} T_1^3 T_2^5 + c_{19} T_1^3 T_2^6 + c_{34} T_1^3 T_2^6 + c_{49} T_1^3 T_2^6 + c_{64} T_1^3 T_2^6)\Big]\Big]$$

(Alt) In[ ]:=

i **Conway** == $i \, T_1^{-2} T_2^{-2}$ **KT**

(Alt) Out[ ]=

True