

Pensieve header: Finding the A2 $d=1$ invariant using undetermined coefficients.

Searching for $Q + p_{xx} + \epsilon(p_{px} + 1 + px + pp_{xx})$ solutions.

Initialization

```
In[1]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank"];
Once[<< KnotTheory` ; << Rot.m];
<< FormalGaussianIntegration.m;
i_+ := i + 1;
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

Loading Rot.m from <http://drorbn.net/AP/Projects/HigherRank> to compute rotation numbers.

```
In[2]:= Features[Knot[8, 17]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[2]=

```
Features[18,
C6[-1] C14[-1] X1,7[1] X3,9[-1] X5,13[-1] X8,16[1] X10,4[-1] X12,18[1] X15,2[-1] X17,11[1]]
```

```
In[3]:= T3 = T1 T2;
S = {x_, p__};
q[s_, i_, j_] := Sum[
  x_{v,i} (p_{v,i^*} - p_{v,i}) + x_{v,j} (p_{v,j^*} - p_{v,j}) + (T_v^s - 1) x_{v,i} (p_{v,i^*} - p_{v,j^*}),
  {v, 3}];
L[X_{i_,j_}[s_]] :=
  T3^s E[q[s, i, j] + B^-1 r0[s, i, j] + \epsilon B r1[s, i, j] + \epsilon r42[s, i, j] + O[\epsilon]^2];
(*\gamma1[\varphi_, k_] := \varphi (3/2 - x_{1,k} p_{1,k} - x_{2,k} p_{2,k} - x_{3,k} p_{3,k}); *)
L[C_k_[0]] := E[Sum[x_{v,k} (p_{v,k^*} - p_{v,k}), {v, 3}] + O[\epsilon]^2];
L[C_k_[\varphi_]] :=
  T3^\varphi E[Sum[x_{v,k} (p_{v,k^*} - p_{v,k}), {v, 3}] + B^-1 \gamma0[\varphi, k] + \epsilon B \gamma1[\varphi, k] + \epsilon \gamma42[\varphi, k] + O[\epsilon]^2];
ps_i_ := Sequence[p1,i, p2,i, p3,i];
xs_i_ := Sequence[x1,i, x2,i, x3,i];
vs_i_ := Sequence[ps_i, xs_i];
F[is___] := E[Sum[\pi_{v,i} p_{v,i}, {i, {is}}], {v, 3}]];
L[K_] := CF[L /@ Features[K][2]];
vs[K_] := Union @@ Table[{vs_i}, {i, Features[K][1]}]
```

```
In[4]:= vs
```

Out[4]=

```
Sequence[p1,i, p2,i, p3,i, x1,i, x2,i, x3,i]
```

The Various Terms (r_0)

The pxx Terms (r_0)

```
In[=]:= x = 0;
r0[1, i_, j_] := Evaluate[Sum[
  a_{++x} p3,k3 x1,k1 x2,k2,
  {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}}

$$\text{ ]];}$$

r0[1, i, j]

Out[=]=

$$\begin{aligned} & a_1 p_{3,i} x_{1,i} x_{2,i} + a_2 p_{3,j} x_{1,i} x_{2,i} + a_5 p_{3,i} x_{1,j} x_{2,i} + a_6 p_{3,j} x_{1,j} x_{2,i} + \\ & a_3 p_{3,i} x_{1,i} x_{2,j} + a_4 p_{3,j} x_{1,i} x_{2,j} + a_7 p_{3,i} x_{1,j} x_{2,j} + a_8 p_{3,j} x_{1,j} x_{2,j} \end{aligned}$$

```



```
In[=]:= x = 0;
r0[-1, i_, j_] := Evaluate[Sum[
  d_{++x} p3,k3 x1,k1 x2,k2,
  {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}}

$$\text{ ]];}$$

r0[-1, i, j]

Out[=]=

$$\begin{aligned} & d_1 p_{3,i} x_{1,i} x_{2,i} + d_2 p_{3,j} x_{1,i} x_{2,i} + d_5 p_{3,i} x_{1,j} x_{2,i} + d_6 p_{3,j} x_{1,j} x_{2,i} + \\ & d_3 p_{3,i} x_{1,i} x_{2,j} + d_4 p_{3,j} x_{1,i} x_{2,j} + d_7 p_{3,i} x_{1,j} x_{2,j} + d_8 p_{3,j} x_{1,j} x_{2,j} \end{aligned}$$

```

The ppx Terms (r_1)

```
In[=]:= x = 0;
r1[1, i_, j_] := Evaluate[Sum[
  b_{++x} x3,k3 p1,k1 p2,k2,
  {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}}

$$\text{ ]];}$$

r1[1, i, j]

Out[=]=

$$\begin{aligned} & b_1 p_{1,i} p_{2,i} x_{3,i} + b_5 p_{1,j} p_{2,i} x_{3,i} + b_3 p_{1,i} p_{2,j} x_{3,i} + b_7 p_{1,j} p_{2,j} x_{3,i} + \\ & b_2 p_{1,i} p_{2,i} x_{3,j} + b_6 p_{1,j} p_{2,i} x_{3,j} + b_4 p_{1,i} p_{2,j} x_{3,j} + b_8 p_{1,j} p_{2,j} x_{3,j} \end{aligned}$$

```



```
In[=]:= x = 0;
r1[-1, i_, j_] := Evaluate[Sum[
  e_{++x} x3,k3 p1,k1 p2,k2,
  {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}}

$$\text{ ]];}$$

r1[-1, i, j]

Out[=]=

$$\begin{aligned} & e_1 p_{1,i} p_{2,i} x_{3,i} + e_5 p_{1,j} p_{2,i} x_{3,i} + e_3 p_{1,i} p_{2,j} x_{3,i} + e_7 p_{1,j} p_{2,j} x_{3,i} + \\ & e_2 p_{1,i} p_{2,i} x_{3,j} + e_6 p_{1,j} p_{2,i} x_{3,j} + e_4 p_{1,i} p_{2,j} x_{3,j} + e_8 p_{1,j} p_{2,j} x_{3,j} \end{aligned}$$

```

The ppxx Terms (r_{42})

```
In[=]:= x = 0;
Short[r42[1, i_, j_]] = Evaluate[Plus[
  Sum[
    C++x Xv1,k1 pv1,k2 Xv2,k3 pv2,k4,
    {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}, {k4, {i, j}}, {v1, 2}, {v2, v1, 3}
  ],
  Sum[
    C++x Xv,k1 pv,k2,
    {k1, {i, j}}, {k2, {i, j}}, {v, 3}
  ],
  C++x
]]]

Out[=]//Short=
C93 + C81 p1,i x1,i + <<90>> + C80 p2,j p3,j x2,j x3,j

In[=]:= x = 0;
Short[r42[-1, i_, j_]] = Evaluate[Plus[
  Sum[
    f++x Xv1,k1 pv1,k2 Xv2,k3 pv2,k4,
    {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}, {k4, {i, j}}, {v1, 2}, {v2, v1, 3}
  ],
  Sum[
    f++x Xv,k1 pv,k2,
    {k1, {i, j}}, {k2, {i, j}}, {v, 3}
  ],
  f++x
]]]

Out[=]//Short=
f93 + f81 p1,i x1,i + <<90>> + f80 p2,j p3,j x2,j x3,j
```

The γ Terms ($\gamma_0, \gamma_1, \gamma_{42}$)

```
In[=]:= x = 0;
γ0[1, k_] := Evaluate[g++x p3,k X1,k X2,k];
γ1[1, k_] := Evaluate[g++x X3,k p1,k p2,k];
γ42[1, k_] := Evaluate[Plus[
  Sum[g++x Xv,k pv,k, {v, 3}],
  Sum[g++x Xv1,k pv1,k Xv2,k pv2,k, {v1, 2}, {v2, v1, 3}]
  ]];
{γ0[1, k], γ0[1, k], γ42[1, k]}

Out[=]=
{g1 p3,k X1,k X2,k, g1 p3,k X1,k X2,k, g3 p1,k X1,k + g6 p1,k2 X1,k2 + g4 p2,k X2,k +
g7 p1,k p2,k X1,k X2,k + g9 p2,k2 X2,k2 + g5 p3,k X3,k + g8 p1,k p3,k X1,k X3,k + g10 p2,k p3,k X2,k X3,k}
```

```
In[=]:= x = 0;
y0[-1, k_] := Evaluate[h++k p3,k x1,k x2,k];
y1[-1, k_] := Evaluate[h++k x3,k p1,k p2,k];
y42[-1, k_] := Evaluate[Plus[
    Sum[h++k xv,k pv,k, {v, 3}],
    Sum[h++k xv1,k pv1,k xv2,k pv2,k, {v1, 2}, {v2, v1, 3}]
  ]];
{y0[-1, k], y0[-1, k], y42[-1, k]}

Out[=]= {h1 p3,k x1,k x2,k, h1 p3,k x1,k x2,k, h3 p1,k x1,k + h6 p1,k2 x1,k2 + h4 p2,k x2,k +
h7 p1,k p2,k x1,k x2,k + h9 p2,k2 x2,k2 + h5 p3,k x3,k + h8 p1,k p3,k x1,k x3,k + h10 p2,k p3,k x2,k x3,k}
```

Reidemeister 3b

```
In[=]:= Timing[LeftR3b] =
Cases[  $\int \mathcal{F}[i, j, k] \times \mathcal{L} /@ (\mathbf{X}_{i,j}[1] \mathbf{X}_{i^+,k}[1] \mathbf{X}_{j^+,k^+}[1]) \text{d}\{\mathbf{vs}_i, \mathbf{vs}_j, \mathbf{vs}_k, \mathbf{vs}_{i^+}, \mathbf{vs}_{j^+}, \mathbf{vs}_{k^+}\},$ 
IE[\mathcal{E}_] \Rightarrow \mathcal{E}, \infty] ]
```

Out[=]=

{19.5156, { ∞ Series[
T₁² p_{1,2+i} π_{1,i} - (-1 + T₁) T₁ p_{1,2+j} π_{1,j} + (1 - T₁) p_{1,2+k} π_{1,i} + T₁ p_{1,2+j} π_{1,j} + (1 - T₁) p_{1,2+k} π_{1,j} + p_{1,2+k} π_{1,k} + ... 44 ... +
T₁² T₂² p_{3,2+i} π_{3,i} - T₁ T₂ (-1 + T₁ T₂) p_{3,2+j} π_{3,i} + (1 - T₁ T₂) p_{3,2+k} π_{3,i} + T₁ T₂ p_{3,2+j} π_{3,j} + (1 - T₁ T₂) p_{3,2+k} π_{3,j} + p_{3,2+k} π_{3,k},
3 (a₁ b₁ + a₂ b₂ + a₃ b₃ + a₄ b₄ + a₅ b₅ + a₆ b₆ + a₇ b₇ + a₈ b₈ + ... 23 ... + C₈₀ + C₈₁ + C₈₂ + C₈₃ + C₉₀ + C₉₁ + C₉₂ + C₉₃) +
... 498 ... + ... 1 ...]]}

Full expression not available (original memory size: 3.8 MB)



```
In[=]:= Timing[RightR3b] =
Cases[  $\int \mathcal{F}[i, j, k] \times \mathcal{L} /@ (\mathbf{X}_{j,k}[1] \mathbf{X}_{i,k^+}[1] \mathbf{X}_{i^+,j^+}[1]) \text{d}\{\mathbf{vs}_i, \mathbf{vs}_j, \mathbf{vs}_k, \mathbf{vs}_{i^+}, \mathbf{vs}_{j^+}, \mathbf{vs}_{k^+}\},$ 
IE[\mathcal{E}_] \Rightarrow \mathcal{E}, \infty] ]
```

Out[=]=

{10.7344, Null}

```
In[=]:= Short[eqn = CF[LeftR3b[[1]] - RightR3b[[1]]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r0[1, i, j], a_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0), 3]
{sol} = Solve[eqns, vars]
```

$$\frac{T_1 T_2 (\pi_{1,i} \pi_{2,i})}{B} - \frac{\pi_{1,i}}{B} + \frac{\pi_{2,i}}{B} + \frac{a_7 \pi_{1,i}}{B}$$

```
Out[=]= {p3,2+i, p3,2+j, p3,2+k, π1,i, π1,j, π1,k, π2,i, π2,j, π2,k}
```

```
Out[=]= {a1, a2, a3, a4, a5, a6, a7, a8}
```

$$\left\{ -\frac{a_3 T_1^2 T_2^2}{B} + \frac{a_3 T_1^2 T_2^3}{B} = 0, \frac{a_3 T_1^2 T_2}{B} - \frac{a_3 T_1^2 T_2^2}{B} = 0, \right. \\ \left. \ll22\gg, -\frac{a_7}{B} - \frac{a_8}{B} + \frac{a_7 T_2}{B} + \frac{a_8 T_2}{B} + \frac{a_7 T_1 T_2}{B} - \frac{a_7 T_1 T_2^2}{B} = 0 \right\}$$

Solve: Equations may not give solutions for all "solve" variables. [i](#)

```
Out[=]= {{a1 → 0, a3 → 0, a5 → 0, a6 → -a2/T1 - a4 T2/T1, a7 → 0, a8 → 0}}
```

```
In[=]:= sol /. (v_ → val_) :> (v = CF[val]);
r0[1, i, j]
```

$$a_2 p_{3,j} x_{1,i} x_{2,i} - \frac{(a_2 + a_4 T_2) p_{3,j} x_{1,j} x_{2,i}}{T_1} + a_4 p_{3,j} x_{1,i} x_{2,j}$$

```
In[=]:= Short[eqn = CF[Coefficient[
  LeftR3b[2] - RightR3b[2] /. v : (\pi | p) \[Rule] \[Mu] v,
  \[Mu]^3
], 5]
cvs = Union@Cases[eqn, p \[Rule] \[Pi] \[Or] \[Infty]]
vars = Union@Cases[r1[1, i, j], b \[Or] \[Infty]]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ \[Rule] c) \[Rule] (c == 0), 3]
{sol} = Solve[eqns, vars]

Out[=]//Short=
B b1 (-1 + T1) T1 T2^2 p1,2+j p2,2+i \[Pi]3,i - B b1 (-1 + T1) T1 T2^2 p1,2+k p2,2+i \[Pi]3,i + <<31>> +
B (-1 + T1) T1 (-b2 - b4 + b2 T2) p1,2+i p2,2+k \[Pi]3,k - B (-1 + T1) T1 (-b2 - b4 + b2 T2) p1,2+j p2,2+k \[Pi]3,k

Out[=]=
{p1,2+i, p1,2+j, p1,2+k, p2,2+i, p2,2+j, p2,2+k, \[Pi]3,i, \[Pi]3,j, \[Pi]3,k}

Out[=]=
{b1, b2, b3, b4, b5, b6, b7, b8}

Out[=]//Short=
{-B b2 T1^2 T2^2 + B b2 T1^3 T2^3 == 0, B b2 T1 T2 - B b2 T1^2 T2^2 == 0, <<1>> == 0, <<19>>, <<1>> == 0,
 B b1 + <<47>> == 0, -B b6 T1 - B b8 T1 - B b4 T2 - B b8 T2 + B b2 T1 T2 + <<11>> + B b2 T1^2 T2^2 == 0}

[[ Solve: Equations may not give solutions for all "solve" variables. ]]

Out[=]=
{{b1 \[Rule] 0, b2 \[Rule] 0, b4 \[Rule] 0, b6 \[Rule] 0, b7 \[Rule] -b3 - b5, b8 \[Rule] 0} }

In[=]:= sol /. (v_ \[Rule] val_) \[Rule] (v = CF[val]);
r1[1, i, j]

Out[=]=
b5 p1,j p2,i x3,i + b3 p1,i p2,j x3,i + (-b3 - b5) p1,j p2,j x3,i
```

```
In[=]:= Short[eqn = CF[LeftR3b[[2]] - RightR3b[[2]], 5]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r42[1, i, j], c_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0), 3]
Short[{{sol} = Solve[eqns, vars]]]

Out[=]//Short=
- ((2 c11 + 2 c41 + c42 + c43 + 2 c56 + c57 + c58 + 2 c71 + c87) (-1 + T1) T1^2 p1,2+j π1,i) -
(-1 + T1) (4 c1 + c2 + c3 + 2 c6 + c16 + c17 + c18 + <<49>>) p1,2+k π1,i + <<1>> +
<<473>> + (-1 + T2) T2 (-c55 - c60 + c55 T1 T2) p2 <<1>> 2 <<1>> <<1>> p3,2+k π2,k π3,k -
(-1 + T2) T2 (-c55 - c60 + c55 T1 T2) p2,2+j p3,2+k π2,k π3,k

Out[=]=
{p1,2+i, p1,2+j, p1,2+k, p2,2+i, p2,2+j, p2,2+k, p3,2+i,
p3,2+j, p3,2+k, π1,i, π1,j, π1,k, π2,i, π2,j, π2,k, π3,i, π3,j, π3,k}

Out[=]=
{c1, c2, c3, c4, c5, c6, c7, c8, c9, c10, c11, c12, c13, c14, c15, c16, c17, c18, c19, c20,
c21, c22, c23, c24, c25, c26, c27, c28, c29, c30, c31, c32, c33, c34, c35, c36, c37, c38, c39,
c40, c41, c42, c43, c44, c45, c46, c47, c48, c49, c50, c51, c52, c53, c54, c55, c56, c57,
c58, c59, c60, c61, c62, c63, c64, c65, c66, c67, c68, c69, c70, c71, c72, c73, c74, c75,
c76, c77, c78, c79, c80, c81, c82, c83, c84, c85, c86, c87, c88, c89, c90, c91, c92, c93}

Out[=]//Short=
{-c11 T1^4 - c41 T1^4 + c11 T1^5 + c41 T1^5 == 0, <<1>> == 0,
<<315>>, c13 T1 T2 + c15 T1 T2 + c73 T1 T2 + c75 T1 T2 + c89 T1 T2 -
<<1>> - c15 T1^2 T2^2 - c73 T1^2 T2^2 - c75 T1^2 T2^2 - c89 T1^2 T2^2 == 0}

*** Solve: Equations may not give solutions for all "solve" variables. ⓘ

Out[=]//Short=
{{c1 → 0, <<60>>, c92 → -c83/T1 T2 - c<<2>>/(<<1>> <<1>> <<1>>) - <<8>> + <<1>>/T1^2 <<1>> (-1 + <<1>>)}}

In[=]:= sol /. (v_ → val_) :> (v = CF[val]);
```

In[$\#$]:= **Short[CF[r42[1, i, j]], 20]**

Out[$\#$]//Short=

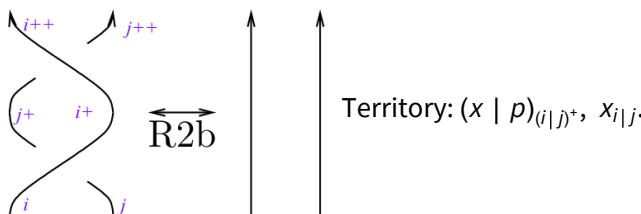
$$\begin{aligned}
 & c_{93} + c_{81} p_{1,i} x_{1,i} + c_{84} p_{1,j} x_{1,i} + (c_6 + c_{21}) p_{1,i} p_{1,j} x_{1,i}^2 + \\
 & \frac{1}{2} (-1 + T_1) (2 c_6 + 2 c_{21} + c_{16} T_1 + c_{31} T_1 + c_{46} T_1 + c_{61} T_1) p_{1,j}^2 x_{1,i}^2 - \\
 & \frac{(c_{81} + c_{84}) p_{1,j} x_{1,j}}{T_1} + (c_{16} + c_{31} + c_{46} + c_{61}) p_{1,i} p_{1,j} x_{1,i} x_{1,j} + \\
 & \frac{1}{2} (-2 c_6 - c_{16} - 2 c_{21} - c_{31} - c_{46} - c_{61} - c_{16} T_1 - c_{31} T_1 - c_{46} T_1 - c_{61} T_1) p_{1,j}^2 x_{1,i} x_{1,j} + \text{<<33>>} + \\
 & \frac{1}{(-1 + T_1) (-1 + T_2)} \left(-a_4 b_5 - c_{25} + c_{25} T_1 + a_4 b_3 T_2 + a_4 b_5 T_2 - a_4 b_3 T_1 T_2 + c_{25} T_1 T_2 - c_{25} T_1^2 T_2 \right) \\
 & p_{2,j} p_{3,j} x_{2,j} x_{3,i} - \frac{1}{T_1^2 T_2 (-1 + T_1 T_2)} \\
 & (-a_2 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 - c_{83} T_1 - c_{86} T_1 - a_4 b_5 T_2 - a_4 b_3 T_1 T_2 + a_4 b_5 T_1 T_2 + c_{83} T_1^2 T_2 + c_{86} T_1^2 T_2) \\
 & p_{3,j} x_{3,j} - \frac{(a_2 b_5 - a_4 b_5 - c_8 + c_{23} + c_8 T_1 + a_4 b_5 T_2 - c_{23} T_1 T_2) p_{1,i} p_{3,j} x_{1,i} x_{3,j}}{(-1 + T_1) (-1 + T_1 T_2)} - \\
 & ((-a_2 b_3 + c_8 + a_2 b_3 T_1 - a_4 b_3 T_1 - c_8 T_1 - a_4 b_3 T_2 - c_8 T_2 + 2 a_4 b_3 T_1 T_2 + c_8 T_1 T_2) p_{1,j} p_{3,j} x_{1,i} x_{3,j}) / \\
 & ((-1 + T_2) (-1 + T_1 T_2)) - \\
 & ((a_2 b_3 - c_{10} T_1 + c_{25} T_1 + a_4 b_3 T_2 - a_4 b_3 T_1 T_2 + c_{10} T_1 T_2 - c_{25} T_1^2 T_2) p_{2,i} p_{3,j} x_{2,i} x_{3,j}) / \\
 & (T_1 (-1 + T_2) (-1 + T_1 T_2)) - \\
 & ((c_{10} T_1 - c_{10} T_1^2 + a_2 b_5 T_2 - a_2 b_5 T_1 T_2 + a_4 b_5 T_1 T_2 - c_{10} T_1 T_2 + c_{10} T_1^2 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2) \\
 & p_{2,j} p_{3,j} x_{2,i} x_{3,j}) / ((-1 + T_1) T_1 (-1 + T_1 T_2))
 \end{aligned}$$

In[$\#$]:= **CF[LeftR3b - RightR3b]**

Out[$\#$]=

\in Series[0, 0]

Reidemeister 2b



In[$\#$]:= **Timing[Short[LeftR2b = $\left(\int \mathcal{F}[i, j] \times \mathcal{L} /@ (X_{i,j}[1] X_{i^+, j^+}[-1]) \text{d}\{\text{vs}_i, \text{vs}_j, \text{vs}_{i^+}, \text{vs}_{j^+}\}\right) [1]\right]$]]**

Out[$\#$]=

{0.671875, \in Series[p_{1,2+i} π_{1,i} + p_{1,2+j} π_{1,j} + <<12>> + p_{3,2+j} π_{3,j}, <<1>>]}

In[$\#$]:= **RightR2b = \in Series[p_{1,2+i} π_{1,i} + p_{1,2+j} π_{1,j} + p_{2,2+i} π_{2,i} + p_{2,2+j} π_{2,j} + p_{3,2+i} π_{3,i} + p_{3,2+j} π_{3,j}, 0]**

Out[$\#$]=

\in Series[p_{1,2+i} π_{1,i} + p_{1,2+j} π_{1,j} + p_{2,2+i} π_{2,i} + p_{2,2+j} π_{2,j} + p_{3,2+i} π_{3,i} + p_{3,2+j} π_{3,j}, 0]

```
In[=]:= Short[eqn = CF[LeftR2b[[1]] - RightR2b[[1]]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r0[-1, i, j], d_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0), 3]
{sol} = Solve[eqns, vars]

Out[=]/.Short=

$$\frac{(d_7 + \dots + d_7 T_1 T_2) \pi_{\dots} + \dots + \frac{\pi_{\dots}}{B \dots T_2}}{B T_1 T_2}$$


Out[=]= {p3,2+i, p3,2+j, π1,i, π1,j, π2,i, π2,j}

Out[=]= {d1, d2, d3, d4, d5, d6, d7, d8}

Out[=]/.Short=

$$\left\{ \frac{d_1}{B} - \frac{d_3}{B} - \frac{d_5}{B} + \frac{d_7}{B} + \frac{d_5}{B T_1} - \frac{d_7}{B T_1} + \frac{d_3}{B T_2} - \frac{d_7}{B T_2} + \frac{d_7}{B T_1 T_2} = 0, \right. \\ \left. \frac{d_3}{B T_2} - \frac{d_7}{B T_2} + \frac{d_7}{B T_1 T_2} = 0, \dots, \frac{d_7}{B} + \frac{d_8}{B} - \frac{d_7}{B T_1 T_2} = 0 \right\}$$


Out[=]= {d1 → 0, d2 → - $\frac{a_2 - a_4 T_1 + a_4 T_2}{T_1^2 T_2}$ , d3 → 0, d4 → - $\frac{a_4}{T_1}$ , d5 → 0, d6 → - $\frac{-a_2 - a_4 T_2}{T_1 T_2}$ , d7 → 0, d8 → 0}

In[=]:= sol /. (v_ → val_) :> (v = CF[val]);
r0[-1, i, j]

Out[=]= 
$$\frac{(-a_2 + a_4 T_1 - a_4 T_2) p_{3,j} x_{1,i} x_{2,i}}{T_1^2 T_2} + \frac{(a_2 + a_4 T_2) p_{3,j} x_{1,j} x_{2,i}}{T_1 T_2} - \frac{a_4 p_{3,j} x_{1,i} x_{2,j}}{T_1}$$

```

```
In[=]:= Short[eqn = CF[LeftR2b[[2]] - RightR2b[[2]]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r1[-1, i, j] + r42[-1, i, j], e_ | f_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0), 3]
Short[{{sol} = Solve[eqns, vars]]

Out[=]/Short=
<<1>> + <<1>> + <<1>>
<<107>> + ----- + -----
<<1>> T1 <<1>>

Out[=]=
{p1,2+i, p1,2+j, p2,2+i, p2,2+j, p3,2+i, p3,2+j, π1,i, π1,j, π2,i, π2,j, π3,i, π3,j}

Out[=]=
{e1, e2, e3, e4, e5, e6, e7, e8, f1, f2, f3, f4, f5, f6, f7, f8, f9, f10, f11, f12, f13, f14, f15, f16,
f17, f18, f19, f20, f21, f22, f23, f24, f25, f26, f27, f28, f29, f30, f31, f32, f33, f34, f35, f36,
f37, f38, f39, f40, f41, f42, f43, f44, f45, f46, f47, f48, f49, f50, f51, f52, f53, f54, f55,
f56, f57, f58, f59, f60, f61, f62, f63, f64, f65, f66, f67, f68, f69, f70, f71, f72, f73, f74,
f75, f76, f77, f78, f79, f80, f81, f82, f83, f84, f85, f86, f87, f88, f89, f90, f91, f92, f93}

Out[=]/Short=
{f1 - f11 - f41 + f51 + f51/T1^2 + f11/T1 + f41/T1 - 2 f51/T1 == 0, <<85>>,
2 a4 b3/(1 - T1) (1 - T2) + c16/(1 - T1) (1 - T2) + <<336>> + f93 T1 T2/(1 - T1) (1 - T2) == 0}

::: Solve: Equations may not give solutions for all "solve" variables. i

Out[=]/Short=
{{e1 → 0, e2 → 0, <<84>>, f93 → -c93} }

In[=]:= sol /. (v_ → val_) :> (v = CF[val]);
```

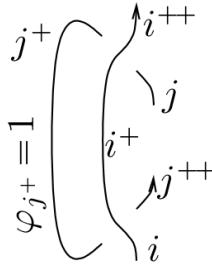
```
In[=]:= r1[-1, i, j]
Short[CF[r42[-1, i, j]], 5]

Out[=]=
- b5 p1,j p2,i x3,i/T1 - b3 p1,i p2,j x3,i/T2 + (b3 T1 + b5 T2) p1,j p2,j x3,i/T1 T2
- c93 - c81 p1,i x1,i + <<53>> + ( <<1>> ) <<3>> x<<1>> /T1 (-1 + <<1>>) ( <<1>> ) +
((a2 b3 - c10 T1 + c25 T1 + a4 b3 T2 - a4 b3 T1 T2 + c10 T1 T2 - c25 T1^2 T2) p2,i p3 <<1>> <<1>> x2,i x3,j) /
(T1 (-1 + T2) (-1 + T1 T2)) + ( <<1>> ) p2,j p3,j x2,i x3,j /(-1 + T1) T1 T2 (-1 + T1 T2)
```

```
In[=]:= CF[LeftR2b - RightR2b]

Out[=]=
εSeries[0, 0]
```

Reidemeister 2c



```
In[=]:= Timing[ Short[{LeftR2c} = Cases[
  Integrate[F[i, j] \[Cross] L /. {X_{i+1,j}[1] X_{i,j+2}[-1] C_{j+1}[1]} d{vs_i, vs_j, vs_{i^+}, vs_{j^+}, vs_{j+2}}, E[\[Epsilon]] \[Rule] \[Epsilon]
  ]]]
Out[=]= {0.71875, {eSeries[p_{1,2+i} \pi_{1,i} + p_{1,3+j} \pi_{1,j} + <<9>> + p_{3,3+j} \pi_{3,j}, g_1 g_2 + <<49>> + <<1>>]}}
In[=]:= Timing[ Short[{RightR2c} =
  Cases[Integrate[F[i, j] \[Cross] L /. {C_i[0] C_{i+1}[0] C_j[0] C_{j+1}[1] C_{j+2}[0]} d{vs_i, vs_j, vs_{i^+}, vs_{j^+}, vs_{j+2}}, E[\[Epsilon]] \[Rule] \[Epsilon]
  ]]]
Out[=]= {0.015625, {eSeries[p_{1,2+i} \pi_{1,i} + p_{1,3+j} \pi_{1,j} + <<4>> + p_{3,3+j} \pi_{3,j}, g_1 g_2 + <<16>> + <<1>>]}}
In[=]:= Short[eqn = CF[LeftR2c[[1]] - RightR2c[[1]]]];
cvs = Union@Cases[eqn, p__ | \[Pi]__, \[Infinity]];
vars = Union@Cases[y_0[1, k], g_, \[Infinity]];
Short[eqns = CoefficientRules[eqn, cvs] /. (_ \[Rule] c_) \[Rule] (c == 0), 3];
{sol} = Solve[eqns, vars];
Out[=]/.Short=
g_1 (-1 + T_1) <<1>> <<1>> <<1>> \pi_1 <<1>> <<1>> \pi_{2,i} / (B T_1 T_2) - <<1>> / (B <<1>> <<1>>) - g_1 <<3>> <<1>> / (B T_1)
Out[=]= {p_{3,3+j}, \pi_{1,i}, \pi_{1,j}, \pi_{2,i}, \pi_{2,j}}
Out[=]= {g_1}
Out[=]/.Short=
{g_1 / B - g_1 / (B T_1) - g_1 / (B T_2) + g_1 / (B T_1 T_2) == 0, -g_1 / B + g_1 / (B T_1) == 0, -g_1 / B + g_1 / (B T_2) == 0}
```

```
In[=]:= sol /. (v_ → val_) ↪ (v = CF[val]);  

Y0[1, k]  

Out[=]= 0  

In[=]:= Short[eqn = CF[LeftR2c[2] - RightR2c[2]]]  

cvs = Union@Cases[eqn, p_ | π_, ∞]  

vars = Union@Cases[Y1[1, k] + Y42[1, k], g_, ∞]  

Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) ↪ (c == 0), 3]  

Short[{sol} = Solve[eqns, vars]]  

Out[=]/Short=  


$$\frac{(C_{16} + C_{31} + \dots) \pi_1 \pi_2 \pi_3}{T_1} + \dots$$
  

Out[=]= {p1,3+j, p2,3+j, p3,3+j, π1,i, π1,j, π2,i, π2,j, π3,i, π3,j}  

Out[=]= {g2, g3, g4, g5, g6, g7, g8, g9, g10}  

Out[=]/Short=  


$$\left\{ g_6 + \frac{g_6}{T_1^2} - \frac{2g_6}{T_1} = 0, -2g_6 + \frac{2g_6}{T_1} = 0, \dots, g_4 + \frac{g_7}{T_2} + \frac{4g_9}{T_2} + \frac{g_{10}}{T_2} = 0, \right.$$
  


$$\left. \frac{2a_4 b_3}{(1-T_1)(1-T_2)} + \frac{a_4 b_3}{(1-T_1)T_1^2(1-T_2)} + \dots = 0 \right\}$$
  

Out[=]/Short=  

{ {g2 → 0, g3 → C16 + C31 + C46 + C61, g5 → 0, g6 → 0, g7 → 0, g8 → 0, g9 → 0, g10 → 0} }  

In[=]:= sol /. (v_ → val_) ↪ (v = CF[val]);  

Y1[1, k]  

Short[CF[Y42[1, k]], 5]  

Out[=]= 0  

Out[=]/Short=  


$$\frac{(C_{16} + C_{31} + C_{46} + C_{61}) p_{1,k} x_{1,k} + (C_{19} + C_{34} + C_{49} + C_{64}) p_{2,k} x_{2,k} + \dots}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)}$$
  

In[=]:= CF[LeftR2c - RightR2c]  

Out[=]= εSeries[0, 0]
```

$C_k[1]$ and $C_k[-1]$ are inverses

```
In[1]:= Timing[ Short[{LeftCC} = Cases[{\int \mathcal{F}[k] \times \mathcal{L} /@ (C_k[1] C_{k+1}[-1]) d{vs_k, vs_{k^*}}}, \mathbb{E}[\mathcal{E}] \Rightarrow \mathcal{E}]]]
```

Out[1]=

$$\left\{ 0.0625, \left\{ \in \text{Series} \left[p_{1,2+k} \pi_{1,k} + p_{2,2+k} \pi_{2,k} + \frac{\lll 1 \rrr}{B} + p_{3,2+k} \pi_{3,k}, \frac{\lll 1 \rrr}{\lll 1 \rrr} + \lll 8 \rrr + \lll 1 \rrr \right] \right\} \right\}$$

```
In[2]:= Timing[ Short[{RightCC} = Cases[{\int \mathcal{F}[k] \times \mathcal{L} /@ (C_k[0] C_{k+1}[0]) d{vs_k, vs_{k^*}}}, \mathbb{E}[\mathcal{E}] \Rightarrow \mathcal{E}]]]
```

Out[2]=

$$\{0., \{ \in \text{Series} [p_{1,2+k} \pi_{1,k} + p_{2,2+k} \pi_{2,k} + p_{3,2+k} \pi_{3,k}, 0] \} \}$$

```
In[3]:= Short[eqn = CF[LeftCC[[1]] - RightCC[[1]]]]
cvs = Union@Cases[eqn, p__ | \pi__, \infty]
vars = Union@Cases[y_0[-1, k], h__, \infty]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ \rightarrow c_) \Rightarrow (c == 0), 3]
{sol} = Solve[eqns, vars]
```

Out[3]//Short=

$$\frac{h_1 p_{3,2+k} \pi_{1,k} \pi_{2,k}}{B}$$

Out[3]=

$$\{p_{3,2+k}, \pi_{1,k}, \pi_{2,k}\}$$

Out[4]=

$$\{h_1\}$$

Out[4]//Short=

$$\left\{ \frac{h_1}{B} == 0 \right\}$$

Out[5]=

$$\{ \{h_1 \rightarrow 0\} \}$$

```
In[5]:= sol /. (v_ \rightarrow val_) \Rightarrow (v = CF[val]);
y_0[-1, k]
```

Out[5]=

$$0$$

```
In[=]:= Short[eqn = CF[LeftCC[[2]] - RightCC[[2]]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[γ1[-1, k] + γ42[-1, k], h_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0), 3]
Short[{{sol} = Solve[eqns, vars]]

Out[=]//Short=
<<1>>

Out[=]=
{p1,2+k, p2,2+k, p3,2+k, π1,k, π2,k, π3,k}

Out[=]=
{h2, h3, h4, h5, h6, h7, h8, h9, h10}

Out[=]//Short=
{h6 == 0, h7 == 0, B h2 == 0, h8 == 0, <<1>> == 0, <<1>>,
 h10 == 0, c19 + c34 + c49 + c64 + h4 + h7 + 4 h9 + h10 == 0, <<1>> == 0,
 
$$\frac{2 a_2 b_3}{(1 - T_1) (1 - T_2) (1 - T_1 T_2)} - \frac{a_4 b_3}{(1 - T_1) (1 - T_2) (1 - T_1 T_2)} - \frac{<<1>>}{<<1>>} + <<165>> == 0\}$$


Out[=]//Short=
{ {h2 → 0, h3 → -c16 - c31 - c46 - c61, <<5>>, h9 → 0, h10 → 0} }

In[=]:= sol /. (v_ → val_) :> (v = CF[val]);
In[=]:= γ1[-1, k]
Short[CF[γ42[-1, k]], 5]

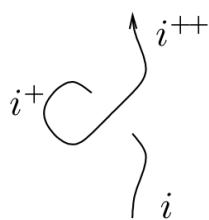
Out[=]=
0

Out[=]//Short=

$$\frac{(-c_{16} - c_{31} - c_{46} - c_{61}) p_{1,k} x_{1,k} + (-c_{19} - c_{34} - c_{49} - c_{64}) p_{2,k} x_{2,k} - (-b_3 + b_5 + b_3 T_1 - b_5 T_2) (-a_2 + a_2 T_1 - a_4 T_1 - a_4 T_2 + 2 a_4 T_1 T_2) p_{3,k} x_{3,k}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)}$$


In[=]:= CF[LeftCC - RightCC]
Out[=]=
Series[0, 0]
```

Invariance Under R1



```

In[*]:= {LeftR11} = Cases[{\int \mathcal{F}[i] \times \mathcal{L} /@ (X_{i+2,i}[1] C_{i+1}[1]) d{vs_i, vs_{i^+}, vs_{i+2}}}, E[\mathcal{E}_-] \rightarrow \mathcal{E}, \infty]
Out[*]= {Series[p_{1,3+i} \pi_{1,i} + p_{2,3+i} \pi_{2,i} + p_{3,3+i} \pi_{3,i}, C93]}

In[*]:= {RightR11} = Cases[{\int \mathcal{F}[i] \times \mathcal{L} /@ (C_i[0] C_{i+1}[0] C_{i+2}[0]) d{vs_i, vs_{i^+}, vs_{i+2}}}, E[\mathcal{E}_-] \rightarrow \mathcal{E}, \infty]
Out[*]= {Series[p_{1,3+i} \pi_{1,i} + p_{2,3+i} \pi_{2,i} + p_{3,3+i} \pi_{3,i}, 0]}

In[*]:= LeftR11[[1]] == RightR11[[1]]
Out[*]= True

In[*]:= Short[eqn = CF[LeftR11[[2]] - RightR11[[2]]]]
cvs = Union@Cases[eqn, p__ | \pi__, \infty]
vars = Union@Cases[eqn, (c | d | e | f | g | h)_, \infty]
Short[eqns = If[cvs === {}, 
  {eqn == 0},
  CoefficientRules[eqn, cvs] /. (_ \rightarrow c_) \rightarrow (c == 0)
], 3]
{sol} = Solve[eqns, vars]

Out[*]//Short=
C93

Out[*]= {}

Out[*]= {}

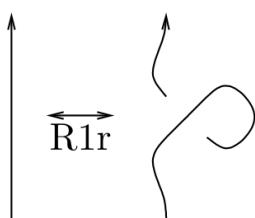
Out[*]//Short=
{C93 == 0}

Out[*]=
{{C93 \rightarrow 0} }

In[*]:= sol /. (v_ \rightarrow val_) \rightarrow (v = CF[val]);
In[*]:= CF[LeftR11 - RightR11]
Out[*]= Series[0, 0]

```

Invariance Under R1r



In[$\#$]:= $\{\text{LeftR1r}\} = \text{Cases}\left[\left\{\int \mathcal{F}[i] \times \mathcal{L} / @ (\mathbf{X}_{i,i+2}[1] \mathbf{C}_{i+1}[-1]) \text{ d}\{\mathbf{vs}_i, \mathbf{vs}_{i^+}, \mathbf{vs}_{i+2}\}\right\}, \text{IE}[\mathcal{E}_-] \Rightarrow \mathcal{E}, \infty\right]$

Out[$\#$]=

$$\begin{aligned} & \in \text{Series}\left[p_{1,3+i} \pi_{1,i} + p_{2,3+i} \pi_{2,i} + p_{3,3+i} \pi_{3,i}, \right. \\ & \frac{1}{T_1^2 T_2 (-1 + T_1 T_2)} \left(a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 + c_{83} T_1 + c_{86} T_1 + c_{82} T_1^2 + c_{85} T_1^2 + a_4 b_5 T_2 + \right. \\ & a_4 b_3 T_1 T_2 - a_4 b_5 T_1 T_2 + c_{81} T_1 T_2 + c_{84} T_1 T_2 - c_{81} T_1^2 T_2 - c_{82} T_1^2 T_2 - 2 c_{83} T_1^2 T_2 - \\ & c_{86} T_1^2 T_2 - c_{82} T_1^3 T_2 - c_{85} T_1^3 T_2 - c_{81} T_1^2 T_2^2 - c_{84} T_1^2 T_2^2 + c_{81} T_1^3 T_2^2 + c_{82} T_1^3 T_2^2 + c_{83} T_1^3 T_2^2 \Big) \Big] \end{aligned}$$

In[$\#$]:= $\{\text{RightR1r}\} = \text{Cases}\left[\left\{\int \mathcal{F}[i] \times \mathcal{L} / @ (\mathbf{C}_i[0] \mathbf{C}_{i+1}[0] \mathbf{C}_{i+2}[0]) \text{ d}\{\mathbf{vs}_i, \mathbf{vs}_{i^+}, \mathbf{vs}_{i+2}\}\right\}, \text{IE}[\mathcal{E}_-] \Rightarrow \mathcal{E}, \infty\right]$

Out[$\#$]=

$$\in \text{Series}[p_{1,3+i} \pi_{1,i} + p_{2,3+i} \pi_{2,i} + p_{3,3+i} \pi_{3,i}, 0]$$

In[$\#$]:= $\text{LeftR1r}[1] == \text{RightR1r}[1]$ Out[$\#$]=

True

In[$\#$]:= $\text{Short}[\text{eqn} = \text{CF}[\text{LeftR1r}[2] - \text{RightR1r}[2]]]$
 $\text{cvs} = \text{Union}@\text{Cases}[\text{eqn}, p_{__} | \pi_{__}, \infty]$
 $\text{vars} = \text{Union}@\text{Cases}[\text{eqn}, (c | d | e | f | g | h)_ , \infty]$
 $\text{Short}[\text{eqns} = \text{CoefficientRules}[\text{eqn}, \text{cvs}] /. (_ \rightarrow c_{_}) \Rightarrow (c_{_} == 0), 3]$
 $\{\text{sol}\} = \text{Solve}[\text{eqns}, \text{vars}]$

Out[$\#$]/Short=

$$\frac{a_2 b_5 + \text{c83 } T_1^3 T_2^2}{T_1^2 T_2 (-1 + T_1 T_2)}$$

Out[$\#$]=

{ }

Out[$\#$]=

{c81, c82, c83, c84, c85, c86}

Out[$\#$]/Short=

$$\left\{ \frac{a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 + \text{c81 } T_1^3 T_2^2 + c_{82} T_1^3 T_2^2 + c_{83} T_1^3 T_2^2}{T_1^2 T_2 (-1 + T_1 T_2)} = 0 \right\}$$

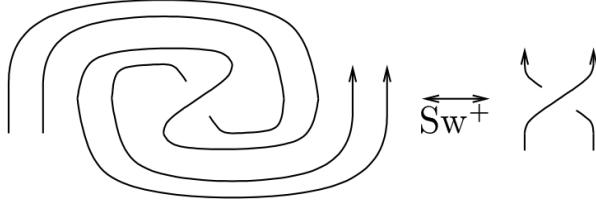
Solve: Equations may not give solutions for all "solve" variables.

Out[$\#$]=

$$\left\{ \begin{aligned} c_{86} \rightarrow -c_{85} T_1 - c_{84} T_2 - c_{83} (1 - T_1 T_2) - c_{82} (T_1 - T_1 T_2) - \\ c_{81} (T_2 - T_1 T_2) - \frac{-a_2 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 - a_4 b_5 T_2 - a_4 b_3 T_1 T_2 + a_4 b_5 T_1 T_2}{T_1 (-1 + T_1 T_2)} \end{aligned} \right\}$$

In[$\#$]:= $\text{sol} /. (v_{_} \rightarrow val_{_}) \Rightarrow (v = \text{CF}[val])$;In[$\#$]:= $\text{CF}[\text{LeftR1r} - \text{RightR1r}]$ Out[$\#$]= $\in \text{Series}[0, 0]$

Invariance Under Sw



```
In[]:= Timing[ Short[{LeftSw} = Cases[{\int \mathcal{F}[i, j] \times \mathcal{L} /@ (X_{i+1, j+1}[1] C_i[-1] C_j[-1] C_{i+2}[1] C_{j+2}[1]) d{vs_i, vs_j, vs_{i^*}, vs_{j^*}, vs_{i+2}, vs_{j+2}}}, \!E[\mathcal{E}_] \Rightarrow \mathcal{E}, \infty]
]]

```

```
Out[]= {0.0625, {Series[T_1 p_{1,3+i} \pi_{1,i} + <<11>> + p_{3,3+j} \pi_{3,j}, \frac{<<1>>}{<<1>>} + <<56>> + \frac{<<1>>}{<<1>>} ]}}

```

```
In[]:= Timing[ Short[{RightSw} = Cases[{\int \mathcal{F}[i, j] \times \mathcal{L} /@ (X_{i+1, j+1}[1] C_i[0] C_j[0] C_{i+2}[0] C_{j+2}[0]) d{vs_i, vs_j, vs_{i^*}, vs_{j^*}, vs_{i+2}, vs_{j+2}}}, \!E[\mathcal{E}_] \Rightarrow \mathcal{E}, \infty]
]]

```

```
Out[]= {0.28125, {Series[T_1 p_{1,3+i} \pi_{1,i} + <<11>> + p_{3,3+j} \pi_{3,j}, \frac{<<1>>}{<<1>>} + <<56>> + \frac{<<1>>}{<<1>>} ]}}

```

```
In[]:= LeftSw == RightSw

```

```
Out[]= True

```

The Solution

```
In[]:= Union@Cases[\mathcal{L}@X_{i,j}[1], (a | b | c | d | e | f | g | h)_, \infty]

```

```
Out[]= {a_2, a_4, b_3, b_5, c_6, c_7, c_8, c_9, c_{10}, c_{16}, c_{19}, c_{21}, c_{22}, c_{23}, c_{24}, c_{25}, c_{31}, c_{34}, c_{36}, c_{39}, c_{46}, c_{49}, c_{61}, c_{64}, c_{81}, c_{82}, c_{83}, c_{84}, c_{85}}

```

```
In[]:= LeafCount@CF[\mathcal{L}@X_{i,j}[1] /. {c_{16|19|31|34|46|49|61|64|81|82|84|85} \rightarrow 0, b_{3|5} \rightarrow 1, a_{2|4} \rightarrow 1, B \rightarrow 1} //.
{c_{21} \rightarrow -c_6, c_{22} \rightarrow -c_7, c_{24} \rightarrow -c_9, c_7 \rightarrow -1 / T_1, c_{8|10|23|25|83} \rightarrow 0}]

```

```
Out[]= 1310

```

```
In[]:= LeafCount@CF[\mathcal{L}@X_{i,j}[1] /. {c_{16|19|31|34|46|49|61|64|81|82|84|85} \rightarrow 0, b_{3|5} \rightarrow 1, a_{2|4} \rightarrow 1, B \rightarrow 1} //.
{c_{21} \rightarrow -c_6, c_{22} \rightarrow -c_7, c_{24} \rightarrow -c_9, c_7 \rightarrow 0, c_{8|10|23|25|83} \rightarrow 0}]

```

```
Out[]= 1205

```

In[=]:= **CF** [$\underline{\mathcal{L}} @ \underline{\mathbf{X}}_{i,j}$ [1] /. { $c_{16|19|31|34|46|49|61|64|81|82|84|85} \rightarrow 0$, $b_{3|5} \rightarrow 1$, $a_{2|4} \rightarrow 1$, $B \rightarrow 1$ } // . { $c_{21} \rightarrow -c_6$, $c_{22} \rightarrow -c_7$, $c_{24} \rightarrow -c_9$, $c_7 \rightarrow 0$, $c_{8|10|23|25|83} \rightarrow 0$ }]

Out[=]=

$$\begin{aligned} & T_1 T_2 \mathbb{E} \left[\in \text{Series} \left[-p_{1,i} x_{1,i} + T_1 p_{1,1+i} x_{1,i} + (1 - T_1) p_{1,1+j} x_{1,i} - p_{1,j} x_{1,j} + p_{1,1+j} x_{1,j} - p_{2,i} x_{2,i} + \right. \right. \\ & T_2 p_{2,1+i} x_{2,i} + (1 - T_2) p_{2,1+j} x_{2,i} + p_{3,j} x_{1,i} x_{2,i} - \frac{(1 + T_2) p_{3,j} x_{1,j} x_{2,i}}{T_1} - p_{2,j} x_{2,j} + p_{2,1+j} x_{2,j} + \\ & p_{3,j} x_{1,i} x_{2,j} - p_{3,i} x_{3,i} + T_1 T_2 p_{3,1+i} x_{3,i} + (1 - T_1 T_2) p_{3,1+j} x_{3,i} - p_{3,j} x_{3,j} + p_{3,1+j} x_{3,j}, \\ & \frac{(-1 - T_2 + T_1 T_2 - T_2^2) p_{1,j} p_{2,j} x_{1,i} x_{2,i}}{-1 + T_1 T_2} - \frac{(T_1 - T_2) (-1 - T_2 + T_1 T_2) p_{1,j} p_{2,i} x_{1,j} x_{2,i}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} - \\ & \frac{(1 + T_2) p_{1,j} p_{2,j} x_{1,j} x_{2,i}}{(-1 + T_1) T_1} - \frac{(T_1 - T_2) T_2 p_{1,i} p_{2,j} x_{1,i} x_{2,j}}{(-1 + T_1) (-1 + T_2) (-1 + T_1 T_2)} + \frac{p_{1,j} p_{2,j} x_{1,i} x_{2,j}}{-1 + T_2} + \\ & p_{1,j} p_{2,i} x_{3,i} + p_{1,i} p_{2,j} x_{3,i} - 2 p_{1,j} p_{2,j} x_{3,i} + \frac{(1 + T_2) p_{3,j} x_{3,i}}{T_1 (-1 + T_1 T_2)} - \\ & \frac{(1 - 2 T_1 T_2 + T_2^2) p_{1,j} p_{3,j} x_{1,i} x_{3,i}}{-1 + T_2} + \frac{(-T_1 + T_2 - 2 T_1 T_2 + 2 T_1^2 T_2 + T_2^2 - T_1 T_2^2) p_{1,j} p_{3,i} x_{1,j} x_{3,i}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} + \\ & \frac{(1 + T_2) p_{1,i} p_{3,j} x_{1,j} x_{3,i}}{T_1 (-1 + T_2)} + \frac{(1 + T_2) (1 - 2 T_1 + T_1 T_2) p_{1,j} p_{3,j} x_{1,j} x_{3,i}}{(-1 + T_1) T_1 (-1 + T_2)} + \\ & \frac{(1 - T_1 + 2 T_2 - 2 T_1 T_2 + T_1^2 T_2 + T_2^2 - 2 T_1 T_2^2) p_{2,j} p_{3,j} x_{2,i} x_{3,i}}{(-1 + T_1) T_1} + \\ & \frac{(-1 + T_1) T_2 + T_1^2 T_2 + T_2^2 - 2 T_1 T_2^2) p_{2,j} p_{3,i} x_{2,j} x_{3,i}}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} - \\ & p_{2,i} p_{3,j} x_{2,j} x_{3,i} - \frac{(1 - 2 T_2 + T_1 T_2) p_{2,j} p_{3,j} x_{2,j} x_{3,i}}{-1 + T_1} - \\ & \frac{T_2 p_{1,i} p_{3,j} x_{1,i} x_{3,j}}{(-1 + T_1) (-1 + T_2)} - \frac{(-1 - T_2 + 2 T_1 T_2) p_{1,j} p_{3,j} x_{1,i} x_{3,j}}{(-1 + T_2) (-1 + T_1 T_2)} + \\ & \frac{(-1 - T_2 + T_1 T_2) p_{2,i} p_{3,j} x_{2,i} x_{3,j}}{T_1 (-1 + T_2)} + \frac{T_2 (-1 - T_2 + 2 T_1 T_2) p_{2,j} p_{3,j} x_{2,i} x_{3,j}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} \Big] \Big] \end{aligned}$$

In[=]:= **CF** [$\underline{\mathcal{L}} @ \underline{\mathbf{X}}_{i,j}$ [-1] /. { $c_{16|19|31|34|46|49|61|64|81|82|84|85} \rightarrow 0$, $b_{3|5} \rightarrow 1$, $a_{2|4} \rightarrow 1$, $B \rightarrow 1$ } // . { $c_{21} \rightarrow -c_6$, $c_{22} \rightarrow -c_7$, $c_{24} \rightarrow -c_9$, $c_7 \rightarrow 0$, $c_{8|10|23|25|83} \rightarrow 0$ }]

Out[=]=

$$\begin{aligned} & \frac{1}{T_1 T_2} \mathbb{E} \left[\in \text{Series} \left[-p_{1,i} x_{1,i} + \frac{p_{1,1+i} x_{1,i}}{T_1} + \frac{(-1 + T_1) p_{1,1+j} x_{1,i}}{T_1} - p_{1,j} x_{1,j} + p_{1,1+j} x_{1,j} - p_{2,i} x_{2,i} + \frac{p_{2,1+i} x_{2,i}}{T_2} + \right. \right. \\ & \frac{(-1 + T_2) p_{2,1+j} x_{2,i}}{T_2} + \frac{(-1 + T_1 - T_2) p_{3,j} x_{1,i} x_{2,i}}{T_1^2 T_2} + \frac{(1 + T_2) p_{3,j} x_{1,j} x_{2,i}}{T_1 T_2} - p_{2,j} x_{2,j} + p_{2,1+j} x_{2,j} - \\ & \frac{p_{3,j} x_{1,i} x_{2,j}}{T_1} - p_{3,i} x_{3,i} + \frac{p_{3,1+i} x_{3,i}}{T_1 T_2} + \frac{(-1 + T_1 T_2) p_{3,1+j} x_{3,i}}{T_1 T_2} - p_{3,j} x_{3,j} + p_{3,1+j} x_{3,j}, \right. \right. \end{aligned}$$

$$\begin{aligned}
& \frac{(\mathbf{T}_1 - \mathbf{T}_2) (-1 - \mathbf{T}_2 + \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{1,j} \mathbf{p}_{2,i} \mathbf{x}_{1,i} \mathbf{x}_{2,i}}{\mathbf{T}_1^2 (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2)} + \frac{(\mathbf{T}_1 - \mathbf{T}_2) \mathbf{p}_{1,i} \mathbf{p}_{2,j} \mathbf{x}_{1,i} \mathbf{x}_{2,i}}{(-1 + \mathbf{T}_1) (-1 + \mathbf{T}_1 \mathbf{T}_2)} - \\
& \frac{(\mathbf{1} - \mathbf{T}_1 + 2 \mathbf{T}_2 - \mathbf{T}_1 \mathbf{T}_2 + 2 \mathbf{T}_1^2 \mathbf{T}_2 + \mathbf{T}_2^2 - 2 \mathbf{T}_1 \mathbf{T}_2^2) \mathbf{p}_{1,j} \mathbf{p}_{2,j} \mathbf{x}_{1,i} \mathbf{x}_{2,i}}{\mathbf{T}_1^2 \mathbf{T}_2 (-1 + \mathbf{T}_1 \mathbf{T}_2)} + \\
& \frac{(\mathbf{T}_1 - \mathbf{T}_2) (-1 - \mathbf{T}_2 + \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{1,j} \mathbf{p}_{2,i} \mathbf{x}_{1,j} \mathbf{x}_{2,i}}{(-1 + \mathbf{T}_1) \mathbf{T}_1 (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2)} - \\
& \frac{(\mathbf{1} - \mathbf{T}_1 + 2 \mathbf{T}_2 - 2 \mathbf{T}_1 \mathbf{T}_2 + \mathbf{T}_1^2 \mathbf{T}_2 + \mathbf{T}_2^2 - 2 \mathbf{T}_1 \mathbf{T}_2^2) \mathbf{p}_{1,j} \mathbf{p}_{2,j} \mathbf{x}_{1,j} \mathbf{x}_{2,i}}{(-1 + \mathbf{T}_1) \mathbf{T}_1 \mathbf{T}_2 (-1 + \mathbf{T}_1 \mathbf{T}_2)} + \frac{(\mathbf{T}_1 - \mathbf{T}_2) \mathbf{T}_2 \mathbf{p}_{1,i} \mathbf{p}_{2,j} \mathbf{x}_{1,i} \mathbf{x}_{2,j}}{(-1 + \mathbf{T}_1) (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2)} - \\
& \frac{(-1 + 2 \mathbf{T}_1 \mathbf{T}_2 - \mathbf{T}_2^2) \mathbf{p}_{1,j} \mathbf{p}_{2,j} \mathbf{x}_{1,i} \mathbf{x}_{2,j}}{\mathbf{T}_1 (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2)} - \frac{\mathbf{p}_{1,j} \mathbf{p}_{2,i} \mathbf{x}_{3,i}}{\mathbf{T}_1} - \frac{\mathbf{p}_{1,i} \mathbf{p}_{2,j} \mathbf{x}_{3,i}}{\mathbf{T}_2} + \frac{(\mathbf{T}_1 + \mathbf{T}_2) \mathbf{p}_{1,j} \mathbf{p}_{2,j} \mathbf{x}_{3,i}}{\mathbf{T}_1 \mathbf{T}_2} + \\
& \frac{(-1 - \mathbf{T}_2 + \mathbf{T}_1 \mathbf{T}_2 - \mathbf{T}_2^2) \mathbf{p}_{3,j} \mathbf{x}_{3,i}}{\mathbf{T}_1 \mathbf{T}_2 (-1 + \mathbf{T}_1 \mathbf{T}_2)} - \frac{(-\mathbf{T}_1 + \mathbf{T}_2 - 2 \mathbf{T}_1 \mathbf{T}_2 + 2 \mathbf{T}_1^2 \mathbf{T}_2 + \mathbf{T}_2^2 - \mathbf{T}_1 \mathbf{T}_2^2) \mathbf{p}_{1,j} \mathbf{p}_{3,i} \mathbf{x}_{1,i} \mathbf{x}_{3,i}}{\mathbf{T}_1^2 (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2)} - \\
& \frac{(\mathbf{1} - \mathbf{T}_1 + \mathbf{T}_2 - 2 \mathbf{T}_1 \mathbf{T}_2 + 2 \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{T}_1 \mathbf{T}_2^2) \mathbf{p}_{1,i} \mathbf{p}_{3,j} \mathbf{x}_{1,i} \mathbf{x}_{3,i}}{(-1 + \mathbf{T}_1) \mathbf{T}_1^2 (-1 + \mathbf{T}_2) \mathbf{T}_2} + \\
& \frac{(\mathbf{1} - 2 \mathbf{T}_1 + \mathbf{T}_2 - 3 \mathbf{T}_1 \mathbf{T}_2 + 4 \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{T}_1 \mathbf{T}_2^2) \mathbf{p}_{1,j} \mathbf{p}_{3,j} \mathbf{x}_{1,i} \mathbf{x}_{3,i}}{\mathbf{T}_1^3 (-1 + \mathbf{T}_2) \mathbf{T}_2} - \\
& \frac{(-\mathbf{T}_1 + \mathbf{T}_2 - 2 \mathbf{T}_1 \mathbf{T}_2 + 2 \mathbf{T}_1^2 \mathbf{T}_2 + \mathbf{T}_2^2 - \mathbf{T}_1 \mathbf{T}_2^2) \mathbf{p}_{1,j} \mathbf{p}_{3,i} \mathbf{x}_{1,j} \mathbf{x}_{3,i}}{(-1 + \mathbf{T}_1) \mathbf{T}_1 (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2)} - \frac{(\mathbf{1} + \mathbf{T}_2) \mathbf{p}_{1,i} \mathbf{p}_{3,j} \mathbf{x}_{1,j} \mathbf{x}_{3,i}}{\mathbf{T}_1 (-1 + \mathbf{T}_2) \mathbf{T}_2} + \\
& \frac{(\mathbf{1} - 2 \mathbf{T}_1 + \mathbf{T}_1^2 + \mathbf{T}_2 - 3 \mathbf{T}_1 \mathbf{T}_2 + 3 \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{T}_1 \mathbf{T}_2^2) \mathbf{p}_{1,j} \mathbf{p}_{3,j} \mathbf{x}_{1,j} \mathbf{x}_{3,i}}{(-1 + \mathbf{T}_1) \mathbf{T}_1^2 (-1 + \mathbf{T}_2) \mathbf{T}_2} - \\
& \frac{(-1 + \mathbf{T}_1 \mathbf{T}_2 + \mathbf{T}_1^2 \mathbf{T}_2 + \mathbf{T}_2^2 - 2 \mathbf{T}_1 \mathbf{T}_2^2) \mathbf{p}_{2,j} \mathbf{p}_{3,i} \mathbf{x}_{2,i} \mathbf{x}_{3,i}}{(-1 + \mathbf{T}_1) \mathbf{T}_1 \mathbf{T}_2 (-1 + \mathbf{T}_1 \mathbf{T}_2)} - \\
& \frac{(-\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_1^2 \mathbf{T}_2 + \mathbf{T}_2^2 - 2 \mathbf{T}_1 \mathbf{T}_2^2) \mathbf{p}_{2,i} \mathbf{p}_{3,j} \mathbf{x}_{2,i} \mathbf{x}_{3,i}}{(-1 + \mathbf{T}_1) \mathbf{T}_1^2 (-1 + \mathbf{T}_2) \mathbf{T}_2} + \\
& \frac{(-\mathbf{T}_1 + 2 \mathbf{T}_2 + \mathbf{T}_1^2 \mathbf{T}_2 + 2 \mathbf{T}_2^2 - 4 \mathbf{T}_1 \mathbf{T}_2^2) \mathbf{p}_{2,j} \mathbf{p}_{3,j} \mathbf{x}_{2,i} \mathbf{x}_{3,i}}{(-1 + \mathbf{T}_1) \mathbf{T}_1^2 \mathbf{T}_2} - \\
& \frac{(-1 + \mathbf{T}_1 \mathbf{T}_2 + \mathbf{T}_1^2 \mathbf{T}_2 + \mathbf{T}_2^2 - 2 \mathbf{T}_1 \mathbf{T}_2^2) \mathbf{p}_{2,j} \mathbf{p}_{3,i} \mathbf{x}_{2,j} \mathbf{x}_{3,i}}{(-1 + \mathbf{T}_1) \mathbf{T}_1 (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2)} + \frac{\mathbf{p}_{2,i} \mathbf{p}_{3,j} \mathbf{x}_{2,j} \mathbf{x}_{3,i}}{(-1 + \mathbf{T}_1) \mathbf{T}_1} + \\
& \frac{(-1 - \mathbf{T}_1 + \mathbf{T}_1^2 + 2 \mathbf{T}_1 \mathbf{T}_2 + \mathbf{T}_1^2 \mathbf{T}_2 + \mathbf{T}_2^2 - 3 \mathbf{T}_1 \mathbf{T}_2^2) \mathbf{p}_{2,j} \mathbf{p}_{3,j} \mathbf{x}_{2,j} \mathbf{x}_{3,i}}{(-1 + \mathbf{T}_1) \mathbf{T}_1^2 (-1 + \mathbf{T}_2) \mathbf{T}_2} + \frac{\mathbf{T}_2 \mathbf{p}_{1,i} \mathbf{p}_{3,j} \mathbf{x}_{1,i} \mathbf{x}_{3,j}}{(-1 + \mathbf{T}_1) (-1 + \mathbf{T}_1 \mathbf{T}_2)} + \\
& \frac{(-1 + 2 \mathbf{T}_1 \mathbf{T}_2 - \mathbf{T}_2^2) \mathbf{p}_{1,j} \mathbf{p}_{3,j} \mathbf{x}_{1,i} \mathbf{x}_{3,j}}{\mathbf{T}_1 (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2)} - \frac{(-1 - \mathbf{T}_2 + \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{2,i} \mathbf{p}_{3,j} \mathbf{x}_{2,i} \mathbf{x}_{3,j}}{\mathbf{T}_1 (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2)} + \\
& \frac{(\mathbf{1} - \mathbf{T}_1 + 2 \mathbf{T}_2 - 2 \mathbf{T}_1 \mathbf{T}_2 + \mathbf{T}_1^2 \mathbf{T}_2 + \mathbf{T}_2^2 - 2 \mathbf{T}_1 \mathbf{T}_2^2) \mathbf{p}_{2,j} \mathbf{p}_{3,j} \mathbf{x}_{2,i} \mathbf{x}_{3,j}}{(-1 + \mathbf{T}_1) \mathbf{T}_1 \mathbf{T}_2 (-1 + \mathbf{T}_1 \mathbf{T}_2)} \Big]
\end{aligned}$$

```
In[=]:= MatrixForm@CF[{  
    {r0[1, i, j], r1[1, i, j]},  
    {r0[-1, i, j], r1[-1, i, j]},  
    {(-a2 + a2 T1 - a4 T1 - a4 T2 + 2 a4 T1 T2), (-b3 + b5 + b3 T1 - b5 T2)}  
} /. {b3 → 0, a4 → 0, b5 → T1, a2 → T1}]  
  
Out[=]//MatrixForm=  
{{T1 p3,j x1,i x2,i - p3,j x1,j x2,i - T1 p1,j p2,i x3,i - T1 p1,j p2,j x3,i  
- p3,j x1,i x2,i + p3,j x1,j x2,i - p1,j p2,i x3,i + p1,j p2,j x3,i  
(-1 + T1) T1 - T1 (-1 + T2)},  
{  
    {r0[1, i, j], r1[1, i, j]},  
    {r0[-1, i, j], r1[-1, i, j]},  
    {(-a2 + a2 T1 - a4 T1 - a4 T2 + 2 a4 T1 T2), (-b3 + b5 + b3 T1 - b5 T2)}  
} /. {b3 → 0, a4 → 0, b5 → 1, a2 → 1}]  
  
Out[=]//MatrixForm=  
{{p3,j x1,i x2,i - p3,j x1,j x2,i - p1,j p2,i x3,i - p1,j p2,j x3,i  
- p3,j x1,i x2,i + p3,j x1,j x2,i - p1,j p2,i x3,i + p1,j p2,j x3,i  
- 1 + T1 1 - T2},  
{  
    {r0[1, i, j], r1[1, i, j]},  
    {r0[-1, i, j], r1[-1, i, j]},  
    {(-a2 + a2 T1 - a4 T1 - a4 T2 + 2 a4 T1 T2), (-b3 + b5 + b3 T1 - b5 T2)}  
} /. {b3 → T2^(1/2), b5 → -T1^(1/2), a4 → T1^(1/2), a2 → -Sqrt[T1] T2}]  
  
Out[=]//MatrixForm=  
{{-Sqrt[T1] T2 p3,j x1,i x2,i + Sqrt[T1] p3,j x1,i x2,j - Sqrt[T1] p1,j p2,i x3,i + Sqrt[T2] p1,i p2,j x3,i + (Sqrt[T1] - Sqrt[T2]) p1,j p2,i x3,i  
- p3,j x1,i x2,i + p3,j x1,j x2,j - p1,j p2,i x3,i + (Sqrt[T1] - Sqrt[T2]) p1,j p2,j x3,i  
Sqrt[T1] T2 Sqrt[T1] - Sqrt[T1]  
T1^(3/2) (-1 + T2) (Sqrt[T1] + Sqrt[T2]) (-1 + Sqrt[T1] Sqrt[T2])},  
{  
    {r0[1, i, j], r1[1, i, j]},  
    {r0[-1, i, j], r1[-1, i, j]},  
    {(-a2 + a2 T1 - a4 T1 - a4 T2 + 2 a4 T1 T2), (-b3 + b5 + b3 T1 - b5 T2)}  
} /. {a2 → -T2 a4, b5 → -b3 T1} /. {b3 → T1 - 1, a4 → 1}]  
  
Out[=]//MatrixForm=  
{{-T2 p3,j x1,i x2,i + p3,j x1,i x2,j - ((-1 + T1) T1 p1,j p2,i x3,i) + (-1 + T1) p1,i p2,j x3,i + (-1 + T1)^2 p1,j |  
- p3,j x1,i x2,i + p3,j x1,j x2,j - (-1 + T1) p1,j p2,i x3,i - ((-1 + T1) p1,i p2,j x3,i - (-1 + T1) (-1 + T2) p1,j p2,j x3,i  
T1 T2 T1 (-1 + T2) T2 (-1 + T1) (-1 + T1 T2)},  
{  
    {r0[1, i, j], r1[1, i, j]},  
    {r0[-1, i, j], r1[-1, i, j]},  
    {(-a2 + a2 T1 - a4 T1 - a4 T2 + 2 a4 T1 T2), (-b3 + b5 + b3 T1 - b5 T2)}  
} /. {a2 → -T2 a4, b5 → -b3 T1} /. {b3 → T1 - 1, a4 → 1}]
```

```
In[=]:= MatrixForm@CF[{r0[1, i, j], r1[1, i, j]}, {r0[-1, i, j], r1[-1, i, j]}, {(-a2 + a2 T1 - a4 T1 - a4 T2 + 2 a4 T1 T2), (-b3 + b5 + b3 T1 - b5 T2)}, {a2 → -T2 a4, b5 → -b3 T1} /. {b3 → 1, a4 → 1 - T1^-1}]

Out[=]//MatrixForm=

$$\begin{pmatrix} -\frac{(-1+T_1) T_2 p_{3,j} x_{1,i} x_{2,i}}{T_1} + \frac{(-1+T_1) p_{3,j} x_{1,i} x_{2,j}}{T_1} & -T_1 p_{1,j} p_{2,i} x_{3,i} + p_{1,i} p_{2,j} x_{3,i} + (-1+T_1) p_{1,j} p_{2,j} x_{3,i} \\ \frac{(-1+T_1) p_{3,j} x_{1,i} x_{2,i}}{T_1^2 T_2} - \frac{(-1+T_1) p_{3,j} x_{1,i} x_{2,j}}{T_1^2} & p_{1,j} p_{2,i} x_{3,i} - \frac{p_{1,i} p_{2,j} x_{3,i}}{T_2} - \frac{(-1+T_2) p_{1,j} p_{2,j} x_{3,i}}{T_2} \\ (-1+T_1) (-1+T_2) & -1+T_1 T_2 \end{pmatrix}$$

{b3 → T2^(1/2) b3, b5 → T1^(1/2) b5}, {b3 → T2, b5 → -T1, a2 → T1 - T2, a4 → 1}

In[=]:= L[Ck[1]]

Out[=]=
T1 T2 E[Series[(-p1,k + p1,1+k) x1,k + (-p2,k + p2,1+k) x2,k + (-p3,k + p3,1+k) x3,k,
(c16 + c31 + c46 + c61) p1,k x1,k + (c19 + c34 + c49 + c64) p2,k x2,k +
(-b3 + b5 + b3 T1 - b5 T2) (-a2 + a2 T1 - a4 T1 - a4 T2 + 2 a4 T1 T2) p3,k x3,k],
(-1+T1) T1 (-1+T2) (-1+T1 T2)]]

In[=]:= Factor[((-b3 + b5 + b3 T1 - b5 T2) (-a2 + a2 T1 - a4 T1 - a4 T2 + 2 a4 T1 T2) p3,k x3,k) /.
{a2 → -T2 a4, b5 → -b3 T1} /. {b3 → T1 - 1, a4 → 1}]

Out[=]=
p3,k x3,k

In[=]:= L[Ck[1]] /. c16|19|31|34|46|49|61|64|81|82|84|85 → 0 /. T2 → 1
Power::infexp: Infinite expression 1/0 encountered. ⓘ
Out[=]=
T1 E[Series[(-p1,k + p1,1+k) x1,k + (-p2,k + p2,1+k) x2,k + (-p3,k + p3,1+k) x3,k, ComplexInfinity]]]

In[=]:= L[Ck[-1]]

Out[=]=

$$\frac{1}{T_1 T_2} E[Series[(-p1,k + p1,1+k) x1,k + (-p2,k + p2,1+k) x2,k + (-p3,k + p3,1+k) x3,k,
(-c16 - c31 - c46 - c61) p1,k x1,k + (-c19 - c34 - c49 - c64) p2,k x2,k -
(-b3 + b5 + b3 T1 - b5 T2) (-a2 + a2 T1 - a4 T1 - a4 T2 + 2 a4 T1 T2) p3,k x3,k],
(-1+T1) T1 (-1+T2) (-1+T1 T2)]]]$$

```

Some Knots

```
In[1]:= tab1 = Last /@ Table[K = Knot[n, 1];
Echo@Timing[K → ∫ L[K] × d vs[K]],
{n, 3, 10}];
```

In[2]:= tab1[[1]]

Out[2]=

```
Knot[3, 1] →
- (( 1/2 T12 T22 E ∈ Series[0, - (( (-c81 - c82 - c84 - c85 + c16 T1 + c31 T1 + c46 T1 + c61 T1 + 5 c81 T1 + 4 c82 T1 +
4 c84 T1 + 4 c85 T1 - 3 c16 T12 - 3 c31 T12 - 3 c46 T12 - 3 c61 T12 - 10 c81 T12 - 8 c82 T12 -
6 c84 T12 - 8 c85 T12 + 5 c16 T13 + 5 c31 T13 + 5 c46 T13 + 5 c61 T13 + 11 c81 T13 + 10 c82 T13 +
5 c84 T13 + 10 c85 T13 - 5 c16 T14 - 5 c31 T14 - 5 c46 T14 - 5 c61 T14 - 7 c81 T14 - 8 c82 T14 -
2 c84 T14 - 8 c85 T14 + 2 c16 T15 + 2 c31 T15 + 2 c46 T15 + 2 c61 T15 + 2 c81 T15 + 4 c82 T15 -
c82 T16 - c85 T16 - a2 b3 T2 + a2 b5 T2 + c19 T2 + c34 T2 + c49 T2 + c64 T2 + 4 c81 T2 + 5 c82 T2 +
4 c84 T2 + 4 c85 T2 + 4 a2 b3 T1 T2 - a4 b3 T1 T2 - 3 a2 b5 T1 T2 + a4 b5 T1 T2 - 3 c16 T1 T2 -
3 c19 T1 T2 - 3 c31 T1 T2 - 3 c34 T1 T2 - 3 c46 T1 T2 - 3 c49 T1 T2 - 3 c61 T1 T2 - 3 c64 T1 T2 -
17 c81 T1 T2 - 17 c82 T1 T2 - 13 c84 T1 T2 - 13 c85 T1 T2 - 8 a2 b3 T12 T2 + 3 a4 b3 T12 T2 +
5 a2 b5 T12 T2 - 2 a4 b5 T12 T2 + 7 c16 T12 T2 + 5 c19 T12 T2 + 7 c31 T12 T2 + 5 c34 T12 T2 +
7 c46 T12 T2 + 5 c49 T12 T2 + 7 c61 T12 T2 + 5 c64 T12 T2 + 28 c81 T12 T2 + 28 c82 T12 T2 + 15 c84 T12 T2 +
20 c85 T12 T2 + 10 a2 b3 T13 T2 - 5 a4 b3 T13 T2 - 5 a2 b5 T13 T2 + 3 a4 b5 T13 T2 - 9 c16 T13 T2 -
5 c19 T13 T2 - 9 c31 T13 T2 - 5 c34 T13 T2 - 9 c46 T13 T2 - 5 c49 T13 T2 - 9 c61 T13 T2 - 5 c64 T13 T2 -
23 c81 T13 T2 - 26 c82 T13 T2 - 8 c84 T13 T2 - 16 c85 T13 T2 - 8 a2 b3 T14 T2 + 5 a4 b3 T14 T2 +
3 a2 b5 T14 T2 - 2 a4 b5 T14 T2 + 5 c16 T14 T2 + 3 c19 T14 T2 + 5 c31 T14 T2 + 3 c34 T14 T2 +
5 c46 T14 T2 + 3 c49 T14 T2 + 5 c61 T14 T2 + 3 c64 T14 T2 + 10 c82 T14 T2 - c84 T14 T2 +
2 c85 T14 T2 + 4 a2 b3 T15 T2 - 3 a4 b3 T15 T2 - a2 b5 T15 T2 + a4 b5 T15 T2 - c16 T15 T2 -
c31 T15 T2 - c34 T15 T2 + 4 c46 T15 T2 - c49 T15 T2 + 4 c61 T15 T2 - c64 T15 T2 + 4 c81 T15 T2 +
4 c82 T15 T2 + 3 c84 T15 T2 + 8 c85 T15 T2 - a2 b3 T16 T2 - 3 a4 b3 T16 T2 - 2 a2 b5 T16 T2 - 3 c19 T16 T2 - 3 c34 T16 T2 - 3 c49 T16 T2 - 3 c64 T16 T2 -
8 c81 T16 T2 - 10 c82 T16 T2 - 8 c84 T16 T2 - 6 c85 T16 T2 - 7 a2 b3 T17 T2 + 7 a4 b3 T17 T2 - 8 a2 b5 T17 T2 - 2 a4 b5 T17 T2 -
7 a4 b5 T18 T2 + 5 c16 T18 T2 + 7 c19 T18 T2 + 5 c31 T18 T2 + 7 c34 T18 T2 + 5 c46 T18 T2 + 7 c49 T18 T2 +
5 c61 T18 T2 + 7 c64 T18 T2 + 28 c81 T18 T2 + 28 c82 T18 T2 + 20 c84 T18 T2 + 15 c85 T18 T2 +
11 a2 b3 T19 T2 - 16 a4 b3 T19 T2 - 11 a2 b5 T19 T2 + 12 a4 b5 T19 T2 - 9 c16 T19 T2 - 9 c31 T19 T2 -
9 c34 T19 T2 - 9 c46 T19 T2 - 9 c49 T19 T2 - 9 c61 T19 T2 - 9 c64 T19 T2 - 34 c81 T19 T2 -
34 c82 T19 T2 - 14 c84 T19 T2 - 14 c85 T19 T2 - 8 a2 b3 T110 T2 + 21 a4 b3 T110 T2 + 7 a2 b5 T110 T2 - 14 a4 b5 T110 T2 +
9 c16 T110 T2 + 5 c19 T110 T2 + 9 c31 T110 T2 + 5 c34 T110 T2 + 9 c46 T110 T2 + 5 c49 T110 T2 +
5 c61 T110 T2 + 9 c64 T110 T2 + 28 c81 T110 T2 + 28 c82 T110 T2 + 12 a4 b5 T111 T2 - 3 c84 T111 T2 -
4 c85 T111 T2 - a2 b3 T112 T2 - 15 a4 b3 T112 T2 + 6 a4 b5 T112 T2 - c16 T112 T2 + c19 T112 T2 - c31 T112 T2 +
c34 T112 T2 - c46 T112 T2 + c49 T112 T2 - c61 T112 T2 + c64 T112 T2 + 17 c81 T112 T2 + 24 c82 T112 T2 +
14 c84 T112 T2 + 26 c85 T112 T2 + 7 a2 b3 T113 T2 + 4 a4 b3 T113 T2 - 3 a2 b5 T113 T2 - a4 b5 T113 T2 -
10 c16 T113 T2 - 3 c19 T113 T2 - 10 c31 T113 T2 - 3 c34 T113 T2 - 10 c46 T113 T2 - 3 c49 T113 T2 -
10 c61 T113 T2 - 3 c64 T113 T2 - 22 c81 T113 T2 - 36 c82 T113 T2 - 8 c84 T113 T2 - 28 c85 T113 T2 +
6 a2 b3 T114 T2 + 2 a4 b3 T114 T2 + 2 a2 b5 T114 T2 - 2 a4 b5 T114 T2 + 2 c16 T114 T2 + 2 c19 T114 T2 +
```

$$\begin{aligned}
& 2 c_{31} T_1^6 T_2^2 + 2 c_{34} T_1^6 T_2^2 + 2 c_{46} T_1^6 T_2^2 + 2 c_{49} T_1^6 T_2^2 + 2 c_{61} T_1^6 T_2^2 + 2 c_{64} T_1^6 T_2^2 + 7 c_{81} T_1^6 T_2^2 + \\
& 22 c_{82} T_1^6 T_2^2 - c_{84} T_1^6 T_2^2 + 14 c_{85} T_1^6 T_2^2 + 2 a_2 b_3 T_1^7 T_2^2 - 2 a_4 b_3 T_1^7 T_2^2 + 4 c_{16} T_1^7 T_2^2 + \\
& 4 c_{31} T_1^7 T_2^2 + 4 c_{46} T_1^7 T_2^2 + 4 c_{61} T_1^7 T_2^2 + c_{81} T_1^7 T_2^2 - 4 c_{82} T_1^7 T_2^2 - c_{85} T_1^7 T_2^2 - 2 c_{82} T_1^8 T_2^2 - \\
& 2 c_{85} T_1^8 T_2^2 - 3 a_2 b_3 T_1^8 T_2^2 + 2 a_4 b_3 T_1^8 T_2^2 + 5 a_2 b_5 T_1^8 T_2^2 - 3 a_4 b_5 T_1^8 T_2^2 + 5 c_{19} T_1^8 T_2^2 + 5 c_{34} T_1^8 T_2^2 + 5 c_{49} T_1^8 T_2^2 + \\
& 5 c_{64} T_1^9 T_2^3 + 10 c_{81} T_1^9 T_2^3 + 11 c_{82} T_1^9 T_2^3 + 10 c_{84} T_1^9 T_2^3 + 5 c_{85} T_1^9 T_2^3 + 9 a_2 b_3 T_1 T_2^3 - 12 a_4 b_3 T_1 T_2^3 - \\
& 11 a_2 b_5 T_1 T_2^3 + 16 a_4 b_5 T_1 T_2^3 - 5 c_{16} T_1 T_2^3 - 9 c_{19} T_1 T_2^3 - 5 c_{31} T_1 T_2^3 - 9 c_{34} T_1 T_2^3 - \\
& 5 c_{46} T_1 T_2^3 - 9 c_{49} T_1 T_2^3 - 5 c_{61} T_1 T_2^3 - 9 c_{64} T_1 T_2^3 - 26 c_{81} T_1 T_2^3 - 23 c_{82} T_1 T_2^3 - \\
& 16 c_{84} T_1 T_2^3 - 8 c_{85} T_1 T_2^3 - 11 a_2 b_3 T_1 T_2^3 + 22 a_4 b_3 T_1 T_2^3 + 11 a_2 b_5 T_1 T_2^3 - 22 a_4 b_5 T_1 T_2^3 + \\
& 5 c_{16} T_1 T_2^3 + 9 c_{19} T_1 T_2^3 + 5 c_{31} T_1 T_2^3 + 9 c_{34} T_1 T_2^3 + 5 c_{46} T_1 T_2^3 + 9 c_{49} T_1 T_2^3 + 5 c_{61} T_1 T_2^3 + \\
& 9 c_{64} T_1 T_2^3 + 12 c_{81} T_1 T_2^3 + 11 c_{82} T_1 T_2^3 - 4 c_{84} T_1 T_2^3 - 3 c_{85} T_1 T_2^3 + 2 a_2 b_3 T_1 T_2^3 - \\
& 19 a_4 b_3 T_1 T_2^3 + a_2 b_5 T_1 T_2^3 + 18 a_4 b_5 T_1 T_2^3 - c_{16} T_1 T_2^3 - c_{19} T_1 T_2^3 - c_{31} T_1 T_2^3 - c_{34} T_1 T_2^3 - \\
& c_{46} T_1 T_2^3 - c_{49} T_1 T_2^3 - c_{61} T_1 T_2^3 - c_{64} T_1 T_2^3 + 30 c_{81} T_1 T_2^3 + 30 c_{82} T_1 T_2^3 + 26 c_{84} T_1 T_2^3 + \\
& 26 c_{85} T_1 T_2^3 + 13 a_2 b_3 T_1 T_2^3 - 2 a_4 b_3 T_1 T_2^3 - 13 a_2 b_5 T_1 T_2^3 + 5 a_4 b_5 T_1 T_2^3 - 8 c_{16} T_1 T_2^3 - \\
& 5 c_{19} T_1 T_2^3 - 8 c_{31} T_1 T_2^3 - 5 c_{34} T_1 T_2^3 - 8 c_{46} T_1 T_2^3 - 5 c_{49} T_1 T_2^3 - 8 c_{61} T_1 T_2^3 - 5 c_{64} T_1 T_2^3 - \\
& 54 c_{81} T_1 T_2^3 - 66 c_{82} T_1 T_2^3 - 28 c_{84} T_1 T_2^3 - 40 c_{85} T_1 T_2^3 - 19 a_2 b_3 T_1 T_2^5 + 20 a_4 b_3 T_1 T_2^5 + \\
& 13 a_2 b_5 T_1 T_2^5 - 11 a_4 b_5 T_1 T_2^5 + 13 c_{16} T_1 T_2^5 + 3 c_{19} T_1 T_2^5 + 13 c_{31} T_1 T_2^5 + 3 c_{34} T_1 T_2^5 + \\
& 13 c_{46} T_1 T_2^5 + 3 c_{49} T_1 T_2^5 + 13 c_{61} T_1 T_2^5 + 3 c_{64} T_1 T_2^5 + 36 c_{81} T_1 T_2^5 + 54 c_{82} T_1 T_2^5 + \\
& 8 c_{84} T_1 T_2^5 + 26 c_{85} T_1 T_2^5 + 13 a_2 b_3 T_1 T_2^6 - 19 a_4 b_3 T_1 T_2^6 - 6 a_2 b_5 T_1 T_2^6 + 10 a_4 b_5 T_1 T_2^6 + \\
& 5 c_{16} T_1 T_2^6 + 5 c_{31} T_1 T_2^6 + 5 c_{46} T_1 T_2^6 + 5 c_{61} T_1 T_2^6 - 4 c_{81} T_1 T_2^6 - 17 c_{82} T_1 T_2^6 + 4 c_{84} T_1 T_2^6 - \\
& 3 c_{85} T_1 T_2^6 - 4 a_2 b_3 T_1 T_2^7 - 8 a_4 b_3 T_1 T_2^7 - 7 c_{16} T_1 T_2^7 - 2 c_{19} T_1 T_2^7 - 7 c_{31} T_1 T_2^7 - \\
& 2 c_{34} T_1 T_2^7 - 7 c_{46} T_1 T_2^7 - 2 c_{49} T_1 T_2^7 - 7 c_{61} T_1 T_2^7 - 2 c_{64} T_1 T_2^7 - 4 c_{81} T_1 T_2^7 - 7 c_{82} T_1 T_2^7 - \\
& 8 c_{85} T_1 T_2^7 - 2 c_{16} T_1 T_2^8 - 2 c_{31} T_1 T_2^8 - 2 c_{46} T_1 T_2^8 - 2 c_{61} T_1 T_2^8 + 7 c_{82} T_1 T_2^8 + 5 c_{85} T_1 T_2^8 + \\
& 2 a_2 b_3 T_1 T_2^8 - 3 a_4 b_3 T_1 T_2^8 - 5 a_2 b_5 T_1 T_2^8 + 5 a_4 b_5 T_1 T_2^8 - 5 c_{19} T_1 T_2^8 - 5 c_{34} T_1 T_2^8 - 5 c_{49} T_1 T_2^8 - 5 c_{64} T_1 T_2^8 - \\
& 8 c_{81} T_1 T_2^8 - 7 c_{82} T_1 T_2^8 - 8 c_{84} T_1 T_2^8 - 2 c_{85} T_1 T_2^8 - 3 a_2 b_3 T_1 T_2^8 + 14 a_4 b_3 T_1 T_2^8 + 7 a_2 b_5 T_1 T_2^8 - \\
& 21 a_4 b_5 T_1 T_2^8 + 3 c_{16} T_1 T_2^8 + 5 c_{19} T_1 T_2^8 + 3 c_{31} T_1 T_2^8 + 5 c_{34} T_1 T_2^8 + 3 c_{46} T_1 T_2^8 + 5 c_{49} T_1 T_2^8 + \\
& 3 c_{61} T_1 T_2^8 + 5 c_{64} T_1 T_2^8 + 10 c_{81} T_1 T_2^8 + 7 c_{82} T_1 T_2^8 + 2 c_{84} T_1 T_2^8 - c_{85} T_1 T_2^8 - 5 a_2 b_3 T_1 T_2^8 - \\
& 18 a_4 b_3 T_1 T_2^8 + a_2 b_5 T_1 T_2^8 + 19 a_4 b_5 T_1 T_2^8 + c_{16} T_1 T_2^8 - c_{19} T_1 T_2^8 + c_{31} T_1 T_2^8 - c_{34} T_1 T_2^8 + \\
& c_{46} T_1 T_2^8 - c_{49} T_1 T_2^8 + c_{61} T_1 T_2^8 - c_{64} T_1 T_2^8 + 24 c_{81} T_1 T_2^8 + 17 c_{82} T_1 T_2^8 + 26 c_{84} T_1 T_2^8 + \\
& 14 c_{85} T_1 T_2^8 + 24 a_2 b_3 T_1 T_2^8 + a_4 b_3 T_1 T_2^8 - 21 a_2 b_5 T_1 T_2^8 - a_4 b_5 T_1 T_2^8 - 5 c_{16} T_1 T_2^8 - \\
& 8 c_{19} T_1 T_2^8 - 5 c_{31} T_1 T_2^8 - 8 c_{34} T_1 T_2^8 - 5 c_{46} T_1 T_2^8 - 8 c_{49} T_1 T_2^8 - 5 c_{61} T_1 T_2^8 - 8 c_{64} T_1 T_2^8 - \\
& 66 c_{81} T_1 T_2^8 - 54 c_{82} T_1 T_2^8 - 40 c_{84} T_1 T_2^8 - 28 c_{85} T_1 T_2^8 - 41 a_2 b_3 T_1 T_2^8 + 34 a_4 b_3 T_1 T_2^8 + \\
& 33 a_2 b_5 T_1 T_2^8 - 37 a_4 b_5 T_1 T_2^8 + 8 c_{16} T_1 T_2^8 + 8 c_{19} T_1 T_2^8 + 8 c_{31} T_1 T_2^8 + 8 c_{34} T_1 T_2^8 + \\
& 8 c_{46} T_1 T_2^8 + 8 c_{49} T_1 T_2^8 + 8 c_{61} T_1 T_2^8 + 8 c_{64} T_1 T_2^8 + 66 c_{81} T_1 T_2^8 + 66 c_{82} T_1 T_2^8 + 26 c_{84} T_1 T_2^8 + \\
& 26 c_{85} T_1 T_2^8 + 39 a_2 b_3 T_1 T_2^8 - 52 a_4 b_3 T_1 T_2^8 - 25 a_2 b_5 T_1 T_2^8 + 35 a_4 b_5 T_1 T_2^8 - 3 c_{16} T_1 T_2^8 - \\
& 3 c_{31} T_1 T_2^8 - 3 c_{46} T_1 T_2^8 - 3 c_{61} T_1 T_2^8 - 24 c_{81} T_1 T_2^8 - 30 c_{82} T_1 T_2^8 + 2 c_{84} T_1 T_2^8 - 4 c_{85} T_1 T_2^8 - \\
& 22 a_2 b_3 T_1 T_2^8 + 38 a_4 b_3 T_1 T_2^8 + 10 a_2 b_5 T_1 T_2^8 - 22 a_4 b_5 T_1 T_2^8 - 15 c_{16} T_1 T_2^8 - 3 c_{19} T_1 T_2^8 - \\
& 15 c_{31} T_1 T_2^8 - 3 c_{34} T_1 T_2^8 - 15 c_{46} T_1 T_2^8 - 3 c_{49} T_1 T_2^8 - 15 c_{61} T_1 T_2^8 - 3 c_{64} T_1 T_2^8 - \\
& 10 c_{81} T_1 T_2^8 - 11 c_{82} T_1 T_2^8 - 8 c_{84} T_1 T_2^8 - 14 c_{85} T_1 T_2^8 + 6 a_2 b_3 T_1 T_2^8 - 14 a_4 b_3 T_1 T_2^8 + \\
& 5 c_{16} T_1 T_2^8 + 3 c_{19} T_1 T_2^8 + 5 c_{31} T_1 T_2^8 + 3 c_{34} T_1 T_2^8 + 5 c_{46} T_1 T_2^8 + 3 c_{49} T_1 T_2^8 + 5 c_{61} T_1 T_2^8 + \\
& 3 c_{64} T_1 T_2^8 + 8 c_{81} T_1 T_2^8 + 23 c_{82} T_1 T_2^8 + 15 c_{85} T_1 T_2^8 + 6 c_{16} T_1 T_2^8 + c_{19} T_1 T_2^8 + 6 c_{31} T_1 T_2^8 + \\
& c_{34} T_1 T_2^8 + 6 c_{46} T_1 T_2^8 + c_{49} T_1 T_2^8 + 6 c_{61} T_1 T_2^8 + c_{64} T_1 T_2^8 - 11 c_{82} T_1 T_2^8 - 6 c_{85} T_1 T_2^8 - \\
& a_2 b_3 T_1 T_2^8 + 2 a_4 b_3 T_1 T_2^8 + 3 a_2 b_5 T_1 T_2^8 - 5 a_4 b_5 T_1 T_2^8 + 2 c_{19} T_1 T_2^8 + 2 c_{34} T_1 T_2^8 + 2 c_{49} T_1 T_2^8 + 2 c_{64} T_1 T_2^8 + \\
& 4 c_{81} T_1 T_2^8 + 2 c_{82} T_1 T_2^8 + 4 c_{84} T_1 T_2^8 - 6 a_4 b_3 T_1 T_2^8 + 15 a_4 b_5 T_1 T_2^8 - c_{16} T_1 T_2^8 + 4 c_{19} T_1 T_2^8 - \\
& c_{31} T_1 T_2^8 + 4 c_{34} T_1 T_2^8 - c_{46} T_1 T_2^8 + 4 c_{49} T_1 T_2^8 - c_{61} T_1 T_2^8 + 4 c_{64} T_1 T_2^8 + 4 c_{81} T_1 T_2^8 + \\
& 4 c_{82} T_1 T_2^8 + 8 c_{84} T_1 T_2^8 + 3 c_{85} T_1 T_2^8 + 8 a_2 b_3 T_1 T_2^8 - 5 a_4 b_3 T_1 T_2^8 - 13 a_2 b_5 T_1 T_2^8 + \\
& 2 a_4 b_5 T_1 T_2^8 - 3 c_{16} T_1 T_2^8 - 10 c_{19} T_1 T_2^8 - 3 c_{31} T_1 T_2^8 - 10 c_{34} T_1 T_2^8 - 3 c_{46} T_1 T_2^8 -
\end{aligned}$$

$$\begin{aligned}
& 10 c_{49} T_1^2 T_2^5 - 3 c_{61} T_1^2 T_2^5 - 10 c_{64} T_1^2 T_2^5 - 36 c_{81} T_1^2 T_2^5 - 22 c_{82} T_1^2 T_2^5 - 28 c_{84} T_1^2 T_2^5 - \\
& 8 c_{85} T_1^2 T_2^5 - 22 a_2 b_3 T_1^3 T_2^5 + 37 a_4 b_3 T_1^3 T_2^5 + 33 a_2 b_5 T_1^3 T_2^5 - 34 a_4 b_5 T_1^3 T_2^5 + 3 c_{16} T_1^3 T_2^5 + \\
& 13 c_{19} T_1^3 T_2^5 + 3 c_{31} T_1^3 T_2^5 + 13 c_{34} T_1^3 T_2^5 + 3 c_{46} T_1^3 T_2^5 + 13 c_{49} T_1^3 T_2^5 + 3 c_{61} T_1^3 T_2^5 + \\
& 13 c_{64} T_1^3 T_2^5 + 54 c_{81} T_1^3 T_2^5 + 36 c_{82} T_1^3 T_2^5 + 26 c_{84} T_1^3 T_2^5 + 8 c_{85} T_1^3 T_2^5 + 32 a_2 b_3 T_1^4 T_2^5 - \\
& 74 a_4 b_3 T_1^4 T_2^5 - 40 a_2 b_5 T_1^4 T_2^5 + 74 a_4 b_5 T_1^4 T_2^5 - 3 c_{19} T_1^4 T_2^5 - 3 c_{34} T_1^4 T_2^5 - 3 c_{49} T_1^4 T_2^5 - \\
& 3 c_{64} T_1^4 T_2^5 - 30 c_{81} T_1^4 T_2^5 - 24 c_{82} T_1^4 T_2^5 - 4 c_{84} T_1^4 T_2^5 + 2 c_{85} T_1^4 T_2^5 - 28 a_2 b_3 T_1^5 T_2^5 + \\
& 79 a_4 b_3 T_1^5 T_2^5 + 27 a_2 b_5 T_1^5 T_2^5 - 57 a_4 b_5 T_1^5 T_2^5 - 7 c_{16} T_1^5 T_2^5 - 7 c_{19} T_1^5 T_2^5 - 7 c_{31} T_1^5 T_2^5 - \\
& 7 c_{34} T_1^5 T_2^5 - 7 c_{46} T_1^5 T_2^5 - 7 c_{49} T_1^5 T_2^5 - 7 c_{61} T_1^5 T_2^5 - 7 c_{64} T_1^5 T_2^5 - 12 c_{81} T_1^5 T_2^5 - \\
& 12 c_{82} T_1^5 T_2^5 - 16 c_{84} T_1^5 T_2^5 - 16 c_{85} T_1^5 T_2^5 + 15 a_2 b_3 T_1^6 T_2^5 - 49 a_4 b_3 T_1^6 T_2^5 - 10 a_2 b_5 T_1^6 T_2^5 + \\
& 30 a_4 b_5 T_1^6 T_2^5 + 13 c_{16} T_1^6 T_2^5 + 5 c_{19} T_1^6 T_2^5 + 13 c_{31} T_1^6 T_2^5 + 5 c_{34} T_1^6 T_2^5 + 13 c_{46} T_1^6 T_2^5 + \\
& 5 c_{49} T_1^6 T_2^5 + 13 c_{61} T_1^6 T_2^5 + 5 c_{64} T_1^6 T_2^5 + 26 c_{81} T_1^6 T_2^5 + 34 c_{82} T_1^6 T_2^5 + 10 c_{84} T_1^6 T_2^5 + \\
& 20 c_{85} T_1^6 T_2^5 - 4 a_2 b_3 T_1^7 T_2^5 + 16 a_4 b_3 T_1^7 T_2^5 + 5 c_{16} T_1^7 T_2^5 - c_{19} T_1^7 T_2^5 + 5 c_{31} T_1^7 T_2^5 - \\
& c_{34} T_1^7 T_2^5 + 5 c_{46} T_1^7 T_2^5 - c_{49} T_1^7 T_2^5 + 5 c_{61} T_1^7 T_2^5 - c_{64} T_1^7 T_2^5 - 10 c_{81} T_1^7 T_2^5 - 28 c_{82} T_1^7 T_2^5 - \\
& 13 c_{85} T_1^7 T_2^5 - 10 c_{16} T_1^8 T_2^5 - 3 c_{19} T_1^8 T_2^5 - 10 c_{31} T_1^8 T_2^5 - 3 c_{34} T_1^8 T_2^5 - 10 c_{46} T_1^8 T_2^5 - \\
& 3 c_{49} T_1^8 T_2^5 - 10 c_{61} T_1^8 T_2^5 - 3 c_{64} T_1^8 T_2^5 + 10 c_{82} T_1^8 T_2^5 + 4 c_{85} T_1^8 T_2^5 - a_4 b_3 T_1^6 - a_2 b_5 T_1^6 + \\
& 3 a_4 b_5 T_1^6 - c_{81} T_1^6 - c_{84} T_1^6 + 2 a_2 b_3 T_1^6 + a_4 b_3 T_1^6 - 3 a_2 b_5 T_1^6 - 4 a_4 b_5 T_1^6 - \\
& 4 c_{19} T_1^6 - 4 c_{34} T_1^6 - 4 c_{49} T_1^6 - 4 c_{64} T_1^6 - 7 c_{81} T_1^6 - 3 c_{82} T_1^6 - 8 c_{84} T_1^6 - \\
& 8 a_2 b_3 T_1^6 + 11 a_4 b_3 T_1^6 + 13 a_2 b_5 T_1^6 - 20 a_4 b_5 T_1^6 + 2 c_{16} T_1^6 + 2 c_{19} T_1^6 + \\
& 2 c_{31} T_1^6 + 2 c_{34} T_1^6 + 2 c_{46} T_1^6 + 2 c_{49} T_1^6 + 2 c_{61} T_1^6 + 2 c_{64} T_1^6 + 22 c_{81} T_1^6 + \\
& 7 c_{82} T_1^6 + 14 c_{84} T_1^6 + c_{85} T_1^6 + 16 a_2 b_3 T_1^6 - 35 a_4 b_3 T_1^6 - 25 a_2 b_5 T_1^6 + \\
& 52 a_4 b_5 T_1^6 + 5 c_{19} T_1^6 + 5 c_{34} T_1^6 + 5 c_{49} T_1^6 + 5 c_{64} T_1^6 - 17 c_{81} T_1^6 - \\
& 4 c_{82} T_1^6 - 3 c_{84} T_1^6 + 4 c_{85} T_1^6 - 20 a_2 b_3 T_1^6 + 57 a_4 b_3 T_1^6 + 27 a_2 b_5 T_1^6 - \\
& 79 a_4 b_5 T_1^6 - 3 c_{16} T_1^6 - 15 c_{19} T_1^6 - 3 c_{31} T_1^6 - 15 c_{34} T_1^6 - 3 c_{46} T_1^6 - \\
& 15 c_{49} T_1^6 - 3 c_{61} T_1^6 - 15 c_{64} T_1^6 - 11 c_{81} T_1^6 - 10 c_{82} T_1^6 - 14 c_{84} T_1^6 - \\
& 8 c_{85} T_1^6 + 16 a_2 b_3 T_1^6 - 55 a_4 b_3 T_1^6 - 17 a_2 b_5 T_1^6 + 55 a_4 b_5 T_1^6 + 5 c_{16} T_1^6 + \\
& 13 c_{19} T_1^6 + 5 c_{31} T_1^6 + 13 c_{34} T_1^6 + 5 c_{46} T_1^6 + 13 c_{49} T_1^6 + 5 c_{61} T_1^6 + \\
& 13 c_{64} T_1^6 + 34 c_{81} T_1^6 + 26 c_{82} T_1^6 + 20 c_{84} T_1^6 + 10 c_{85} T_1^6 - 8 a_2 b_3 T_1^6 + \\
& 32 a_4 b_3 T_1^6 + 6 a_2 b_5 T_1^6 - 26 a_4 b_5 T_1^6 - c_{16} T_1^6 - c_{19} T_1^6 - c_{31} T_1^6 - c_{34} T_1^6 - \\
& c_{46} T_1^6 - c_{49} T_1^6 - c_{61} T_1^6 - c_{64} T_1^6 - 28 c_{81} T_1^6 - 28 c_{82} T_1^6 - 8 c_{84} T_1^6 - \\
& 8 c_{85} T_1^6 + 2 a_2 b_3 T_1^6 - 10 a_4 b_3 T_1^6 - 13 c_{16} T_1^6 - 5 c_{19} T_1^6 - 13 c_{31} T_1^6 - \\
& 5 c_{34} T_1^6 - 13 c_{46} T_1^6 - 5 c_{49} T_1^6 - 13 c_{61} T_1^6 - 5 c_{64} T_1^6 + 8 c_{81} T_1^6 + \\
& 17 c_{82} T_1^6 + 4 c_{85} T_1^6 + 10 c_{16} T_1^6 + 5 c_{19} T_1^6 + 10 c_{31} T_1^6 + 5 c_{34} T_1^6 + \\
& 10 c_{46} T_1^6 + 5 c_{49} T_1^6 + 10 c_{61} T_1^6 + 5 c_{64} T_1^6 - 5 c_{82} T_1^6 - c_{85} T_1^6 - a_4 b_5 T_1^6 + \\
& 2 a_4 b_3 T_1^7 + 2 a_2 b_5 T_1^7 - 2 a_4 b_5 T_1^7 + 3 c_{81} T_1^7 + 3 c_{84} T_1^7 - 10 a_4 b_3 T_1^7 - \\
& 6 a_2 b_5 T_1^7 + 19 a_4 b_5 T_1^7 + 4 c_{19} T_1^7 + 4 c_{34} T_1^7 + 4 c_{49} T_1^7 + 4 c_{64} T_1^7 - \\
& 4 c_{81} T_1^7 + c_{82} T_1^7 - c_{84} T_1^7 + 22 a_4 b_3 T_1^7 + 10 a_2 b_5 T_1^7 - 38 a_4 b_5 T_1^7 - \\
& 2 c_{16} T_1^7 - 7 c_{19} T_1^7 - 2 c_{31} T_1^7 - 7 c_{34} T_1^7 - 2 c_{46} T_1^7 - 7 c_{49} T_1^7 - 2 c_{61} T_1^7 - \\
& 7 c_{64} T_1^7 - 7 c_{81} T_1^7 - 4 c_{82} T_1^7 - 8 c_{84} T_1^7 - 30 a_4 b_3 T_1^7 - 10 a_2 b_5 T_1^7 + \\
& 49 a_4 b_5 T_1^7 + 3 c_{16} T_1^7 + 5 c_{19} T_1^7 + 3 c_{31} T_1^7 + 5 c_{34} T_1^7 + 3 c_{46} T_1^7 + \\
& 5 c_{49} T_1^7 + 3 c_{61} T_1^7 + 5 c_{64} T_1^7 + 23 c_{81} T_1^7 + 8 c_{82} T_1^7 + 15 c_{84} T_1^7 + \\
& 26 a_4 b_3 T_1^7 + 6 a_2 b_5 T_1^7 - 32 a_4 b_5 T_1^7 - c_{16} T_1^7 + 5 c_{19} T_1^7 - c_{31} T_1^7 + \\
& 5 c_{34} T_1^7 - c_{46} T_1^7 + 5 c_{49} T_1^7 - c_{61} T_1^7 + 5 c_{64} T_1^7 - 28 c_{81} T_1^7 - 10 c_{82} T_1^7 - \\
& 13 c_{84} T_1^7 - 14 a_4 b_3 T_1^7 - 2 a_2 b_5 T_1^7 + 14 a_4 b_5 T_1^7 - 5 c_{16} T_1^7 - 13 c_{19} T_1^7 - \\
& 5 c_{31} T_1^7 - 13 c_{34} T_1^7 - 5 c_{46} T_1^7 - 13 c_{49} T_1^7 - 5 c_{61} T_1^7 - 13 c_{64} T_1^7 + \\
& 17 c_{81} T_1^7 + 8 c_{82} T_1^7 + 4 c_{84} T_1^7 + 4 a_4 b_3 T_1^7 + 11 c_{16} T_1^7 + 11 c_{19} T_1^7 + \\
& 11 c_{31} T_1^7 + 11 c_{34} T_1^7 + 11 c_{46} T_1^7 + 11 c_{49} T_1^7 + 11 c_{61} T_1^7 + 11 c_{64} T_1^7
\end{aligned}$$

$$\begin{aligned}
& 4 c_{81} T_1^7 T_2^7 - 4 c_{82} T_1^7 T_2^7 - 6 c_{16} T_1^8 T_2^7 - 5 c_{19} T_1^8 T_2^7 - 6 c_{31} T_1^8 T_2^7 - 5 c_{34} T_1^8 T_2^7 - 6 c_{46} T_1^8 T_2^7 - \\
& 5 c_{49} T_1^8 T_2^7 - 6 c_{61} T_1^8 T_2^7 - 5 c_{64} T_1^8 T_2^7 + c_{82} T_1^8 T_2^7 + 2 a_4 b_5 T_1 T_2^8 - 8 a_4 b_5 T_1^2 T_2^8 - \\
& 2 c_{81} T_1^2 T_2^8 - 2 c_{84} T_1^2 T_2^8 + 14 a_4 b_5 T_1^3 T_2^8 - 2 c_{19} T_1^3 T_2^8 - 2 c_{34} T_1^3 T_2^8 - 2 c_{49} T_1^3 T_2^8 - \\
& 2 c_{64} T_1^3 T_2^8 + 7 c_{81} T_1^3 T_2^8 + 5 c_{84} T_1^3 T_2^8 - 16 a_4 b_5 T_1^4 T_2^8 + c_{16} T_1^4 T_2^8 + 6 c_{19} T_1^4 T_2^8 + c_{31} T_1^4 T_2^8 + \\
& 6 c_{34} T_1^4 T_2^8 + c_{46} T_1^4 T_2^8 + 6 c_{49} T_1^4 T_2^8 + c_{61} T_1^4 T_2^8 + 6 c_{64} T_1^4 T_2^8 - 11 c_{81} T_1^4 T_2^8 - 6 c_{84} T_1^4 T_2^8 + \\
& 10 a_4 b_5 T_1^5 T_2^8 - 3 c_{16} T_1^5 T_2^8 - 10 c_{19} T_1^5 T_2^8 - 3 c_{31} T_1^5 T_2^8 - 10 c_{34} T_1^5 T_2^8 - 3 c_{46} T_1^5 T_2^8 - \\
& 10 c_{49} T_1^5 T_2^8 - 3 c_{61} T_1^5 T_2^8 - 10 c_{64} T_1^5 T_2^8 + 10 c_{81} T_1^5 T_2^8 + 4 c_{84} T_1^5 T_2^8 - 4 a_4 b_5 T_1^6 T_2^8 + \\
& 5 c_{16} T_1^6 T_2^8 + 10 c_{19} T_1^6 T_2^8 + 5 c_{31} T_1^6 T_2^8 + 10 c_{34} T_1^6 T_2^8 + 5 c_{46} T_1^6 T_2^8 + 10 c_{49} T_1^6 T_2^8 + \\
& 5 c_{61} T_1^6 T_2^8 + 10 c_{64} T_1^6 T_2^8 - 5 c_{81} T_1^6 T_2^8 - c_{84} T_1^6 T_2^8 - 5 c_{16} T_1^7 T_2^8 - 6 c_{19} T_1^7 T_2^8 - 5 c_{31} T_1^7 T_2^8 - \\
& 6 c_{34} T_1^7 T_2^8 - 5 c_{46} T_1^7 T_2^8 - 6 c_{49} T_1^7 T_2^8 - 5 c_{61} T_1^7 T_2^8 - 6 c_{64} T_1^7 T_2^8 + c_{81} T_1^7 T_2^8 + 2 c_{16} T_1^7 T_2^8 + \\
& 2 c_{19} T_1^8 T_2^8 + 2 c_{31} T_1^8 T_2^8 + 2 c_{34} T_1^8 T_2^8 + 2 c_{46} T_1^8 T_2^8 + 2 c_{49} T_1^8 T_2^8 + 2 c_{61} T_1^8 T_2^8 + 2 c_{64} T_1^8 T_2^8) / \\
& ((-1 + T_1) (1 - T_1 + T_1^2)^2 (-1 + T_2) (-1 + T_1 T_2) (1 - T_2 + T_2^2)^2 (1 - T_1 T_2 + T_1^2 T_2^2)))] / \\
& ((1 - T_1 + T_1^2) (1 - T_2 + T_2^2) (1 - T_1 T_2 + T_1^2 T_2^2)))
\end{aligned}$$

Factor[Cases[tab1, eSeries[θ, ε_]] :> ε, ∞] / . c_{16|19|31|34|46|49|61|64|81|82|84|85} → θ]

In[1]:=

$$(-b_3 + b_5 + b_3 T_1 - b_5 T_2) (-a_2 + a_2 T_1 - a_4 T_1 - a_4 T_2 + 2 a_4 T_1 T_2)$$

Out[1]:=

$$\begin{aligned}
& \left\{ -\frac{T_2 (-1 + T_1 - T_1^2 + T_2 - T_1^2 T_2 + 2 T_1^3 T_2 - T_2^2 - T_1 T_2^2 + T_1^2 T_2^2 - 2 T_1^3 T_2^2 + 2 T_1 T_2^3 - 2 T_1^2 T_2^3 + 2 T_1^3 T_2^3)}{(-1 + T_1) (1 - T_1 + T_1^2) (-1 + T_2) (-1 + T_1 T_2) (1 - T_2 + T_2^2) (1 - T_1 T_2 + T_1^2 T_2^2)}, \right. \\
& -\frac{1 + T_1 T_2}{(-1 + T_1) T_1 (-1 + T_2) (1 - 3 T_1 T_2 + T_1^2 T_2^2)}, \\
& -\left((T_2 (-1 + T_1 - T_1^2 + T_1^3 - T_1^4 + T_2 - T_1^4 T_2 + 2 T_1^5 T_2 - T_2^2 - 2 T_1^2 T_2^2 + 2 T_1^3 T_2^2 - T_1^4 T_2^2 + T_1^5 T_2^2 - 3 T_1^6 T_2^2 + T_2^3 + 2 T_1^2 T_2^3 - 2 T_1^4 T_2^3 + 2 T_1^5 T_2^3 - T_1^6 T_2^3 + 4 T_1^7 T_2^3 - T_2^4 - T_1 T_2^4 - T_1^2 T_2^4 - 2 T_1^3 T_2^4 + 2 T_1^4 T_2^4 - 2 T_1^5 T_2^4 + T_1^6 T_2^4 - 4 T_1^7 T_2^4 + 2 T_1 T_2^5 + T_1^2 T_2^5 + 2 T_1^3 T_2^5 - 2 T_1^4 T_2^5 + 2 T_1^5 T_2^5 - T_1^6 T_2^5 + 4 T_1^7 T_2^5 - 3 T_1^8 T_2^5 - T_1^9 T_2^5 + T_1^10 T_2^5) / \right. \\
& \left. ((-1 + T_1) (1 - T_1 + T_1^2 - T_1^3 + T_1^4) (-1 + T_2) (-1 + T_1 T_2) (1 - T_2 + T_2^2 - T_2^3 + T_2^4) \right. \\
& \left. (1 - T_1 T_2 + T_1^2 T_2^2 - T_1^3 T_2^3 + T_1^4 T_2^4)) \right), \\
& -\left((-7 + 17 T_1 - 7 T_1^2 + 17 T_2 - 44 T_1 T_2 + 26 T_1^2 T_2 - 3 T_1^3 T_2 - 7 T_2^2 + 26 T_1 T_2^2 - 24 T_1^2 T_2^2 - 14 T_1^3 T_2^2 + \right. \\
& \left. 9 T_1^4 T_2^2 - 3 T_1 T_2^3 - 14 T_1^2 T_2^3 + 56 T_1^3 T_2^3 - 23 T_1^4 T_2^3 + 9 T_1^2 T_2^4 - 23 T_1^3 T_2^4 + 9 T_1^4 T_2^4) / ((-2 + T_1) (-1 + T_1) \right. \\
& \left. T_1 (-1 + 2 T_1) (-2 + T_2) (-1 + T_2) (-1 + 2 T_2) (-2 + T_1 T_2) (-1 + T_1 T_2) (-1 + 2 T_1 T_2)) \right), \\
& -\left((T_2 (-1 + T_1 - T_1^2 + T_1^3 - T_1^4 + T_1^5 - T_1^6 + T_2 - T_1^6 T_2 + 2 T_1^7 T_2 - T_2^2 - 2 T_1^2 T_2^2 + 2 T_1^3 T_2^2 - 2 T_1^4 T_2^2 + \right. \\
& \left. T_1^5 T_2^2 + T_1^6 T_2^2 - 3 T_1^8 T_2^2 + T_2^3 + 2 T_1^2 T_2^3 - 2 T_1^6 T_2^3 + 2 T_1^7 T_2^3 - T_1^8 T_2^3 + 4 T_1^9 T_2^3 - T_2^4 - 2 T_1^2 T_2^4 - 3 T_1^4 T_2^4 + \right. \\
& \left. 3 T_1^5 T_2^4 - T_1^6 T_2^4 + 2 T_1^7 T_2^4 - 3 T_1^8 T_2^4 + T_1^9 T_2^4 - 5 T_1^{10} T_2^4 + T_1^5 + 2 T_1^2 T_2^5 + 3 T_1^4 T_2^5 - 3 T_1^6 T_2^5 + 2 T_1^7 T_2^5 - \right. \\
& \left. 2 T_1^8 T_2^5 + 4 T_1^9 T_2^5 - T_1^{10} T_2^5 + 6 T_1^{11} T_2^5 - T_1^6 - T_1 T_2^6 - T_1^2 T_2^6 - 2 T_1^3 T_2^6 - T_1^4 T_2^6 - 3 T_1^5 T_2^6 + 3 T_1^6 T_2^6 - \right. \\
& \left. 2 T_1^7 T_2^6 + 2 T_1^8 T_2^6 - 4 T_1^9 T_2^6 + T_1^{10} T_2^6 - 6 T_1^{11} T_2^6 + 2 T_1 T_2^7 + T_1^2 T_2^7 + 2 T_1^3 T_2^7 + 2 T_1^4 T_2^7 + 2 T_1^5 T_2^7 - \right. \\
& \left. 2 T_1^6 T_2^7 + 2 T_1^7 T_2^7 - 2 T_1^8 T_2^7 + 4 T_1^9 T_2^7 - T_1^{10} T_2^7 + 6 T_1^{11} T_2^7 - 3 T_1^2 T_2^8 - T_1^3 T_2^8 - 3 T_1^4 T_2^8 - 2 T_1^5 T_2^8 + \right. \\
& \left. 2 T_1^6 T_2^8 - 2 T_1^7 T_2^8 + 2 T_1^8 T_2^8 - 4 T_1^9 T_2^8 + T_1^{10} T_2^8 - 6 T_1^{11} T_2^8 + 4 T_1^3 T_2^9 + T_1^4 T_2^9 + 4 T_1^5 T_2^9 - 4 T_1^6 T_2^9 + \right. \\
& \left. 4 T_1^7 T_2^9 - 4 T_1^8 T_2^9 + 4 T_1^9 T_2^9 - T_1^{10} T_2^9 + 6 T_1^{11} T_2^9 - 5 T_1^4 T_2^{10} - T_1^5 T_2^{10} + T_1^6 T_2^{10} - T_1^7 T_2^{10} + T_1^8 T_2^{10} - T_1^9 T_2^{10} + \right. \\
& \left. T_1^{10} T_2^{10} - 6 T_1^{11} T_2^{10} + 6 T_1^5 T_2^{11} - 6 T_1^6 T_2^{11} + 6 T_1^7 T_2^{11} - 6 T_1^8 T_2^{11} + 6 T_1^9 T_2^{11} - 6 T_1^{10} T_2^{11} + 6 T_1^{11} T_2^{11}) / \right. \\
& \left. ((-1 + T_1) (1 - T_1 + T_1^2 - T_1^3 + T_1^4 - T_1^5 + T_1^6) (-1 + T_2) (-1 + T_1 T_2) (1 - T_2 + T_2^2 - T_2^3 + T_2^4 - T_2^5 + T_2^6) \right. \\
& \left. (1 - T_1 T_2 + T_1^2 T_2^2 - T_1^3 T_2^3 + T_1^4 T_2^4 - T_1^5 T_2^5 + T_1^6 T_2^6)) \right), \\
& -\left((-18 + 40 T_1 - 18 T_1^2 + 40 T_2 - 111 T_1 T_2 + 99 T_1^2 T_2 - 23 T_1^3 T_2 - 18 T_2^2 + 99 T_1 T_2^2 - 132 T_1^2 T_2^2 - \right. \\
& \left. 27 T_1^3 T_2^2 + 36 T_1^4 T_2^2 - 23 T_1 T_2^3 - 27 T_1^2 T_2^3 + 183 T_1^3 T_2^3 - 86 T_1^4 T_2^3 + 36 T_1^2 T_2^4 - 86 T_1^3 T_2^4 + 36 T_1^4 T_2^4) /
\right.
\end{aligned}$$

$$\begin{aligned}
& \left((-1 + T_1) T_1 (3 - 7 T_1 + 3 T_1^2) (-1 + T_2) (-1 + T_1 T_2) (3 - 7 T_2 + 3 T_2^2) (3 - 7 T_1 T_2 + 3 T_1^2 T_2^2) \right) , \\
& - \left((T_2 (-1 + T_1 - T_1^2 + T_1^3 - T_1^4 + T_1^5 - T_1^6 + T_1^7 - T_1^8 + T_2 - T_1^8 T_2 + 2 T_1^9 T_2 - T_2^2 - 2 T_1^2 T_2^2 + 2 T_1^3 T_2^2 - 2 T_1^4 T_2^2 + \right. \\
& \quad 2 T_1^5 T_2^2 - 2 T_1^6 T_2^2 + 2 T_1^7 T_2^2 - T_1^8 T_2^2 + T_1^9 T_2^2 - 3 T_1^{10} T_2^2 + T_1^3 + 2 T_1^2 T_2^3 - 2 T_1^8 T_2^3 + 2 T_1^9 T_2^3 - T_1^{10} T_2^3 + \\
& \quad 4 T_1^{11} T_2^3 - T_1^{12} T_2^4 - 2 T_1^2 T_2^4 - 3 T_1^4 T_2^4 + 3 T_1^5 T_2^4 - 3 T_1^6 T_2^4 + 3 T_1^7 T_2^4 - T_1^8 T_2^4 + 2 T_1^9 T_2^4 - 3 T_1^{10} T_2^4 + T_1^{11} T_2^4 - \\
& \quad 5 T_1^{12} T_2^4 + T_1^5 + 2 T_1^2 T_2^5 + 3 T_1^4 T_2^5 - 3 T_1^8 T_2^5 + 2 T_1^9 T_2^5 - 2 T_1^{10} T_2^5 + 4 T_1^{11} T_2^5 - T_1^{12} T_2^5 + 6 T_1^{13} T_2^5 - T_2^6 - \\
& \quad 2 T_1^2 T_2^6 - 3 T_1^4 T_2^6 - 4 T_1^6 T_2^6 + 4 T_1^7 T_2^6 - T_1^8 T_2^6 + 3 T_1^9 T_2^6 - 3 T_1^{10} T_2^6 + 2 T_1^{11} T_2^6 - 5 T_1^{12} T_2^6 + T_1^{13} T_2^6 - \\
& \quad 7 T_1^{14} T_2^6 + T_1^7 + 2 T_1^2 T_2^7 + 3 T_1^4 T_2^7 - 4 T_1^8 T_2^7 + 2 T_1^9 T_2^7 - 3 T_1^{10} T_2^7 + 4 T_1^{11} T_2^7 - 2 T_1^{12} T_2^7 + \\
& \quad 6 T_1^{13} T_2^7 - T_1^{14} T_2^7 + 8 T_1^{15} T_2^7 - T_1^8 - T_1^2 T_2^8 - 2 T_1^3 T_2^8 - T_1^4 T_2^8 - 3 T_1^5 T_2^8 - T_1^6 T_2^8 - 4 T_1^7 T_2^8 + \\
& \quad 4 T_1^8 T_2^8 - 2 T_1^9 T_2^8 + 3 T_1^{10} T_2^8 - 4 T_1^{11} T_2^8 + 2 T_1^{12} T_2^8 - 6 T_1^{13} T_2^8 + T_1^{14} T_2^8 - 8 T_1^{15} T_2^8 + 2 T_1 T_2^9 + T_1^2 T_2^9 + \\
& \quad 2 T_1^3 T_2^9 + 2 T_1^4 T_2^9 + 2 T_1^5 T_2^9 + 3 T_1^6 T_2^9 + 2 T_1^7 T_2^9 - 2 T_1^8 T_2^9 + 2 T_1^9 T_2^9 - 3 T_1^{10} T_2^9 + 4 T_1^{11} T_2^9 - 2 T_1^{12} T_2^9 + \\
& \quad 6 T_1^{13} T_2^9 - T_1^{14} T_2^9 + 8 T_1^{15} T_2^9 - 3 T_1^2 T_2^{10} - T_1^3 T_2^{10} - 3 T_1^4 T_2^{10} - 2 T_1^5 T_2^{10} - 3 T_1^6 T_2^{10} - 3 T_1^7 T_2^{10} + \\
& \quad 3 T_1^8 T_2^{10} - 3 T_1^9 T_2^{10} + 3 T_1^{10} T_2^{10} - 4 T_1^{11} T_2^{10} + 2 T_1^{12} T_2^{10} - 6 T_1^{13} T_2^{10} + T_1^{14} T_2^{10} - 8 T_1^{15} T_2^{10} + 4 T_1^3 T_2^{11} + \\
& \quad T_1^4 T_2^{11} + 4 T_1^5 T_2^{11} + 2 T_1^6 T_2^{11} + 4 T_1^7 T_2^{11} - 4 T_1^8 T_2^{11} + 4 T_1^9 T_2^{11} - 4 T_1^{10} T_2^{11} + 4 T_1^{11} T_2^{11} - 2 T_1^{12} T_2^{11} + \\
& \quad 6 T_1^{13} T_2^{11} - T_1^{14} T_2^{11} + 8 T_1^{15} T_2^{11} - 5 T_1^4 T_2^{12} - T_1^5 T_2^{12} - 5 T_1^6 T_2^{12} - 2 T_1^7 T_2^{12} + 2 T_1^8 T_2^{12} - 2 T_1^9 T_2^{12} + \\
& \quad 2 T_1^{10} T_2^{12} - 2 T_1^{11} T_2^{12} + 2 T_1^{12} T_2^{12} - 6 T_1^{13} T_2^{12} + T_1^{14} T_2^{12} - 8 T_1^{15} T_2^{12} + 6 T_1^5 T_2^{13} + T_1^6 T_2^{13} + 6 T_1^7 T_2^{13} - \\
& \quad 6 T_1^8 T_2^{13} + 6 T_1^9 T_2^{13} - 6 T_1^{10} T_2^{13} + 6 T_1^{11} T_2^{13} - 6 T_1^{12} T_2^{13} + 6 T_1^{13} T_2^{13} - T_1^{14} T_2^{13} + 8 T_1^{15} T_2^{13} - 7 T_1^6 T_2^{14} - \\
& \quad T_1^7 T_2^{14} + T_1^8 T_2^{14} - T_1^9 T_2^{14} + T_1^{10} T_2^{14} - T_1^{11} T_2^{14} + T_1^{12} T_2^{14} - T_1^{13} T_2^{14} + T_1^{14} T_2^{14} - 8 T_1^{15} T_2^{14} + 8 T_1^7 T_2^{15} - \\
& \quad 8 T_1^8 T_2^{15} + 8 T_1^9 T_2^{15} - 8 T_1^{10} T_2^{15} + 8 T_1^{11} T_2^{15} - 8 T_1^{12} T_2^{15} + 8 T_1^{13} T_2^{15} - 8 T_1^{14} T_2^{15} + 8 T_1^{15} T_2^{15}) / \\
& \left((-1 + T_1) (1 - T_1 + T_1^2) (1 - T_1^3 + T_1^6) (-1 + T_2) (-1 + T_1 T_2) (1 - T_2 + T_2^2) \right. \\
& \quad \left. (1 - T_1 T_2 + T_1^2 T_2^2) (1 - T_1^3 T_2^3 + T_1^6 T_2^6) \right) , \\
& - \left((2 (-14 + 29 T_1 - 14 T_1^2 + 29 T_2 - 102 T_1 T_2 + 132 T_2^2 T_2 - 43 T_1^3 T_2 - 14 T_2^2 + 132 T_1 T_2^2 - 210 T_1^2 T_2^2 - \right. \\
& \quad 12 T_1^3 T_2^2 + 50 T_1^4 T_2^2 - 43 T_1 T_2^3 - 12 T_1^2 T_2^3 + 222 T_1^3 T_2^3 - 115 T_1^4 T_2^3 + 50 T_1^2 T_2^4 - 115 T_1^3 T_2^4 + 50 T_1^4 T_2^4) / \\
& \quad \left. \left((-1 + T_1) T_1 (4 - 9 T_1 + 4 T_1^2) (-1 + T_2) (-1 + T_1 T_2) (4 - 9 T_2 + 4 T_2^2) (4 - 9 T_1 T_2 + 4 T_1^2 T_2^2) \right) \right\}
\end{aligned}$$

`In[]:= K = Knot["K11n34"]; Conway = ⊕[K] × d vs[K]`

`KnotTheory`: Loading precomputed data in DTCODE4KNOTSTO11`.

`KnotTheory`: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

`Out[]=`

$$\begin{aligned}
& - \frac{1}{16} \\
& \mathbb{E} \left[\text{Series}[0, - \left((a_2 b_3 T_1^2 - a_2 b_5 T_1^2 - 4 a_2 b_3 T_1^3 + a_4 b_3 T_1^3 + 3 a_2 b_5 T_1^3 - a_4 b_5 T_1^3 + 6 a_2 b_3 T_1^4 - 3 a_4 b_3 T_1^4 - \right. \right. \\
& \quad 3 a_2 b_5 T_1^4 + 2 a_4 b_5 T_1^4 - 4 a_2 b_3 T_1^5 + 3 a_4 b_3 T_1^5 + a_2 b_5 T_1^5 - a_4 b_5 T_1^5 + a_2 b_3 T_1^6 - a_4 b_3 T_1^6 - \\
& \quad 2 a_2 b_3 T_1 T_2 + 2 a_2 b_5 T_1 T_2 + 6 a_2 b_3 T_1^2 T_2 - a_4 b_3 T_1^2 T_2 - 3 a_2 b_5 T_1^2 T_2 + a_4 b_5 T_1^2 T_2 - 6 a_2 b_3 T_1^3 T_2 - \\
& \quad a_4 b_3 T_1^3 T_2 - a_2 b_5 T_1^3 T_2 + 3 a_4 b_5 T_1^3 T_2 + 2 a_2 b_3 T_1^4 T_2 + 7 a_4 b_3 T_1^4 T_2 + 3 a_2 b_5 T_1^4 T_2 - 7 a_4 b_5 T_1^4 T_2 + \\
& \quad 2 a_2 b_3 T_1^5 T_2 - 7 a_4 b_3 T_1^5 T_2 - 3 a_2 b_5 T_1^5 T_2 + 3 a_4 b_5 T_1^5 T_2 - 6 a_2 b_3 T_1^6 T_2 + 4 a_4 b_3 T_1^6 T_2 + \\
& \quad 4 a_2 b_5 T_1^6 T_2 - 2 a_4 b_5 T_1^6 T_2 + 6 a_2 b_3 T_1^7 T_2 - 4 a_4 b_3 T_1^7 T_2 - 2 a_2 b_5 T_1^7 T_2 + 2 a_4 b_5 T_1^7 T_2 - \\
& \quad 2 a_2 b_3 T_1^8 T_2 + 2 a_4 b_3 T_1^8 T_2 + a_2 b_3 T_1^9 T_2 - a_2 b_5 T_1^9 T_2 - a_4 b_3 T_1^9 T_2 - 3 a_2 b_5 T_1^9 T_2 + a_4 b_5 T_1^9 T_2 - \\
& \quad 5 a_2 b_3 T_1^2 T_2^2 + 9 a_4 b_3 T_1^2 T_2^2 + 8 a_2 b_5 T_1^2 T_2^2 - 9 a_4 b_5 T_1^2 T_2^2 + 6 a_2 b_3 T_1^3 T_2^2 - 14 a_4 b_3 T_1^3 T_2^2 - \\
& \quad 4 a_2 b_5 T_1^3 T_2^2 + 4 a_4 b_5 T_1^3 T_2^2 - 4 a_2 b_3 T_1^4 T_2^2 + 6 a_4 b_3 T_1^4 T_2^2 + 2 a_2 b_5 T_1^4 T_2^2 + 5 a_4 b_5 T_1^4 T_2^2 + \\
& \quad 4 a_2 b_3 T_1^5 T_2^2 - 2 a_4 b_5 T_1^5 T_2^2 - 4 a_2 b_3 T_1^6 T_2^2 - 6 a_4 b_3 T_1^6 T_2^2 - 2 a_2 b_5 T_1^6 T_2^2 + 8 a_4 b_5 T_1^6 T_2^2 + \\
& \quad 6 a_2 b_3 T_1^7 T_2^2 + 8 a_4 b_3 T_1^7 T_2^2 - 2 a_2 b_5 T_1^7 T_2^2 - 4 a_4 b_5 T_1^7 T_2^2 - 5 a_2 b_3 T_1^8 T_2^2 + a_2 b_5 T_1^8 T_2^2 - \\
& \quad 2 a_4 b_5 T_1^8 T_2^2 - a_4 b_3 T_1^9 T_2^2 + a_2 b_5 T_1^9 T_2^2 - a_4 b_5 T_1^9 T_2^2 + a_2 b_3 T_1^{10} T_2^2 - a_4 b_3 T_1^{10} T_2^2 - 2 a_2 b_3 T_1^3 + \\
& \quad a_4 b_3 T_1^3 + 3 a_2 b_5 T_1^3 - a_4 b_5 T_1^3 + 4 a_2 b_3 T_1^2 T_2 - 3 a_4 b_3 T_1^2 T_2 - a_2 b_5 T_1^2 T_2 + a_4 b_5 T_1^2 T_2 - \\
& \quad 2 a_2 b_3 T_1^2 T_2^3 - 4 a_4 b_3 T_1^2 T_2^3 - 4 a_2 b_5 T_1^2 T_2^3 + 14 a_4 b_5 T_1^2 T_2^3 + 10 a_4 b_3 T_1^3 T_2^3 + 2 a_2 b_5 T_1^3 T_2^3 -
\end{aligned}$$

$$\begin{aligned}
& 10 a_4 b_5 T_1^3 T_2^3 + a_2 b_3 T_1^4 T_2^3 - 6 a_4 b_3 T_1^4 T_2^3 - 3 a_2 b_5 T_1^4 T_2^3 + 2 a_4 b_5 T_1^4 T_2^3 - a_2 b_3 T_1^5 T_2^3 + 7 a_4 b_3 T_1^5 T_2^3 + \\
& 2 a_2 b_5 T_1^5 T_2^3 - 5 a_4 b_5 T_1^5 T_2^3 - a_2 b_3 T_1^6 T_2^3 - 6 a_4 b_3 T_1^6 T_2^3 - a_2 b_5 T_1^6 T_2^3 - 5 a_4 b_5 T_1^6 T_2^3 + a_2 b_3 T_1^7 T_2^3 + \\
& 7 a_4 b_3 T_1^7 T_2^3 + 4 a_2 b_5 T_1^7 T_2^3 - 4 a_4 b_5 T_1^7 T_2^3 + 2 c_{19} T_1^7 T_2^3 + 2 c_{34} T_1^7 T_2^3 + 2 c_{49} T_1^7 T_2^3 + 2 c_{64} T_1^7 T_2^3 - \\
& 9 a_4 b_3 T_1^8 T_2^3 - a_2 b_5 T_1^8 T_2^3 + 5 a_4 b_5 T_1^8 T_2^3 - 2 c_{19} T_1^8 T_2^3 - 2 c_{34} T_1^8 T_2^3 - 2 c_{49} T_1^8 T_2^3 - 2 c_{64} T_1^8 T_2^3 - \\
& 2 a_2 b_3 T_1^9 T_2^3 + a_4 b_3 T_1^9 T_2^3 + a_2 b_5 T_1^9 T_2^3 + 3 a_4 b_5 T_1^9 T_2^3 + 4 a_2 b_3 T_1^{10} T_2^3 - 2 a_2 b_5 T_1^{10} T_2^3 + \\
& 2 a_4 b_5 T_1^{10} T_2^3 - 2 a_2 b_3 T_1^{11} T_2^3 + 2 a_4 b_3 T_1^{11} T_2^3 - 2 a_4 b_5 T_1^4 T_2^4 - 3 a_2 b_5 T_1^4 T_2^4 + 3 a_4 b_5 T_1^4 T_2^4 - \\
& 2 a_2 b_3 T_1 T_2^4 + 7 a_4 b_3 T_1 T_2^4 + 3 a_2 b_5 T_1 T_2^4 - 7 a_4 b_5 T_1 T_2^4 - a_2 b_3 T_1^2 T_2^4 - 5 a_4 b_3 T_1^2 T_2^4 + \\
& 2 a_2 b_5 T_1^2 T_2^4 - 6 a_4 b_5 T_1^2 T_2^4 + 5 a_2 b_3 T_1^3 T_2^4 - 2 a_4 b_3 T_1^3 T_2^4 - 3 a_2 b_5 T_1^3 T_2^4 + 6 a_4 b_5 T_1^3 T_2^4 - \\
& 2 a_2 b_3 T_1^4 T_2^4 + 4 a_4 b_3 T_1^4 T_2^4 - 4 a_4 b_5 T_1^4 T_2^4 - 3 a_2 b_3 T_1^5 T_2^4 - a_4 b_3 T_1^5 T_2^4 + 2 a_2 b_5 T_1^5 T_2^4 + \\
& 4 a_4 b_5 T_1^5 T_2^4 + 4 a_2 b_3 T_1^6 T_2^4 - 3 a_4 b_3 T_1^6 T_2^4 - 3 a_2 b_5 T_1^6 T_2^4 + a_4 b_5 T_1^6 T_2^4 - 3 a_2 b_3 T_1^7 T_2^4 + \\
& 4 a_4 b_3 T_1^7 T_2^4 + a_2 b_5 T_1^7 T_2^4 + 4 a_4 b_5 T_1^7 T_2^4 - 6 c_{19} T_1^7 T_2^4 - 6 c_{34} T_1^7 T_2^4 - 6 c_{49} T_1^7 T_2^4 - 6 c_{64} T_1^7 T_2^4 - \\
& 2 a_2 b_3 T_1^8 T_2^4 - a_4 b_3 T_1^8 T_2^4 + 3 a_2 b_5 T_1^8 T_2^4 - 4 a_4 b_5 T_1^8 T_2^4 + 4 c_{19} T_1^8 T_2^4 + 4 c_{34} T_1^8 T_2^4 + 4 c_{49} T_1^8 T_2^4 + \\
& 4 c_{64} T_1^8 T_2^4 + 5 a_2 b_3 T_1^9 T_2^4 - 5 a_4 b_3 T_1^9 T_2^4 - 4 a_2 b_5 T_1^9 T_2^4 + 2 a_4 b_5 T_1^9 T_2^4 + 2 c_{19} T_1^9 T_2^4 + 2 c_{34} T_1^9 T_2^4 + \\
& 2 c_{49} T_1^9 T_2^4 + 2 c_{64} T_1^9 T_2^4 - a_2 b_3 T_1^{10} T_2^4 + 8 a_4 b_3 T_1^{10} T_2^4 + a_2 b_5 T_1^{10} T_2^4 - 6 a_4 b_5 T_1^{10} T_2^4 - 2 a_2 b_3 T_1^{11} T_2^4 - \\
& 3 a_4 b_3 T_1^{11} T_2^4 + a_2 b_5 T_1^{11} T_2^4 - a_4 b_5 T_1^{11} T_2^4 + a_2 b_3 T_1^{12} T_2^4 - a_4 b_3 T_1^{12} T_2^4 + a_2 b_5 T_1^5 T_2^5 - \\
& 3 a_4 b_5 T_1^5 T_2^5 + 2 a_2 b_3 T_1^5 T_2^5 - 3 a_4 b_3 T_1^5 T_2^5 - 3 a_2 b_5 T_1^5 T_2^5 + 7 a_4 b_5 T_1^5 T_2^5 - 4 a_2 b_3 T_1^2 T_2^5 + 2 a_4 b_3 T_1^2 T_2^5 + \\
& 3 a_2 b_3 T_1^3 T_2^5 + 5 a_4 b_3 T_1^3 T_2^5 + 2 a_2 b_5 T_1^3 T_2^5 - 7 a_4 b_5 T_1^3 T_2^5 - 2 a_2 b_3 T_1^4 T_2^5 - 4 a_4 b_3 T_1^4 T_2^5 + \\
& 2 a_2 b_5 T_1^4 T_2^5 + a_4 b_5 T_1^4 T_2^5 - 3 a_2 b_3 T_1^5 T_2^5 - 5 a_4 b_3 T_1^5 T_2^5 + 2 a_2 b_5 T_1^5 T_2^5 + 5 a_4 b_5 T_1^5 T_2^5 + \\
& 4 a_2 b_3 T_1^6 T_2^5 + 2 a_4 b_3 T_1^6 T_2^5 + 2 a_2 b_5 T_1^6 T_2^5 + 4 a_2 b_3 T_1^7 T_2^5 - 5 a_4 b_3 T_1^7 T_2^5 - 5 a_2 b_5 T_1^7 T_2^5 + \\
& 9 a_4 b_5 T_1^7 T_2^5 + 6 c_{19} T_1^7 T_2^5 + 6 c_{34} T_1^7 T_2^5 + 6 c_{49} T_1^7 T_2^5 + 6 c_{64} T_1^7 T_2^5 - 3 a_2 b_3 T_1^8 T_2^5 + 3 a_4 b_3 T_1^8 T_2^5 - \\
& 4 a_2 b_5 T_1^8 T_2^5 + 5 a_4 b_5 T_1^8 T_2^5 - 2 a_2 b_3 T_1^9 T_2^5 + 9 a_4 b_3 T_1^9 T_2^5 + 3 a_2 b_5 T_1^9 T_2^5 - 9 a_4 b_5 T_1^9 T_2^5 - \\
& 6 c_{19} T_1^9 T_2^5 - 6 c_{34} T_1^9 T_2^5 - 6 c_{49} T_1^9 T_2^5 - 6 c_{64} T_1^9 T_2^5 + 3 a_2 b_3 T_1^{10} T_2^5 - 4 a_4 b_3 T_1^{10} T_2^5 - a_2 b_5 T_1^{10} T_2^5 + \\
& 3 a_4 b_5 T_1^{10} T_2^5 - 4 a_2 b_3 T_1^{11} T_2^5 - a_4 b_3 T_1^{11} T_2^5 + a_2 b_5 T_1^{11} T_2^5 + a_4 b_5 T_1^{11} T_2^5 + 2 a_2 b_3 T_1^{12} T_2^5 + a_4 b_5 T_1^{12} T_2^5 - \\
& 2 a_2 b_3 T_1 T_2^6 + 2 a_4 b_3 T_1 T_2^6 + 4 a_2 b_5 T_1 T_2^6 - 4 a_4 b_5 T_1 T_2^6 + 2 a_2 b_3 T_1^2 T_2^6 - 8 a_4 b_3 T_1^2 T_2^6 - \\
& 2 a_2 b_5 T_1^2 T_2^6 + 6 a_4 b_5 T_1^2 T_2^6 + 2 a_2 b_3 T_1^3 T_2^6 + 5 a_4 b_3 T_1^3 T_2^6 - a_2 b_5 T_1^3 T_2^6 + 6 a_4 b_5 T_1^3 T_2^6 - \\
& a_4 b_3 T_1^4 T_2^6 - 3 a_2 b_5 T_1^4 T_2^6 + 3 a_4 b_5 T_1^4 T_2^6 + 2 c_{16} T_1^4 T_2^6 + 2 c_{31} T_1^4 T_2^6 + 2 c_{46} T_1^4 T_2^6 + 2 c_{61} T_1^4 T_2^6 - \\
& 8 a_2 b_3 T_1^5 T_2^6 + 2 a_2 b_5 T_1^5 T_2^6 - 2 a_4 b_3 T_1^5 T_2^6 - 6 c_{16} T_1^5 T_2^6 - 6 c_{31} T_1^5 T_2^6 - 6 c_{46} T_1^5 T_2^6 - 6 c_{61} T_1^5 T_2^6 + \\
& 22 a_2 b_3 T_1^6 T_2^6 + 2 a_4 b_3 T_1^6 T_2^6 - 16 a_2 b_5 T_1^6 T_2^6 - 2 a_4 b_5 T_1^6 T_2^6 + 6 c_{16} T_1^6 T_2^6 + 6 c_{31} T_1^6 T_2^6 + \\
& 6 c_{46} T_1^6 T_2^6 + 6 c_{61} T_1^6 T_2^6 - 32 a_2 b_3 T_1^7 T_2^6 + 20 a_4 b_3 T_1^7 T_2^6 + 20 a_2 b_5 T_1^7 T_2^6 - 27 a_4 b_5 T_1^7 T_2^6 - \\
& 2 c_{16} T_1^7 T_2^6 - 2 c_{19} T_1^7 T_2^6 - 2 c_{31} T_1^7 T_2^6 - 2 c_{34} T_1^7 T_2^6 - 2 c_{46} T_1^7 T_2^6 - 2 c_{49} T_1^7 T_2^6 - 2 c_{61} T_1^7 T_2^6 - \\
& 2 c_{64} T_1^7 T_2^6 + 22 a_2 b_3 T_1^8 T_2^6 - 23 a_4 b_3 T_1^8 T_2^6 - 5 a_2 b_5 T_1^8 T_2^6 - a_4 b_5 T_1^8 T_2^6 + 2 c_{16} T_1^8 T_2^6 - 4 c_{19} T_1^8 T_2^6 + \\
& 2 c_{31} T_1^8 T_2^6 - 4 c_{34} T_1^8 T_2^6 + 2 c_{46} T_1^8 T_2^6 - 4 c_{49} T_1^8 T_2^6 + 2 c_{61} T_1^8 T_2^6 - 4 c_{64} T_1^8 T_2^6 - 8 a_2 b_3 T_1^9 T_2^6 + \\
& 3 a_4 b_3 T_1^9 T_2^6 + a_2 b_5 T_1^9 T_2^6 + 5 a_4 b_5 T_1^9 T_2^6 - 6 c_{16} T_1^9 T_2^6 + 6 c_{19} T_1^9 T_2^6 - 6 c_{31} T_1^9 T_2^6 + 6 c_{34} T_1^9 T_2^6 - \\
& 6 c_{46} T_1^9 T_2^6 + 6 c_{49} T_1^9 T_2^6 - 6 c_{61} T_1^9 T_2^6 + 6 c_{64} T_1^9 T_2^6 + 2 a_4 b_3 T_1^{10} T_2^6 + 4 a_2 b_5 T_1^{10} T_2^6 - a_4 b_5 T_1^{10} T_2^6 + \\
& 6 c_{16} T_1^{10} T_2^6 + 6 c_{31} T_1^{10} T_2^6 + 6 c_{46} T_1^{10} T_2^6 + 6 c_{61} T_1^{10} T_2^6 + 2 a_2 b_3 T_1^{11} T_2^6 - 8 a_4 b_3 T_1^{11} T_2^6 - \\
& 2 a_2 b_5 T_1^{11} T_2^6 + 6 a_4 b_5 T_1^{11} T_2^6 - 2 c_{16} T_1^{11} T_2^6 - 2 c_{31} T_1^{11} T_2^6 - 2 c_{46} T_1^{11} T_2^6 - 2 c_{61} T_1^{11} T_2^6 + \\
& 2 a_2 b_3 T_1^{12} T_2^6 + 4 a_4 b_3 T_1^{12} T_2^6 - 2 a_2 b_5 T_1^{12} T_2^6 + 2 a_4 b_5 T_1^{12} T_2^6 - 2 a_2 b_3 T_1^{13} T_2^6 + 2 a_4 b_3 T_1^{13} T_2^6 - \\
& 2 a_4 b_3 T_1 T_2^7 - 2 a_2 b_5 T_1 T_2^7 + 4 a_4 b_5 T_1 T_2^7 + 2 a_2 b_3 T_1^2 T_2^7 + 4 a_4 b_3 T_1^2 T_2^7 - 2 a_2 b_5 T_1^2 T_2^7 - \\
& 8 a_4 b_5 T_1^2 T_2^7 - 4 a_2 b_3 T_1^3 T_2^7 + 4 a_4 b_3 T_1^3 T_2^7 + 4 a_2 b_5 T_1^3 T_2^7 - 7 a_4 b_5 T_1^3 T_2^7 + 3 a_2 b_3 T_1^4 T_2^7 - \\
& 4 a_4 b_3 T_1^4 T_2^7 + a_2 b_5 T_1^4 T_2^7 - 4 a_4 b_5 T_1^4 T_2^7 - 2 c_{16} T_1^4 T_2^7 - 2 c_{31} T_1^4 T_2^7 - 2 c_{46} T_1^4 T_2^7 - 2 c_{61} T_1^4 T_2^7 - \\
& 2 a_2 b_3 T_1^5 T_2^7 - 9 a_4 b_3 T_1^5 T_2^7 - 5 a_2 b_5 T_1^5 T_2^7 + 5 a_4 b_5 T_1^5 T_2^7 + 4 c_{16} T_1^5 T_2^7 + 4 c_{31} T_1^5 T_2^7 + 4 c_{46} T_1^5 T_2^7 + \\
& 4 c_{61} T_1^5 T_2^7 - 3 a_2 b_3 T_1^6 T_2^7 + 27 a_4 b_3 T_1^6 T_2^7 + 20 a_2 b_5 T_1^6 T_2^7 - 20 a_4 b_5 T_1^6 T_2^7 + 4 a_2 b_3 T_1^7 T_2^7 - \\
& 52 a_4 b_3 T_1^7 T_2^7 - 16 a_2 b_5 T_1^7 T_2^7 + 52 a_4 b_5 T_1^7 T_2^7 - 4 c_{16} T_1^7 T_2^7 + 2 c_{19} T_1^7 T_2^7 - 4 c_{31} T_1^7 T_2^7 + \\
& 2 c_{34} T_1^7 T_2^7 - 4 c_{46} T_1^7 T_2^7 + 2 c_{49} T_1^7 T_2^7 - 4 c_{61} T_1^7 T_2^7 + 2 c_{64} T_1^7 T_2^7 + 4 a_2 b_3 T_1^8 T_2^7 + 38 a_4 b_3 T_1^8 T_2^7 + \\
& 2 a_2 b_5 T_1^8 T_2^7 - 9 a_4 b_5 T_1^8 T_2^7 - 3 a_2 b_3 T_1^9 T_2^7 - 10 a_4 b_3 T_1^9 T_2^7 - 3 a_2 b_5 T_1^9 T_2^7 + 4 a_4 b_5 T_1^9 T_2^7 + \\
& 4 c_{16} T_1^9 T_2^7 - 2 c_{19} T_1^9 T_2^7 + 4 c_{31} T_1^9 T_2^7 - 2 c_{34} T_1^9 T_2^7 + 4 c_{46} T_1^9 T_2^7 - 2 c_{49} T_1^9 T_2^7 + 4 c_{61} T_1^9 T_2^7 -
\end{aligned}$$

$$\begin{aligned}
& 2 c_{64} T_1^9 T_2^7 - 2 a_2 b_3 T_1^{10} T_2^7 + 3 a_4 b_3 T_1^{10} T_2^7 - a_2 b_5 T_1^{10} T_2^7 + 3 a_4 b_5 T_1^{10} T_2^7 + 3 a_2 b_3 T_1^{11} T_2^7 + \\
& 3 a_4 b_3 T_1^{11} T_2^7 - 2 a_2 b_5 T_1^{11} T_2^7 - 8 a_4 b_5 T_1^{11} T_2^7 - 4 c_{16} T_1^{11} T_2^7 - 4 c_{31} T_1^{11} T_2^7 - 4 c_{46} T_1^{11} T_2^7 - \\
& 4 c_{61} T_1^{11} T_2^7 - 4 a_2 b_3 T_1^{12} T_2^7 + 4 a_4 b_3 T_1^{12} T_2^7 + 4 a_2 b_5 T_1^{12} T_2^7 - 8 a_4 b_5 T_1^{12} T_2^7 + 2 c_{16} T_1^{12} T_2^7 + \\
& 2 c_{31} T_1^{12} T_2^7 + 2 c_{46} T_1^{12} T_2^7 + 2 c_{61} T_1^{12} T_2^7 + 2 a_2 b_3 T_1^{13} T_2^7 - 6 a_4 b_3 T_1^{13} T_2^7 - 2 a_4 b_5 T_1 T_2^8 + \\
& a_2 b_3 T_1^2 T_2^8 + 2 a_4 b_3 T_1^2 T_2^8 + a_2 b_5 T_1^2 T_2^8 - 2 a_2 b_3 T_1^3 T_2^8 - 5 a_4 b_3 T_1^3 T_2^8 - a_2 b_5 T_1^3 T_2^8 + 9 a_4 b_5 T_1^3 T_2^8 - \\
& a_2 b_3 T_1^4 T_2^8 + 4 a_4 b_3 T_1^4 T_2^8 + 3 a_2 b_5 T_1^4 T_2^8 + a_4 b_5 T_1^4 T_2^8 + 5 a_2 b_3 T_1^5 T_2^8 - 5 a_4 b_3 T_1^5 T_2^8 - 4 a_2 b_5 T_1^5 T_2^8 - \\
& 3 a_4 b_5 T_1^5 T_2^8 + 2 c_{16} T_1^5 T_2^8 + 2 c_{31} T_1^5 T_2^8 + 2 c_{46} T_1^5 T_2^8 + 2 c_{61} T_1^5 T_2^8 - 2 a_2 b_3 T_1^6 T_2^8 + a_4 b_3 T_1^6 T_2^8 - \\
& 5 a_2 b_5 T_1^6 T_2^8 + 23 a_4 b_5 T_1^6 T_2^8 - 6 c_{16} T_1^6 T_2^8 - 6 c_{31} T_1^6 T_2^8 - 6 c_{46} T_1^6 T_2^8 - 6 c_{61} T_1^6 T_2^8 - 3 a_2 b_3 T_1^7 T_2^8 + \\
& 9 a_4 b_3 T_1^7 T_2^8 + 2 a_2 b_5 T_1^7 T_2^8 - 38 a_4 b_5 T_1^7 T_2^8 + 6 c_{16} T_1^7 T_2^8 - 6 c_{19} T_1^7 T_2^8 + 6 c_{31} T_1^7 T_2^8 - 6 c_{34} T_1^7 T_2^8 + \\
& 6 c_{46} T_1^7 T_2^8 - 6 c_{49} T_1^7 T_2^8 + 6 c_{61} T_1^7 T_2^8 - 6 c_{64} T_1^7 T_2^8 + 4 a_2 b_3 T_1^8 T_2^8 + 2 a_4 b_3 T_1^8 T_2^8 + 2 a_2 b_5 T_1^8 T_2^8 - \\
& 2 a_4 b_5 T_1^8 T_2^8 - 2 c_{16} T_1^8 T_2^8 + 4 c_{19} T_1^8 T_2^8 - 2 c_{31} T_1^8 T_2^8 + 4 c_{34} T_1^8 T_2^8 - 2 c_{46} T_1^8 T_2^8 + 4 c_{49} T_1^8 T_2^8 - \\
& 2 c_{61} T_1^8 T_2^8 + 4 c_{64} T_1^8 T_2^8 - 3 a_2 b_3 T_1^9 T_2^8 - 5 a_4 b_3 T_1^9 T_2^8 + 2 a_2 b_5 T_1^9 T_2^8 - 7 a_4 b_5 T_1^9 T_2^8 + 2 c_{16} T_1^9 T_2^8 + \\
& 2 c_{19} T_1^9 T_2^8 + 2 c_{31} T_1^9 T_2^8 + 2 c_{34} T_1^9 T_2^8 + 2 c_{46} T_1^9 T_2^8 + 2 c_{49} T_1^9 T_2^8 + 2 c_{61} T_1^9 T_2^8 + 2 c_{64} T_1^9 T_2^8 - \\
& 2 a_2 b_3 T_1^{10} T_2^8 - 4 a_4 b_3 T_1^{10} T_2^8 + 2 a_2 b_5 T_1^{10} T_2^8 - 6 c_{31} T_1^{10} T_2^8 - 6 c_{46} T_1^{10} T_2^8 - \\
& 6 c_{61} T_1^{10} T_2^8 + 5 a_2 b_3 T_1^{11} T_2^8 + a_4 b_3 T_1^{11} T_2^8 + 2 a_4 b_5 T_1^{11} T_2^8 + 6 c_{16} T_1^{11} T_2^8 + 6 c_{31} T_1^{11} T_2^8 + \\
& 6 c_{46} T_1^{11} T_2^8 + 6 c_{61} T_1^{11} T_2^8 - a_2 b_3 T_1^{12} T_2^8 - 4 a_4 b_3 T_1^{12} T_2^8 - 3 a_2 b_5 T_1^{12} T_2^8 + 10 a_4 b_5 T_1^{12} T_2^8 - \\
& 2 c_{16} T_1^{12} T_2^8 - 2 c_{31} T_1^{12} T_2^8 - 2 c_{46} T_1^{12} T_2^8 - 2 c_{61} T_1^{12} T_2^8 - 2 a_2 b_3 T_1^{13} T_2^8 + 5 a_4 b_3 T_1^{13} T_2^8 + \\
& a_2 b_5 T_1^{13} T_2^8 - a_4 b_5 T_1^{13} T_2^8 + a_2 b_3 T_1^{14} T_2^8 - a_4 b_3 T_1^{14} T_2^8 + a_4 b_5 T_1^2 T_2^9 + a_2 b_5 T_1^2 T_2^9 + a_4 b_5 T_1^2 T_2^9 - \\
& 2 a_2 b_3 T_1^3 T_2^9 - 3 a_4 b_3 T_1^3 T_2^9 + a_2 b_5 T_1^3 T_2^9 - a_4 b_5 T_1^3 T_2^9 + 4 a_2 b_3 T_1^4 T_2^9 - 2 a_4 b_3 T_1^4 T_2^9 - \\
& 4 a_2 b_5 T_1^4 T_2^9 + 5 a_4 b_5 T_1^4 T_2^9 - 2 a_2 b_3 T_1^5 T_2^9 + 9 a_4 b_3 T_1^5 T_2^9 + 3 a_2 b_5 T_1^5 T_2^9 - 9 a_4 b_5 T_1^5 T_2^9 - \\
& 5 a_4 b_3 T_1^6 T_2^9 + a_2 b_5 T_1^6 T_2^9 - 3 a_4 b_5 T_1^6 T_2^9 + a_2 b_3 T_1^7 T_2^9 - 4 a_4 b_3 T_1^7 T_2^9 - 3 a_2 b_5 T_1^7 T_2^9 + \\
& 10 a_4 b_5 T_1^7 T_2^9 + 6 c_{19} T_1^7 T_2^9 + 6 c_{34} T_1^7 T_2^9 + 6 c_{49} T_1^7 T_2^9 + 6 c_{64} T_1^7 T_2^9 - a_2 b_3 T_1^8 T_2^9 + 7 a_4 b_3 T_1^8 T_2^9 + \\
& 2 a_2 b_5 T_1^8 T_2^9 + 5 a_4 b_5 T_1^8 T_2^9 - a_2 b_3 T_1^9 T_2^9 - 5 a_4 b_3 T_1^9 T_2^9 + 5 a_4 b_5 T_1^9 T_2^9 - 6 c_{19} T_1^9 T_2^9 - 6 c_{34} T_1^9 T_2^9 - \\
& 6 c_{49} T_1^9 T_2^9 - 6 c_{64} T_1^9 T_2^9 + a_2 b_3 T_1^{10} T_2^9 - 2 a_4 b_3 T_1^{10} T_2^9 - 3 a_2 b_5 T_1^{10} T_2^9 + 3 a_4 b_5 T_1^{10} T_2^9 + \\
& 8 a_4 b_3 T_1^{11} T_2^9 + 2 a_2 b_5 T_1^{11} T_2^9 - 4 a_4 b_5 T_1^{11} T_2^9 - 2 a_2 b_3 T_1^{12} T_2^9 - 3 a_4 b_3 T_1^{12} T_2^9 + 3 a_2 b_5 T_1^{12} T_2^9 - \\
& 5 a_4 b_5 T_1^{12} T_2^9 + 4 a_2 b_3 T_1^{13} T_2^9 - 5 a_4 b_3 T_1^{13} T_2^9 - 3 a_2 b_5 T_1^{13} T_2^9 + 5 a_4 b_5 T_1^{13} T_2^9 - 2 a_2 b_3 T_1^{14} T_2^9 + \\
& 4 a_4 b_3 T_1^{14} T_2^9 + a_4 b_5 T_1^{14} T_2^9 - 2 a_4 b_3 T_1^{15} T_2^9 - 2 a_2 b_5 T_1^{15} T_2^9 + a_2 b_3 T_1^4 T_2^{10} + 6 a_4 b_3 T_1^4 T_2^{10} + \\
& a_2 b_5 T_1^4 T_2^{10} - 8 a_4 b_5 T_1^4 T_2^{10} - 3 a_4 b_3 T_1^5 T_2^{10} - a_2 b_5 T_1^5 T_2^{10} + 4 a_4 b_5 T_1^5 T_2^{10} - 5 a_2 b_3 T_1^6 T_2^{10} + \\
& a_4 b_3 T_1^6 T_2^{10} + 4 a_2 b_5 T_1^6 T_2^{10} - 2 a_4 b_5 T_1^7 T_2^{10} + 6 a_2 b_3 T_1^7 T_2^{10} - 3 a_4 b_3 T_1^7 T_2^{10} - a_2 b_5 T_1^7 T_2^{10} - \\
& 3 a_4 b_5 T_1^7 T_2^{10} - 2 c_{19} T_1^7 T_2^{10} - 2 c_{34} T_1^7 T_2^{10} - 2 c_{49} T_1^7 T_2^{10} - 2 c_{64} T_1^7 T_2^{10} - 4 a_2 b_3 T_1^8 T_2^{10} + \\
& 2 a_2 b_5 T_1^8 T_2^{10} + 4 a_4 b_5 T_1^8 T_2^{10} - 4 c_{19} T_1^8 T_2^{10} - 4 c_{34} T_1^8 T_2^{10} - 4 c_{49} T_1^8 T_2^{10} - 4 c_{64} T_1^8 T_2^{10} + \\
& 4 a_2 b_3 T_1^9 T_2^{10} - 3 a_4 b_3 T_1^9 T_2^{10} - 3 a_2 b_5 T_1^9 T_2^{10} + 2 a_4 b_5 T_1^9 T_2^{10} + 6 c_{19} T_1^9 T_2^{10} + 6 c_{34} T_1^9 T_2^{10} + \\
& 6 c_{49} T_1^9 T_2^{10} + 6 c_{64} T_1^9 T_2^{10} - 4 a_2 b_3 T_1^{10} T_2^{10} + 4 a_4 b_3 T_1^{10} T_2^{10} + 2 a_2 b_5 T_1^{10} T_2^{10} - 4 a_4 b_5 T_1^{10} T_2^{10} + \\
& 6 a_2 b_3 T_1^{11} T_2^{10} - 2 a_4 b_3 T_1^{11} T_2^{10} - 4 a_2 b_5 T_1^{11} T_2^{10} + 6 a_4 b_3 T_1^{11} T_2^{10} - 5 a_2 b_3 T_1^{12} T_2^{10} + \\
& 2 a_4 b_3 T_1^{12} T_2^{10} - a_2 b_5 T_1^{12} T_2^{10} + a_4 b_5 T_1^{12} T_2^{10} + 5 a_4 b_3 T_1^{13} T_2^{10} + 3 a_2 b_5 T_1^{13} T_2^{10} - 9 a_4 b_5 T_1^{13} T_2^{10} + \\
& a_2 b_3 T_1^{14} T_2^{10} - 5 a_4 b_3 T_1^{14} T_2^{10} - 2 a_4 b_5 T_1^{14} T_2^{10} + a_4 b_3 T_1^{15} T_2^{10} + a_2 b_5 T_1^{15} T_2^{11} + 3 a_4 b_5 T_1^{15} T_2^{11} - \\
& a_4 b_3 T_1^5 T_2^{11} + a_2 b_5 T_1^5 T_2^{11} + a_4 b_5 T_1^5 T_2^{11} - 2 a_2 b_3 T_1^6 T_2^{11} - 6 a_4 b_3 T_1^6 T_2^{11} - 2 a_2 b_5 T_1^6 T_2^{11} + \\
& 8 a_4 b_5 T_1^6 T_2^{11} + 6 a_2 b_3 T_1^7 T_2^{11} + 8 a_4 b_3 T_1^7 T_2^{11} - 2 a_2 b_5 T_1^7 T_2^{11} - 3 a_4 b_3 T_1^7 T_2^{11} - 6 a_2 b_3 T_1^8 T_2^{11} - \\
& 2 a_4 b_3 T_1^8 T_2^{11} - a_4 b_5 T_1^8 T_2^{11} + 2 c_{19} T_1^8 T_2^{11} + 2 c_{34} T_1^8 T_2^{11} + 2 c_{49} T_1^8 T_2^{11} + 2 c_{64} T_1^8 T_2^{11} + \\
& 2 a_2 b_3 T_1^9 T_2^{11} + 4 a_4 b_3 T_1^9 T_2^{11} + 2 a_2 b_5 T_1^9 T_2^{11} - 8 a_4 b_5 T_1^9 T_2^{11} - 2 c_{19} T_1^9 T_2^{11} - 2 c_{34} T_1^9 T_2^{11} - \\
& 2 c_{49} T_1^9 T_2^{11} - 2 c_{64} T_1^9 T_2^{11} + 2 a_2 b_3 T_1^{10} T_2^{11} - 6 a_4 b_3 T_1^{10} T_2^{11} - 4 a_2 b_5 T_1^{10} T_2^{11} + 2 a_4 b_5 T_1^{10} T_2^{11} - \\
& 6 a_2 b_3 T_1^{11} T_2^{11} + 10 a_4 b_3 T_1^{11} T_2^{11} + 8 a_2 b_5 T_1^{11} T_2^{11} - 10 a_4 b_5 T_1^{11} T_2^{11} + 6 a_2 b_3 T_1^{12} T_2^{11} - \\
& 13 a_4 b_3 T_1^{12} T_2^{11} - 3 a_2 b_5 T_1^{12} T_2^{11} + 5 a_4 b_5 T_1^{12} T_2^{11} - 2 a_2 b_3 T_1^{13} T_2^{11} + 3 a_4 b_3 T_1^{13} T_2^{11} - a_2 b_5 T_1^{13} T_2^{11} + \\
& 7 a_4 b_5 T_1^{13} T_2^{11} + 2 a_4 b_3 T_1^{14} T_2^{11} + a_4 b_5 T_1^{14} T_2^{11} - 2 a_4 b_3 T_1^5 T_2^{12} - 2 a_2 b_5 T_1^6 T_2^{12} - 4 a_4 b_5 T_1^6 T_2^{12} + \\
& 8 a_4 b_3 T_1^7 T_2^{12} + 4 a_2 b_5 T_1^7 T_2^{12} - 4 a_4 b_5 T_1^7 T_2^{12} + a_2 b_3 T_1^8 T_2^{12} - 10 a_4 b_3 T_1^8 T_2^{12} - 3 a_2 b_5 T_1^8 T_2^{12} + \\
& 4 a_4 b_5 T_1^8 T_2^{12} - 4 a_2 b_3 T_1^9 T_2^{12} + 5 a_4 b_3 T_1^9 T_2^{12} + 3 a_2 b_5 T_1^9 T_2^{12} + 3 a_4 b_5 T_1^9 T_2^{12} + 6 a_2 b_3 T_1^{10} T_2^{12} -
\end{aligned}$$

$$\begin{aligned}
& a_4 b_3 T_1^{10} T_2^{12} - a_2 b_5 T_1^{10} T_2^{12} - 2 a_4 b_5 T_1^{10} T_2^{12} - 4 a_2 b_3 T_1^{11} T_2^{12} - 5 a_4 b_3 T_1^{11} T_2^{12} - 3 a_2 b_5 T_1^{11} T_2^{12} + \\
& 13 a_4 b_5 T_1^{11} T_2^{12} + a_2 b_3 T_1^{12} T_2^{12} + 9 a_4 b_3 T_1^{12} T_2^{12} + 2 a_2 b_5 T_1^{12} T_2^{12} - 9 a_4 b_5 T_1^{12} T_2^{12} - 4 a_4 b_3 T_1^{13} T_2^{12} - \\
& 2 a_4 b_5 T_1^{13} T_2^{12} - 2 a_4 b_5 T_1^6 T_2^{13} + 6 a_4 b_5 T_1^7 T_2^{13} + a_4 b_3 T_1^8 T_2^{13} + a_2 b_5 T_1^8 T_2^{13} - 5 a_4 b_5 T_1^8 T_2^{13} - \\
& 5 a_4 b_3 T_1^9 T_2^{13} - 3 a_2 b_5 T_1^9 T_2^{13} + 5 a_4 b_5 T_1^9 T_2^{13} + 9 a_4 b_3 T_1^{10} T_2^{13} + 3 a_2 b_5 T_1^{10} T_2^{13} - 5 a_4 b_5 T_1^{10} T_2^{13} - \\
& 7 a_4 b_3 T_1^{11} T_2^{13} - a_2 b_5 T_1^{11} T_2^{13} - 3 a_4 b_5 T_1^{11} T_2^{13} + 2 a_4 b_3 T_1^{12} T_2^{13} + 4 a_4 b_5 T_1^{12} T_2^{13} + a_4 b_5 T_1^8 T_2^{14} - \\
& 4 a_4 b_5 T_1^9 T_2^{14} + 5 a_4 b_5 T_1^{10} T_2^{14} - 2 a_4 b_5 T_1^{11} T_2^{14} \Big) / \Big((-1 + T_1) T_1^7 (-1 + T_2) T_2^6 (-1 + T_1 T_2) \Big) \Big]
\end{aligned}$$

In[]:= $\mathbf{K} = \text{Knot}["K11n42"]$; $\mathbf{KT} = \int \mathcal{L}[\mathbf{K}] \times \text{dil} \mathbf{vs}[\mathbf{K}]$

Out[]:=

$\frac{1}{16}$

$$\begin{aligned}
& \mathbb{E} \left[\in \text{Series}[0, (a_2 b_3 T_1 - a_2 b_5 T_1 - a_2 b_3 T_1^2 + a_4 b_3 T_1^2 - a_4 b_5 T_1^2 - a_2 b_3 T_1^3 + a_2 b_5 T_1^3 - a_4 b_5 T_1^3 + a_2 b_3 T_1^4 - \right. \\
& a_4 b_3 T_1^4 - 2 c_{19} T_1^4 - 2 c_{34} T_1^4 - 2 c_{49} T_1^4 - 2 c_{64} T_1^4 + 2 c_{19} T_1^5 + 2 c_{34} T_1^5 + 2 c_{49} T_1^5 + 2 c_{64} T_1^5 + \\
& a_2 b_3 T_2 - a_2 b_5 T_2 - 4 a_2 b_3 T_1 T_2 + 2 a_4 b_3 T_1 T_2 + 4 a_2 b_5 T_1 T_2 - 2 a_4 b_5 T_1 T_2 + 3 a_2 b_3 T_1^2 T_2 - \\
& 5 a_4 b_3 T_1^2 T_2 + 4 a_4 b_5 T_1^2 T_2 - a_4 b_3 T_1^3 T_2 - a_2 b_5 T_1^3 T_2 + 5 a_4 b_5 T_1^3 T_2 + 3 a_2 b_3 T_1^4 T_2 + 2 a_4 b_3 T_1^4 T_2 - \\
& 3 a_2 b_5 T_1^4 T_2 + 2 a_4 b_5 T_1^4 T_2 + 6 c_{19} T_1^4 T_2 + 6 c_{34} T_1^4 T_2 + 6 c_{49} T_1^4 T_2 + 6 c_{64} T_1^4 T_2 - 4 a_2 b_3 T_1^5 T_2 + \\
& 3 a_4 b_3 T_1^5 T_2 + a_2 b_5 T_1^5 T_2 - a_4 b_5 T_1^5 T_2 - 4 c_{19} T_1^5 T_2 - 4 c_{34} T_1^5 T_2 - 4 c_{49} T_1^5 T_2 - 4 c_{64} T_1^5 T_2 + \\
& a_2 b_3 T_1^6 T_2 - a_4 b_3 T_1^6 T_2 - 2 c_{19} T_1^6 T_2 - 2 c_{34} T_1^6 T_2 - 2 c_{49} T_1^6 T_2 - 2 c_{64} T_1^6 T_2 + a_2 b_3 T_2^2 + a_4 b_3 T_2^2 - \\
& a_4 b_5 T_2^2 - 4 a_2 b_3 T_1 T_2^2 - 4 a_4 b_3 T_1 T_2^2 + 5 a_4 b_5 T_1 T_2^2 + 7 a_2 b_3 T_1^2 T_2^2 + 3 a_4 b_3 T_1^2 T_2^2 - 4 a_2 b_5 T_1 T_2^2 - \\
& 3 a_4 b_5 T_1^2 T_2^2 - 4 a_2 b_3 T_1^3 T_2^2 + 4 a_4 b_3 T_1^3 T_2^2 - 8 a_4 b_5 T_1^3 T_2^2 - 4 a_2 b_3 T_1^4 T_2^2 + 3 a_4 b_3 T_1^4 T_2^2 + 7 a_2 b_5 T_1^4 T_2^2 - \\
& 9 a_4 b_5 T_1^4 T_2^2 - 6 c_{19} T_1^4 T_2^2 - 6 c_{34} T_1^4 T_2^2 - 6 c_{49} T_1^4 T_2^2 - 6 c_{64} T_1^4 T_2^2 + 7 a_2 b_3 T_1^5 T_2^2 - 11 a_4 b_3 T_1^5 T_2^2 - \\
& 4 a_2 b_5 T_1^5 T_2^2 + 5 a_4 b_5 T_1^5 T_2^2 - 4 a_2 b_3 T_1^6 T_2^2 + 5 a_4 b_3 T_1^6 T_2^2 + a_2 b_5 T_1^6 T_2^2 - a_4 b_5 T_1^6 T_2^2 + 6 c_{19} T_1^6 T_2^2 + \\
& 6 c_{34} T_1^6 T_2^2 + 6 c_{49} T_1^6 T_2^2 + 6 c_{64} T_1^6 T_2^2 + a_2 b_3 T_1^7 T_2^2 - a_4 b_3 T_1^7 T_2^2 + a_4 b_3 T_1^8 T_2^2 - 2 a_2 b_3 T_1 T_2^3 - \\
& 5 a_4 b_3 T_1 T_2^3 - a_2 b_5 T_1 T_2^3 + a_4 b_5 T_1 T_2^3 - 2 c_{16} T_1 T_2^3 - 2 c_{31} T_1 T_2^3 - 2 c_{46} T_1 T_2^3 - 2 c_{61} T_1 T_2^3 + \\
& 6 a_2 b_3 T_1^2 T_2^3 + 8 a_4 b_3 T_1^2 T_2^3 - 4 a_4 b_5 T_1^2 T_2^3 + 6 c_{16} T_1^2 T_2^3 + 6 c_{31} T_1^2 T_2^3 + 6 c_{46} T_1^2 T_2^3 - 6 c_{61} T_1^2 T_2^3 - \\
& 6 a_2 b_3 T_1^3 T_2^3 - 4 a_4 b_3 T_1^3 T_2^3 + 2 a_2 b_5 T_1^3 T_2^3 + 4 a_4 b_5 T_1^3 T_2^3 - 6 c_{16} T_1^3 T_2^3 - 6 c_{31} T_1^3 T_2^3 - 6 c_{46} T_1^3 T_2^3 - \\
& 6 c_{61} T_1^3 T_2^3 + 4 a_2 b_3 T_1^4 T_2^3 - 6 a_4 b_3 T_1^4 T_2^3 - 6 a_2 b_5 T_1^4 T_2^3 + 13 a_4 b_5 T_1^4 T_2^3 + 2 c_{16} T_1^4 T_2^3 + 2 c_{19} T_1^4 T_2^3 + \\
& 2 c_{31} T_1^4 T_2^3 + 2 c_{34} T_1^4 T_2^3 + 2 c_{46} T_1^4 T_2^3 + 2 c_{49} T_1^4 T_2^3 + 2 c_{61} T_1^4 T_2^3 + 2 c_{64} T_1^4 T_2^3 - 6 a_2 b_3 T_1^5 T_2^3 + \\
& 13 a_4 b_3 T_1^5 T_2^3 + 7 a_2 b_5 T_1^5 T_2^3 - 11 a_4 b_5 T_1^5 T_2^3 - 2 c_{16} T_1^5 T_2^3 + 4 c_{19} T_1^5 T_2^3 - 2 c_{31} T_1^5 T_2^3 + 4 c_{34} T_1^5 T_2^3 - \\
& 2 c_{46} T_1^5 T_2^3 + 4 c_{49} T_1^5 T_2^3 - 2 c_{61} T_1^5 T_2^3 + 4 c_{64} T_1^5 T_2^3 + 6 a_2 b_3 T_1^6 T_2^3 - 11 a_4 b_3 T_1^6 T_2^3 - 3 a_2 b_5 T_1^6 T_2^3 + \\
& 5 a_4 b_5 T_1^6 T_2^3 + 6 c_{16} T_1^6 T_2^3 - 6 c_{19} T_1^6 T_2^3 + 6 c_{31} T_1^6 T_2^3 - 6 c_{34} T_1^6 T_2^3 + 6 c_{46} T_1^6 T_2^3 - 6 c_{49} T_1^6 T_2^3 + \\
& 6 c_{61} T_1^6 T_2^3 - 6 c_{64} T_1^6 T_2^3 - 2 a_2 b_3 T_1^7 T_2^3 + 4 a_4 b_3 T_1^7 T_2^3 - 6 c_{16} T_1^7 T_2^3 - 6 c_{31} T_1^7 T_2^3 - 6 c_{46} T_1^7 T_2^3 - \\
& 6 c_{61} T_1^7 T_2^3 + 2 c_{16} T_1^8 T_2^3 + 2 c_{31} T_1^8 T_2^3 + 2 c_{46} T_1^8 T_2^3 + 2 c_{61} T_1^8 T_2^3 + a_2 b_3 T_1 T_2^4 - 2 a_4 b_3 T_1 T_2^4 + \\
& 3 a_2 b_5 T_1 T_2^4 - 2 a_4 b_5 T_1 T_2^4 + 2 c_{16} T_1 T_2^4 + 2 c_{31} T_1 T_2^4 + 2 c_{46} T_1 T_2^4 + 2 c_{61} T_1 T_2^4 - 4 a_2 b_3 T_1^2 T_2^4 + \\
& 9 a_4 b_3 T_1^2 T_2^4 + 7 a_2 b_5 T_1^2 T_2^4 - 3 a_4 b_5 T_1^2 T_2^4 - 4 c_{16} T_1^2 T_2^4 - 4 c_{31} T_1^2 T_2^4 - 4 c_{46} T_1^2 T_2^4 - 4 c_{61} T_1^2 T_2^4 + \\
& 7 a_2 b_3 T_1^3 T_2^4 - 13 a_4 b_3 T_1^3 T_2^4 - 6 a_2 b_5 T_1^3 T_2^4 + 6 a_4 b_5 T_1^3 T_2^4 - 4 a_2 b_3 T_1^4 T_2^4 + 10 a_4 b_3 T_1^4 T_2^4 + \\
& 2 a_2 b_5 T_1^4 T_2^4 - 10 a_4 b_5 T_1^4 T_2^4 + 4 c_{16} T_1^4 T_2^4 - 2 c_{19} T_1^4 T_2^4 + 4 c_{31} T_1^4 T_2^4 - 2 c_{34} T_1^4 T_2^4 + 4 c_{46} T_1^4 T_2^4 - \\
& 2 c_{49} T_1^4 T_2^4 + 4 c_{61} T_1^4 T_2^4 - 2 c_{64} T_1^4 T_2^4 - 4 a_2 b_3 T_1^5 T_2^4 - 8 a_4 b_3 T_1^5 T_2^4 + 11 a_4 b_5 T_1^5 T_2^4 + 7 a_2 b_3 T_1 T_2^5 + \\
& 6 a_4 b_3 T_1^6 T_2^4 - a_2 b_5 T_1^6 T_2^4 - 6 a_4 b_5 T_1^6 T_2^4 - 4 c_{16} T_1^6 T_2^4 + 2 c_{19} T_1^6 T_2^4 - 4 c_{31} T_1^6 T_2^4 + 2 c_{34} T_1^6 T_2^4 - \\
& 4 c_{46} T_1^6 T_2^4 + 2 c_{49} T_1^6 T_2^4 - 4 c_{61} T_1^6 T_2^4 + 2 c_{64} T_1^6 T_2^4 - 4 a_2 b_3 T_1^7 T_2^4 - a_4 b_3 T_1^7 T_2^4 + a_2 b_5 T_1^7 T_2^4 - \\
& a_4 b_5 T_1^7 T_2^4 + a_2 b_3 T_1^8 T_2^4 - a_4 b_3 T_1^8 T_2^4 + 4 c_{16} T_1^8 T_2^4 + 4 c_{31} T_1^8 T_2^4 + 4 c_{46} T_1^8 T_2^4 + 4 c_{61} T_1^8 T_2^4 - 2 c_{16} T_1^9 T_2^4 - \\
& 2 c_{31} T_1^9 T_2^4 - 2 c_{46} T_1^9 T_2^4 - 2 c_{61} T_1^9 T_2^4 + a_4 b_3 T_1 T_2^5 + a_2 b_5 T_1 T_2^5 - 3 a_4 b_5 T_1 T_2^5 + a_2 b_3 T_1^2 T_2^5 - \\
& 5 a_4 b_3 T_1^2 T_2^5 - 4 a_2 b_5 T_1^2 T_2^5 + 11 a_4 b_5 T_1^2 T_2^5 - 2 c_{16} T_1^2 T_2^5 - 2 c_{31} T_1^2 T_2^5 - 2 c_{46} T_1^2 T_2^5 - 2 c_{61} T_1^2 T_2^5 - \\
& 4 a_2 b_3 T_1^3 T_2^5 + 11 a_4 b_3 T_1^3 T_2^5 + 7 a_2 b_5 T_1^3 T_2^5 - 13 a_4 b_5 T_1^3 T_2^5 + 6 c_{16} T_1^3 T_2^5 + 6 c_{31} T_1^3 T_2^5 + 6 c_{46} T_1^3 T_2^5 + \\
& 6 c_{61} T_1^3 T_2^5 + 3 a_2 b_3 T_1^4 T_2^5 - 11 a_4 b_3 T_1^4 T_2^5 + 8 a_2 b_5 T_1^4 T_2^5 - 6 c_{16} T_1^4 T_2^5 + 6 c_{19} T_1^4 T_2^5 - 6 c_{31} T_1^4 T_2^5 + \\
& 6 c_{34} T_1^4 T_2^5 - 6 c_{46} T_1^4 T_2^5 + 6 c_{49} T_1^4 T_2^5 - 6 c_{61} T_1^4 T_2^5 + 6 c_{64} T_1^4 T_2^5 - 4 a_4 b_3 T_1^5 T_2^5 - 4 a_2 b_5 T_1^5 T_2^5 + \\
& 4 a_4 b_5 T_1^5 T_2^5 + 2 c_{16} T_1^5 T_2^5 - 4 c_{19} T_1^5 T_2^5 + 2 c_{31} T_1^5 T_2^5 - 4 c_{34} T_1^5 T_2^5 + 2 c_{46} T_1^5 T_2^5 - 4 c_{49} T_1^5 T_2^5 +
\end{aligned}$$

$$\begin{aligned}
& 2 c_{61} T_1^5 T_2^5 - 4 c_{64} T_1^5 T_2^5 + 3 a_2 b_3 T_1^6 T_2^5 + 11 a_4 b_3 T_1^6 T_2^5 - a_4 b_5 T_1^6 T_2^5 - 2 c_{16} T_1^6 T_2^5 - 2 c_{19} T_1^6 T_2^5 - \\
& 2 c_{31} T_1^6 T_2^5 - 2 c_{34} T_1^6 T_2^5 - 2 c_{46} T_1^6 T_2^5 - 2 c_{49} T_1^6 T_2^5 - 2 c_{61} T_1^6 T_2^5 - 2 c_{64} T_1^6 T_2^5 - 4 a_2 b_3 T_1^7 T_2^5 - \\
& 4 a_4 b_3 T_1^7 T_2^5 + 2 a_4 b_5 T_1^7 T_2^5 + 6 c_{16} T_1^7 T_2^5 + 6 c_{31} T_1^7 T_2^5 + 6 c_{46} T_1^7 T_2^5 + 6 c_{61} T_1^7 T_2^5 + a_2 b_3 T_1^8 T_2^5 + \\
& a_4 b_3 T_1^8 T_2^5 - 6 c_{16} T_1^8 T_2^5 - 6 c_{31} T_1^8 T_2^5 - 6 c_{46} T_1^8 T_2^5 - 6 c_{61} T_1^8 T_2^5 + 2 c_{16} T_1^9 T_2^5 + 2 c_{31} T_1^9 T_2^5 + 2 c_{46} T_1^9 T_2^5 + \\
& 2 c_{61} T_1^9 T_2^5 + a_4 b_5 T_1^9 T_2^5 + a_4 b_3 T_1^9 T_2^5 + a_2 b_5 T_1^9 T_2^5 - 5 a_4 b_5 T_1^9 T_2^5 - 5 a_4 b_3 T_1^9 T_2^5 - 3 a_2 b_5 T_1^9 T_2^5 + \\
& 11 a_4 b_5 T_1^9 T_2^5 + a_2 b_3 T_1^9 T_2^5 + 6 a_4 b_3 T_1^9 T_2^5 - a_2 b_5 T_1^9 T_2^5 - 6 a_4 b_5 T_1^9 T_2^5 - 6 c_{19} T_1^9 T_2^5 - 6 c_{34} T_1^9 T_2^5 - \\
& 6 c_{49} T_1^9 T_2^5 - 6 c_{64} T_1^9 T_2^5 - a_2 b_3 T_1^9 T_2^5 + a_4 b_3 T_1^9 T_2^5 - 11 a_4 b_5 T_1^9 T_2^5 - a_2 b_3 T_1^9 T_2^5 + 3 a_4 b_3 T_1^9 T_2^5 + \\
& 4 a_2 b_5 T_1^6 T_2^6 - 3 a_4 b_5 T_1^6 T_2^6 + 6 c_{19} T_1^6 T_2^6 + 6 c_{34} T_1^6 T_2^6 + 6 c_{49} T_1^6 T_2^6 + 6 c_{64} T_1^6 T_2^6 + a_2 b_3 T_1^7 T_2^6 - \\
& 8 a_4 b_3 T_1^7 T_2^6 - a_2 b_5 T_1^7 T_2^6 + a_4 b_5 T_1^7 T_2^6 + 2 a_4 b_3 T_1^8 T_2^6 + a_4 b_5 T_1^8 T_2^6 - 4 a_4 b_5 T_1^9 T_2^6 + a_4 b_3 T_1^9 T_2^6 + \\
& a_2 b_5 T_1^9 T_2^6 + a_4 b_5 T_1^9 T_2^6 + 2 c_{19} T_1^9 T_2^6 + 2 c_{34} T_1^9 T_2^6 + 2 c_{49} T_1^9 T_2^6 + 2 c_{64} T_1^9 T_2^6 - 2 a_4 b_3 T_1^9 T_2^6 + \\
& 4 a_4 b_5 T_1^9 T_2^7 + 4 c_{19} T_1^9 T_2^7 + 4 c_{34} T_1^9 T_2^7 + 4 c_{49} T_1^9 T_2^7 + 4 c_{64} T_1^9 T_2^7 - a_4 b_3 T_1^9 T_2^7 - a_2 b_5 T_1^9 T_2^7 + \\
& 8 a_4 b_5 T_1^9 T_2^7 - 6 c_{19} T_1^9 T_2^7 - 6 c_{34} T_1^9 T_2^7 - 6 c_{49} T_1^9 T_2^7 - 6 c_{64} T_1^9 T_2^7 + 2 a_4 b_3 T_1^9 T_2^7 - 2 a_4 b_5 T_1^9 T_2^7 + \\
& a_4 b_5 T_1^9 T_2^8 - a_4 b_5 T_1^9 T_2^8 - 2 c_{19} T_1^9 T_2^8 - 2 c_{34} T_1^9 T_2^8 - 2 c_{49} T_1^9 T_2^8 - 2 c_{64} T_1^9 T_2^8 - 2 a_4 b_5 T_1^9 T_2^8 + \\
& 2 c_{19} T_1^9 T_2^8 + 2 c_{34} T_1^9 T_2^8 + 2 c_{49} T_1^9 T_2^8 + 2 c_{64} T_1^9 T_2^8 \Big) / \left((-1 + T_1) T_1^4 (-1 + T_2) T_2^3 (-1 + T_1 T_2) \right) \Big]
\end{aligned}$$

$$\begin{aligned}
In[]:= & \text{CKT} = \text{Factor} \left[(-1 + T_1) (-1 + T_2) (-1 + T_1 T_2) \right. \\
& \left. (\text{Cases} [\{\text{Conway}, \text{KT}\}, \text{eSeries} [0, \underline{\mathcal{E}}, \infty] /. \text{c}_{16|19|31|34|46|49|61|64|81|82|84|85} \rightarrow 0]) \right] \\
& (-b_3 + b_5 + b_3 T_1 - b_5 T_2) (-a_2 + a_2 T_1 - a_4 T_1 - a_4 T_2 + 2 a_4 T_1 T_2)
\end{aligned}$$

Out[]:=

$$\begin{aligned}
& \left\{ -\frac{1}{T_1^7 T_2^6} \right. \\
& \left(T_1^2 - 2 T_1^3 + T_1^4 - 2 T_1 T_2 + 2 T_1^2 T_2 + 2 T_1^5 T_2 - 2 T_1^6 T_2 + T_1^2 + 2 T_1 T_2^2 - 2 T_1^2 T_2^2 - 2 T_1^4 T_2^2 - 2 T_1^6 T_2^2 + 2 T_1^7 T_2^2 + \right. \\
& T_1^8 T_2^2 - 2 T_1^3 + T_1^4 T_2^3 + T_1^5 T_2^3 - 2 T_1^9 T_2^3 + T_1^4 - 2 T_1^2 T_2^4 + T_1^3 T_2^4 + 2 T_1^4 T_2^4 + 2 T_1^6 T_2^4 + T_1^7 T_2^4 - 2 T_1^8 T_2^4 + T_1^{10} T_2^4 + \\
& 2 T_1 T_2^5 + T_1^3 T_2^5 - 4 T_1^5 T_2^5 - 4 T_1^6 T_2^5 + T_1^8 T_2^5 + 2 T_1^{10} T_2^5 - 2 T_1 T_2^6 - 2 T_1^2 T_2^6 + 2 T_1^4 T_2^6 - 4 T_1^5 T_2^6 + 12 T_1^6 T_2^6 - \\
& 4 T_1^7 T_2^6 + 2 T_1^8 T_2^6 - 2 T_1^{10} T_2^6 - 2 T_1^{11} T_2^6 + 2 T_1^2 T_2^7 + T_1^4 T_2^7 - 4 T_1^6 T_2^7 - 4 T_1^7 T_2^7 + T_1^9 T_2^7 + \\
& 2 T_1^{11} T_2^7 + T_1^2 T_2^8 - 2 T_1^4 T_2^8 + T_1^5 T_2^8 + 2 T_1^6 T_2^8 + 2 T_1^8 T_2^8 + T_1^9 T_2^8 - 2 T_1^{10} T_2^8 + T_1^{12} T_2^8 - 2 T_1^3 T_2^9 + \\
& T_1^7 T_2^9 + T_1^8 T_2^9 - 2 T_1^{12} T_2^9 + T_1^4 T_2^{10} + 2 T_1^5 T_2^{10} - 2 T_1^6 T_2^{10} - 2 T_1^8 T_2^{10} - 2 T_1^{10} T_2^{10} + 2 T_1^{11} T_2^{10} + \\
& T_1^{12} T_2^{10} - 2 T_1^6 T_2^{11} + 2 T_1^7 T_2^{11} + 2 T_1^{10} T_2^{11} - 2 T_1^{11} T_2^{11} + T_1^8 T_2^{12} - 2 T_1^9 T_2^{12} + T_1^{10} T_2^{12} \Big), \\
& \left. \frac{1}{T_1^4 T_2^3} \right(T_1 + T_1^2 + T_2 - 2 T_1 T_2 - 2 T_1^2 T_2 - 2 T_1^3 T_2 + T_1^4 T_2 + T_2^2 - 2 T_1 T_2^2 + 2 T_1^2 T_2^2 + 2 T_1^3 T_2^2 - \\
& 2 T_1^4 T_2^2 + T_1^5 T_2^2 - 2 T_1 T_2^3 + 2 T_1^2 T_2^3 + 2 T_1^4 T_2^3 - 2 T_1^5 T_2^3 + T_1 T_2^4 - 2 T_1^2 T_2^4 + 2 T_1^3 T_2^4 + \\
& 2 T_1^4 T_2^4 - 2 T_1^5 T_2^4 + T_1^6 T_2^4 + T_1^2 T_2^5 - 2 T_1^3 T_2^5 - 2 T_1^4 T_2^5 - 2 T_1^5 T_2^5 + T_1^6 T_2^5 + T_1^4 T_2^6 + T_1^5 T_2^6 \Big) \Big\}
\end{aligned}$$

$$In[]:= \text{Factor} [\text{CKT} /. T_1 \rightarrow 1]$$

Out[]:=

$$\left\{ \frac{2 (-1 + T_2)^2 (1 + T_2^4)}{T_2^3}, \frac{2 (-1 + T_2)^2 (1 + T_2^4)}{T_2^3} \right\}$$

$$In[]:= \text{Factor} [\text{CKT} /. T_2 \rightarrow 1]$$

Out[]:=

$$\left\{ \frac{2 (-1 + T_1)^2 (1 + T_1^4)}{T_1^4}, \frac{2 (-1 + T_1)^2 (1 + T_1^4)}{T_1^4} \right\}$$

$$\text{In}[#]:= \text{Factor} \left[\frac{(\text{CKT} / . \{ \text{T}_1 \rightarrow \text{T}_2, \text{T}_2 \rightarrow \text{T}_1 \})}{\text{CKT}} \right]$$

Out[#]=

$$\left\{ \frac{\text{T}_1}{\text{T}_2}, \frac{\text{T}_1}{\text{T}_2} \right\}$$

$$\text{In}[#]:= \text{ntab1} = \text{Factor} \left[\frac{(-1 + \text{T}_1) (-1 + \text{T}_2) (-1 + \text{T}_1 \text{T}_2)}{(-\text{b}_3 + \text{b}_5 + \text{b}_3 \text{T}_1 - \text{b}_5 \text{T}_2) (-\text{a}_2 + \text{a}_2 \text{T}_1 - \text{a}_4 \text{T}_1 - \text{a}_4 \text{T}_2 + 2 \text{a}_4 \text{T}_1 \text{T}_2)} \right]$$

Out[#]=

$$\begin{aligned} & - \frac{\text{T}_2 (-1 + \text{T}_1 - \text{T}_1^2 + \text{T}_2 - \text{T}_1^2 \text{T}_2 + 2 \text{T}_1^3 \text{T}_2 - \text{T}_2^2 - \text{T}_1 \text{T}_2^2 + \text{T}_1^2 \text{T}_2^2 - 2 \text{T}_1^3 \text{T}_2^2 + 2 \text{T}_1 \text{T}_2^3 - 2 \text{T}_1^2 \text{T}_2^3 + 2 \text{T}_1^3 \text{T}_2^3)}{(1 - \text{T}_1 + \text{T}_1^2) (1 - \text{T}_2 + \text{T}_2^2) (1 - \text{T}_1 \text{T}_2 + \text{T}_1^2 \text{T}_2^2)}, \\ & - \frac{(-1 + \text{T}_1 \text{T}_2) (1 + \text{T}_1 \text{T}_2)}{\text{T}_1 (1 - 3 \text{T}_1 \text{T}_2 + \text{T}_1^2 \text{T}_2^2)}, \\ & - \left(\left(\text{T}_2 (-1 + \text{T}_1 - \text{T}_1^2 + \text{T}_1^3 - \text{T}_1^4 + \text{T}_2 - \text{T}_1^4 \text{T}_2 + 2 \text{T}_1^5 \text{T}_2 - \text{T}_2^2 - 2 \text{T}_1^2 \text{T}_2^2 + 2 \text{T}_1^3 \text{T}_2^2 - \text{T}_1^4 \text{T}_2^2 + \text{T}_1^5 \text{T}_2^2 - 3 \text{T}_1^6 \text{T}_2^2 + \text{T}_1^3 + 2 \text{T}_1^2 \text{T}_2^3 - 2 \text{T}_1^4 \text{T}_2^3 + 2 \text{T}_1^6 \text{T}_2^3 - \text{T}_1^7 \text{T}_2^3 + 4 \text{T}_1^7 \text{T}_2^3 - \text{T}_1^4 \text{T}_2^4 - \text{T}_1^2 \text{T}_2^4 - 2 \text{T}_1^3 \text{T}_2^4 + 2 \text{T}_1^4 \text{T}_2^4 - 2 \text{T}_1^5 \text{T}_2^4 + \text{T}_1^6 \text{T}_2^4 - 4 \text{T}_1^7 \text{T}_2^4 + 2 \text{T}_1 \text{T}_2^5 + \text{T}_1^2 \text{T}_2^5 + 2 \text{T}_1^3 \text{T}_2^5 - 2 \text{T}_1^4 \text{T}_2^5 + 2 \text{T}_1^5 \text{T}_2^5 - \text{T}_1^6 \text{T}_2^5 + 4 \text{T}_1^7 \text{T}_2^5 - 3 \text{T}_1^8 \text{T}_2^5 - \text{T}_1^3 \text{T}_2^6 + \text{T}_1^4 \text{T}_2^6 - \text{T}_1^5 \text{T}_2^6 + 4 \text{T}_1^7 \text{T}_2^6 - 4 \text{T}_1^8 \text{T}_2^6 + 4 \text{T}_1^9 \text{T}_2^6 - 4 \text{T}_1^{10} \text{T}_2^6) \right) / \\ & \quad \left((1 - \text{T}_1 + \text{T}_1^2 - \text{T}_1^3 + \text{T}_1^4) (1 - \text{T}_2 + \text{T}_2^2 - \text{T}_2^3 + \text{T}_1^4) (1 - \text{T}_1 \text{T}_2 + \text{T}_1^2 \text{T}_2^2 - \text{T}_1^3 \text{T}_2^3 + \text{T}_1^4 \text{T}_2^4) \right), \\ & - \left((-7 + 17 \text{T}_1 - 7 \text{T}_1^2 + 17 \text{T}_2 - 44 \text{T}_1 \text{T}_2 + 26 \text{T}_1^2 \text{T}_2 - 3 \text{T}_1^3 \text{T}_2 - 7 \text{T}_2^2 + 26 \text{T}_1 \text{T}_2^2 - 24 \text{T}_1^2 \text{T}_2^2 - 14 \text{T}_1^3 \text{T}_2^2 + 9 \text{T}_1^4 \text{T}_2^2 - 3 \text{T}_1 \text{T}_2^3 - 14 \text{T}_1^2 \text{T}_2^3 + 56 \text{T}_1^3 \text{T}_2^3 - 23 \text{T}_1^4 \text{T}_2^3 + 9 \text{T}_1^2 \text{T}_2^4 - 23 \text{T}_1^3 \text{T}_2^4 + 9 \text{T}_1^4 \text{T}_2^4) \right) / \\ & \quad \left((-2 + \text{T}_1) \text{T}_1 (-1 + 2 \text{T}_1) (-2 + 2 \text{T}_2) (-1 + 2 \text{T}_1 \text{T}_2) (-1 + 2 \text{T}_1 \text{T}_2) \right), \\ & - \left(\left(\text{T}_2 (-1 + \text{T}_1 - \text{T}_1^2 + \text{T}_1^3 - \text{T}_1^4 + \text{T}_1^5 - \text{T}_1^6 + \text{T}_2 - \text{T}_1^6 \text{T}_2 + 2 \text{T}_1^7 \text{T}_2 - \text{T}_2^2 - 2 \text{T}_1^2 \text{T}_2^2 + 2 \text{T}_1^3 \text{T}_2^2 - 2 \text{T}_1^4 \text{T}_2^2 + 2 \text{T}_1^5 \text{T}_2^2 - \text{T}_1^6 \text{T}_2^2 + \text{T}_1^7 \text{T}_2^2 - 3 \text{T}_1^8 \text{T}_2^2 + 2 \text{T}_1^2 \text{T}_2^3 - 2 \text{T}_1^6 \text{T}_2^3 + 2 \text{T}_1^7 \text{T}_2^3 - \text{T}_1^8 \text{T}_2^3 + 4 \text{T}_1^9 \text{T}_2^3 - \text{T}_1^4 \text{T}_2^4 - 2 \text{T}_1^2 \text{T}_2^4 - 3 \text{T}_1^4 \text{T}_2^4 + 3 \text{T}_1^5 \text{T}_2^4 - 2 \text{T}_1^6 \text{T}_2^4 + 2 \text{T}_1^7 \text{T}_2^4 - 3 \text{T}_1^5 \text{T}_2^5 + 2 \text{T}_1^6 \text{T}_2^5 - 3 \text{T}_1^5 \text{T}_2^6 + 2 \text{T}_1^6 \text{T}_2^6 + 2 \text{T}_1^7 \text{T}_2^6 - 3 \text{T}_1^8 \text{T}_2^6 + 2 \text{T}_1^9 \text{T}_2^6 - \text{T}_1^4 \text{T}_2^7 - 2 \text{T}_1^2 \text{T}_2^7 + 6 \text{T}_1^{11} \text{T}_2^7 - \text{T}_1^6 \text{T}_2^7 - \text{T}_1^2 \text{T}_2^8 - 2 \text{T}_1^3 \text{T}_2^8 - 2 \text{T}_1^4 \text{T}_2^8 - 2 \text{T}_1^5 \text{T}_2^8 - 2 \text{T}_1^6 \text{T}_2^8 + 2 \text{T}_1^7 \text{T}_2^8 - 2 \text{T}_1^8 \text{T}_2^8 + 2 \text{T}_1^9 \text{T}_2^8 - \text{T}_1^{10} \text{T}_2^8 + 6 \text{T}_1^{11} \text{T}_2^8 - \text{T}_1^{12} \text{T}_2^8 + 4 \text{T}_1^3 \text{T}_2^9 + \text{T}_1^4 \text{T}_2^9 + 4 \text{T}_1^5 \text{T}_2^9 - 4 \text{T}_1^6 \text{T}_2^9 + 4 \text{T}_1^7 \text{T}_2^9 - 2 \text{T}_1^8 \text{T}_2^9 + 4 \text{T}_1^9 \text{T}_2^9 - 4 \text{T}_1^{10} \text{T}_2^9 + 6 \text{T}_1^{11} \text{T}_2^9 - 6 \text{T}_1^{12} \text{T}_2^9 + 6 \text{T}_1^3 \text{T}_2^{10} + 6 \text{T}_1^4 \text{T}_2^{10} - 6 \text{T}_1^5 \text{T}_2^{10} + \text{T}_1^6 \text{T}_2^{10} - \text{T}_1^7 \text{T}_2^{10} + \text{T}_1^8 \text{T}_2^{10} - \text{T}_1^9 \text{T}_2^{10} + \text{T}_1^{10} \text{T}_2^{10} - 6 \text{T}_1^{11} \text{T}_2^{10} + 6 \text{T}_1^{12} \text{T}_2^{10} - 6 \text{T}_1^6 \text{T}_2^{11} + 6 \text{T}_1^7 \text{T}_2^{11} - 6 \text{T}_1^8 \text{T}_2^{11} + 6 \text{T}_1^9 \text{T}_2^{11} - 6 \text{T}_1^{10} \text{T}_2^{11} + 6 \text{T}_1^{11} \text{T}_2^{11}) \right) / \\ & \quad \left((1 - \text{T}_1 + \text{T}_1^2 - \text{T}_1^3 + \text{T}_1^4 - \text{T}_1^5 + \text{T}_1^6) (1 - \text{T}_2 + \text{T}_2^2 - \text{T}_2^3 + \text{T}_2^4 - \text{T}_2^5 + \text{T}_2^6) \right. \\ & \quad \left. (1 - \text{T}_1 \text{T}_2 + \text{T}_1^2 \text{T}_2^2 - \text{T}_1^3 \text{T}_2^3 + \text{T}_1^4 \text{T}_2^4 - \text{T}_1^5 \text{T}_2^5 + \text{T}_1^6 \text{T}_2^6) \right), \\ & - \left((-18 + 40 \text{T}_1 - 18 \text{T}_1^2 + 40 \text{T}_2 - 111 \text{T}_1 \text{T}_2 + 99 \text{T}_1^2 \text{T}_2 - 23 \text{T}_1^3 \text{T}_2 - 18 \text{T}_2^2 + 99 \text{T}_1 \text{T}_2^2 - 132 \text{T}_1^2 \text{T}_2^2 - 27 \text{T}_1^3 \text{T}_2^2 + 36 \text{T}_1^4 \text{T}_2^2 - 23 \text{T}_1 \text{T}_2^3 - 27 \text{T}_1^2 \text{T}_2^3 + 183 \text{T}_1^3 \text{T}_2^3 - 86 \text{T}_1^4 \text{T}_2^3 + 36 \text{T}_1^2 \text{T}_2^4 - 86 \text{T}_1^3 \text{T}_2^4 + 36 \text{T}_1^4 \text{T}_2^4) \right) / \\ & \quad \left(\text{T}_1 (3 - 7 \text{T}_1 + 3 \text{T}_1^2) (3 - 7 \text{T}_2 + 3 \text{T}_2^2) (3 - 7 \text{T}_1 \text{T}_2 + 3 \text{T}_1^2 \text{T}_2^2) \right), \\ & - \left(\left(\text{T}_2 (-1 + \text{T}_1 - \text{T}_1^2 + \text{T}_1^3 - \text{T}_1^4 + \text{T}_1^5 - \text{T}_1^6 + \text{T}_1^7 - \text{T}_1^8 + \text{T}_2 - \text{T}_1^8 \text{T}_2 + 2 \text{T}_1^9 \text{T}_2 - \text{T}_2^2 - 2 \text{T}_1^2 \text{T}_2^2 + 2 \text{T}_1^3 \text{T}_2^2 - 2 \text{T}_1^4 \text{T}_2^2 + 2 \text{T}_1^5 \text{T}_2^2 - 2 \text{T}_1^6 \text{T}_2^2 + 2 \text{T}_1^7 \text{T}_2^2 + 2 \text{T}_1^8 \text{T}_2^2 - \text{T}_1^9 \text{T}_2^2 + 3 \text{T}_1^{10} \text{T}_2^2 + \text{T}_1^3 + 2 \text{T}_1^2 \text{T}_2^3 - 2 \text{T}_1^8 \text{T}_2^3 + 2 \text{T}_1^9 \text{T}_2^3 - \text{T}_1^{10} \text{T}_2^3 + 4 \text{T}_1^{11} \text{T}_2^3 - \text{T}_1^4 \text{T}_2^4 - 2 \text{T}_1^2 \text{T}_2^4 - 3 \text{T}_1^4 \text{T}_2^4 + 3 \text{T}_1^5 \text{T}_2^4 + 3 \text{T}_1^6 \text{T}_2^4 + 3 \text{T}_1^7 \text{T}_2^4 - \text{T}_1^8 \text{T}_2^4 + 2 \text{T}_1^9 \text{T}_2^4 - 3 \text{T}_1^{10} \text{T}_2^4 + 3 \text{T}_1^4 \text{T}_2^5 + 5 \text{T}_1^{12} \text{T}_2^5 + \text{T}_1^5 \text{T}_2^5 + 2 \text{T}_1^2 \text{T}_2^5 + 3 \text{T}_1^4 \text{T}_2^5 - 3 \text{T}_1^8 \text{T}_2^5 + 2 \text{T}_1^9 \text{T}_2^5 - 2 \text{T}_1^{10} \text{T}_2^5 + 4 \text{T}_1^{11} \text{T}_2^5 - \text{T}_1^{12} \text{T}_2^5 + 6 \text{T}_1^{13} \text{T}_2^5 - \text{T}_1^{14} \text{T}_2^5 + 2 \text{T}_1^2 \text{T}_2^6 - 3 \text{T}_1^4 \text{T}_2^6 - 4 \text{T}_1^6 \text{T}_2^6 + 4 \text{T}_1^7 \text{T}_2^6 - \text{T}_1^8 \text{T}_2^6 + 3 \text{T}_1^9 \text{T}_2^6 - 3 \text{T}_1^{10} \text{T}_2^6 + 2 \text{T}_1^{11} \text{T}_2^6 - 5 \text{T}_1^{12} \text{T}_2^6 + \text{T}_1^{13} \text{T}_2^6 + 7 \text{T}_1^{14} \text{T}_2^6 + \text{T}_1^7 + 2 \text{T}_1^2 \text{T}_2^7 + 3 \text{T}_1^4 \text{T}_2^7 + 4 \text{T}_1^6 \text{T}_2^7 - 4 \text{T}_1^8 \text{T}_2^7 + 2 \text{T}_1^9 \text{T}_2^7 - 3 \text{T}_1^{10} \text{T}_2^7 + 4 \text{T}_1^{11} \text{T}_2^7 - 2 \text{T}_1^{12} \text{T}_2^7 + 6 \text{T}_1^{13} \text{T}_2^7 - \text{T}_1^{14} \text{T}_2^7 + 8 \text{T}_1^{15} \text{T}_2^7 - \text{T}_1^8 - \text{T}_1 \text{T}_2^8 - 2 \text{T}_1^2 \text{T}_2^8 - \text{T}_1^4 \text{T}_2^8 - 3 \text{T}_1^5 \text{T}_2^8 - \text{T}_1^6 \text{T}_2^8 - 4 \text{T}_1^7 \text{T}_2^8 + \right) \right) / \end{aligned}$$

$$\begin{aligned}
& 4 T_1^8 T_2^8 - 2 T_1^9 T_2^8 + 3 T_1^{10} T_2^8 - 4 T_1^{11} T_2^8 + 2 T_1^{12} T_2^8 - 6 T_1^{13} T_2^8 + T_1^{14} T_2^8 - 8 T_1^{15} T_2^8 + 2 T_1 T_2^9 + T_1^2 T_2^9 + \\
& 2 T_1^3 T_2^9 + 2 T_1^4 T_2^9 + 2 T_1^5 T_2^9 + 3 T_1^6 T_2^9 + 2 T_1^7 T_2^9 - 2 T_1^8 T_2^9 + 2 T_1^9 T_2^9 - 3 T_1^{10} T_2^9 + 4 T_1^{11} T_2^9 - 2 T_1^{12} T_2^9 + \\
& 6 T_1^{13} T_2^9 - T_1^{14} T_2^9 + 8 T_1^{15} T_2^9 - 3 T_1^2 T_2^{10} - T_1^3 T_2^{10} - 3 T_1^4 T_2^{10} - 2 T_1^5 T_2^{10} - 3 T_1^6 T_2^{10} - 3 T_1^7 T_2^{10} + \\
& 3 T_1^8 T_2^{10} - 3 T_1^9 T_2^{10} + 3 T_1^{10} T_2^{10} - 4 T_1^{11} T_2^{10} + 2 T_1^{12} T_2^{10} - 6 T_1^{13} T_2^{10} + T_1^{14} T_2^{10} - 8 T_1^{15} T_2^{10} + 4 T_1^3 T_2^{11} + \\
& T_1^4 T_2^{11} + 4 T_1^5 T_2^{11} + 2 T_1^6 T_2^{11} + 4 T_1^7 T_2^{11} - 4 T_1^8 T_2^{11} + 4 T_1^9 T_2^{11} - 4 T_1^{10} T_2^{11} + 4 T_1^{11} T_2^{11} - 2 T_1^{12} T_2^{11} + \\
& 6 T_1^{13} T_2^{11} - T_1^{14} T_2^{11} + 8 T_1^{15} T_2^{11} - 5 T_1^4 T_2^{12} - T_1^5 T_2^{12} - 5 T_1^6 T_2^{12} - 2 T_1^7 T_2^{12} + 2 T_1^8 T_2^{12} - 2 T_1^9 T_2^{12} + \\
& 2 T_1^{10} T_2^{12} - 2 T_1^{11} T_2^{12} + 2 T_1^{12} T_2^{12} - 6 T_1^{13} T_2^{12} + T_1^{14} T_2^{12} - 8 T_1^{15} T_2^{12} + 6 T_1^5 T_2^{13} + T_1^6 T_2^{13} + 6 T_1^7 T_2^{13} - \\
& 6 T_1^8 T_2^{13} + 6 T_1^9 T_2^{13} - 6 T_1^{10} T_2^{13} + 6 T_1^{11} T_2^{13} - 6 T_1^{12} T_2^{13} + 6 T_1^{13} T_2^{13} - T_1^{14} T_2^{13} + 8 T_1^{15} T_2^{13} - 7 T_1^6 T_2^{14} - \\
& T_1^7 T_2^{14} + T_1^8 T_2^{14} - T_1^9 T_2^{14} + T_1^{10} T_2^{14} - T_1^{11} T_2^{14} + T_1^{12} T_2^{14} - T_1^{13} T_2^{14} + T_1^{14} T_2^{14} - 8 T_1^{15} T_2^{14} + 8 T_1^7 T_2^{15} - \\
& 8 T_1^8 T_2^{15} + 8 T_1^9 T_2^{15} - 8 T_1^{10} T_2^{15} + 8 T_1^{11} T_2^{15} - 8 T_1^{12} T_2^{15} + 8 T_1^{13} T_2^{15} - 8 T_1^{14} T_2^{15} + 8 T_1^{15} T_2^{15} \Big)) / \\
& \left((1 - T_1 + T_1^2) (1 - T_1^3 + T_1^6) (1 - T_2 + T_2^2) (1 - T_1 T_2 + T_1^2 T_2^2) (1 - T_2^3 + T_2^6) (1 - T_1^3 T_2^3 + T_1^6 T_2^6) \right) , \\
& - \left(2 \left(-14 + 29 T_1 - 14 T_1^2 + 29 T_2 - 102 T_1 T_2 + 132 T_1^2 T_2 - 43 T_1^3 T_2 - 14 T_2^2 + 132 T_1 T_2^2 - 210 T_1^2 T_2^2 - \right. \right. \\
& \left. \left. 12 T_1^3 T_2^2 + 50 T_1^4 T_2^2 - 43 T_1 T_2^3 - 12 T_1^2 T_2^3 + 222 T_1^3 T_2^3 - 115 T_1^4 T_2^3 + 50 T_1^2 T_2^4 - 115 T_1^3 T_2^4 + 50 T_1^4 T_2^4 \right) \right) / \\
& \left. \left(T_1 (4 - 9 T_1 + 4 T_1^2) (4 - 9 T_2 + 4 T_2^2) (4 - 9 T_1 T_2 + 4 T_1^2 T_2^2) \right) \right\}
\end{aligned}$$

In[1]:= **Factor** $\left[\frac{(\text{ntab1} / . \{T_1 \rightarrow T_2, T_2 \rightarrow T_1\})}{\text{ntab1}} \right]$

Out[1]=

$$\left\{ \frac{T_1}{T_2}, \frac{T_1}{T_2}, \frac{T_1}{T_2}, \frac{T_1}{T_2}, \frac{T_1}{T_2}, \frac{T_1}{T_2}, \frac{T_1}{T_2}, \frac{T_1}{T_2} \right\}$$