Pensieve header: Finding the A2 $d=1$ invariant using undetermined coefficients.

Searching for Q + pxx + ϵ(ppx + 1 + px + ppxx) solutions.

Initialization

```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank"];
        Once[<< KnotTheory`; << Rot.m];
        << FormalGaussianIntegration.m;
        i_⁺ := i + 1;
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
Loading Rot.m from http://drorbn.net/AP/Projects/HigherRank to compute rotation numbers.
```

```
In[•]:= Features[Knot[8, 17]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

```
Out[•]=
```

$$\text{Features}[18,\ C_6[-1]\,C_{14}[-1]\,X_{1,7}[1]\,X_{3,9}[-1]\,X_{5,13}[-1]\,X_{8,16}[1]\,X_{10,4}[-1]\,X_{12,18}[1]\,X_{15,2}[-1]\,X_{17,11}[1]]$$

```
In[•]:= T3 = T1 T2;
        S = {x__, p__};
        q[s_, i_, j_] := Sum[
            x_{v,i} (p_{v,i⁺} - p_{v,i}) + x_{v,j} (p_{v,j⁺} - p_{v,j}) + (T_v^s - 1) x_{v,i} (p_{v,i⁺} - p_{v,j⁺}),
            {v, 3}];
        ℒ[X_{i_,j_}[s_]] := T3^s 𝔼[q[s, i, j] + r0[s, i, j] + ϵ r1[s, i, j] + O[ϵ]^1];
        γ1[φ_, k_] := φ (3 / 2 - x_{1,k} p_{1,k} - x_{2,k} p_{2,k} - x_{3,k} p_{3,k});
        ℒ[C_k_[φ_]] := T3^φ 𝔼[Sum[x_{v,k} (p_{v,k⁺} - p_{v,k}), {v, 3}] + ϵ γ1[φ, k] + O[ϵ]^1];
        ps_i_ := Sequence[p_{1,i}, p_{2,i}, p_{3,i}];
        xs_i_ := Sequence[x_{1,i}, x_{2,i}, x_{3,i}];
        vs_i_ := Sequence[ps_i, xs_i];
        ℱ[is___] := 𝔼[Sum[π_{v,i} p_{v,i}, {i, {is}}, {v, 3}]];
        ℒ[K_] := (2 π)^-Features[K][[1]] CF[ℒ /@ Features[K][[2]]];
        vs[K_] := Union @@ Table[{vs_i}, {i, Features[K][[1]]}]
```

```
In[•]:= vs_i
```

```
Out[•]=
```

$$\text{Sequence}[p_{1,i},\ p_{2,i},\ p_{3,i},\ x_{1,i},\ x_{2,i},\ x_{3,i}]$$

The pxx Terms (r_0)

```
In[•]:= κ = 0;
        r₀[1, i_, j_] := Evaluate[Sum[
            a_{++κ} p_{3,k3} x_{1,k1} x_{2,k2},
            {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
           ]];
        r₀[1, i, j]
Out[•]=
        a1 p3,i x1,i x2,i + a2 p3,j x1,i x2,i + a5 p3,i x1,j x2,i + a6 p3,j x1,j x2,i +
         a3 p3,i x1,i x2,j + a4 p3,j x1,i x2,j + a7 p3,i x1,j x2,j + a8 p3,j x1,j x2,j

In[•]:= ℒ[X_{i_,j_}[s_]] := T₃^s 𝔼[q[s, i, j] + r₀[s, i, j]];
        ℒ[X_{i,j}[1]]
Out[•]=
        T1 T2
         𝔼[(-p1,i + p1,1+i) x1,i + (-1 + T1) (p1,1+i - p1,1+j) x1,i + (-p1,j + p1,1+j) x1,j + (-p2,i + p2,1+i) x2,i +
           (-1 + T2) (p2,1+i - p2,1+j) x2,i + a1 p3,i x1,i x2,i + a2 p3,j x1,i x2,i + a5 p3,i x1,j x2,i +
           a6 p3,j x1,j x2,i + (-p2,j + p2,1+j) x2,j + a3 p3,i x1,i x2,j + a4 p3,j x1,i x2,j + a7 p3,i x1,j x2,j +
           a8 p3,j x1,j x2,j + (-p3,i + p3,1+i) x3,i + (-1 + T1 T2) (p3,1+i - p3,1+j) x3,i + (-p3,j + p3,1+j) x3,j]
```

Reidemeister 3 for pxx (r_0)

```
In[•]:= {lhs} = Cases[∫ℱ[i, j, k] ℒ /@ (X_{i,j}[1] X_{i⁺,k}[1] X_{j⁺,k⁺}[1]) đ{vs_i, vs_j, vs_k, vs_{i⁺}, vs_{j⁺}, vs_{k⁺}},
           𝔼[ℰ_] :↦ ℰ, ∞];

In[•]:= {rhs} = Cases[∫ℱ[i, j, k] ℒ /@ (X_{j,k}[1] X_{i,k⁺}[1] X_{i⁺,j⁺}[1]) đ{vs_i, vs_j, vs_k, vs_{i⁺}, vs_{j⁺}, vs_{k⁺}},
           𝔼[ℰ_] :↦ ℰ, ∞];

In[•]:= eqn = CF[lhs - rhs];

In[•]:= cvs = Union@Cases[eqn, p__ | π__, ∞]
Out[•]=
        {p3,2+i, p3,2+j, p3,2+k, π1,i, π1,j, π1,k, π2,i, π2,j, π2,k}

In[•]:= eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :↦ (c == 0);

In[•]:= vars = Union@Cases[r₀[1, i, j], a_, ∞]
Out[•]=
        {a1, a2, a3, a4, a5, a6, a7, a8}
```

```
In[•]:= {sol} = Solve[eqns, vars]
```

Solve: Equations may not give solutions for all "solve" variables.

Out[•]=

$$\left\{\left\{a_1 \to 0,\ a_3 \to 0,\ a_5 \to 0,\ a_6 \to -\frac{a_2}{T_1} - \frac{a_4\, T_2}{T_1},\ a_7 \to 0,\ a_8 \to 0\right\}\right\}$$

```
In[•]:= sol /. (v_ → val_) :> (v = CF[val]);
```

```
In[•]:= r0[1, i, j]
```

Out[•]=

$$a_2\, p_{3,j}\, x_{1,i}\, x_{2,i} - \frac{(a_2 + a_4\, T_2)\, p_{3,j}\, x_{1,j}\, x_{2,i}}{T_1} + a_4\, p_{3,j}\, x_{1,i}\, x_{2,j}$$

The ppx Terms (r_1)

```
In[•]:= κ = 0;
        r1[1, i_, j_] := Evaluate[Sum[
            b_{++κ} x_{3,k3} p_{1,k1} p_{2,k2},
            {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
           ]];
        r1[1, i, j]
```

Out[•]=

$$b_1\, p_{1,i}\, p_{2,i}\, x_{3,i} + b_5\, p_{1,j}\, p_{2,i}\, x_{3,i} + b_3\, p_{1,i}\, p_{2,j}\, x_{3,i} + b_7\, p_{1,j}\, p_{2,j}\, x_{3,i} + b_2\, p_{1,i}\, p_{2,i}\, x_{3,j} + b_6\, p_{1,j}\, p_{2,i}\, x_{3,j} + b_4\, p_{1,i}\, p_{2,j}\, x_{3,j} + b_8\, p_{1,j}\, p_{2,j}\, x_{3,j}$$

```
In[•]:= ℒ[X_{i_,j_}[s_]] := T3^s 𝔼[q[s, i, j] + ϵ r1[s, i, j] + O[ϵ]^2];
        ℒ[X_{i,j}[1]]
```

Out[•]=

$$T_1\, T_2\, \mathbb{E}[\epsilon\text{Series}[(-p_{1,i} + p_{1,1+i})\, x_{1,i} + (-1 + T_1)\, (p_{1,1+i} - p_{1,1+j})\, x_{1,i} + (-p_{1,j} + p_{1,1+j})\, x_{1,j} + (-p_{2,i} + p_{2,1+i})\, x_{2,i} + (-1 + T_2)\, (p_{2,1+i} - p_{2,1+j})\, x_{2,i} + (-p_{2,j} + p_{2,1+j})\, x_{2,j} + (-p_{3,i} + p_{3,1+i})\, x_{3,i} + (-1 + T_1\, T_2)\, (p_{3,1+i} - p_{3,1+j})\, x_{3,i} + (-p_{3,j} + p_{3,1+j})\, x_{3,j},\ b_1\, p_{1,i}\, p_{2,i}\, x_{3,i} + b_5\, p_{1,j}\, p_{2,i}\, x_{3,i} + b_3\, p_{1,i}\, p_{2,j}\, x_{3,i} + b_7\, p_{1,j}\, p_{2,j}\, x_{3,i} + b_2\, p_{1,i}\, p_{2,i}\, x_{3,j} + b_6\, p_{1,j}\, p_{2,i}\, x_{3,j} + b_4\, p_{1,i}\, p_{2,j}\, x_{3,j} + b_8\, p_{1,j}\, p_{2,j}\, x_{3,j}]]$$

Reidemeister 3 for ppx (r_1)

```
In[•]:= {lhs} = Cases[∫ℱ[i, j, k] ℒ /@ (X_{i,j}[1] X_{i⁺,k}[1] X_{j⁺,k⁺}[1]) đ{vs_i, vs_j, vs_k, vs_{i⁺}, vs_{j⁺}, vs_{k⁺}},
            ϵSeries[_, ℰ_] :> ℰ, ∞];
```

```
In[•]:= {rhs} = Cases[∫ℱ[i, j, k] ℒ /@ (X_{j,k}[1] X_{i,k⁺}[1] X_{i⁺,j⁺}[1]) đ{vs_i, vs_j, vs_k, vs_{i⁺}, vs_{j⁺}, vs_{k⁺}},
            ϵSeries[_, ℰ_] :> ℰ, ∞];
```

```
In[•]:= eqn = CF[lhs - rhs];
```

```
In[•]:= cvs = Union@Cases[eqn, p__ | π__, ∞]
```

```
Out[•]=
{p1,2+i, p1,2+j, p1,2+k, p2,2+i, p2,2+j, p2,2+k, π3,i, π3,j, π3,k}
```

```
In[•]:= eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0);
```

```
In[•]:= vars = Union@Cases[r1[1, i, j], b_, ∞]
```

```
Out[•]=
{b1, b2, b3, b4, b5, b6, b7, b8}
```

```
In[•]:= {sol} = Solve[eqns, vars]
```

```
Solve: Equations may not give solutions for all "solve" variables.
```

```
Out[•]=
{{b1 → 0, b2 → 0, b4 → 0, b6 → 0, b7 → -b3 - b5, b8 → 0}}
```

```
In[•]:= sol /. (v_ → val_) :> (v = CF[val]);
```

```
In[•]:= r1[1, i, j]
```

```
Out[•]=
b5 p1,j p2,i x3,i + b3 p1,i p2,j x3,i + (-b3 - b5) p1,j p2,j x3,i
```

Reidemeister 3 with pxx and ppx

```
In[•]:= ℒ[Xi_,j_[s_]] := T3^s 𝔼[q[s, i, j] + r0[s, i, j] + ϵ r1[s, i, j] + O[ϵ]^2];
        ℒ[Xi,j[1]]
```

```
Out[•]=
T1 T2 𝔼[ϵSeries[
  (-p1,i + p1,1+i) x1,i + (-1 + T1) (p1,1+i - p1,1+j) x1,i + (-p1,j + p1,1+j) x1,j + (-p2,i + p2,1+i) x2,i +
   (-1 + T2) (p2,1+i - p2,1+j) x2,i + a2 p3,j x1,i x2,i - (a2 + a4 T2) p3,j x1,j x2,i / T1 + (-p2,j + p2,1+j) x2,j +
   a4 p3,j x1,i x2,j + (-p3,i + p3,1+i) x3,i + (-1 + T1 T2) (p3,1+i - p3,1+j) x3,i + (-p3,j + p3,1+j) x3,j,
  b5 p1,j p2,i x3,i + b3 p1,i p2,j x3,i + (-b3 - b5) p1,j p2,j x3,i]]
```

```
In[•]:= {lhs} = Cases[∫ℱ[i, j, k] ℒ /@ (Xi,j[1] Xi+,k[1] Xj+,k+[1]) 𝕕{vsi, vsj, vsk, vsi+, vsj+, vsk+},
          𝔼[ℰ_] :> ℰ, ∞];
```

```
In[•]:= {rhs} = Cases[∫ℱ[i, j, k] ℒ /@ (Xj,k[1] Xi,k+[1] Xi+,j+[1]) 𝕕{vsi, vsj, vsk, vsi+, vsj+, vsk+},
          𝔼[ℰ_] :> ℰ, ∞];
```

```
In[•]:= Echo /@ Short /@ (CF@CoefficientList[CF[lhs - rhs][[2]] /. {ai_ :> λ ai, bi_ :> λ bi}, λ]);
```

» 0

» 0

» $a_2\, b_5\, T_2\, p_{1,2+k}\, p_{2,2+j}\, \pi_{1,i}\, \pi_{2,i} + \ll 45 \gg + \dfrac{b_5\ \ll 5 \gg\ \pi_{\ll 1 \gg}}{T_1} -$

$\dfrac{(a_2 + a_4\, T_2)\left(-b_3 + b_3\, T_2 + b_3\, T_1\, T_2 + b_5\, T_2^2\right)\ \ll 1 \gg\ \ll 1 \gg\ \pi_{\ll 1 \gg}\ \pi_{3,j}}{T_1}$

» $-\dfrac{a_2\,(a_2\, b_5 - a_4\, b_3\, T_1 + a_4\, b_5\, T_2)\, p_{3,2+k}\, \pi_{1,i}\, \pi_{2,i}}{T_1} -$

$a_2\, a_4\, b_3\,(-1 + T_1)\, T_1\, p_{1,2+j}\, p_{3,2+k}\, \pi_{1,i}^2\, \pi_{2,i} + a_2\ \ll 6 \gg\ \pi_{\ll 1 \gg} + \ll 72 \gg$

» $\dfrac{a_2\, a_4\,(-1 + T_1)\,(-1 + T_2)\,(a_2 + \ll 1 \gg)\,(b_3 + b_5 + b_3\, T_1 + b_5\, T_2)\, p_{3,2+k}^2\, \pi_{1,i}^2\, \pi_{2,i}^2}{T_1} +$

$\ll 26 \gg + a_4^3\, b_3\, p_{3,2+k}^2\, \pi_{1,i}\, \pi_{1,j}\, \pi_{2,j}\, \pi_{2,k}$

In[•]:=
```
err = CF@Coefficient[CF[lhs - rhs][[2]] /. {a_i_ :> λ a_i, b_i_ :> λ b_i}, λ^2]
```

Out[•]=

$$a_2 b_5 T_2 p_{1,2+k} p_{2,2+j} \pi_{1,i} \pi_{2,i} + a_2 b_3 T_1 p_{1,2+j} p_{2,2+k} \pi_{1,i} \pi_{2,i} - a_2 (b_3 T_1 + b_5 T_2) p_{1,2+k} p_{2,2+k} \pi_{1,i} \pi_{2,i} -$$
$$\frac{b_5 T_2 (a_2 + a_4 T_2) p_{1,2+k} p_{2,2+j} \pi_{1,j} \pi_{2,i}}{T_1} - b_3 (a_2 + a_4 T_2) p_{1,2+j} p_{2,2+k} \pi_{1,j} \pi_{2,i} +$$
$$\frac{(a_2 + a_4 T_2) (b_3 T_1 + b_5 T_2) p_{1,2+k} p_{2,2+k} \pi_{1,j} \pi_{2,i}}{T_1} + a_4 b_5 T_2 p_{1,2+k} p_{2,2+j} \pi_{1,i} \pi_{2,j} +$$
$$a_4 b_3 T_1 p_{1,2+j} p_{2,2+k} \pi_{1,i} \pi_{2,j} - a_4 (b_3 T_1 + b_5 T_2) p_{1,2+k} p_{2,2+k} \pi_{1,i} \pi_{2,j} -$$
$$\frac{(a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 + a_4 b_5 T_2 + a_4 b_3 T_1 T_2 - a_4 b_5 T_1 T_2) p_{3,2+k} \pi_{3,i}}{T_1} +$$
$$b_5 T_1 (-a_4 - a_2 T_1 + a_4 T_1) T_2 p_{1,2+k} p_{3,2+j} \pi_{1,i} \pi_{3,i} +$$
$$T_1 \left(a_2 b_5 - a_4 b_5 - a_4 b_3 T_1 + a_4 b_5 T_2 + a_4 b_3 T_1^2 T_2\right) p_{1,2+j} p_{3,2+k} \pi_{1,i} \pi_{3,i} -$$
$$\left(-a_4 b_5 - a_2 b_5 T_1 + 2 a_4 b_5 T_1 + a_4 b_3 T_1^2\right) (-1 + T_1 T_2) p_{1,2+k} p_{3,2+k} \pi_{1,i} \pi_{3,i} +$$
$$b_5 T_1 T_2 (a_2 + a_4 T_2) p_{1,2+k} p_{3,2+j} \pi_{1,j} \pi_{3,i} + b_3 T_1^2 (a_2 + a_4 T_2) p_{1,2+i} p_{3,2+k} \pi_{1,j} \pi_{3,i} +$$
$$\left(-a_2 b_5 + a_4 b_3 T_1 - a_2 b_3 T_1^2 - a_4 b_5 T_2 + a_4 b_5 T_1 T_2 - a_4 b_3 T_1^2 T_2\right) p_{1,2+j} p_{3,2+k} \pi_{1,j} \pi_{3,i} +$$
$$\left(a_2 b_5 - a_4 b_5 - a_4 b_3 T_1 + a_4 b_5 T_2 - a_2 b_5 T_1 T_2 - a_4 b_5 T_1 T_2^2\right) p_{1,2+k} p_{3,2+k} \pi_{1,j} \pi_{3,i} -$$
$$b_3 T_1 (a_2 + a_4 T_2) p_{1,2+i} p_{3,2+k} \pi_{1,k} \pi_{3,i} + b_3 T_1 (a_2 + a_4 T_2) p_{1,2+j} p_{3,2+k} \pi_{1,k} \pi_{3,i} -$$
$$b_3 T_2 \left(-a_2 + a_2 T_2 - a_4 T_2 + a_2 T_1 T_2 + a_4 T_2^2\right) p_{2,2+k} p_{3,2+j} \pi_{2,i} \pi_{3,i} +$$
$$\frac{T_2 \left(a_2 b_3 + a_4 b_3 T_2 + a_2 b_5 T_2 - a_4 b_3 T_1 T_2 + a_4 b_5 T_2^2 - a_2 b_5 T_1 T_2^2 - a_4 b_5 T_1 T_2^3\right) p_{2,2+j} p_{3,2+k} \pi_{2,i} \pi_{3,i}}{T_1} +$$
$$\frac{1}{T_1} (-1 + T_1 T_2) \left(-a_2 b_3 + 2 a_2 b_3 T_2 - a_4 b_3 T_2 + a_2 b_3 T_1 T_2 + 2 a_4 b_3 T_2^2 + a_2 b_5 T_2^2 + a_4 b_5 T_2^3\right)$$
$$p_{2,2+k} p_{3,2+k} \pi_{2,i} \pi_{3,i} - a_4 b_3 T_1 T_2^2 p_{2,2+k} p_{3,2+j} \pi_{2,j} \pi_{3,i} - a_4 b_5 T_2^3 p_{2,2+i} p_{3,2+k} \pi_{2,j} \pi_{3,i} -$$
$$\frac{T_2 \left(a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 + a_4 b_5 T_2 + a_4 b_3 T_1 T_2 - a_4 b_5 T_1 T_2^2\right) p_{2,2+j} p_{3,2+k} \pi_{2,j} \pi_{3,i}}{T_1} +$$
$$\frac{\left(a_2 b_3 + a_4 b_3 T_2 + a_2 b_5 T_2 - a_4 b_3 T_1 T_2 + a_4 b_5 T_2^2 + a_4 b_3 T_1^2 T_2^2\right) p_{2,2+k} p_{3,2+k} \pi_{2,j} \pi_{3,i}}{T_1} +$$
$$a_4 b_5 T_2^2 p_{2,2+i} p_{3,2+k} \pi_{2,k} \pi_{3,i} - a_4 b_5 T_2^2 p_{2,2+j} p_{3,2+k} \pi_{2,k} \pi_{3,i} - a_4 b_5 T_1 T_2 p_{1,2+k} p_{3,2+j} \pi_{1,i} \pi_{3,j} -$$
$$a_4 b_3 T_1^2 p_{1,2+j} p_{3,2+k} \pi_{1,i} \pi_{3,j} + a_4 \left(-b_5 + b_5 T_1 + b_3 T_1^2 + b_5 T_1 T_2\right) p_{1,2+k} p_{3,2+k} \pi_{1,i} \pi_{3,j} +$$
$$b_3 T_2 (a_2 + a_4 T_2) p_{2,2+k} p_{3,2+j} \pi_{2,i} \pi_{3,j} + \frac{b_5 T_2^2 (a_2 + a_4 T_2) p_{2,2+j} p_{3,2+k} \pi_{2,i} \pi_{3,j}}{T_1} -$$
$$\frac{(a_2 + a_4 T_2) \left(-b_3 + b_3 T_2 + b_3 T_1 T_2 + b_5 T_2^2\right) p_{2,2+k} p_{3,2+k} \pi_{2,i} \pi_{3,j}}{T_1}$$

Reidemeister 3 with ppxx and px added

In[•]:=
```
κ = 0;
r42[1, i_, j_] = Evaluate[Plus[
   Sum[
    c++κ xv1,k1 pv1,k2 xv2,k3 pv2,k4,
    {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}, {k4, {i, j}}, {v1, 2}, {v2, v1 + 1, 3}
   ],
   Sum[
    c++κ xv,k1 pv,k2,
    {k1, {i, j}}, {k2, {i, j}}, {v, 3}
   ]
  ]]
```

Out[•]=

$$
\begin{aligned}
&c_{49}\,p_{1,i}\,x_{1,i} + c_{52}\,p_{1,j}\,x_{1,i} + c_{55}\,p_{1,i}\,x_{1,j} + c_{58}\,p_{1,j}\,x_{1,j} + c_{50}\,p_{2,i}\,x_{2,i} + c_{53}\,p_{2,j}\,x_{2,i} + \\
&\ c_{1}\,p_{1,i}\,p_{2,i}\,x_{1,i}\,x_{2,i} + c_{13}\,p_{1,j}\,p_{2,i}\,x_{1,i}\,x_{2,i} + c_{4}\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,i} + c_{16}\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,i} + \\
&\ c_{25}\,p_{1,i}\,p_{2,i}\,x_{1,j}\,x_{2,i} + c_{37}\,p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,i} + c_{28}\,p_{1,i}\,p_{2,j}\,x_{1,j}\,x_{2,i} + c_{40}\,p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,i} + \\
&\ c_{56}\,p_{2,i}\,x_{2,j} + c_{59}\,p_{2,j}\,x_{2,j} + c_{7}\,p_{1,i}\,p_{2,i}\,x_{1,i}\,x_{2,j} + c_{19}\,p_{1,j}\,p_{2,i}\,x_{1,i}\,x_{2,j} + c_{10}\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,j} + \\
&\ c_{22}\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,j} + c_{31}\,p_{1,i}\,p_{2,i}\,x_{1,j}\,x_{2,j} + c_{43}\,p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,j} + c_{34}\,p_{1,i}\,p_{2,j}\,x_{1,j}\,x_{2,j} + \\
&\ c_{46}\,p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,j} + c_{51}\,p_{3,i}\,x_{3,i} + c_{54}\,p_{3,j}\,x_{3,i} + c_{2}\,p_{1,i}\,p_{3,i}\,x_{1,i}\,x_{3,i} + c_{14}\,p_{1,j}\,p_{3,i}\,x_{1,i}\,x_{3,i} + \\
&\ c_{5}\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,i} + c_{17}\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,i} + c_{26}\,p_{1,i}\,p_{3,i}\,x_{1,j}\,x_{3,i} + c_{38}\,p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,i} + \\
&\ c_{29}\,p_{1,i}\,p_{3,j}\,x_{1,j}\,x_{3,i} + c_{41}\,p_{1,j}\,p_{3,j}\,x_{1,j}\,x_{3,i} + c_{3}\,p_{2,i}\,p_{3,i}\,x_{2,i}\,x_{3,i} + c_{15}\,p_{2,j}\,p_{3,i}\,x_{2,i}\,x_{3,i} + \\
&\ c_{6}\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,i} + c_{18}\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,i} + c_{27}\,p_{2,i}\,p_{3,i}\,x_{2,j}\,x_{3,i} + c_{39}\,p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,i} + \\
&\ c_{30}\,p_{2,i}\,p_{3,j}\,x_{2,j}\,x_{3,i} + c_{42}\,p_{2,j}\,p_{3,j}\,x_{2,j}\,x_{3,i} + c_{57}\,p_{3,i}\,x_{3,j} + c_{60}\,p_{3,j}\,x_{3,j} + \\
&\ c_{8}\,p_{1,i}\,p_{3,i}\,x_{1,i}\,x_{3,j} + c_{20}\,p_{1,j}\,p_{3,i}\,x_{1,i}\,x_{3,j} + c_{11}\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,j} + c_{23}\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,j} + \\
&\ c_{32}\,p_{1,i}\,p_{3,i}\,x_{1,j}\,x_{3,j} + c_{44}\,p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,j} + c_{35}\,p_{1,i}\,p_{3,j}\,x_{1,j}\,x_{3,j} + c_{47}\,p_{1,j}\,p_{3,j}\,x_{1,j}\,x_{3,j} + \\
&\ c_{9}\,p_{2,i}\,p_{3,i}\,x_{2,i}\,x_{3,j} + c_{21}\,p_{2,j}\,p_{3,i}\,x_{2,i}\,x_{3,j} + c_{12}\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,j} + c_{24}\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,j} + \\
&\ c_{33}\,p_{2,i}\,p_{3,i}\,x_{2,j}\,x_{3,j} + c_{45}\,p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,j} + c_{36}\,p_{2,i}\,p_{3,j}\,x_{2,j}\,x_{3,j} + c_{48}\,p_{2,j}\,p_{3,j}\,x_{2,j}\,x_{3,j}
\end{aligned}
$$

In[•]:=
```
ℒ[X_i_,j_[s_]] := T3^s 𝔼[q[s, i, j] + ϵ r42[s, i, j] + O[ϵ]^2];
ℒ[Xi,j[1]]
```

Out[•]=
```
T1 T2 𝔼[∈Series[(-p1,i + p1,1+i) x1,i + (-1 + T1) (p1,1+i - p1,1+j) x1,i + (-p1,j + p1,1+j) x1,j +
      (-p2,i + p2,1+i) x2,i + (-1 + T2) (p2,1+i - p2,1+j) x2,i + (-p2,j + p2,1+j) x2,j +
      (-p3,i + p3,1+i) x3,i + (-1 + T1 T2) (p3,1+i - p3,1+j) x3,i + (-p3,j + p3,1+j) x3,j,
    c49 p1,i x1,i + c52 p1,j x1,i + c55 p1,i x1,j + c58 p1,j x1,j + c50 p2,i x2,i + c53 p2,j x2,i +
     c1 p1,i p2,i x1,i x2,i + c13 p1,j p2,i x1,i x2,i + c4 p1,i p2,j x1,i x2,i + c16 p1,j p2,j x1,i x2,i +
     c25 p1,i p2,i x1,j x2,i + c37 p1,j p2,i x1,j x2,i + c28 p1,i p2,j x1,j x2,i + c40 p1,j p2,j x1,j x2,i +
     c56 p2,i x2,j + c59 p2,j x2,j + c7 p1,i p2,i x1,i x2,j + c19 p1,j p2,i x1,i x2,j + c10 p1,i p2,j x1,i x2,j +
     c22 p1,j p2,j x1,i x2,j + c31 p1,i p2,i x1,j x2,j + c43 p1,j p2,i x1,j x2,j + c34 p1,i p2,j x1,j x2,j +
     c46 p1,j p2,j x1,j x2,j + c51 p3,i x3,i + c54 p3,j x3,i + c2 p1,i p3,i x1,i x3,i + c14 p1,j p3,i x1,i x3,i +
     c5 p1,i p3,j x1,i x3,i + c17 p1,j p3,j x1,i x3,i + c26 p1,i p3,i x1,j x3,i + c38 p1,j p3,i x1,j x3,i +
     c29 p1,i p3,j x1,j x3,i + c41 p1,j p3,j x1,j x3,i + c3 p2,i p3,i x2,i x3,i + c15 p2,j p3,i x2,i x3,i +
     c6 p2,i p3,j x2,i x3,i + c18 p2,j p3,j x2,i x3,i + c27 p2,i p3,i x2,j x3,i + c39 p2,j p3,i x2,j x3,i +
     c30 p2,i p3,j x2,j x3,i + c42 p2,j p3,j x2,j x3,i + c57 p3,i x3,j + c60 p3,j x3,j +
     c8 p1,i p3,i x1,i x3,j + c20 p1,j p3,i x1,i x3,j + c11 p1,i p3,j x1,i x3,j + c23 p1,j p3,j x1,i x3,j +
     c32 p1,i p3,i x1,j x3,j + c44 p1,j p3,i x1,j x3,j + c35 p1,i p3,j x1,j x3,j + c47 p1,j p3,j x1,j x3,j +
     c9 p2,i p3,i x2,i x3,j + c21 p2,j p3,i x2,i x3,j + c12 p2,i p3,j x2,i x3,j + c24 p2,j p3,j x2,i x3,j +
     c33 p2,i p3,i x2,j x3,j + c45 p2,j p3,i x2,j x3,j + c36 p2,i p3,j x2,j x3,j + c48 p2,j p3,j x2,j x3,j]]
```

In[•]:=
```
{lhs} = Cases[
  ∫ℱ[i, j, k] ℒ /@ (Xi,j[1] Xi⁺,k[1] Xj⁺,k⁺[1]) 𝕕{vsi, vsj, vsk, vsi⁺, vsj⁺, vsk⁺}, 𝔼[ℰ_] :> ℰ, ∞]
```

Out[•]=
```
{∈Series[T1^2 p1,2+i π1,i - (-1 + T1) T1 p1,2+j π1,i + (1 - T1) p1,2+k π1,i + T1 p1,2+j π1,j + (1 - T1) p1,2+k π1,j + p1,2+k π1,k +
    T2^2 p2,2+i π2,i - (-1 + T2) T2 p2,2+j π2,i + (1 - T2) p2,2+k π2,i + T2 p2,2+j π2,j + (1 - T2) p2,2+k π2,j + p2,2+k π2,k +
    T1^2 T2^2 p3,2+i π3,i - T1 T2 (-1 + T1 T2) p3,2+j π3,i + (1 - T1 T2) p3,2+k π3,i + T1 T2 p3,2+j π3,j + (1 - T1 T2) p3,2+k π3,j + p3,2+k π3,k,
  3 (c1 + c2 + c3 + c10 + c11 + c12 + c37 + c38 + c39 + c46 + c47 + c48 + c49 + c50 + c51 + c58 + c59 + c60) +
   2 (c1 + c2 + c10 + c11 + c49) T1^2 p1,2+i π1,i - T1 (-2 c1 - 2 c2 - 2 c10 - 2 c11 - c13 - c14 - c22 - c23 - 2 c49 - c52 + 2 c1 T1 + 2 c2 T1 +
      2 c10 T1 + 2 c11 T1 - c25 T1 - c26 T1 - c34 T1 - c35 T1 + 2 c49 T1 - c55 T1 + c25 T1^2 + c26 T1^2 + c34 T1^2 + c35 T1^2 + c55 T1^2) p1,2+j π1,i +
   ··· 338 ··· + 2 (c33 + c36 + c45 + c48 - c33 T2 - c36 T2 - c33 T1 T2 - c45 T1 T2 + c33 T1 T2^2) p2,2+k p3,2+k π2,k π3,k]}
```

Full expression not available (original memory size: 0.7 MB)

In[•]:=
```
{rhs} = Cases[
  ∫ℱ[i, j, k] ℒ /@ (X_{j,k}[1] X_{i,k^+}[1] X_{i^+,j^+}[1]) 𝕕{vs_i, vs_j, vs_k, vs_{i^+}, vs_{j^+}, vs_{k^+}}, 𝔼[ℰ_] :> ℰ, ∞]
```

Out[•]=
```
{∈Series[T1^2 p1,2+i π1,i - (-1 + T1) T1 p1,2+j π1,i + (1 - T1) p1,2+k π1,i + T1 p1,2+j π1,j + (1 - T1) p1,2+k π1,j + p1,2+k π1,k +
    T2^2 p2,2+i π2,i - (-1 + T2) T2 p2,2+j π2,i + (1 - T2) p2,2+k π2,i + T2 p2,2+j π2,j + (1 - T2) p2,2+k π2,j + p2,2+k π2,k +
    T1^2 T2^2 p3,2+i π3,i - T1 T2 (-1 + T1 T2) p3,2+j π3,i + (1 - T1 T2) p3,2+k π3,i + T1 T2 p3,2+j π3,j + (1 - T1 T2) p3,2+k π3,j + p3,2+k π3,k,
  3 (c1 + c2 + c3 + c10 + c11 + c12 + c37 + c38 + c39 + c46 + c47 + c48 + c49 + c50 + c51 + c58 + c59 + c60) +
   2 (c1 + c2 + c10 + c11 + c49) T1^2 p1,2+i π1,i -
   T1 (-2 c1 - 2 c2 - 2 c10 - 2 c11 - c13 - c14 - c22 - c23 - 2 c49 - c52 + 2 c1 T1 + 2 c2 T1 + 2 c10 T1 + 2 c11 T1 + 2 c49 T1) p1,2+j π1,i +
   ··· 391 ··· + (-2 + T2) T2 (-c33 - c36 + c33 T1 T2) p2,2+j p3,2+k π2,k π3,k +
   2 (c33 + c36 + c45 + c48 - c33 T2 - c36 T2 - c33 T1 T2 - c45 T1 T2 + c33 T1 T2^2) p2,2+k p3,2+k π2,k π3,k]}
```

Full expression not available (original memory size: 0.6 MB)

In[•]:=
```
eqn = CF[(lhs - rhs)[[2]] - err]
```

Out[•]=
```
-((c25 + c26 + c34 + c35 + c55) (-1 + T1) T1^2 p1,2+j π1,i) -
 (-1 + T1) (c1 + c2 + c10 + c11 + c13 + c14 + c22 + c23 + c49 + c52 + c25 T1 + c26 T1 + c34 T1 + c35 T1 +
    c37 T1 + c38 T1 + c46 T1 + c47 T1 + c55 T1 + c58 T1 - c25 T1^2 - c26 T1^2 - c34 T1^2 - c35 T1^2 - c55 T1^2) p1,2+k π1,i +
 (c25 + c26 + c34 + c35 + c55) (-1 + T1) T1^2 p1,2+i π1,j - (c25 + c26 + c34 + c35 + c55) (-1 + T1)^2 T1 p1,2+j π1,j -
 (c25 + c26 + c34 + c35 + c55) (-1 + T1) T1 p1,2+k π1,j - (c25 + c26 + c34 + c35 + c55) (-1 + T1) T1 p1,2+i π1,k +
 ··· 365 ··· + c33 T1^2 (-1 + T2) T2^3 p2,2+j p3,2+i π2,k π3,k + T1 T2 (-c33 - c45 + c33 T2) (-1 + T1 T2) p2,2+k p3,2+i π2,k π3,k +
 c33 T1 T2^3 (-1 + T1 T2) p2,2+i p3,2+j π2,k π3,k - c33 T1 (-1 + T2) T2^2 (-1 + T1 T2) p2,2+j p3,2+j π2,k π3,k -
 T1 T2 (-c33 - c45 + c33 T2) (-1 + T1 T2) p2,2+k p3,2+j π2,k π3,k +
 (-1 + T2) T2 (-c33 - c36 + c33 T1 T2) p2,2+i p3,2+k π2,k π3,k - (-1 + T2) T2 (-c33 - c36 + c33 T1 T2) p2,2+j p3,2+k π2,k π3,k
```

Full expression not available (original memory size: 0.8 MB)

In[•]:=
```
cvs = Union@Cases[eqn, p__ | π__, ∞]
```

Out[•]=
```
{p1,2+i, p1,2+j, p1,2+k, p2,2+i, p2,2+j, p2,2+k, p3,2+i,
 p3,2+j, p3,2+k, π1,i, π1,j, π1,k, π2,i, π2,j, π2,k, π3,i, π3,j, π3,k}
```

In[•]:=
```
eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0)
```

Out[•]=
```
{-c7 T1^2 T2^2 + c7 T1^2 T2^3 == 0, c7 T1^2 T2 - c7 T1^2 T2^2 == 0, -c25 T1^2 T2^2 + c25 T1^3 T2^2 == 0, -c31 T1^2 T2^2 + c31 T1^3 T2^2 + c31 T1^2 T2^3 - c31 T1^3 T2^3 == 0,
 ··· 245 ···, -c8 T1 T2 - c9 T1 T2 - c44 T1 T2 - c45 T1 T2 - c57 T1 T2 + c8 T1^2 T2^2 + c9 T1^2 T2^2 + c44 T1^2 T2^2 + c45 T1^2 T2^2 + c57 T1^2 T2^2 == 0,
 a2 b3 - a4 b3 + c2 + c3 + c5 + c6 + c38 + c39 + c41 + c42 + c51 + c54 + (a2 b5)/T1 + a4 b3 T2 - a4 b5 T2 + (a4 b5 T2)/T1 - c2 T1 T2 - c3 T1 T2 - c5 T1 T2 -
   c6 T1 T2 + c8 T1 T2 + c9 T1 T2 + c11 T1 T2 + c12 T1 T2 - c38 T1 T2 - c39 T1 T2 - c41 T1 T2 - c42 T1 T2 + c44 T1 T2 + c45 T1 T2 +
   c47 T1 T2 + c48 T1 T2 - c51 T1 T2 - c54 T1 T2 + c57 T1 T2 + c60 T1 T2 - 2 c8 T1^2 T2^2 - 2 c9 T1^2 T2^2 - c11 T1^2 T2^2 - c12 T1^2 T2^2 - 2 c44 T1^2 T2^2 -
   2 c45 T1^2 T2^2 - c47 T1^2 T2^2 - c48 T1^2 T2^2 - 2 c57 T1^2 T2^2 - c60 T1^2 T2^2 + c8 T1^3 T2^2 + c9 T1^3 T2^2 + c44 T1^3 T2^2 + c45 T1^3 T2^2 + c57 T1^3 T2^2 == 0,
 c8 T1 T2 + c9 T1 T2 + c44 T1 T2 + c45 T1 T2 + c57 T1 T2 - c8 T1^2 T2^2 - c9 T1^2 T2^2 - c44 T1^2 T2^2 - c45 T1^2 T2^2 - c57 T1^2 T2^2 == 0}
```

Full expression not available (original memory size: 1 MB)

In[•]:=
```
vars = Union@Cases[r42[1, i, j], c_, ∞]
```

Out[•]=

$$\{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12}, c_{13}, c_{14}, c_{15}, c_{16}, c_{17}, c_{18}, c_{19}, c_{20}, c_{21}, c_{22}, c_{23}, c_{24}, c_{25}, c_{26}, c_{27}, c_{28}, c_{29}, c_{30}, c_{31}, c_{32}, c_{33}, c_{34}, c_{35}, c_{36}, c_{37}, c_{38}, c_{39}, c_{40}, c_{41}, c_{42}, c_{43}, c_{44}, c_{45}, c_{46}, c_{47}, c_{48}, c_{49}, c_{50}, c_{51}, c_{52}, c_{53}, c_{54}, c_{55}, c_{56}, c_{57}, c_{58}, c_{59}, c_{60}\}$$

In[•]:=
```
{sol} = Solve[eqns, vars]
```

Solve: Equations may not give solutions for all "solve" variables.

Out[•]=

$$\Big\{\Big\{c_1 \to 0,\ c_2 \to 0,\ c_3 \to 0,\ c_7 \to 0,\ c_8 \to 0,\ c_9 \to 0,$$

$$c_{10} \to -\frac{c_{13}}{1-T_1} - \frac{c_4}{-1+T_2} - \left(a_2 b_3 - a_4 b_3 - a_2 b_5 + a_4 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 + a_4 b_3 T_2 + a_2 b_5 T_2 - 2 a_4 b_5 T_2 - a_4 b_3 T_1 T_2 + a_4 b_5 T_2^2\right) \Big/ \left((-1+T_1)(-1+T_2)(-1+T_1 T_2)\right),$$

$$c_{11} \to -\frac{c_{14}}{1-T_1} - \frac{c_5}{-1+T_1 T_2} - \frac{-a_2 b_5 + a_4 b_5 - a_4 b_5 T_2}{(-1+T_1)(-1+T_1 T_2)},$$

$$c_{12} \to -\frac{c_{15}}{1-T_2} - \frac{c_6}{-1+T_1 T_2} - \frac{-a_2 b_3 - a_4 b_3 T_2 + a_4 b_3 T_1 T_2}{T_1 (-1+T_2)(-1+T_1 T_2)},$$

$$c_{16} \to -c_4 (1-T_1) - c_{13} (1-T_2) - \frac{a_2 b_3 + a_4 b_3 T_2 + a_2 b_5 T_2 - a_4 b_5 T_2 - a_4 b_3 T_1 T_2 + a_4 b_5 T_2^2}{-1+T_1 T_2},$$

$$c_{17} \to -c_5 (1-T_1) - c_{14} (1-T_1 T_2) - \frac{1}{-1+T_2} \left(-a_2 b_3 + a_2 b_5 - a_4 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 - a_4 b_3 T_2 - a_2 b_5 T_2 + 2 a_4 b_5 T_2 + 2 a_4 b_3 T_1 T_2 - a_4 b_5 T_2^2\right),$$

$$c_{18} \to -c_6 (1-T_2) - c_{15} (1-T_1 T_2) - \frac{1}{(-1+T_1) T_1} \left(a_2 b_3 - a_2 b_3 T_1 + a_4 b_3 T_2 + a_2 b_5 T_2 - 2 a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + a_4 b_5 T_1 T_2 + a_4 b_3 T_1^2 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2\right),$$

$$c_{19} \to 0,\ c_{20} \to 0,\ c_{21} \to 0,\ c_{22} \to -\frac{a_4 b_3}{-1+T_2} - \frac{c_4 (-1+T_1)}{-1+T_2},$$

$$c_{23} \to -\frac{c_5 (-1+T_1)}{-1+T_1 T_2} - \frac{a_2 b_3 - a_2 b_3 T_1 + a_4 b_3 T_1 + a_4 b_3 T_2 - 2 a_4 b_3 T_1 T_2}{(-1+T_2)(-1+T_1 T_2)},$$

$$c_{24} \to -\frac{c_6 (-1+T_2)}{-1+T_1 T_2} - \frac{T_2 (a_2 b_5 + a_4 b_5 T_2)}{T_1 (-1+T_1 T_2)} - \frac{a_4 b_5 \left(-T_2 + T_2^2\right)}{(-1+T_1)(-1+T_1 T_2)},$$

$$c_{25} \to 0,\ c_{26} \to 0,\ c_{27} \to 0,\ c_{28} \to 0,\ c_{29} \to -\frac{a_2 b_3 + a_4 b_3 T_2}{T_1 (-1+T_2)},$$

$$c_{30} \to \frac{a_4 b_5}{-1+T_1},\ c_{31} \to 0,\ c_{32} \to 0,\ c_{33} \to 0,\ c_{34} \to 0,$$

$$c_{35} \to 0,\ c_{36} \to 0,\ c_{37} \to -\frac{c_{13}}{-1+T_1} - \frac{c_4}{1-T_2} - \left(-a_2 b_3 + a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_2 - a_2 b_5 T_2 + a_4 b_5 T_2 + 2 a_4 b_3 T_1 T_2 - a_4 b_5 T_1 T_2 - a_4 b_3 T_1^2 T_2 - a_4 b_5 T_2^2 + a_4 b_5 T_1 T_2^2\right) \Big/ \left((-1+T_1) T_1 (-1+T_2)(-1+T_1 T_2)\right),\ c_{38} \to -\frac{c_{14}}{-1+T_1} - \frac{c_5}{1-T_1 T_2} -$$

$$\left(a_2 b_3 - a_2 b_5 - 2 a_2 b_3 T_1 + a_4 b_3 T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + a_4 b_3 T_2 + a_2 b_5 T_2 - a_4 b_5 T_2 - 3 a_4 b_3 T_1 T_2 +$$

$$a_4\,b_5\,T_1\,T_2 + 2\,a_4\,b_3\,T_1^2\,T_2 + a_4\,b_5\,T_2^2 - a_4\,b_5\,T_1\,T_2^2\big) \Big/ \big((-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2)\big),$$

$$c_{39} \to -\frac{c_{15}}{-1+T_2} - \frac{c_6}{1-T_1\,T_2} - \big(-a_2\,b_5 - a_2\,b_3\,T_1 + a_4\,b_3\,T_1 + a_2\,b_5\,T_1 - a_4\,b_5\,T_1 + a_2\,b_3\,T_1^2 - a_4\,b_3\,T_1^2 +$$

$$a_2\,b_5\,T_2 - a_4\,b_5\,T_2 - a_4\,b_3\,T_1\,T_2 - a_2\,b_5\,T_1\,T_2 + 3\,a_4\,b_5\,T_1\,T_2 + a_4\,b_3\,T_1^2\,T_2 + a_4\,b_5\,T_2^2 - 2\,a_4\,b_5\,T_1\,T_2^2\big) \Big/$$

$$\big((-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2)\big),\ c_{40} \to -\frac{c_{13}\,(-1+T_2)}{-1+T_1} - \frac{-a_2\,b_5 - a_4\,b_5\,T_2}{(-1+T_1)\,T_1},$$

$$c_{41} \to \frac{a_2\,b_3 + a_4\,b_3\,T_2}{T_1\,(-1+T_2)} - \frac{a_2\,b_5 + a_4\,b_5\,T_2}{-1+T_1} - \frac{c_{14}\,(-1+T_1\,T_2)}{-1+T_1},$$

$$c_{42} \to -\frac{a_4\,b_5}{-1+T_1} + \frac{a_4\,b_3\,T_2}{-1+T_2} - \frac{c_{15}\,(-1+T_1\,T_2)}{-1+T_2},\ c_{43} \to 0,$$

$$c_{44} \to 0,\ c_{45} \to 0,\ c_{46} \to 0,\ c_{47} \to 0,\ c_{48} \to 0,\ c_{55} \to 0,$$

$$c_{56} \to 0,\ c_{57} \to 0,\ c_{58} \to -\frac{c_{49}}{T_1} - \frac{c_{52}}{T_1},\ c_{59} \to -\frac{c_{50}}{T_2} - \frac{c_{53}}{T_2},$$

$$c_{60} \to -\frac{c_{51}}{T_1\,T_2} - \frac{c_{54}}{T_1\,T_2} - \frac{a_2\,b_5 + a_2\,b_3\,T_1 - a_4\,b_3\,T_1 + a_4\,b_5\,T_2 + a_4\,b_3\,T_1\,T_2 - a_4\,b_5\,T_1\,T_2}{T_1^2\,T_2\,(-1+T_1\,T_2)}\Big\}\Big\}$$

In[•]:=
```
sol /. (v_ → val_) :> (v = CF[val]);
```

In[•]:=
```
CF[r42[1, i, j]]
```

Out[•]=

$$c_{49}\,p_{1,i}\,x_{1,i} + c_{52}\,p_{1,j}\,x_{1,i} - \frac{(c_{49}+c_{52})\,p_{1,j}\,x_{1,j}}{T_1} +$$

$$c_{50}\,p_{2,i}\,x_{2,i} + c_{53}\,p_{2,j}\,x_{2,i} + c_{13}\,p_{1,j}\,p_{2,i}\,x_{1,i}\,x_{2,i} + c_4\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,i} -$$

$$\frac{1}{-1+T_1\,T_2}\big(a_2\,b_3 - c_4 - c_{13} + c_4\,T_1 + a_4\,b_3\,T_2 + a_2\,b_5\,T_2 - a_4\,b_5\,T_2 + c_{13}\,T_2 - a_4\,b_3\,T_1\,T_2 + c_4\,T_1\,T_2 + c_{13}\,T_1\,T_2 -$$

$$c_4\,T_1^2\,T_2 + a_4\,b_5\,T_2^2 - c_{13}\,T_1\,T_2^2\big)\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,i} - \frac{1}{(-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2)}$$

$$\big(-a_2\,b_3 + a_2\,b_5 + a_2\,b_3\,T_1 - c_4\,T_1 + c_{13}\,T_1 + c_4\,T_1^2 - a_4\,b_3\,T_2 - a_2\,b_5\,T_2 + a_4\,b_5\,T_2 + 2\,a_4\,b_3\,T_1\,T_2 -$$

$$a_4\,b_5\,T_1\,T_2 - c_{13}\,T_1\,T_2 - a_4\,b_3\,T_1^2\,T_2 + c_4\,T_1^2\,T_2 - c_{13}\,T_1^2\,T_2 - c_4\,T_1^3\,T_2 - a_4\,b_5\,T_2^2 + a_4\,b_5\,T_1\,T_2^2 + c_{13}\,T_1^2\,T_2^2\big)$$

$$p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,i} - \frac{(-a_2\,b_5 - c_{13}\,T_1 - a_4\,b_5\,T_2 + c_{13}\,T_1\,T_2)\,p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,i}}{(-1+T_1)\,T_1} -$$

$$\frac{(c_{50}+c_{53})\,p_{2,j}\,x_{2,j}}{T_2} + \frac{1}{(-1+T_1)\,(-1+T_2)\,(-1+T_1\,T_2)}$$

$$\big(-a_2\,b_3 + a_4\,b_3 + a_2\,b_5 - a_4\,b_5 - c_4 + c_{13} + a_2\,b_3\,T_1 - a_4\,b_3\,T_1 + c_4\,T_1 - a_4\,b_3\,T_2 - a_2\,b_5\,T_2 + 2\,a_4\,b_5\,T_2 -$$

$$c_{13}\,T_2 + a_4\,b_3\,T_1\,T_2 + c_4\,T_1\,T_2 - c_{13}\,T_1\,T_2 - c_4\,T_1^2\,T_2 - a_4\,b_5\,T_2^2 + c_{13}\,T_1\,T_2^2\big)\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,j} -$$

$$\frac{(a_4\,b_3 - c_4 + c_4\,T_1)\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,j}}{-1+T_2} + c_{51}\,p_{3,i}\,x_{3,i} + c_{54}\,p_{3,j}\,x_{3,i} +$$

$$c_{14}\,p_{1,j}\,p_{3,i}\,x_{1,i}\,x_{3,i} + c_5\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,i} -$$

$$\frac{1}{-1+T_2}\big(-a_2\,b_3 + a_2\,b_5 - a_4\,b_5 - c_5 - c_{14} + a_2\,b_3\,T_1 - a_4\,b_3\,T_1 + c_5\,T_1 - a_4\,b_3\,T_2 - a_2\,b_5\,T_2 + 2\,a_4\,b_5\,T_2 +$$

$$c_5\,T_2 + c_{14}\,T_2 + 2\,a_4\,b_3\,T_1\,T_2 - c_5\,T_1\,T_2 + c_{14}\,T_1\,T_2 - a_4\,b_5\,T_2^2 - c_{14}\,T_1\,T_2^2\big)\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,i} -$$

$$\frac{1}{(-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2)}\big(a_2\,b_3 - a_2\,b_5 - 2\,a_2\,b_3\,T_1 + a_4\,b_3\,T_1 - c_5\,T_1 + c_{14}\,T_1 +$$

$$a_2\,b_3\,T_1^2-a_4\,b_3\,T_1^2+c_5\,T_1^2+a_4\,b_3\,T_2+a_2\,b_5\,T_2-a_4\,b_5\,T_2-3\,a_4\,b_3\,T_1\,T_2-a_4\,b_5\,T_1\,T_2+c_5\,T_1\,T_2-c_{14}\,T_1\,T_2+2\,a_4\,b_3\,T_1^2\,T_2-c_5\,T_1^2\,T_2-c_{14}\,T_1^2\,T_2+a_4\,b_5\,T_2^2-a_4\,b_5\,T_1\,T_2^2+c_{14}\,T_1^2\,T_2^2\big)\,p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,i}-\frac{b_3\,(a_2+a_4\,T_2)\,p_{1,i}\,p_{3,j}\,x_{1,j}\,x_{3,i}}{T_1\,(-1+T_2)}+\Big(\big(-a_2\,b_3+a_2\,b_3\,T_1+a_2\,b_5\,T_1-c_{14}\,T_1-a_4\,b_3\,T_2+a_4\,b_3\,T_1\,T_2-a_2\,b_5\,T_1\,T_2+a_4\,b_5\,T_1\,T_2+c_{14}\,T_1\,T_2+c_{14}\,T_1^2\,T_2-a_4\,b_5\,T_1\,T_2^2-c_{14}\,T_1^2\,T_2^2\big)\,p_{1,j}\,p_{3,j}\,x_{1,j}\,x_{3,i}\Big)\Big/\big((-1+T_1)\,T_1\,(-1+T_2)\big)+c_{15}\,p_{2,j}\,p_{3,i}\,x_{2,i}\,x_{3,i}+c_6\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,i}+\frac{1}{(-1+T_1)\,T_1}\Big(-a_2\,b_3+a_2\,b_3\,T_1+c_6\,T_1+c_{15}\,T_1-c_6\,T_1^2-c_{15}\,T_1^2-a_4\,b_3\,T_2-a_2\,b_5\,T_2+2\,a_4\,b_3\,T_1\,T_2+a_2\,b_5\,T_1\,T_2-a_4\,b_5\,T_1\,T_2-c_6\,T_1\,T_2-a_4\,b_3\,T_1^2\,T_2+c_6\,T_1^2\,T_2-c_{15}\,T_1^2\,T_2+c_{15}\,T_1^3\,T_2-a_4\,b_5\,T_2^2+2\,a_4\,b_5\,T_1\,T_2^2\Big)\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,i}-\frac{1}{(-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2)}\Big(-a_2\,b_5-a_2\,b_3\,T_1+a_4\,b_3\,T_1+a_2\,b_5\,T_1-a_4\,b_5\,T_1-c_6\,T_1+c_{15}\,T_1+a_2\,b_3\,T_1^2-a_4\,b_3\,T_1^2+c_6\,T_1^2-c_{15}\,T_1^2+a_2\,b_5\,T_2-a_4\,b_5\,T_2-a_4\,b_3\,T_1\,T_2-a_2\,b_5\,T_1\,T_2+3\,a_4\,b_5\,T_1\,T_2+c_6\,T_1\,T_2+a_4\,b_3\,T_1^2\,T_2-c_6\,T_1^2\,T_2-c_{15}\,T_1^2\,T_2+c_{15}\,T_1^3\,T_2+a_4\,b_5\,T_2^2-2\,a_4\,b_5\,T_1\,T_2^2\Big)\,p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,i}+\frac{a_4\,b_5\,p_{2,i}\,p_{3,j}\,x_{2,j}\,x_{3,i}}{-1+T_1}-\frac{1}{(-1+T_1)\,(-1+T_2)}\Big(-a_4\,b_5+c_{15}-c_{15}\,T_1+a_4\,b_3\,T_2+a_4\,b_5\,T_2-a_4\,b_3\,T_1\,T_2-c_{15}\,T_1\,T_2+c_{15}\,T_1^2\,T_2\Big)\,p_{2,j}\,p_{3,j}\,x_{2,j}\,x_{3,i}-\frac{1}{T_1^2\,T_2\,(-1+T_1\,T_2)}\Big(a_2\,b_5+a_2\,b_3\,T_1-a_4\,b_3\,T_1-c_{51}\,T_1-c_{54}\,T_1+a_4\,b_5\,T_2+a_4\,b_3\,T_1\,T_2-a_4\,b_5\,T_1\,T_2+c_{51}\,T_1^2\,T_2+c_{54}\,T_1^2\,T_2\Big)\,p_{3,j}\,x_{3,j}+\frac{(a_2\,b_5-a_4\,b_5+c_5-c_{14}-c_5\,T_1+a_4\,b_5\,T_2+c_{14}\,T_1\,T_2)\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,j}}{(-1+T_1)\,(-1+T_1\,T_2)}+\big((-a_2\,b_3-c_5+a_2\,b_3\,T_1-a_4\,b_3\,T_1+c_5\,T_1-a_4\,b_3\,T_2+c_5\,T_2+2\,a_4\,b_3\,T_1\,T_2-c_5\,T_1\,T_2)\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,j}\big)\Big/\big((-1+T_2)\,(-1+T_1\,T_2)\big)-\Big(\big(-a_2\,b_3-c_6\,T_1+c_{15}\,T_1-a_4\,b_3\,T_2+a_4\,b_3\,T_1\,T_2+c_6\,T_1\,T_2-c_{15}\,T_1^2\,T_2\big)\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,j}\Big)\Big/\big(T_1\,(-1+T_2)\,(-1+T_1\,T_2)\big)-\Big(\big(c_6\,T_1-c_6\,T_1^2-a_2\,b_5\,T_2+a_2\,b_5\,T_1\,T_2-a_4\,b_5\,T_1\,T_2-c_6\,T_1\,T_2+c_6\,T_1^2\,T_2-a_4\,b_5\,T_2^2+2\,a_4\,b_5\,T_1\,T_2^2\big)\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,j}\Big)\Big/\big((-1+T_1)\,T_1\,(-1+T_1\,T_2)\big)$$

In[•]:=
```
CF[r42[1, i, j] /.
  {a4 → 0, b3 → 0, b5 → (T1 - 1) (T2 - 1) (T3 - 1) a2^-1, c4|5|6|13|14|15|49|50|51|52|53|54 → 0}]
```

Out[•]=

$$-\left((-1+T_1)\,(-1+T_2)\,T_2\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,i}\right)+\frac{(-1+T_2)\,p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,i}}{T_1}+$$
$$\frac{(-1+T_2)\,(-1+T_1\,T_2)\,p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,i}}{T_1}+(1-T_2)\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,j}+$$
$$(-1+T_1)\,(-1+T_2)\,(-1+T_1\,T_2)\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,i}-\frac{(-1+T_2)\,p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,i}}{T_1}-$$
$$(-1+T_2)\,(-1+T_1\,T_2)\,p_{1,j}\,p_{3,j}\,x_{1,j}\,x_{3,i}+\frac{(-1+T_1)\,(-1+T_2)\,T_2\,(-1+T_1\,T_2)\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,i}}{T_1}+$$
$$\frac{(-1+T_1)\,(-1+T_2)\,p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,i}}{T_1}-\frac{(-1+T_1)\,(-1+T_2)\,p_{3,j}\,x_{3,j}}{T_1^2\,T_2}+$$
$$(-1+T_2)\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,j}-\frac{(-1+T_1)\,(-1+T_2)\,T_2\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,j}}{T_1}$$

In[•]:=
```
CF[(r0[1, i, j] - (r0[1, i, j] /. {T1 → T2, T2 → T1,
        p1,i_ :↦ p2,i, p2,i_ :↦ p1,i, x1,i_ :↦ x2,i, x2,i_ :↦ x1,i})) /. a2 → -(T1 + T2) a4]
```

Out[•]=

0

In[•]:=
```
CF[(r1[1, i, j] - (r1[1, i, j] /.
       {T1 → T2, T2 → T1, p1,i_ :↦ p2,i, p2,i_ :↦ p1,i, x1,i_ :↦ x2,i, x2,i_ :↦ x1,i})) /. b5 → b3]
```

Out[•]=

0

In[•]:=
```
CF[(r42[1, i, j] -
    (r42[1, i, j] /. {T1 → T2, T2 → T1, p1,i_ :→ p2,i, p2,i_ :→ p1,i, x1,i_ :→ x2,i, x2,i_ :→ x1,i})) /.
  {a2 → -(T1 + T2) a4, b5 → b3}]
```

Out[•]=

$$(c_{49}-c_{50})\,p_{1,i}\,x_{1,i}+(c_{52}-c_{53})\,p_{1,j}\,x_{1,i}-\frac{(c_{49}-c_{50}+c_{52}-c_{53})\,p_{1,j}\,x_{1,j}}{T_1}+$$
$$(-c_{49}+c_{50})\,p_{2,i}\,x_{2,i}+(-c_{52}+c_{53})\,p_{2,j}\,x_{2,i}+(-c_4+c_{13})\,p_{1,j}\,p_{2,i}\,x_{1,i}\,x_{2,i}+$$
$$(c_4-c_{13})\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,i}+(c_4-c_{13})\,(T_1-T_2)\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,i}+$$
$$\frac{(c_4-c_{13})\,(-2+T_1+T_2)\,p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,i}}{(-1+T_1)\,(-1+T_2)}+\frac{(c_4-c_{13})\,(-1+T_2)\,p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,i}}{-1+T_1}+$$
$$\frac{(c_{49}-c_{50}+c_{52}-c_{53})\,p_{2,j}\,x_{2,j}}{T_2}-\frac{(c_4-c_{13})\,(-2+T_1+T_2)\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,j}}{(-1+T_1)\,(-1+T_2)}-$$
$$\frac{(c_4-c_{13})\,(-1+T_1)\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,j}}{-1+T_2}+(c_{14}-c_{15})\,p_{1,j}\,p_{3,i}\,x_{1,i}\,x_{3,i}+(c_5-c_6)\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,i}+$$
$$(-c_5+c_6-c_{14}+c_{15}+c_5\,T_1-c_6\,T_1+c_{14}\,T_1\,T_2-c_{15}\,T_1\,T_2)\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,i}+$$
$$\frac{(-c_5+c_6+c_{14}-c_{15}+c_5\,T_1-c_6\,T_1-c_{14}\,T_1\,T_2+c_{15}\,T_1\,T_2)\,p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,i}}{(-1+T_1)\,(-1+T_1\,T_2)}-$$
$$\frac{(c_{14}-c_{15})\,(-1+T_1\,T_2)\,p_{1,j}\,p_{3,j}\,x_{1,j}\,x_{3,i}}{-1+T_1}+(-c_{14}+c_{15})\,p_{2,j}\,p_{3,i}\,x_{2,i}\,x_{3,i}+$$
$$(-c_5+c_6)\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,i}+(c_5-c_6+c_{14}-c_{15}-c_5\,T_2+c_6\,T_2-c_{14}\,T_1\,T_2+c_{15}\,T_1\,T_2)\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,i}-$$
$$\frac{(-c_5+c_6+c_{14}-c_{15}+c_5\,T_2-c_6\,T_2-c_{14}\,T_1\,T_2+c_{15}\,T_1\,T_2)\,p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,i}}{(-1+T_2)\,(-1+T_1\,T_2)}+$$
$$\frac{(c_{14}-c_{15})\,(-1+T_1\,T_2)\,p_{2,j}\,p_{3,j}\,x_{2,j}\,x_{3,i}}{-1+T_2}-$$
$$\frac{(-c_5+c_6+c_{14}-c_{15}+c_5\,T_1-c_6\,T_1-c_{14}\,T_1\,T_2+c_{15}\,T_1\,T_2)\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,j}}{(-1+T_1)\,(-1+T_1\,T_2)}-$$
$$\frac{(c_5-c_6)\,(-1+T_1)\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,j}}{-1+T_1\,T_2}+$$
$$\frac{(-c_5+c_6+c_{14}-c_{15}+c_5\,T_2-c_6\,T_2-c_{14}\,T_1\,T_2+c_{15}\,T_1\,T_2)\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,j}}{(-1+T_2)\,(-1+T_1\,T_2)}+$$
$$\frac{(c_5-c_6)\,(-1+T_2)\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,j}}{-1+T_1\,T_2}$$

In[•]:=
```
CF[(r42[1, i, j] -
    (r42[1, i, j] /. {T1 → T2, T2 → T1, p1,i_ :→ p2,i, p2,i_ :→ p1,i, x1,i_ :→ x2,i, x2,i_ :→ x1,i})) /.
  {a2 → -(T1 + T2) a4, b5 → b3, c50 → c49, c53 → c52, c13 → c4, c6 → c5, c15 → c14}]
```

Out[•]=

$$0$$

In[•]:=
```
CF[r42[1, i, j] /. {a2 → -(T1 + T2) a4, b5 → b3, c50 → c49, c53 → c52, c13 → c4, c6 → c5, c15 → c14} /.
  {c4|5|14|49|51|52|54 → 0}]
```

Out[•]=

$$\frac{a_4\,b_3\,(T_1+T_2+2\,T_1\,T_2)\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,i}}{-1+T_1\,T_2}+\frac{a_4\,b_3\,(T_1-T_2)\,(1+T_2)\,p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,i}}{(-1+T_1)\,(-1+T_2)\,(-1+T_1\,T_2)}-$$
$$\frac{a_4\,b_3\,p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,i}}{-1+T_1}-\frac{a_4\,b_3\,(1+T_1)\,(T_1-T_2)\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,j}}{(-1+T_1)\,(-1+T_2)\,(-1+T_1\,T_2)}-$$
$$\frac{a_4\,b_3\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,j}}{-1+T_2}-\frac{a_4\,b_3\,\left(-1-T_1-T_1^2+T_2+2\,T_1\,T_2\right)\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,i}}{-1+T_2}+$$
$$\frac{a_4\,b_3\,\left(-1-T_1+T_1^2+T_2-T_1\,T_2+T_2^2\right)\,p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,i}}{(-1+T_1)\,(-1+T_2)\,(-1+T_1\,T_2)}+\frac{a_4\,b_3\,p_{1,i}\,p_{3,j}\,x_{1,j}\,x_{3,i}}{-1+T_2}+$$
$$\frac{a_4\,b_3\,(1-2\,T_1+T_1\,T_2)\,p_{1,j}\,p_{3,j}\,x_{1,j}\,x_{3,i}}{(-1+T_1)\,(-1+T_2)}-\frac{a_4\,b_3\,\left(-1+T_1-T_2+2\,T_1\,T_2-T_2^2\right)\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,i}}{-1+T_1}+$$
$$\frac{a_4\,b_3\,\left(-1+T_1+T_1^2-T_2-T_1\,T_2+T_2^2\right)\,p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,i}}{(-1+T_1)\,(-1+T_2)\,(-1+T_1\,T_2)}+\frac{a_4\,b_3\,p_{2,i}\,p_{3,j}\,x_{2,j}\,x_{3,i}}{-1+T_1}+$$
$$\frac{a_4\,b_3\,(1-2\,T_2+T_1\,T_2)\,p_{2,j}\,p_{3,j}\,x_{2,j}\,x_{3,i}}{(-1+T_1)\,(-1+T_2)}+\frac{a_4\,b_3\,(2+T_1+T_2)\,p_{3,j}\,x_{3,j}}{T_1\,T_2\,(-1+T_1\,T_2)}-$$
$$\frac{a_4\,b_3\,(1+T_1)\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,j}}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{a_4\,b_3\,T_1\,(T_1-T_2)\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,j}}{(-1+T_2)\,(-1+T_1\,T_2)}-$$
$$\frac{a_4\,b_3\,(1+T_2)\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,j}}{(-1+T_2)\,(-1+T_1\,T_2)}+\frac{a_4\,b_3\,(T_1-T_2)\,T_2\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,j}}{(-1+T_1)\,(-1+T_1\,T_2)}$$

In[•]:=
```
CF[r42[1, i, j] /. {a2 → -(T1 + T2) a4, b5 → b3, c50 → c49, c53 → c52, c13 → c4, c6 → c5, c15 → c14} /.
  {c4|5|14|49|51|52|54 → 0} /. {a4 → b3^-1 (-1 + T1) (-1 + T2) (-1 + T1 T2)}]
```

Out[•]=

$$(-1+T_1)\,(-1+T_2)\,(T_1+T_2+2\,T_1\,T_2)\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,i}+$$
$$(T_1-T_2)\,(1+T_2)\,p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,i}-(-1+T_2)\,(-1+T_1\,T_2)\,p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,i}-$$
$$(1+T_1)\,(T_1-T_2)\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,j}-(-1+T_1)\,(-1+T_1\,T_2)\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,j}-$$
$$(-1+T_1)\,(-1+T_1\,T_2)\,\left(-1-T_1-T_1^2+T_2+2\,T_1\,T_2\right)\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,i}+$$
$$\left(-1-T_1+T_1^2+T_2-T_1\,T_2+T_2^2\right)\,p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,i}+$$
$$(-1+T_1)\,(-1+T_1\,T_2)\,p_{1,i}\,p_{3,j}\,x_{1,j}\,x_{3,i}+(-1+T_1\,T_2)\,(1-2\,T_1+T_1\,T_2)\,p_{1,j}\,p_{3,j}\,x_{1,j}\,x_{3,i}-$$
$$(-1+T_2)\,(-1+T_1\,T_2)\,\left(-1+T_1-T_2+2\,T_1\,T_2-T_2^2\right)\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,i}+$$
$$\left(-1+T_1+T_1^2-T_2-T_1\,T_2+T_2^2\right)\,p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,i}+(-1+T_2)\,(-1+T_1\,T_2)\,p_{2,i}\,p_{3,j}\,x_{2,j}\,x_{3,i}+$$
$$(-1+T_1\,T_2)\,(1-2\,T_2+T_1\,T_2)\,p_{2,j}\,p_{3,j}\,x_{2,j}\,x_{3,i}+\frac{(-1+T_1)\,(-1+T_2)\,(2+T_1+T_2)\,p_{3,j}\,x_{3,j}}{T_1\,T_2}-$$
$$(1+T_1)\,(-1+T_2)\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,j}-(-1+T_1)\,T_1\,(T_1-T_2)\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,j}-$$
$$(-1+T_1)\,(1+T_2)\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,j}+(T_1-T_2)\,(-1+T_2)\,T_2\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,j}$$

Reidemeister 2b for r_0

In[•]:=
```
κ = 0;
r0[-1, i_, j_] := Evaluate[Sum[
    d++κ p3,k3 x1,k1 x2,k2,
    {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
   ]];
r0[-1, i, j]
```

Out[•]=

$$d_1\, p_{3,i}\, x_{1,i}\, x_{2,i} + d_2\, p_{3,j}\, x_{1,i}\, x_{2,i} + d_5\, p_{3,i}\, x_{1,j}\, x_{2,i} + d_6\, p_{3,j}\, x_{1,j}\, x_{2,i} + d_3\, p_{3,i}\, x_{1,i}\, x_{2,j} + d_4\, p_{3,j}\, x_{1,i}\, x_{2,j} + d_7\, p_{3,i}\, x_{1,j}\, x_{2,j} + d_8\, p_{3,j}\, x_{1,j}\, x_{2,j}$$

In[•]:=
```
ℒ[Xi_,j_[s_]] := T3^s 𝔼[q[s, i, j] + r0[s, i, j] + O[ϵ]];
ℒ[Xi,j[-1]]
```

Out[•]=

$$\frac{1}{T_1\, T_2}\, \mathbb{E}\Big[\epsilon \text{Series}\Big[(-p_{1,i} + p_{1,1+i})\, x_{1,i} + \left(-1 + \frac{1}{T_1}\right)(p_{1,1+i} - p_{1,1+j})\, x_{1,i} + (-p_{1,j} + p_{1,1+j})\, x_{1,j} + (-p_{2,i} + p_{2,1+i})\, x_{2,i} + \left(-1 + \frac{1}{T_2}\right)(p_{2,1+i} - p_{2,1+j})\, x_{2,i} + d_1\, p_{3,i}\, x_{1,i}\, x_{2,i} + d_2\, p_{3,j}\, x_{1,i}\, x_{2,i} + d_5\, p_{3,i}\, x_{1,j}\, x_{2,i} + d_6\, p_{3,j}\, x_{1,j}\, x_{2,i} + (-p_{2,j} + p_{2,1+j})\, x_{2,j} + d_3\, p_{3,i}\, x_{1,i}\, x_{2,j} + d_4\, p_{3,j}\, x_{1,i}\, x_{2,j} + d_7\, p_{3,i}\, x_{1,j}\, x_{2,j} + d_8\, p_{3,j}\, x_{1,j}\, x_{2,j} + (-p_{3,i} + p_{3,1+i})\, x_{3,i} + \left(-1 + \frac{1}{T_1\, T_2}\right)(p_{3,1+i} - p_{3,1+j})\, x_{3,i} + (-p_{3,j} + p_{3,1+j})\, x_{3,j}\Big]\Big]$$

In[•]:=
```
{lhs} = Cases[∫ℱ[i, j] ℒ /@ (X_{i,j}[1] X_{i⁺,j⁺}[-1]) 𝕕{vs_i, vs_j, vs_{i⁺}, vs_{j⁺}}, ϵSeries[ℰ_] :> ℰ, ∞]
```

Out[•]=

$$\begin{aligned}
&\Big\{p_{1,2+i}\,\pi_{1,i} + p_{1,2+j}\,\pi_{1,j} + p_{2,2+i}\,\pi_{2,i} + \\
&\frac{1}{T_1\,T_2}\,(d_7 + d_3\,T_1 - d_7\,T_1 + d_5\,T_2 - d_7\,T_2 + d_1\,T_1\,T_2 - d_3\,T_1\,T_2 - d_5\,T_1\,T_2 + d_7\,T_1\,T_2)\,p_{3,2+i}\,\pi_{1,i}\,\pi_{2,i} + \\
&\frac{1}{T_1\,T_2}\,\Big(-d_7 - d_3\,T_1 + d_7\,T_1 - d_5\,T_2 + d_7\,T_2 + a_2\,T_1\,T_2 - d_1\,T_1\,T_2 + d_3\,T_1\,T_2 + d_5\,T_1\,T_2 + d_8\,T_1\,T_2 + \\
&\qquad d_3\,T_1^2\,T_2 + d_4\,T_1^2\,T_2 - d_7\,T_1^2\,T_2 - d_8\,T_1^2\,T_2 + d_5\,T_1\,T_2^2 + d_6\,T_1\,T_2^2 - d_7\,T_1\,T_2^2 - d_8\,T_1\,T_2^2 + \\
&\qquad d_1\,T_1^2\,T_2^2 + d_2\,T_1^2\,T_2^2 - d_3\,T_1^2\,T_2^2 - d_4\,T_1^2\,T_2^2 - d_5\,T_1^2\,T_2^2 - d_6\,T_1^2\,T_2^2 + d_7\,T_1^2\,T_2^2 + d_8\,T_1^2\,T_2^2\Big) \\
&\quad p_{3,2+j}\,\pi_{1,i}\,\pi_{2,i} + \frac{(d_7 + d_5\,T_2 - d_7\,T_2)\,p_{3,2+i}\,\pi_{1,j}\,\pi_{2,i}}{T_1\,T_2} - \\
&\frac{1}{T_1\,T_2}\,\Big(d_7 + a_2\,T_2 + d_5\,T_2 - d_7\,T_2 - d_7\,T_1\,T_2 - d_8\,T_1\,T_2 + a_4\,T_2^2 - d_5\,T_1\,T_2^2 - d_6\,T_1\,T_2^2 + d_7\,T_1\,T_2^2 + d_8\,T_1\,T_2^2\Big) \\
&\quad p_{3,2+j}\,\pi_{1,j}\,\pi_{2,i} + p_{2,2+j}\,\pi_{2,j} + \frac{(d_7 + d_3\,T_1 - d_7\,T_1)\,p_{3,2+i}\,\pi_{1,i}\,\pi_{2,j}}{T_1\,T_2} + \\
&\frac{1}{T_1\,T_2}\,\Big(-d_7 - d_3\,T_1 + d_7\,T_1 + a_4\,T_1\,T_2 + d_7\,T_1\,T_2 + d_8\,T_1\,T_2 + d_3\,T_1^2\,T_2 + d_4\,T_1^2\,T_2 - d_7\,T_1^2\,T_2 - d_8\,T_1^2\,T_2\Big) \\
&\quad p_{3,2+j}\,\pi_{1,i}\,\pi_{2,j} + \frac{d_7\,p_{3,2+i}\,\pi_{1,j}\,\pi_{2,j}}{T_1\,T_2} + \\
&\frac{(-d_7 + d_7\,T_1\,T_2 + d_8\,T_1\,T_2)\,p_{3,2+j}\,\pi_{1,j}\,\pi_{2,j}}{T_1\,T_2} + p_{3,2+i}\,\pi_{3,i} + p_{3,2+j}\,\pi_{3,j}\Big\}
\end{aligned}$$

In[•]:=
```
eqn = CF[lhs - (p_{1,2+i} π_{1,i} + p_{1,2+j} π_{1,j} + p_{2,2+i} π_{2,i} + p_{2,2+j} π_{2,j} + p_{3,2+i} π_{3,i} + p_{3,2+j} π_{3,j})]
```

Out[•]=

$$\begin{aligned}
&\frac{1}{T_1\,T_2}\,(d_7 + d_3\,T_1 - d_7\,T_1 + d_5\,T_2 - d_7\,T_2 + d_1\,T_1\,T_2 - d_3\,T_1\,T_2 - d_5\,T_1\,T_2 + d_7\,T_1\,T_2)\,p_{3,2+i}\,\pi_{1,i}\,\pi_{2,i} + \\
&\frac{1}{T_1\,T_2}\,\Big(-d_7 - d_3\,T_1 + d_7\,T_1 - d_5\,T_2 + d_7\,T_2 + a_2\,T_1\,T_2 - d_1\,T_1\,T_2 + d_3\,T_1\,T_2 + d_5\,T_1\,T_2 + \\
&\qquad d_8\,T_1\,T_2 + d_3\,T_1^2\,T_2 + d_4\,T_1^2\,T_2 - d_7\,T_1^2\,T_2 - d_8\,T_1^2\,T_2 + d_5\,T_1\,T_2^2 + d_6\,T_1\,T_2^2 - d_7\,T_1\,T_2^2 - d_8\,T_1\,T_2^2 + \\
&\qquad d_1\,T_1^2\,T_2^2 + d_2\,T_1^2\,T_2^2 - d_3\,T_1^2\,T_2^2 - d_4\,T_1^2\,T_2^2 - d_5\,T_1^2\,T_2^2 - d_6\,T_1^2\,T_2^2 + d_7\,T_1^2\,T_2^2 + d_8\,T_1^2\,T_2^2\Big) \\
&\quad p_{3,2+j}\,\pi_{1,i}\,\pi_{2,i} + \frac{(d_7 + d_5\,T_2 - d_7\,T_2)\,p_{3,2+i}\,\pi_{1,j}\,\pi_{2,i}}{T_1\,T_2} - \\
&\frac{1}{T_1\,T_2}\,\Big(d_7 + a_2\,T_2 + d_5\,T_2 - d_7\,T_2 - d_7\,T_1\,T_2 - d_8\,T_1\,T_2 + a_4\,T_2^2 - d_5\,T_1\,T_2^2 - d_6\,T_1\,T_2^2 + d_7\,T_1\,T_2^2 + d_8\,T_1\,T_2^2\Big) \\
&\quad p_{3,2+j}\,\pi_{1,j}\,\pi_{2,i} + \frac{(d_7 + d_3\,T_1 - d_7\,T_1)\,p_{3,2+i}\,\pi_{1,i}\,\pi_{2,j}}{T_1\,T_2} + \\
&\frac{1}{T_1\,T_2}\,\Big(-d_7 - d_3\,T_1 + d_7\,T_1 + a_4\,T_1\,T_2 + d_7\,T_1\,T_2 + d_8\,T_1\,T_2 + d_3\,T_1^2\,T_2 + d_4\,T_1^2\,T_2 - d_7\,T_1^2\,T_2 - d_8\,T_1^2\,T_2\Big) \\
&\quad p_{3,2+j}\,\pi_{1,i}\,\pi_{2,j} + \frac{d_7\,p_{3,2+i}\,\pi_{1,j}\,\pi_{2,j}}{T_1\,T_2} + \frac{(-d_7 + d_7\,T_1\,T_2 + d_8\,T_1\,T_2)\,p_{3,2+j}\,\pi_{1,j}\,\pi_{2,j}}{T_1\,T_2}
\end{aligned}$$

```
In[•]:= cvs = Union@Cases[eqn, p__ | π__, ∞]
```

```
Out[•]=
{p3,2+i, p3,2+j, π1,i, π1,j, π2,i, π2,j}
```

```
In[•]:= eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0)
```

```
Out[•]=
{d1 − d3 − d5 + d7 + d5/T1 − d7/T1 + d3/T2 − d7/T2 + d7/(T1 T2) == 0, d3/T2 − d7/T2 + d7/(T1 T2) == 0, d5/T1 − d7/T1 + d7/(T1 T2) == 0, d7/(T1 T2) == 0,
 a2 − d1 + d3 + d5 + d8 − d5/T1 + d7/T1 + d3 T1 + d4 T1 − d7 T1 − d8 T1 − d3/T2 + d7/T2 − d7/(T1 T2) + d5 T2 + d6 T2 − d7 T2 −
   d8 T2 + d1 T1 T2 + d2 T1 T2 − d3 T1 T2 − d4 T1 T2 − d5 T1 T2 − d6 T1 T2 + d7 T1 T2 + d8 T1 T2 == 0,
 a4 + d7 + d8 + d3 T1 + d4 T1 − d7 T1 − d8 T1 − d3/T2 + d7/T2 − d7/(T1 T2) == 0,
 d7 + d8 − a2/T1 − d5/T1 + d7/T1 − d7/(T1 T2) + d5 T2 + d6 T2 − d7 T2 − d8 T2 − a4 T2/T1 == 0, d7 + d8 − d7/(T1 T2) == 0}
```

```
In[•]:= vars = Union@Cases[r0[-1, i, j], d_, ∞]
```

```
Out[•]=
{d1, d2, d3, d4, d5, d6, d7, d8}
```

```
In[•]:= {sol} = Solve[eqns, vars]
```

```
Out[•]=
{{d1 → 0, d2 → −(a2 − a4 T1 + a4 T2)/(T1^2 T2), d3 → 0, d4 → −a4/T1, d5 → 0, d6 → −(−a2 − a4 T2)/(T1 T2), d7 → 0, d8 → 0}}
```

```
In[•]:= sol /. (v_ → val_) :> (v = CF[val]);
```

```
In[•]:= r0[-1, i, j]
```

```
Out[•]=
(−a2 + a4 T1 − a4 T2) p3,j x1,i x2,i/(T1^2 T2) + (a2 + a4 T2) p3,j x1,j x2,i/(T1 T2) − a4 p3,j x1,i x2,j/T1
```

Reidemeister 2b for r_1

```
In[•]:= κ = 0;
        r1[-1, i_, j_] := Evaluate[Sum[
            e++κ x3,k3 p1,k1 p2,k2,
            {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
           ]];
        r1[-1, i, j]
```

```
Out[•]=
e1 p1,i p2,i x3,i + e5 p1,j p2,i x3,i + e3 p1,i p2,j x3,i + e7 p1,j p2,j x3,i +
 e2 p1,i p2,i x3,j + e6 p1,j p2,i x3,j + e4 p1,i p2,j x3,j + e8 p1,j p2,j x3,j
```

In[•]:=
```
ℒ[X_i_,j_[s_]] := T3^s 𝔼[q[s, i, j] + ϵ r1[s, i, j] + O[ϵ]^2];
ℒ[Xi,j[1]]
ℒ[Xi,j[-1]]
```

Out[•]=
$$T_1\,T_2\,\mathbb{E}\Big[\epsilon\text{Series}\Big[\big(-p_{1,i}+p_{1,1+i}\big)\,x_{1,i}+\big(-1+T_1\big)\,\big(p_{1,1+i}-p_{1,1+j}\big)\,x_{1,i}+\big(-p_{1,j}+p_{1,1+j}\big)\,x_{1,j}+\big(-p_{2,i}+p_{2,1+i}\big)\,x_{2,i}+\big(-1+T_2\big)\,\big(p_{2,1+i}-p_{2,1+j}\big)\,x_{2,i}+\big(-p_{2,j}+p_{2,1+j}\big)\,x_{2,j}+\big(-p_{3,i}+p_{3,1+i}\big)\,x_{3,i}+\big(-1+T_1\,T_2\big)\,\big(p_{3,1+i}-p_{3,1+j}\big)\,x_{3,i}+\big(-p_{3,j}+p_{3,1+j}\big)\,x_{3,j},\; b_5\,p_{1,j}\,p_{2,i}\,x_{3,i}+b_3\,p_{1,i}\,p_{2,j}\,x_{3,i}+\big(-b_3-b_5\big)\,p_{1,j}\,p_{2,j}\,x_{3,i}\Big]\Big]$$

Out[•]=
$$\frac{1}{T_1\,T_2}\,\mathbb{E}\Big[\epsilon\text{Series}\Big[\big(-p_{1,i}+p_{1,1+i}\big)\,x_{1,i}+\Big(-1+\frac{1}{T_1}\Big)\,\big(p_{1,1+i}-p_{1,1+j}\big)\,x_{1,i}+\big(-p_{1,j}+p_{1,1+j}\big)\,x_{1,j}+\big(-p_{2,i}+p_{2,1+i}\big)\,x_{2,i}+\Big(-1+\frac{1}{T_2}\Big)\,\big(p_{2,1+i}-p_{2,1+j}\big)\,x_{2,i}+\big(-p_{2,j}+p_{2,1+j}\big)\,x_{2,j}+\big(-p_{3,i}+p_{3,1+i}\big)\,x_{3,i}+\Big(-1+\frac{1}{T_1\,T_2}\Big)\,\big(p_{3,1+i}-p_{3,1+j}\big)\,x_{3,i}+\big(-p_{3,j}+p_{3,1+j}\big)\,x_{3,j},\; e_1\,p_{1,i}\,p_{2,i}\,x_{3,i}+e_5\,p_{1,j}\,p_{2,i}\,x_{3,i}+e_3\,p_{1,i}\,p_{2,j}\,x_{3,i}+e_7\,p_{1,j}\,p_{2,j}\,x_{3,i}+e_2\,p_{1,i}\,p_{2,i}\,x_{3,j}+e_6\,p_{1,j}\,p_{2,i}\,x_{3,j}+e_4\,p_{1,i}\,p_{2,j}\,x_{3,j}+e_8\,p_{1,j}\,p_{2,j}\,x_{3,j}\Big]\Big]$$

In[•]:=
```
{eqn} = {lhs} =
  Cases[∫ℱ[i, j] ℒ /@ (Xi,j[1] Xi⁺,j⁺[-1]) 𝕕{vsi, vsj, vsi⁺, vsj⁺}, ϵSeries[_, ℰ_] :> ℰ, ∞]
```

Out[•]=
$$\Big\{\frac{\big(e_2+e_1\,T_1\,T_2-e_2\,T_1\,T_2\big)\,p_{1,2+i}\,p_{2,2+i}\,\pi_{3,i}}{T_1\,T_2}+\frac{1}{T_1\,T_2}\Big(-e_2+e_2\,T_1+e_6\,T_1+b_5\,T_1\,T_2-e_1\,T_1\,T_2+e_2\,T_1\,T_2+e_1\,T_1^2\,T_2-e_2\,T_1^2\,T_2+e_5\,T_1^2\,T_2-e_6\,T_1^2\,T_2\Big)\,p_{1,2+j}\,p_{2,2+i}\,\pi_{3,i}+\frac{1}{T_1\,T_2}\Big(-e_2+e_2\,T_2+e_4\,T_2+b_3\,T_1\,T_2-e_1\,T_1\,T_2+e_2\,T_1\,T_2+e_1\,T_1\,T_2^2-e_2\,T_1\,T_2^2+e_3\,T_1\,T_2^2-e_4\,T_1\,T_2^2\Big)\,p_{1,2+i}\,p_{2,2+j}\,\pi_{3,i}-\frac{1}{T_1\,T_2}\Big(-e_2+e_2\,T_1+e_6\,T_1+e_2\,T_2+e_4\,T_2+b_3\,T_1\,T_2+b_5\,T_1\,T_2-e_1\,T_1\,T_2-e_4\,T_1\,T_2-e_6\,T_1\,T_2-e_8\,T_1\,T_2+e_1\,T_1^2\,T_2-e_2\,T_1^2\,T_2+e_5\,T_1^2\,T_2-e_6\,T_1^2\,T_2+e_1\,T_1\,T_2^2-e_2\,T_1\,T_2^2+e_3\,T_1\,T_2^2-e_4\,T_1\,T_2^2-e_1\,T_1^2\,T_2^2+e_2\,T_1^2\,T_2^2-e_3\,T_1^2\,T_2^2+e_4\,T_1^2\,T_2^2-e_5\,T_1^2\,T_2^2+e_6\,T_1^2\,T_2^2-e_7\,T_1^2\,T_2^2+e_8\,T_1^2\,T_2^2\Big)\,p_{1,2+j}\,p_{2,2+j}\,\pi_{3,i}+\frac{e_2\,p_{1,2+i}\,p_{2,2+i}\,\pi_{3,j}}{T_1\,T_2}+\frac{\big(-e_2+e_2\,T_1+e_6\,T_1\big)\,p_{1,2+j}\,p_{2,2+i}\,\pi_{3,j}}{T_1\,T_2}+\frac{\big(-e_2+e_2\,T_2+e_4\,T_2\big)\,p_{1,2+i}\,p_{2,2+j}\,\pi_{3,j}}{T_1\,T_2}+\frac{1}{T_1\,T_2}\big(e_2-e_2\,T_1-e_6\,T_1-e_2\,T_2-e_4\,T_2+e_2\,T_1\,T_2+e_4\,T_1\,T_2+e_6\,T_1\,T_2+e_8\,T_1\,T_2\big)\,p_{1,2+j}\,p_{2,2+j}\,\pi_{3,j}\Big\}$$

In[•]:=
```
cvs = Union@Cases[eqn, p__ | π__, ∞]
```

Out[•]=
$$\{p_{1,2+i},\,p_{1,2+j},\,p_{2,2+i},\,p_{2,2+j},\,\pi_{3,i},\,\pi_{3,j}\}$$

In[•]:=
```
eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0)
```

Out[•]=

$$\left\{e_1 - e_2 + \frac{e_2}{T_1\,T_2} == 0,\ \frac{e_2}{T_1\,T_2} == 0,\right.$$
$$b_3 - e_1 + e_2 + \frac{e_2}{T_1} + \frac{e_4}{T_1} - \frac{e_2}{T_1\,T_2} + e_1\,T_2 - e_2\,T_2 + e_3\,T_2 - e_4\,T_2 == 0,\ \frac{e_2}{T_1} + \frac{e_4}{T_1} - \frac{e_2}{T_1\,T_2} == 0,$$
$$b_5 - e_1 + e_2 + e_1\,T_1 - e_2\,T_1 + e_5\,T_1 - e_6\,T_1 + \frac{e_2}{T_2} + \frac{e_6}{T_2} - \frac{e_2}{T_1\,T_2} == 0,\ \frac{e_2}{T_2} + \frac{e_6}{T_2} - \frac{e_2}{T_1\,T_2} == 0,$$
$$-b_3 - b_5 + e_1 + e_4 + e_6 + e_8 - \frac{e_2}{T_1} - \frac{e_4}{T_1} - e_1\,T_1 + e_2\,T_1 - e_5\,T_1 + e_6\,T_1 - \frac{e_2}{T_2} - \frac{e_6}{T_2} + \frac{e_2}{T_1\,T_2} - e_1\,T_2 + e_2\,T_2 -$$
$$e_3\,T_2 + e_4\,T_2 + e_1\,T_1\,T_2 - e_2\,T_1\,T_2 + e_3\,T_1\,T_2 - e_4\,T_1\,T_2 + e_5\,T_1\,T_2 - e_6\,T_1\,T_2 + e_7\,T_1\,T_2 - e_8\,T_1\,T_2 == 0,$$
$$\left.e_2 + e_4 + e_6 + e_8 - \frac{e_2}{T_1} - \frac{e_4}{T_1} - \frac{e_2}{T_2} - \frac{e_6}{T_2} + \frac{e_2}{T_1\,T_2} == 0\right\}$$

In[•]:=
```
vars = Union@Cases[r1[-1, i, j], e_, ∞]
```

Out[•]=

$$\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$$

In[•]:=
```
{sol} = Solve[eqns, vars]
```

Out[•]=

$$\left\{\left\{e_1 \to 0,\ e_2 \to 0,\ e_3 \to -\frac{b_3}{T_2},\ e_4 \to 0,\ e_5 \to -\frac{b_5}{T_1},\ e_6 \to 0,\ e_7 \to -\frac{-b_3\,T_1 - b_5\,T_2}{T_1\,T_2},\ e_8 \to 0\right\}\right\}$$

In[•]:=
```
sol /. (v_ → val_) :> (v = CF[val]);
```

In[•]:=
```
r1[-1, i, j]
```

Out[•]=

$$-\frac{b_5\,p_{1,j}\,p_{2,i}\,x_{3,i}}{T_1} - \frac{b_3\,p_{1,i}\,p_{2,j}\,x_{3,i}}{T_2} + \frac{(b_3\,T_1 + b_5\,T_2)\,p_{1,j}\,p_{2,j}\,x_{3,i}}{T_1\,T_2}$$

Reidemeister 2b in full

In[•]:=
```
ℒ[X_i_,j_[s_]] := T3^s 𝔼[q[s, i, j] + r0[s, i, j] + ϵ r1[s, i, j] + O[ϵ]^2];
ℒ[Xi,j[1]]
ℒ[Xi,j[-1]]
```

Out[•]=

$$T_1\,T_2\,\mathbb{E}\Big[\epsilon\text{Series}\Big[(-p_{1,i}+p_{1,1+i})\,x_{1,i}+(-1+T_1)\,(p_{1,1+i}-p_{1,1+j})\,x_{1,i}+(-p_{1,j}+p_{1,1+j})\,x_{1,j}+(-p_{2,i}+p_{2,1+i})\,x_{2,i}+(-1+T_2)\,(p_{2,1+i}-p_{2,1+j})\,x_{2,i}+a_2\,p_{3,j}\,x_{1,i}\,x_{2,i}-\frac{(a_2+a_4\,T_2)\,p_{3,j}\,x_{1,j}\,x_{2,i}}{T_1}+(-p_{2,j}+p_{2,1+j})\,x_{2,j}+a_4\,p_{3,j}\,x_{1,i}\,x_{2,j}+(-p_{3,i}+p_{3,1+i})\,x_{3,i}+(-1+T_1\,T_2)\,(p_{3,1+i}-p_{3,1+j})\,x_{3,i}+(-p_{3,j}+p_{3,1+j})\,x_{3,j},\ b_5\,p_{1,j}\,p_{2,i}\,x_{3,i}+b_3\,p_{1,i}\,p_{2,j}\,x_{3,i}+(-b_3-b_5)\,p_{1,j}\,p_{2,j}\,x_{3,i}\Big]\Big]$$

Out[•]=

$$\frac{1}{T_1\,T_2}\,\mathbb{E}\Big[\epsilon\text{Series}\Big[(-p_{1,i}+p_{1,1+i})\,x_{1,i}+\Big(-1+\frac{1}{T_1}\Big)\,(p_{1,1+i}-p_{1,1+j})\,x_{1,i}+(-p_{1,j}+p_{1,1+j})\,x_{1,j}+(-p_{2,i}+p_{2,1+i})\,x_{2,i}+\Big(-1+\frac{1}{T_2}\Big)\,(p_{2,1+i}-p_{2,1+j})\,x_{2,i}+\frac{(-a_2+a_4\,T_1-a_4\,T_2)\,p_{3,j}\,x_{1,i}\,x_{2,i}}{T_1^2\,T_2}+\frac{(a_2+a_4\,T_2)\,p_{3,j}\,x_{1,j}\,x_{2,i}}{T_1\,T_2}+(-p_{2,j}+p_{2,1+j})\,x_{2,j}-\frac{a_4\,p_{3,j}\,x_{1,i}\,x_{2,j}}{T_1}+(-p_{3,i}+p_{3,1+i})\,x_{3,i}+\Big(-1+\frac{1}{T_1\,T_2}\Big)\,(p_{3,1+i}-p_{3,1+j})\,x_{3,i}+(-p_{3,j}+p_{3,1+j})\,x_{3,j},\ \frac{(-b_5\,T_2\,p_{1,j}\,p_{2,i}-b_3\,T_1\,p_{1,i}\,p_{2,j}+b_3\,T_1\,p_{1,j}\,p_{2,j}+b_5\,T_2\,p_{1,j}\,p_{2,j})\,x_{3,i}}{T_1\,T_2}\Big]\Big]$$

In[•]:=
```
∫ℱ[i, j] ℒ /@ (Xi,j[1] Xi',j'[-1]) 𝕕{vsi, vsj, vsi', vsj'}
```

Out[•]=

$$\mathbb{E}\Big[\epsilon\text{Series}\Big[p_{1,2+i}\,\pi_{1,i}+p_{1,2+j}\,\pi_{1,j}+p_{2,2+i}\,\pi_{2,i}+p_{2,2+j}\,\pi_{2,j}+p_{3,2+i}\,\pi_{3,i}+p_{3,2+j}\,\pi_{3,j},\ \frac{(-a_2\,b_5+a_4\,b_3\,T_1-a_4\,b_5\,T_2)\,p_{3,2+j}\,\pi_{3,i}}{T_1}+a_4\,b_3\,p_{1,2+i}\,p_{3,2+j}\,\pi_{1,i}\,\pi_{3,i}+(-a_4\,b_3+a_2\,b_5-a_4\,b_5)\,p_{1,2+j}\,p_{3,2+j}\,\pi_{1,i}\,\pi_{3,i}-\frac{b_5\,(a_2+a_4\,T_2)\,p_{1,2+j}\,p_{3,2+j}\,\pi_{1,j}\,\pi_{3,i}}{T_1}-\frac{b_5\,(a_2+a_4\,T_2)\,p_{2,2+i}\,p_{3,2+j}\,\pi_{2,i}\,\pi_{3,i}}{T_1}+\frac{(a_2\,b_3+a_2\,b_5+a_2\,b_3\,T_1+a_4\,b_3\,T_2+a_4\,b_5\,T_2)\,p_{2,2+j}\,p_{3,2+j}\,\pi_{2,i}\,\pi_{3,i}}{T_1}+\frac{1}{T_1}\,\big(-a_2\,a_4\,b_3+a_2^2\,b_5-a_2\,a_4\,b_5-a_2\,a_4\,b_3\,T_1-a_4^2\,b_3\,T_2+a_2\,a_4\,b_5\,T_2-a_4^2\,b_5\,T_2\big)\,p_{3,2+j}^2\,\pi_{1,i}\,\pi_{2,i}\,\pi_{3,i}-\frac{b_5\,(a_2+a_4\,T_2)^2\,p_{3,2+j}^2\,\pi_{1,j}\,\pi_{2,i}\,\pi_{3,i}}{T_1^2}+a_4\,b_3\,p_{2,2+j}\,p_{3,2+j}\,\pi_{2,j}\,\pi_{3,i}-a_4^2\,b_3\,p_{3,2+j}^2\,\pi_{1,i}\,\pi_{2,j}\,\pi_{3,i}\Big]\Big]$$

In[•]:=
```
Echo /@ (CF@CoefficientList[Cases[∫ℱ[i, j] ℒ /@ (X_{i,j}[1] X_{i⁺,j⁺}[-1]) 𝕕{vs_i, vs_j, vs_{i⁺}, vs_{j⁺}},
    ϵSeries[_, ℰ_] :↦ ℰ, ∞][[1]] /. {a_i_ :↦ λ a_i, b_i_ :↦ λ b_i}, λ]);
```

» 0

» 0

» $\frac{(-a_2 b_5 + a_4 b_3 T_1 - a_4 b_5 T_2)\, p_{3,2+j}\, \pi_{3,i}}{T_1} + a_4 b_3 p_{1,2+i}\, p_{3,2+j}\, \pi_{1,i}\, \pi_{3,i} + (-a_4 b_3 + a_2 b_5 - a_4 b_5)\, p_{1,2+j}\, p_{3,2+j}\, \pi_{1,i}\, \pi_{3,i} -$
$\frac{b_5 (a_2 + a_4 T_2)\, p_{1,2+j}\, p_{3,2+j}\, \pi_{1,j}\, \pi_{3,i}}{T_1} - \frac{b_5 (a_2 + a_4 T_2)\, p_{2,2+i}\, p_{3,2+j}\, \pi_{2,i}\, \pi_{3,i}}{T_1} +$
$\frac{(a_2 b_3 + a_2 b_5 + a_2 b_3 T_1 + a_4 b_3 T_2 + a_4 b_5 T_2)\, p_{2,2+j}\, p_{3,2+j}\, \pi_{2,i}\, \pi_{3,i}}{T_1} + a_4 b_3 p_{2,2+j}\, p_{3,2+j}\, \pi_{2,j}\, \pi_{3,i}$

» $\frac{1}{T_1}\left(-a_2 a_4 b_3 + a_2^2 b_5 - a_2 a_4 b_5 - a_2 a_4 b_3 T_1 - a_4^2 b_3 T_2 + a_2 a_4 b_5 T_2 - a_4^2 b_5 T_2\right) p_{3,2+j}^2\, \pi_{1,i}\, \pi_{2,i}\, \pi_{3,i} -$
$\frac{b_5 (a_2 + a_4 T_2)^2\, p_{3,2+j}^2\, \pi_{1,j}\, \pi_{2,i}\, \pi_{3,i}}{T_1^2} - a_4^2 b_3 p_{3,2+j}^2\, \pi_{1,i}\, \pi_{2,j}\, \pi_{3,i}$

In[•]:=
```
err = CF@Coefficient[
   Cases[∫ℱ[i, j] ℒ /@ (X_{i,j}[1] X_{i⁺,j⁺}[-1]) 𝕕{vs_i, vs_j, vs_{i⁺}, vs_{j⁺}}, ϵSeries[_, ℰ_] :↦ ℰ, ∞][[
     1]] /. {a_i_ :↦ λ a_i, b_i_ :↦ λ b_i}, λ^2]
```

Out[•]=
$\frac{(-a_2 b_5 + a_4 b_3 T_1 - a_4 b_5 T_2)\, p_{3,2+j}\, \pi_{3,i}}{T_1} +$
$a_4 b_3 p_{1,2+i}\, p_{3,2+j}\, \pi_{1,i}\, \pi_{3,i} + (-a_4 b_3 + a_2 b_5 - a_4 b_5)\, p_{1,2+j}\, p_{3,2+j}\, \pi_{1,i}\, \pi_{3,i} -$
$\frac{b_5 (a_2 + a_4 T_2)\, p_{1,2+j}\, p_{3,2+j}\, \pi_{1,j}\, \pi_{3,i}}{T_1} - \frac{b_5 (a_2 + a_4 T_2)\, p_{2,2+i}\, p_{3,2+j}\, \pi_{2,i}\, \pi_{3,i}}{T_1} +$
$\frac{(a_2 b_3 + a_2 b_5 + a_2 b_3 T_1 + a_4 b_3 T_2 + a_4 b_5 T_2)\, p_{2,2+j}\, p_{3,2+j}\, \pi_{2,i}\, \pi_{3,i}}{T_1} + a_4 b_3 p_{2,2+j}\, p_{3,2+j}\, \pi_{2,j}\, \pi_{3,i}$

In[•]:=
```
κ = 0;
r42[-1, i_, j_] = Evaluate[Plus[
   Sum[
    f++κ xν1,k1 pν1,k2 xν2,k3 pν2,k4,
    {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}, {k4, {i, j}}, {ν1, 2}, {ν2, ν1 + 1, 3}
   ],
   Sum[
    f++κ xν,k1 pν,k2,
    {k1, {i, j}}, {k2, {i, j}}, {ν, 3}
   ]
  ]]
```

Out[•]=

$$
\begin{aligned}
&f_{49}\,p_{1,i}\,x_{1,i}+f_{52}\,p_{1,j}\,x_{1,i}+f_{55}\,p_{1,i}\,x_{1,j}+f_{58}\,p_{1,j}\,x_{1,j}+f_{50}\,p_{2,i}\,x_{2,i}+f_{53}\,p_{2,j}\,x_{2,i}+\\
&\ f_1\,p_{1,i}\,p_{2,i}\,x_{1,i}\,x_{2,i}+f_{13}\,p_{1,j}\,p_{2,i}\,x_{1,i}\,x_{2,i}+f_4\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,i}+f_{16}\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,i}+\\
&\ f_{25}\,p_{1,i}\,p_{2,i}\,x_{1,j}\,x_{2,i}+f_{37}\,p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,i}+f_{28}\,p_{1,i}\,p_{2,j}\,x_{1,j}\,x_{2,i}+f_{40}\,p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,i}+\\
&\ f_{56}\,p_{2,i}\,x_{2,j}+f_{59}\,p_{2,j}\,x_{2,j}+f_7\,p_{1,i}\,p_{2,i}\,x_{1,i}\,x_{2,j}+f_{19}\,p_{1,j}\,p_{2,i}\,x_{1,i}\,x_{2,j}+f_{10}\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,j}+\\
&\ f_{22}\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,j}+f_{31}\,p_{1,i}\,p_{2,i}\,x_{1,j}\,x_{2,j}+f_{43}\,p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,j}+f_{34}\,p_{1,i}\,p_{2,j}\,x_{1,j}\,x_{2,j}+\\
&\ f_{46}\,p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,j}+f_{51}\,p_{3,i}\,x_{3,i}+f_{54}\,p_{3,j}\,x_{3,i}+f_2\,p_{1,i}\,p_{3,i}\,x_{1,i}\,x_{3,i}+f_{14}\,p_{1,j}\,p_{3,i}\,x_{1,i}\,x_{3,i}+\\
&\ f_5\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,i}+f_{17}\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,i}+f_{26}\,p_{1,i}\,p_{3,i}\,x_{1,j}\,x_{3,i}+f_{38}\,p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,i}+\\
&\ f_{29}\,p_{1,i}\,p_{3,j}\,x_{1,j}\,x_{3,i}+f_{41}\,p_{1,j}\,p_{3,j}\,x_{1,j}\,x_{3,i}+f_3\,p_{2,i}\,p_{3,i}\,x_{2,i}\,x_{3,i}+f_{15}\,p_{2,j}\,p_{3,i}\,x_{2,i}\,x_{3,i}+\\
&\ f_6\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,i}+f_{18}\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,i}+f_{27}\,p_{2,i}\,p_{3,i}\,x_{2,j}\,x_{3,i}+f_{39}\,p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,i}+\\
&\ f_{30}\,p_{2,i}\,p_{3,j}\,x_{2,j}\,x_{3,i}+f_{42}\,p_{2,j}\,p_{3,j}\,x_{2,j}\,x_{3,i}+f_{57}\,p_{3,i}\,x_{3,j}+f_{60}\,p_{3,j}\,x_{3,j}+\\
&\ f_8\,p_{1,i}\,p_{3,i}\,x_{1,i}\,x_{3,j}+f_{20}\,p_{1,j}\,p_{3,i}\,x_{1,i}\,x_{3,j}+f_{11}\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,j}+f_{23}\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,j}+\\
&\ f_{32}\,p_{1,i}\,p_{3,i}\,x_{1,j}\,x_{3,j}+f_{44}\,p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,j}+f_{35}\,p_{1,i}\,p_{3,j}\,x_{1,j}\,x_{3,j}+f_{47}\,p_{1,j}\,p_{3,j}\,x_{1,j}\,x_{3,j}+\\
&\ f_9\,p_{2,i}\,p_{3,i}\,x_{2,i}\,x_{3,j}+f_{21}\,p_{2,j}\,p_{3,i}\,x_{2,i}\,x_{3,j}+f_{12}\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,j}+f_{24}\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,j}+\\
&\ f_{33}\,p_{2,i}\,p_{3,i}\,x_{2,j}\,x_{3,j}+f_{45}\,p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,j}+f_{36}\,p_{2,i}\,p_{3,j}\,x_{2,j}\,x_{3,j}+f_{48}\,p_{2,j}\,p_{3,j}\,x_{2,j}\,x_{3,j}
\end{aligned}
$$

In[•]:=
```
ℒ[Xi_,j_[s_]] := T3^s 𝔼[q[s, i, j] + ϵ r42[s, i, j] + O[ϵ]^2];
ℒ[Xi,j[1]]
ℒ[Xi,j[-1]]
```

Out[•]=

$$
\begin{aligned}
&T_1\,T_2\,\mathbb{E}\Big[\epsilon\text{Series}\Big[(-p_{1,i}+p_{1,1+i})\,x_{1,i}+(-1+T_1)\,(p_{1,1+i}-p_{1,1+j})\,x_{1,i}+(-p_{1,j}+p_{1,1+j})\,x_{1,j}+\\
&\qquad(-p_{2,i}+p_{2,1+i})\,x_{2,i}+(-1+T_2)\,(p_{2,1+i}-p_{2,1+j})\,x_{2,i}+(-p_{2,j}+p_{2,1+j})\,x_{2,j}+\\
&\qquad(-p_{3,i}+p_{3,1+i})\,x_{3,i}+(-1+T_1\,T_2)\,(p_{3,1+i}-p_{3,1+j})\,x_{3,i}+(-p_{3,j}+p_{3,1+j})\,x_{3,j},\\
&\quad c_{49}\,p_{1,i}\,x_{1,i}+c_{52}\,p_{1,j}\,x_{1,i}-\frac{(c_{49}+c_{52})\,p_{1,j}\,x_{1,j}}{T_1}+c_{50}\,p_{2,i}\,x_{2,i}+c_{53}\,p_{2,j}\,x_{2,i}+c_{13}\,p_{1,j}\,p_{2,i}\,x_{1,i}\,x_{2,i}+\\
&\quad c_4\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,i}+\frac{1}{-1+T_1\,T_2}\Big(-a_2\,b_3+c_4+c_{13}-c_4\,T_1-a_4\,b_3\,T_2-a_2\,b_5\,T_2+a_4\,b_5\,T_2-c_{13}\,T_2+\\
&\qquad a_4\,b_3\,T_1\,T_2-c_4\,T_1\,T_2-c_{13}\,T_1\,T_2+c_4\,T_1^2\,T_2-a_4\,b_5\,T_2^2+c_{13}\,T_1\,T_2^2\Big)\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,i}-\\
&\quad\Big(\Big(-a_2\,b_3+a_2\,b_5+a_2\,b_3\,T_1-c_4\,T_1+c_{13}\,T_1+c_4\,T_1^2-a_4\,b_3\,T_2-a_2\,b_5\,T_2+a_4\,b_5\,T_2+2\,a_4\,b_3\,T_1\,T_2-a_4\,b_5\\
&\qquad T_1\,T_2-c_{13}\,T_1\,T_2-a_4\,b_3\,T_1^2\,T_2+c_4\,T_1^2\,T_2-c_{13}\,T_1^2\,T_2-c_4\,T_1^3\,T_2-a_4\,b_5\,T_2^2+a_4\,b_5\,T_1\,T_2^2+c_{13}\,T_1^2\,T_2^2\Big)\\
&\qquad p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,i}\Big)\Big/\big((-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2)\big)-\\
&\quad\frac{(-a_2\,b_5-c_{13}\,T_1-a_4\,b_5\,T_2+c_{13}\,T_1\,T_2)\,p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,i}}{(-1+T_1)\,T_1}-\frac{(c_{50}+c_{53})\,p_{2,j}\,x_{2,j}}{T_2}+
\end{aligned}
$$

$$\Big(\big(-a_2\,b_3+a_4\,b_3+a_2\,b_5-a_4\,b_5-c_4+c_{13}+a_2\,b_3\,T_1-a_4\,b_3\,T_1+c_4\,T_1-a_4\,b_3\,T_2-a_2\,b_5\,T_2+2\,a_4\,b_5\,T_2-c_{13}\,T_2+a_4\,b_3\,T_1\,T_2+c_4\,T_1\,T_2-c_{13}\,T_1\,T_2-c_4\,T_1^2\,T_2-a_4\,b_5\,T_2^2+c_{13}\,T_1\,T_2^2\big)\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,j}\Big)\Big/\big((-1+T_1)\,(-1+T_2)\,(-1+T_1\,T_2)\big)-\frac{(a_4\,b_3-c_4+c_4\,T_1)\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,j}}{-1+T_2}+$$

$$c_{51}\,p_{3,i}\,x_{3,i}+c_{54}\,p_{3,j}\,x_{3,i}+c_{14}\,p_{1,j}\,p_{3,i}\,x_{1,i}\,x_{3,i}+c_5\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,i}+$$

$$\frac{1}{-1+T_2}\Big(a_2\,b_3-a_2\,b_5+a_4\,b_5+c_5+c_{14}-a_2\,b_3\,T_1+a_4\,b_3\,T_1-c_5\,T_1+a_4\,b_3\,T_2+a_2\,b_5\,T_2-2\,a_4\,b_5\,T_2-c_5\,T_2-c_{14}\,T_2-2\,a_4\,b_3\,T_1\,T_2+c_5\,T_1\,T_2-c_{14}\,T_1\,T_2+a_4\,b_5\,T_2^2+c_{14}\,T_1\,T_2^2\Big)\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,i}-$$

$$\Big(\big(a_2\,b_3-a_2\,b_5-2\,a_2\,b_3\,T_1+a_4\,b_3\,T_1-c_5\,T_1+c_{14}\,T_1+a_2\,b_3\,T_1^2-a_4\,b_3\,T_1^2+c_5\,T_1^2+a_4\,b_3\,T_2+a_2\,b_5\,T_2-a_4\,b_5\,T_2-3\,a_4\,b_3\,T_1\,T_2+a_4\,b_5\,T_1\,T_2+c_5\,T_1\,T_2-c_{14}\,T_1\,T_2+2\,a_4\,b_3\,T_1^2\,T_2-c_5\,T_1^2\,T_2-c_{14}\,T_1^2\,T_2+a_4\,b_5\,T_2^2-a_4\,b_5\,T_1\,T_2^2+c_{14}\,T_1^2\,T_2^2\big)\,p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,i}\Big)\Big/\big((-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2)\big)-\frac{b_3\,(a_2+a_4\,T_2)\,p_{1,i}\,p_{3,j}\,x_{1,j}\,x_{3,i}}{T_1\,(-1+T_2)}-$$

$$\Big(\big(a_2\,b_3-a_2\,b_3\,T_1-a_2\,b_5\,T_1+c_{14}\,T_1+a_4\,b_3\,T_2-a_4\,b_3\,T_1\,T_2+a_2\,b_5\,T_1\,T_2-a_4\,b_5\,T_1\,T_2-c_{14}\,T_1\,T_2-c_{14}\,T_1^2\,T_2+a_4\,b_5\,T_1\,T_2^2+c_{14}\,T_1^2\,T_2^2\big)\,p_{1,j}\,p_{3,j}\,x_{1,j}\,x_{3,i}\Big)\Big/\big((-1+T_1)\,T_1\,(-1+T_2)\big)+c_{15}\,p_{2,j}\,p_{3,i}\,x_{2,i}\,x_{3,i}+c_6\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,i}+$$

$$\frac{1}{(-1+T_1)\,T_1}\Big(-a_2\,b_3+a_2\,b_3\,T_1+c_6\,T_1+c_{15}\,T_1-c_6\,T_1^2-c_{15}\,T_1^2-a_4\,b_3\,T_2-a_2\,b_5\,T_2+2\,a_4\,b_3\,T_1\,T_2+a_2\,b_5\,T_1\,T_2-a_4\,b_5\,T_1\,T_2-c_6\,T_1\,T_2-a_4\,b_3\,T_1^2\,T_2+c_6\,T_1^2\,T_2-c_{15}\,T_1^2\,T_2+c_{15}\,T_1^3\,T_2-a_4\,b_5\,T_2^2+2\,a_4\,b_5\,T_1\,T_2^2\Big)\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,i}-$$

$$\Big(\big(-a_2\,b_5-a_2\,b_3\,T_1+a_4\,b_3\,T_1+a_2\,b_5\,T_1-a_4\,b_5\,T_1-c_6\,T_1+c_{15}\,T_1+a_2\,b_3\,T_1^2-a_4\,b_3\,T_1^2+c_6\,T_1^2-c_{15}\,T_1^2+a_2\,b_5\,T_2-a_4\,b_5\,T_2-a_4\,b_3\,T_1\,T_2-a_2\,b_5\,T_1\,T_2+3\,a_4\,b_5\,T_1\,T_2+c_6\,T_1\,T_2+a_4\,b_3\,T_1^2\,T_2-c_6\,T_1^2\,T_2-c_{15}\,T_1^2\,T_2+c_{15}\,T_1^3\,T_2+a_4\,b_5\,T_2^2-2\,a_4\,b_5\,T_1\,T_2^2\big)\,p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,i}\Big)\Big/\big((-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2)\big)+\frac{a_4\,b_5\,p_{2,i}\,p_{3,j}\,x_{2,j}\,x_{3,i}}{-1+T_1}-\frac{1}{(-1+T_1)\,(-1+T_2)}$$

$$\Big(-a_4\,b_5+c_{15}-c_{15}\,T_1+a_4\,b_3\,T_2+a_4\,b_5\,T_2-a_4\,b_3\,T_1\,T_2-c_{15}\,T_1\,T_2+c_{15}\,T_1^2\,T_2\Big)\,p_{2,j}\,p_{3,j}\,x_{2,j}\,x_{3,i}-$$

$$\frac{1}{T_1^2\,T_2\,(-1+T_1\,T_2)}\Big(a_2\,b_5+a_2\,b_3\,T_1-a_4\,b_3\,T_1-c_{51}\,T_1-c_{54}\,T_1+a_4\,b_5\,T_2+a_4\,b_3\,T_1\,T_2-a_4\,b_5\,T_1\,T_2+c_{51}\,T_1^2\,T_2+c_{54}\,T_1^2\,T_2\Big)\,p_{3,j}\,x_{3,j}+$$

$$\frac{(a_2\,b_5-a_4\,b_5+c_5-c_{14}-c_5\,T_1+a_4\,b_5\,T_2+c_{14}\,T_1\,T_2)\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,j}}{(-1+T_1)\,(-1+T_1\,T_2)}-$$

$$\big(a_2\,b_3+c_5-a_2\,b_3\,T_1+a_4\,b_3\,T_1-c_5\,T_1+a_4\,b_3\,T_2-c_5\,T_2-2\,a_4\,b_3\,T_1\,T_2+c_5\,T_1\,T_2\big)\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,j}\big)\Big/\big((-1+T_2)\,(-1+T_1\,T_2)\big)+$$

$$\Big(\big(a_2\,b_3+c_6\,T_1-c_{15}\,T_1+a_4\,b_3\,T_2-a_4\,b_3\,T_1\,T_2-c_6\,T_1\,T_2+c_{15}\,T_1^2\,T_2\big)\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,j}\Big)\Big/\big(T_1\,(-1+T_2)\,(-1+T_1\,T_2)\big)-$$

$$\Big(\big(c_6\,T_1-c_6\,T_1^2-a_2\,b_5\,T_2+a_2\,b_5\,T_1\,T_2-a_4\,b_5\,T_1\,T_2-c_6\,T_1\,T_2+c_6\,T_1^2\,T_2-a_4\,b_5\,T_2^2+2\,a_4\,b_5\,T_1\,T_2^2\big)\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,j}\Big)\Big/\big((-1+T_1)\,T_1\,(-1+T_1\,T_2)\big)\Big]\Big]$$

Out[•]=

$$\frac{1}{T_1\,T_2}\,\mathbb{E}\Big[\epsilon\text{Series}\Big[\,(-p_{1,i}+p_{1,1+i})\,x_{1,i}+\Big(-1+\frac{1}{T_1}\Big)\,(p_{1,1+i}-p_{1,1+j})\,x_{1,i}+(-p_{1,j}+p_{1,1+j})\,x_{1,j}+$$
$$(-p_{2,i}+p_{2,1+i})\,x_{2,i}+\Big(-1+\frac{1}{T_2}\Big)\,(p_{2,1+i}-p_{2,1+j})\,x_{2,i}+(-p_{2,j}+p_{2,1+j})\,x_{2,j}+$$
$$(-p_{3,i}+p_{3,1+i})\,x_{3,i}+\Big(-1+\frac{1}{T_1\,T_2}\Big)\,(p_{3,1+i}-p_{3,1+j})\,x_{3,i}+(-p_{3,j}+p_{3,1+j})\,x_{3,j},$$
$$f_{49}\,p_{1,i}\,x_{1,i}+f_{52}\,p_{1,j}\,x_{1,i}+f_{55}\,p_{1,i}\,x_{1,j}+f_{58}\,p_{1,j}\,x_{1,j}+f_{50}\,p_{2,i}\,x_{2,i}+f_{53}\,p_{2,j}\,x_{2,i}+$$
$$f_1\,p_{1,i}\,p_{2,i}\,x_{1,i}\,x_{2,i}+f_{13}\,p_{1,j}\,p_{2,i}\,x_{1,i}\,x_{2,i}+f_4\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,i}+f_{16}\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,i}+$$
$$f_{25}\,p_{1,i}\,p_{2,i}\,x_{1,j}\,x_{2,i}+f_{37}\,p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,i}+f_{28}\,p_{1,i}\,p_{2,j}\,x_{1,j}\,x_{2,i}+f_{40}\,p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,i}+$$
$$f_{56}\,p_{2,i}\,x_{2,j}+f_{59}\,p_{2,j}\,x_{2,j}+f_7\,p_{1,i}\,p_{2,i}\,x_{1,i}\,x_{2,j}+f_{19}\,p_{1,j}\,p_{2,i}\,x_{1,i}\,x_{2,j}+f_{10}\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,j}+$$
$$f_{22}\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,j}+f_{31}\,p_{1,i}\,p_{2,i}\,x_{1,j}\,x_{2,j}+f_{43}\,p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,j}+f_{34}\,p_{1,i}\,p_{2,j}\,x_{1,j}\,x_{2,j}+$$
$$f_{46}\,p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,j}+f_{51}\,p_{3,i}\,x_{3,i}+f_{54}\,p_{3,j}\,x_{3,i}+f_2\,p_{1,i}\,p_{3,i}\,x_{1,i}\,x_{3,i}+f_{14}\,p_{1,j}\,p_{3,i}\,x_{1,i}\,x_{3,i}+$$
$$f_5\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,i}+f_{17}\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,i}+f_{26}\,p_{1,i}\,p_{3,i}\,x_{1,j}\,x_{3,i}+f_{38}\,p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,i}+$$
$$f_{29}\,p_{1,i}\,p_{3,j}\,x_{1,j}\,x_{3,i}+f_{41}\,p_{1,j}\,p_{3,j}\,x_{1,j}\,x_{3,i}+f_3\,p_{2,i}\,p_{3,i}\,x_{2,i}\,x_{3,i}+f_{15}\,p_{2,j}\,p_{3,i}\,x_{2,i}\,x_{3,i}+$$
$$f_6\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,i}+f_{18}\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,i}+f_{27}\,p_{2,i}\,p_{3,i}\,x_{2,j}\,x_{3,i}+f_{39}\,p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,i}+$$
$$f_{30}\,p_{2,i}\,p_{3,j}\,x_{2,j}\,x_{3,i}+f_{42}\,p_{2,j}\,p_{3,j}\,x_{2,j}\,x_{3,i}+f_{57}\,p_{3,i}\,x_{3,j}+f_{60}\,p_{3,j}\,x_{3,j}+$$
$$f_8\,p_{1,i}\,p_{3,i}\,x_{1,i}\,x_{3,j}+f_{20}\,p_{1,j}\,p_{3,i}\,x_{1,i}\,x_{3,j}+f_{11}\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,j}+f_{23}\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,j}+$$
$$f_{32}\,p_{1,i}\,p_{3,i}\,x_{1,j}\,x_{3,j}+f_{44}\,p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,j}+f_{35}\,p_{1,i}\,p_{3,j}\,x_{1,j}\,x_{3,j}+f_{47}\,p_{1,j}\,p_{3,j}\,x_{1,j}\,x_{3,j}+$$
$$f_9\,p_{2,i}\,p_{3,i}\,x_{2,i}\,x_{3,j}+f_{21}\,p_{2,j}\,p_{3,i}\,x_{2,i}\,x_{3,j}+f_{12}\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,j}+f_{24}\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,j}+$$
$$f_{33}\,p_{2,i}\,p_{3,i}\,x_{2,j}\,x_{3,j}+f_{45}\,p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,j}+f_{36}\,p_{2,i}\,p_{3,j}\,x_{2,j}\,x_{3,j}+f_{48}\,p_{2,j}\,p_{3,j}\,x_{2,j}\,x_{3,j}\Big]\Big]$$

In[•]:=
```
{lhs} =
 Cases[∫ℱ[i, j] ℒ /@ (X_{i,j}[1] X_{i⁺,j⁺}[-1]) 𝕕{vs_i, vs_j, vs_{i⁺}, vs_{j⁺}}, ϵSeries[_, ℰ_] :↦ ℰ, ∞]
```

Out[•]=

$$\Big\{\frac{a_2\,b_5+a_2\,b_3\,T_1-a_4\,b_3\,T_1-a_2\,b_5\,T_1-c_{51}\,T_1-c_{54}\,T_1-a_2\,b_3\,T_1^2+a_4\,b_3\,T_1^2-c_{50}\,T_1^2+\langle\!\langle 151\rangle\!\rangle+f_{47}\,T_1^3\,T_2^2+f_{48}\,T_1^3\,T_2^2+f_{49}\,T_1^3\,T_2^2+f_{50}\,T_1^3\,T_2^2+f_{51}\,T_1^3\,T_2^2+f_{58}\,T_1^3\,T_2^2+f_{59}\,T_1^3\,T_2^2+f_{60}\,T_1^3\,T_2^2}{(-1+T_1)\,T_1^2\,(-1+T_2)\,T_2}+$$
$$\frac{\big(\langle\!\langle 163\rangle\!\rangle+f_{49}\,T_1^3\,T_2^2-f_{55}\,T_1^3\,T_2^2\big)\,p_{\langle\!\langle 1\rangle\!\rangle}\,\pi_{\langle\!\langle 1\rangle\!\rangle}}{(-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2)}+\frac{\langle\!\langle 1\rangle\!\rangle}{\langle\!\langle 1\rangle\!\rangle}+\langle\!\langle 70\rangle\!\rangle+\langle\!\langle 1\rangle\!\rangle+\frac{\langle\!\langle 1\rangle\!\rangle}{T_1\,T_2^2}+\frac{\big(\langle\!\langle 1\rangle\!\rangle\big)\,p_{2,2+j}\,p_{3,2+j}\,\pi_{2,j}\,\pi_{3,j}}{T_1\,T_2^2}\Big\}$$

Full expression not available (original memory size: 1 MB)

In[•]:=
```
eqn = CF[lhs - err]
```

Out[•]=

$$-\frac{-a_2\,b_5-a_2\,b_3\,T_1+a_4\,b_3\,T_1+a_2\,b_5\,T_1+c_{51}\,T_1+c_{54}\,T_1+a_2\,b_3\,T_1^2-a_4\,b_3\,T_1^2+\langle\!\langle 163\rangle\!\rangle}{(-1+T_1)\,T_1^2\,(-1+T_2)\,T_2}+\frac{\big(\langle\!\langle 163\rangle\!\rangle+f_{49}\,T_1^3\,T_2^2-f_{55}\,T_1^3\,T_2^2\big)\,p_{1,\langle\!\langle 1\rangle\!\rangle}\,\pi_{1,i}}{(-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2)}+\frac{\langle\!\langle 1\rangle\!\rangle}{\langle\!\langle 1\rangle\!\rangle}+\langle\!\langle 75\rangle\!\rangle+$$
$$\langle\!\langle 1\rangle\!\rangle+\frac{\big(\langle\!\langle 1\rangle\!\rangle\big)\,\langle\!\langle 3\rangle\!\rangle\,\pi_{\langle\!\langle 1\rangle\!\rangle}}{T_1\,T_2^2}+\frac{\big(f_{33}-f_{33}\,T_2-f_{45}\,T_2-f_{33}\,T_1\,T_2-f_{36}\,T_1\,T_2+f_{33}\,T_1\,T_2^2+f_{36}\,T_1\,T_2^2+f_{45}\,T_1\,T_2^2+f_{48}\,T_1\,T_2^2\big)\,p_{2,\langle\!\langle 1\rangle\!\rangle}\,p_{3,\langle\!\langle 1\rangle\!\rangle}\,\pi_{2,j}\,\pi_{3,j}}{T_1\,T_2^2}$$

Full expression not available (original memory size: 1 MB)

In[•]:=
```
cvs = Union@Cases[eqn, p__ | π__, ∞]
```

Out[•]=

$$\{p_{1,2+i},\,p_{1,2+j},\,p_{2,2+i},\,p_{2,2+j},\,p_{3,2+i},\,p_{3,2+j},\,\pi_{1,i},\,\pi_{1,j},\,\pi_{2,i},\,\pi_{2,j},\,\pi_{3,i},\,\pi_{3,j}\}$$

In[•]:=
```
eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0)
```

Out[•]=

$$\left\{f_1 - f_7 - f_{25} + f_{31} + \frac{f_{25}}{T_1} - \frac{f_{31}}{T_1} + \frac{f_7}{T_2} - \frac{f_{31}}{T_2} + \frac{f_{31}}{T_1\,T_2} == 0,\ \frac{f_7}{T_2} - \frac{f_{31}}{T_2} + \frac{f_{31}}{T_1\,T_2} == 0,\ \langle\!\langle 1 \rangle\!\rangle == 0,\ \langle\!\langle 55 \rangle\!\rangle,\right.$$
$$\langle\!\langle 1 \rangle\!\rangle == 0,\ \langle\!\langle 1 \rangle\!\rangle == 0,\ -\frac{2\,a_4\,b_3}{(1-T_1)\,(1-T_2)} + \frac{2\,c_{49}}{(1-T_1)\,(1-T_2)} + \frac{2\,c_{50}}{(1-T_1)\,(1-T_2)} + \frac{c_{52}}{(1-T_1)\,(1-T_2)} + \frac{c_{53}}{(1-T_1)\,(1-T_2)} - \frac{c_{54}}{(1-T_1)\,(1-T_2)} +$$
$$\left.\frac{f_1}{(1-T_1)\,(1-T_2)} + \langle\!\langle 159 \rangle\!\rangle + \frac{f_{49}\,T_1\,T_2}{(1-T_1)\,(1-T_2)} + \frac{f_{50}\,T_1\,T_2}{(1-T_1)\,(1-T_2)} + \frac{f_{51}\,T_1\,T_2}{(1-T_1)\,(1-T_2)} + \frac{f_{58}\,T_1\,T_2}{(1-T_1)\,(1-T_2)} + \frac{f_{59}\,T_1\,T_2}{(1-T_1)\,(1-T_2)} + \frac{f_{60}\,T_1\,T_2}{(1-T_1)\,(1-T_2)} == 0\right\}$$

Full expression not available (original memory size: 2.5 MB)

In[•]:=
```
vars = Union@Cases[r42[-1, i, j], f_, ∞]
```

Out[•]=

$$\{f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_{12}, f_{13}, f_{14}, f_{15}, f_{16}, f_{17}, f_{18}, f_{19}, f_{20}, f_{21}, f_{22},$$
$$f_{23}, f_{24}, f_{25}, f_{26}, f_{27}, f_{28}, f_{29}, f_{30}, f_{31}, f_{32}, f_{33}, f_{34}, f_{35}, f_{36}, f_{37}, f_{38}, f_{39}, f_{40}, f_{41},$$
$$f_{42}, f_{43}, f_{44}, f_{45}, f_{46}, f_{47}, f_{48}, f_{49}, f_{50}, f_{51}, f_{52}, f_{53}, f_{54}, f_{55}, f_{56}, f_{57}, f_{58}, f_{59}, f_{60}\}$$

In[•]:=
```
{sol} = Solve[eqns, vars]
```

Out[•]=

$$\left\{\left\{f_1 \to 0,\ f_2 \to 0,\ f_3 \to 0,\right.\right.$$
$$f_4 \to -\left(\left(-a_2\,b_3 + a_4\,b_3 + a_2\,b_5 - a_4\,b_5 + c_{13} + a_2\,b_3\,T_1 - a_4\,b_3\,T_1 - a_4\,b_3\,T_2 - a_2\,b_5\,T_2 + 2\,a_4\,b_5\,T_2 - c_{13}\,T_2 + a_4\,b_3\,T_1\,T_2 - c_{13}\,T_1\,T_2 - a_4\,b_5\,T_2^2 + c_{13}\,T_1\,T_2^2\right) \Big/ \left((-1+T_1)\,T_2\,(-1+T_1\,T_2)\right)\right),$$
$$f_5 \to -\left(\left(-a_2\,b_3 + 2\,a_2\,b_3\,T_1 - a_4\,b_3\,T_1 - a_2\,b_5\,T_1 + a_4\,b_5\,T_1 + c_{14}\,T_1 - a_2\,b_3\,T_1^2 + a_4\,b_3\,T_1^2 - a_4\,b_3\,T_2 + 3\,a_4\,b_3\,T_1\,T_2 + a_2\,b_5\,T_1\,T_2 - 2\,a_4\,b_5\,T_1\,T_2 - c_{14}\,T_1\,T_2 - 2\,a_4\,b_3\,T_1^2\,T_2 - c_{14}\,T_1^2\,T_2 + a_4\,b_5\,T_1\,T_2^2 + c_{14}\,T_1^2\,T_2^2\right) \Big/ \left((-1+T_1)\,T_1^2\,(-1+T_2)\,T_2\right)\right),$$
$$f_6 \to -\left(\left(-a_2\,b_3 + a_2\,b_5 + a_2\,b_3\,T_1 - a_2\,b_5\,T_1 + a_4\,b_5\,T_1 + c_{15}\,T_1 - c_{15}\,T_1^2 - a_4\,b_3\,T_2 - a_2\,b_5\,T_2 + a_4\,b_5\,T_2 + 2\,a_4\,b_3\,T_1\,T_2 + a_2\,b_5\,T_1\,T_2 - 3\,a_4\,b_5\,T_1\,T_2 - a_4\,b_3\,T_1^2\,T_2 - c_{15}\,T_1^2\,T_2 + c_{15}\,T_1^3\,T_2 - a_4\,b_5\,T_2^2 + 2\,a_4\,b_5\,T_1\,T_2^2\right) \Big/ \left((-1+T_1)\,T_1^2\,(-1+T_2)\,T_2\right)\right),\ f_7 \to 0,\ f_8 \to 0,\ f_9 \to 0,$$
$$f_{10} \to -\left(\left(-a_2\,b_3 + a_4\,b_3 + a_2\,b_5 - a_4\,b_5 - c_4 + c_{13} + a_2\,b_3\,T_1 - a_4\,b_3\,T_1 + c_4\,T_1 - a_4\,b_3\,T_2 - a_2\,b_5\,T_2 + 2\,a_4\,b_5\,T_2 - c_{13}\,T_2 + a_4\,b_3\,T_1\,T_2 + c_4\,T_1\,T_2 - c_{13}\,T_1\,T_2 - c_4\,T_1^2\,T_2 - a_4\,b_5\,T_2^2 + c_{13}\,T_1\,T_2^2\right) \Big/ \left((-1+T_1)\,(-1+T_2)\,(-1+T_1\,T_2)\right)\right),\ f_{11} \to -\frac{a_2\,b_5 - a_4\,b_5 + c_5 - c_{14} - c_5\,T_1 + a_4\,b_5\,T_2 + c_{14}\,T_1\,T_2}{(-1+T_1)\,(-1+T_1\,T_2)},$$
$$f_{12} \to -\frac{a_2\,b_3 + c_6\,T_1 - c_{15}\,T_1 + a_4\,b_3\,T_2 - a_4\,b_3\,T_1\,T_2 - c_6\,T_1\,T_2 + c_{15}\,T_1^2\,T_2}{T_1\,(-1+T_2)\,(-1+T_1\,T_2)},$$
$$f_{13} \to -\left(\left(a_2\,b_3 - a_2\,b_5 - a_2\,b_3\,T_1 + c_4\,T_1 - c_4\,T_1^2 + a_4\,b_3\,T_2 + a_2\,b_5\,T_2 - a_4\,b_5\,T_2 - 2\,a_4\,b_3\,T_1\,T_2 + a_4\,b_5\,T_1\,T_2 + a_4\,b_3\,T_1^2\,T_2 - c_4\,T_1^2\,T_2 + c_4\,T_1^3\,T_2 + a_4\,b_5\,T_2^2 - a_4\,b_5\,T_1\,T_2^2\right) \Big/ \left(T_1^2\,(-1+T_2)\,(-1+T_1\,T_2)\right)\right),$$
$$f_{14} \to -\left(\left(-a_2\,b_3 + a_2\,b_5 + 2\,a_2\,b_3\,T_1 - a_4\,b_3\,T_1 + c_5\,T_1 - a_2\,b_3\,T_1^2 + a_4\,b_3\,T_1^2 - c_5\,T_1^2 - a_4\,b_3\,T_2 - a_2\,b_5\,T_2 + a_4\,b_5\,T_2 + 3\,a_4\,b_3\,T_1\,T_2 - a_4\,b_5\,T_1\,T_2 - c_5\,T_1\,T_2 - 2\,a_4\,b_3\,T_1^2\,T_2 + c_5\,T_1^2\,T_2 - a_4\,b_5\,T_2^2 + a_4\,b_5\,T_1\,T_2^2\right) \Big/ \left(T_1^2\,(-1+T_2)\,(-1+T_1\,T_2)\right)\right),$$
$$f_{15} \to -\left(\left(a_2\,b_5 + a_2\,b_3\,T_1 - a_4\,b_3\,T_1 - a_2\,b_5\,T_1 + a_4\,b_5\,T_1 + c_6\,T_1 - a_2\,b_3\,T_1^2 + a_4\,b_3\,T_1^2 - c_6\,T_1^2 - a_2\,b_5\,T_2 + a_4\,b_5\,T_2 + a_4\,b_3\,T_1\,T_2 + a_2\,b_5\,T_1\,T_2 - 3\,a_4\,b_5\,T_1\,T_2 - c_6\,T_1\,T_2 - a_4\,b_3\,T_1^2\,T_2 + c_6\,T_1^2\,T_2 - a_4\,b_5\,T_2^2 + 2\,a_4\,b_5\,T_1\,T_2^2\right) \Big/ \left((-1+T_1)\,T_1\,T_2\,(-1+T_1\,T_2)\right)\right),$$
$$f_{16} \to -\frac{1}{T_1^2\,T_2\,(-1+T_1\,T_2)}\left(-a_2\,b_3 + a_2\,b_3\,T_1 - a_2\,b_5\,T_1 + a_4\,b_5\,T_1 - c_4\,T_1 - c_{13}\,T_1 - a_2\,b_3\,T_1^2 + a_4\,b_3\,T_1^2 + c_4\,T_1^2 - a_4\,b_3\,T_2 - a_2\,b_5\,T_2 + 2\,a_4\,b_3\,T_1\,T_2 + a_2\,b_5\,T_1\,T_2 - 2\,a_4\,b_5\,T_1\,T_2 + c_{13}\,T_1\,T_2 -\right.$$

$$2\,a_4\,b_3\,T_1^2\,T_2+c_4\,T_1^2\,T_2+c_{13}\,T_1^2\,T_2-c_4\,T_1^3\,T_2-a_4\,b_5\,T_2^2+2\,a_4\,b_5\,T_1\,T_2^2-c_{13}\,T_1^2\,T_2^2\big),$$

$$f_{17}\to-\frac{1}{T_1^3\,(-1+T_2)\,T_2}\,\big(a_2\,b_3-3\,a_2\,b_3\,T_1+a_4\,b_3\,T_1+a_2\,b_5\,T_1-a_4\,b_5\,T_1-c_5\,T_1-c_{14}\,T_1+$$
$$2\,a_2\,b_3\,T_1^2-2\,a_4\,b_3\,T_1^2+c_5\,T_1^2+a_4\,b_3\,T_2-4\,a_4\,b_3\,T_1\,T_2-a_2\,b_5\,T_1\,T_2+2\,a_4\,b_5\,T_1\,T_2+$$
$$c_5\,T_1\,T_2+c_{14}\,T_1\,T_2+4\,a_4\,b_3\,T_1^2\,T_2-c_5\,T_1^2\,T_2+c_{14}\,T_1^2\,T_2-a_4\,b_5\,T_1\,T_2^2-c_{14}\,T_1^2\,T_2^2\big),$$

$$f_{18}\to-\frac{1}{(-1+T_1)\,T_1^2\,T_2^2}\,\big(a_2\,b_3-a_2\,b_5-a_2\,b_3\,T_1+a_2\,b_5\,T_1-a_4\,b_5\,T_1-c_6\,T_1-c_{15}\,T_1+c_6\,T_1^2+$$
$$c_{15}\,T_1^2+a_4\,b_3\,T_2+2\,a_2\,b_5\,T_2-a_4\,b_5\,T_2-2\,a_4\,b_3\,T_1\,T_2-2\,a_2\,b_5\,T_1\,T_2+4\,a_4\,b_5\,T_1\,T_2+c_6\,T_1\,T_2+$$
$$a_4\,b_3\,T_1^2\,T_2-c_6\,T_1^2\,T_2+c_{15}\,T_1^2\,T_2-c_{15}\,T_1^3\,T_2+2\,a_4\,b_5\,T_2^2-4\,a_4\,b_5\,T_1\,T_2^2\big),\ f_{19}\to0,\ f_{20}\to0,$$

$$f_{21}\to0,\ f_{22}\to-\Big(\big(a_2\,b_3-a_2\,b_5+a_4\,b_5-c_{13}-a_2\,b_3\,T_1+a_4\,b_3\,T_1+a_4\,b_3\,T_2+a_2\,b_5\,T_2-2\,a_4\,b_5\,T_2+$$
$$c_{13}\,T_2-2\,a_4\,b_3\,T_1\,T_2+c_{13}\,T_1\,T_2+a_4\,b_5\,T_2^2-c_{13}\,T_1\,T_2^2\big)\,/\,(T_1\,(-1+T_2)\,(-1+T_1\,T_2))\Big),$$

$$f_{23}\to-\Big(\big(-a_2\,b_3+a_2\,b_5-a_4\,b_5-c_{14}+a_2\,b_3\,T_1-a_4\,b_3\,T_1-a_4\,b_3\,T_2-a_2\,b_5\,T_2+2\,a_4\,b_5\,T_2+$$
$$c_{14}\,T_2+2\,a_4\,b_3\,T_1\,T_2+c_{14}\,T_1\,T_2-a_4\,b_5\,T_2^2-c_{14}\,T_1\,T_2^2\big)\,/\,(T_1\,(-1+T_2)\,(-1+T_1\,T_2))\Big),$$

$$f_{24}\to-\Big(\big(a_2\,b_3-a_2\,b_3\,T_1-c_{15}\,T_1+c_{15}\,T_1^2+a_4\,b_3\,T_2+a_2\,b_5\,T_2-2\,a_4\,b_3\,T_1\,T_2-a_2\,b_5\,T_1\,T_2+a_4\,b_5\,T_1\,T_2+$$
$$a_4\,b_3\,T_1^2\,T_2+c_{15}\,T_1^2\,T_2-c_{15}\,T_1^3\,T_2+a_4\,b_5\,T_2^2-2\,a_4\,b_5\,T_1\,T_2^2\big)\,/\,((-1+T_1)\,T_1\,T_2\,(-1+T_1\,T_2))\Big),$$

$$f_{25}\to0,\ f_{26}\to0,\ f_{27}\to0,\ f_{28}\to0,\ f_{29}\to-\frac{-a_2\,b_3-a_4\,b_3\,T_2}{T_1\,(-1+T_2)\,T_2},$$

$$f_{30}\to-\frac{a_4\,b_5}{(-1+T_1)\,T_1},$$

$$f_{31}\to0,$$

$$f_{32}\to0,$$

$$f_{33}\to0,$$

$$f_{34}\to0,$$

$$f_{35}\to0,$$

$$f_{36}\to0,$$

$$f_{37}\to-\Big(\big(a_2\,b_3-a_2\,b_5-a_2\,b_3\,T_1+c_4\,T_1-c_{13}\,T_1-c_4\,T_1^2+a_4\,b_3\,T_2+a_2\,b_5\,T_2-a_4\,b_5\,T_2-$$
$$2\,a_4\,b_3\,T_1\,T_2+a_4\,b_5\,T_1\,T_2+c_{13}\,T_1\,T_2+a_4\,b_3\,T_1^2\,T_2-c_4\,T_1^2\,T_2+c_{13}\,T_1^2\,T_2+c_4\,T_1^3\,T_2+$$
$$a_4\,b_5\,T_2^2-a_4\,b_5\,T_1\,T_2^2-c_{13}\,T_1^2\,T_2^2\big)\,/\,((-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2))\Big),$$

$$f_{38}\to-\Big(\big(-a_2\,b_3+a_2\,b_5+2\,a_2\,b_3\,T_1-a_4\,b_3\,T_1+c_5\,T_1-c_{14}\,T_1-a_2\,b_3\,T_1^2+a_4\,b_3\,T_1^2-c_5\,T_1^2-$$
$$a_4\,b_3\,T_2-a_2\,b_5\,T_2+a_4\,b_5\,T_2+3\,a_4\,b_3\,T_1\,T_2-a_4\,b_5\,T_1\,T_2-c_5\,T_1\,T_2+c_{14}\,T_1\,T_2-2\,a_4\,b_3\,T_1^2\,T_2+$$
$$c_5\,T_1^2\,T_2+c_{14}\,T_1^2\,T_2-a_4\,b_5\,T_2^2+a_4\,b_5\,T_1\,T_2^2-c_{14}\,T_1^2\,T_2^2\big)\,/\,((-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2))\Big),$$

$$f_{39}\to-\Big(\big(a_2\,b_5+a_2\,b_3\,T_1-a_4\,b_3\,T_1-a_2\,b_5\,T_1+a_4\,b_5\,T_1+c_6\,T_1-c_{15}\,T_1-a_2\,b_3\,T_1^2+a_4\,b_3\,T_1^2-c_6\,T_1^2+$$
$$c_{15}\,T_1^2-a_2\,b_5\,T_2+a_4\,b_5\,T_2+a_4\,b_3\,T_1\,T_2+a_2\,b_5\,T_1\,T_2-3\,a_4\,b_5\,T_1\,T_2-c_6\,T_1\,T_2-a_4\,b_3\,T_1^2\,T_2+c_6\,T_1^2$$
$$T_2+c_{15}\,T_1^2\,T_2-c_{15}\,T_1^3\,T_2-a_4\,b_5\,T_2^2+2\,a_4\,b_5\,T_1\,T_2^2\big)\,/\,((-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2))\Big),$$

$$f_{40}\to-\Big(\big(-a_2\,b_3+a_2\,b_3\,T_1-c_4\,T_1+c_4\,T_1^2-a_4\,b_3\,T_2-a_2\,b_5\,T_2+2\,a_4\,b_3\,T_1\,T_2+a_2\,b_5\,T_1\,T_2-a_4\,b_5\,T_1\,T_2-$$
$$a_4\,b_3\,T_1^2\,T_2+c_4\,T_1^2\,T_2-c_4\,T_1^3\,T_2-a_4\,b_5\,T_2^2+2\,a_4\,b_5\,T_1\,T_2^2\big)\,/\,((-1+T_1)\,T_1\,T_2\,(-1+T_1\,T_2))\Big),$$

$$f_{41}\to-\Big(\big(a_2\,b_3-3\,a_2\,b_3\,T_1+a_4\,b_3\,T_1-c_5\,T_1+2\,a_2\,b_3\,T_1^2-a_4\,b_3\,T_1^2+c_5\,T_1^2+a_4\,b_3\,T_2-4\,a_4\,b_3\,T_1\,T_2+$$
$$a_4\,b_5\,T_1\,T_2+c_5\,T_1\,T_2+3\,a_4\,b_3\,T_1^2\,T_2-c_5\,T_1^2\,T_2-a_4\,b_5\,T_1\,T_2^2\big)\,/\,\big((-1+T_1)\,T_1^2\,(-1+T_2)\,T_2\big)\Big),$$

$$f_{42}\to-\Big(\big(-a_2\,b_5-a_2\,b_3\,T_1+a_2\,b_5\,T_1-a_4\,b_5\,T_1-c_6\,T_1+a_2\,b_3\,T_1^2+c_6\,T_1^2+a_2\,b_5\,T_2-$$
$$a_4\,b_5\,T_2-a_4\,b_3\,T_1\,T_2-a_2\,b_5\,T_1\,T_2+4\,a_4\,b_5\,T_1\,T_2+c_6\,T_1\,T_2+a_4\,b_3\,T_1^2\,T_2-$$
$$c_6\,T_1^2\,T_2+a_4\,b_5\,T_2^2-3\,a_4\,b_5\,T_1\,T_2^2\big)\,/\,\big((-1+T_1)\,T_1^2\,(-1+T_2)\,T_2\big)\Big),$$

$$f_{43}\to0,\ f_{44}\to0,\ f_{45}\to0,\ f_{46}\to0,\ f_{47}\to0,\ f_{48}\to0,\ f_{49}\to-c_{49},$$

$$f_{50}\to-c_{50},$$

$$f_{51} \to -c_{51},$$

$$f_{52} \to -\frac{c_{49}+c_{52}-c_{49}\,T_1^2}{T_1^2},$$

$$f_{53} \to -\frac{c_{50}+c_{53}-c_{50}\,T_2^2}{T_2^2},$$

$$f_{54} \to -\frac{1}{T_1^3\,T_2^2}\left(-a_2\,b_5-a_2\,b_3\,T_1+a_4\,b_3\,T_1+c_{51}\,T_1+c_{54}\,T_1-a_4\,b_5\,T_2-\right.$$
$$\left.a_4\,b_3\,T_1\,T_2-a_2\,b_5\,T_1\,T_2+a_4\,b_5\,T_1\,T_2+a_4\,b_3\,T_1^2\,T_2-a_4\,b_5\,T_1\,T_2^2-c_{51}\,T_1^3\,T_2^2\right),$$

$$f_{55} \to 0,\ f_{56} \to 0,\ f_{57} \to 0,\ f_{58} \to -\frac{-c_{49}-c_{52}}{T_1},\ f_{59} \to -\frac{-c_{50}-c_{53}}{T_2},$$

$$f_{60} \to -\frac{1}{T_1^2\,T_2\,(-1+T_1\,T_2)}\left(-a_2\,b_5-a_2\,b_3\,T_1+a_4\,b_3\,T_1+c_{51}\,T_1+\right.$$
$$\left.\left.c_{54}\,T_1-a_4\,b_5\,T_2-a_4\,b_3\,T_1\,T_2+a_4\,b_5\,T_1\,T_2-c_{51}\,T_1^2\,T_2-c_{54}\,T_1^2\,T_2\right)\right\}\}$$

In[•]:=
```
Length[sol]
```

Out[•]=

60

In[•]:=
```
sol /. (v_ → val_) :> (v = CF[val]);
```

In[•]:=
```
CF[r42[-1, i, j]]
```

Out[•]=

$$-c_{49}\,p_{1,i}\,x_{1,i}+\frac{\left(-c_{49}-c_{52}+c_{49}\,T_1^2\right)\,p_{1,j}\,x_{1,i}}{T_1^2}+$$
$$\frac{(c_{49}+c_{52})\,p_{1,j}\,x_{1,j}}{T_1}-c_{50}\,p_{2,i}\,x_{2,i}+\frac{\left(-c_{50}-c_{53}+c_{50}\,T_2^2\right)\,p_{2,j}\,x_{2,i}}{T_2^2}-$$
$$\left(\left(a_2\,b_3-a_2\,b_5-a_2\,b_3\,T_1+c_4\,T_1-c_4\,T_1^2+a_4\,b_3\,T_2+a_2\,b_5\,T_2-a_4\,b_5\,T_2-2\,a_4\,b_3\,T_1\,T_2+a_4\,b_5\,T_1\,T_2+a_4\,b_3\,T_1^2\right.\right.$$
$$\left.\left.T_2-c_4\,T_1^2\,T_2+c_4\,T_1^3\,T_2+a_4\,b_5\,T_2^2-a_4\,b_5\,T_1\,T_2^2\right)\,p_{1,j}\,p_{2,i}\,x_{1,i}\,x_{2,i}\right)\Big/\left(T_1^2\,(-1+T_2)\,(-1+T_1\,T_2)\right)-$$
$$\left(\left(-a_2\,b_3+a_4\,b_3+a_2\,b_5-a_4\,b_5+c_{13}+a_2\,b_3\,T_1-a_4\,b_3\,T_1-a_4\,b_3\,T_2-a_2\,b_5\,T_2+2\,a_4\,b_5\,T_2-c_{13}\,T_2+\right.\right.$$
$$\left.\left.a_4\,b_3\,T_1\,T_2-c_{13}\,T_1\,T_2-a_4\,b_5\,T_2^2+c_{13}\,T_1\,T_2^2\right)\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,i}\right)\Big/\left((-1+T_1)\,T_2\,(-1+T_1\,T_2)\right)+$$
$$\frac{1}{T_1^2\,T_2\,(-1+T_1\,T_2)}\left(a_2\,b_3-a_2\,b_3\,T_1+a_2\,b_5\,T_1-a_4\,b_5\,T_1+c_4\,T_1+c_{13}\,T_1+a_2\,b_3\,T_1^2-a_4\,b_3\,T_1^2-\right.$$
$$c_4\,T_1^2+a_4\,b_3\,T_2+a_2\,b_5\,T_2-2\,a_4\,b_3\,T_1\,T_2-a_2\,b_5\,T_1\,T_2+2\,a_4\,b_5\,T_1\,T_2-c_{13}\,T_1\,T_2+2\,a_4\,b_3\,T_1^2\,T_2-$$
$$\left.c_4\,T_1^2\,T_2-c_{13}\,T_1^2\,T_2+c_4\,T_1^3\,T_2+a_4\,b_5\,T_2^2-2\,a_4\,b_5\,T_1\,T_2^2+c_{13}\,T_1^2\,T_2^2\right)\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,i}+$$
$$\left(\left(-a_2\,b_3+a_2\,b_5+a_2\,b_3\,T_1-c_4\,T_1+c_{13}\,T_1+c_4\,T_1^2-a_4\,b_3\,T_2-a_2\,b_5\,T_2+a_4\,b_5\,T_2+2\,a_4\,b_3\,T_1\,T_2-\right.\right.$$
$$\left.\left.a_4\,b_5\,T_1\,T_2-c_{13}\,T_1\,T_2-a_4\,b_3\,T_1^2\,T_2+c_4\,T_1^2\,T_2-c_{13}\,T_1^2\,T_2-c_4\,T_1^3\,T_2-a_4\,b_5\,T_2^2+a_4\,b_5\,T_1\,T_2^2+c_{13}\,T_1^2\,T_2^2\right)\right.$$
$$\left.p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,i}\right)\Big/\left((-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2)\right)+$$
$$\left(\left(a_2\,b_3-a_2\,b_3\,T_1+c_4\,T_1-c_4\,T_1^2+a_4\,b_3\,T_2+a_2\,b_5\,T_2-2\,a_4\,b_3\,T_1\,T_2-a_2\,b_5\,T_1\,T_2+a_4\,b_5\,T_1\,T_2+\right.\right.$$
$$\left.\left.a_4\,b_3\,T_1^2\,T_2-c_4\,T_1^2\,T_2+c_4\,T_1^3\,T_2+a_4\,b_5\,T_2^2-2\,a_4\,b_5\,T_1\,T_2^2\right)\,p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,i}\right)\Big/$$
$$\left((-1+T_1)\,T_1\,T_2\,(-1+T_1\,T_2)\right)+\frac{(c_{50}+c_{53})\,p_{2,j}\,x_{2,j}}{T_2}-$$
$$\left(\left(-a_2\,b_3+a_4\,b_3+a_2\,b_5-a_4\,b_5-c_4+c_{13}+a_2\,b_3\,T_1-a_4\,b_3\,T_1+c_4\,T_1-a_4\,b_3\,T_2-a_2\,b_5\,T_2+\right.\right.$$

$$2\,a_4\,b_5\,T_2 - c_{13}\,T_2 + a_4\,b_3\,T_1\,T_2 + c_4\,T_1\,T_2 - c_{13}\,T_1\,T_2 - c_4\,T_1^2\,T_2 - a_4\,b_5\,T_2^2 + c_{13}\,T_1\,T_2^2\big)$$
$$p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,j}\big) \Big/ \big((-1+T_1)\,(-1+T_2)\,(-1+T_1\,T_2)\big) +$$
$$\big(\big(-a_2\,b_3 + a_2\,b_5 - a_4\,b_5 + c_{13} + a_2\,b_3\,T_1 - a_4\,b_3\,T_1 - a_4\,b_3\,T_2 - a_2\,b_5\,T_2 + 2\,a_4\,b_5\,T_2 - c_{13}\,T_2 +$$
$$2\,a_4\,b_3\,T_1\,T_2 - c_{13}\,T_1\,T_2 - a_4\,b_5\,T_2^2 + c_{13}\,T_1\,T_2^2\big)\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,j}\big) \Big/ \big(T_1\,(-1+T_2)\,(-1+T_1\,T_2)\big) -$$
$$c_{51}\,p_{3,i}\,x_{3,i} + \frac{1}{T_1^3\,T_2^2}\,\big(a_2\,b_5 + a_2\,b_3\,T_1 - a_4\,b_3\,T_1 - c_{51}\,T_1 - c_{54}\,T_1 + a_4\,b_5\,T_2 + a_4\,b_3\,T_1\,T_2 +$$
$$a_2\,b_5\,T_1\,T_2 - a_4\,b_5\,T_1\,T_2 - a_4\,b_3\,T_1^2\,T_2 + a_4\,b_5\,T_1\,T_2^2 + c_{51}\,T_1^3\,T_2^2\big)\,p_{3,j}\,x_{3,i} +$$
$$\big(\big(a_2\,b_3 - a_2\,b_5 - 2\,a_2\,b_3\,T_1 + a_4\,b_3\,T_1 - c_5\,T_1 + a_2\,b_3\,T_1^2 - a_4\,b_3\,T_1^2 + c_5\,T_1^2 + a_4\,b_3\,T_2 + a_2\,b_5\,T_2 -$$
$$a_4\,b_5\,T_2 - 3\,a_4\,b_3\,T_1\,T_2 + a_4\,b_5\,T_1\,T_2 + c_5\,T_1\,T_2 + 2\,a_4\,b_3\,T_1^2\,T_2 - c_5\,T_1^2\,T_2 + a_4\,b_5\,T_2^2 - a_4\,b_5\,T_1\,T_2^2\big)$$
$$p_{1,j}\,p_{3,i}\,x_{1,i}\,x_{3,i}\big) \Big/ \big(T_1^2\,(-1+T_2)\,(-1+T_1\,T_2)\big) +$$
$$\big(\big(a_2\,b_3 - 2\,a_2\,b_3\,T_1 + a_4\,b_3\,T_1 + a_2\,b_5\,T_1 - a_4\,b_5\,T_1 - c_{14}\,T_1 + a_2\,b_3\,T_1^2 - a_4\,b_3\,T_1^2 + a_4\,b_3\,T_2 - 3\,a_4\,b_3\,T_1\,T_2 -$$
$$a_2\,b_5\,T_1\,T_2 + 2\,a_4\,b_5\,T_1\,T_2 + c_{14}\,T_1\,T_2 + 2\,a_4\,b_3\,T_1^2\,T_2 + c_{14}\,T_1^2\,T_2 - a_4\,b_5\,T_1\,T_2^2 - c_{14}\,T_1^2\,T_2^2\big)$$
$$p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,i}\big) \Big/ \big((-1+T_1)\,T_1^2\,(-1+T_2)\,T_2\big) - \frac{1}{T_1^3\,(-1+T_2)\,T_2}$$
$$\big(a_2\,b_3 - 3\,a_2\,b_3\,T_1 + a_4\,b_3\,T_1 + a_2\,b_5\,T_1 - a_4\,b_5\,T_1 - c_5\,T_1 - c_{14}\,T_1 + 2\,a_2\,b_3\,T_1^2 - 2\,a_4\,b_3\,T_1^2 +$$
$$c_5\,T_1^2 + a_4\,b_3\,T_2 - 4\,a_4\,b_3\,T_1\,T_2 - a_2\,b_5\,T_1\,T_2 + 2\,a_4\,b_5\,T_1\,T_2 + c_5\,T_1\,T_2 + c_{14}\,T_1\,T_2 +$$
$$4\,a_4\,b_3\,T_1^2\,T_2 - c_5\,T_1^2\,T_2 + c_{14}\,T_1^2\,T_2 - a_4\,b_5\,T_1\,T_2^2 - c_{14}\,T_1^2\,T_2^2\big)\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,i} +$$
$$\big(\big(a_2\,b_3 - a_2\,b_5 - 2\,a_2\,b_3\,T_1 + a_4\,b_3\,T_1 - c_5\,T_1 + c_{14}\,T_1 + a_2\,b_3\,T_1^2 - a_4\,b_3\,T_1^2 + c_5\,T_1^2 + a_4\,b_3\,T_2 +$$
$$a_2\,b_5\,T_2 - a_4\,b_5\,T_2 - 3\,a_4\,b_3\,T_1\,T_2 + a_4\,b_5\,T_1\,T_2 + c_5\,T_1\,T_2 - c_{14}\,T_1\,T_2 + 2\,a_4\,b_3\,T_1^2\,T_2 -$$
$$c_5\,T_1^2\,T_2 - c_{14}\,T_1^2\,T_2 + a_4\,b_5\,T_2^2 - a_4\,b_5\,T_1\,T_2^2 + c_{14}\,T_1^2\,T_2^2\big)\,p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,i}\big) \Big/$$
$$\big((-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2)\big) + \frac{b_3\,(a_2 + a_4\,T_2)\,p_{1,i}\,p_{3,j}\,x_{1,j}\,x_{3,i}}{T_1\,(-1+T_2)\,T_2} -$$
$$\big(\big(a_2\,b_3 - 3\,a_2\,b_3\,T_1 + a_4\,b_3\,T_1 - c_5\,T_1 + 2\,a_2\,b_3\,T_1^2 - a_4\,b_3\,T_1^2 + c_5\,T_1^2 + a_4\,b_3\,T_2 - 4\,a_4\,b_3\,T_1\,T_2 + a_4\,b_5\,T_1\,T_2 +$$
$$c_5\,T_1\,T_2 + 3\,a_4\,b_3\,T_1^2\,T_2 - c_5\,T_1^2\,T_2 - a_4\,b_5\,T_1\,T_2^2\big)\,p_{1,j}\,p_{3,j}\,x_{1,j}\,x_{3,i}\big) \Big/ \big((-1+T_1)\,T_1^2\,(-1+T_2)\,T_2\big) +$$
$$\big(\big(-a_2\,b_5 - a_2\,b_3\,T_1 + a_4\,b_3\,T_1 + a_2\,b_5\,T_1 - a_4\,b_5\,T_1 - c_6\,T_1 + a_2\,b_3\,T_1^2 - a_4\,b_3\,T_1^2 + c_6\,T_1^2 + a_2\,b_5\,T_2 - a_4\,b_5\,T_2 -$$
$$a_4\,b_3\,T_1\,T_2 - a_2\,b_5\,T_1\,T_2 + 3\,a_4\,b_5\,T_1\,T_2 + c_6\,T_1\,T_2 + a_4\,b_3\,T_1^2\,T_2 - c_6\,T_1^2\,T_2 + a_4\,b_5\,T_2^2 - 2\,a_4\,b_5\,T_1\,T_2^2\big)$$
$$p_{2,j}\,p_{3,i}\,x_{2,i}\,x_{3,i}\big) \Big/ \big((-1+T_1)\,T_1\,T_2\,(-1+T_1\,T_2)\big) +$$
$$\big(\big(a_2\,b_3 - a_2\,b_5 - a_2\,b_3\,T_1 + a_2\,b_5\,T_1 - a_4\,b_5\,T_1 - c_{15}\,T_1 + c_{15}\,T_1^2 + a_4\,b_3\,T_2 + a_2\,b_5\,T_2 - a_4\,b_5\,T_2 -$$
$$2\,a_4\,b_3\,T_1\,T_2 - a_2\,b_5\,T_1\,T_2 + 3\,a_4\,b_5\,T_1\,T_2 + a_4\,b_3\,T_1^2\,T_2 + c_{15}\,T_1^2\,T_2 - c_{15}\,T_1^3\,T_2 + a_4\,b_5\,T_2^2 - 2\,a_4\,b_5\,T_1\,T_2^2\big)$$
$$p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,i}\big) \Big/ \big((-1+T_1)\,T_1^2\,(-1+T_2)\,T_2\big) + \frac{1}{(-1+T_1)\,T_1^2\,T_2^2}$$
$$\big(-a_2\,b_3 + a_2\,b_5 + a_2\,b_3\,T_1 - a_2\,b_5\,T_1 + a_4\,b_5\,T_1 + c_6\,T_1 + c_{15}\,T_1 - c_6\,T_1^2 - c_{15}\,T_1^2 - a_4\,b_3\,T_2 -$$
$$2\,a_2\,b_5\,T_2 + a_4\,b_5\,T_2 + 2\,a_4\,b_3\,T_1\,T_2 + 2\,a_2\,b_5\,T_1\,T_2 - 4\,a_4\,b_5\,T_1\,T_2 - c_6\,T_1\,T_2 - a_4\,b_3\,T_1^2\,T_2 +$$
$$c_6\,T_1^2\,T_2 - c_{15}\,T_1^2\,T_2 + c_{15}\,T_1^3\,T_2 - 2\,a_4\,b_5\,T_2^2 + 4\,a_4\,b_5\,T_1\,T_2^2\big)\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,i} +$$
$$\big(\big(-a_2\,b_5 - a_2\,b_3\,T_1 + a_4\,b_3\,T_1 + a_2\,b_5\,T_1 - a_4\,b_5\,T_1 - c_6\,T_1 + c_{15}\,T_1 + a_2\,b_3\,T_1^2 - a_4\,b_3\,T_1^2 + c_6\,T_1^2 -$$
$$c_{15}\,T_1^2 + a_2\,b_5\,T_2 - a_4\,b_5\,T_2 - a_4\,b_3\,T_1\,T_2 - a_2\,b_5\,T_1\,T_2 + 3\,a_4\,b_5\,T_1\,T_2 + c_6\,T_1\,T_2 +$$
$$a_4\,b_3\,T_1^2\,T_2 - c_6\,T_1^2\,T_2 - c_{15}\,T_1^2\,T_2 + c_{15}\,T_1^3\,T_2 + a_4\,b_5\,T_2^2 - 2\,a_4\,b_5\,T_1\,T_2^2\big)\,p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,i}\big) \Big/$$
$$\big((-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2)\big) - \frac{a_4\,b_5\,p_{2,i}\,p_{3,j}\,x_{2,j}\,x_{3,i}}{(-1+T_1)\,T_1} -$$
$$\big(\big(-a_2\,b_5 - a_2\,b_3\,T_1 + a_2\,b_5\,T_1 - a_4\,b_5\,T_1 - c_6\,T_1 + a_2\,b_3\,T_1^2 + c_6\,T_1^2 + a_2\,b_5\,T_2 - a_4\,b_5\,T_2 - a_4\,b_3\,T_1\,T_2 -$$
$$a_2\,b_5\,T_1\,T_2 + 4\,a_4\,b_5\,T_1\,T_2 + c_6\,T_1\,T_2 + a_4\,b_3\,T_1^2\,T_2 - c_6\,T_1^2\,T_2 + a_4\,b_5\,T_2^2 - 3\,a_4\,b_5\,T_1\,T_2^2\big)$$

$$p_{2,j}\,p_{3,j}\,x_{2,j}\,x_{3,i}\Big)\Big/\Big((-1+T_1)\,T_1^2\,(-1+T_2)\,T_2\Big)+\frac{1}{T_1^2\,T_2\,(-1+T_1\,T_2)}$$
$$\Big(a_2\,b_5+a_2\,b_3\,T_1-a_4\,b_3\,T_1-c_{51}\,T_1-c_{54}\,T_1+a_4\,b_5\,T_2+a_4\,b_3\,T_1\,T_2-a_4\,b_5\,T_1\,T_2+c_{51}\,T_1^2\,T_2+c_{54}\,T_1^2\,T_2\Big)$$
$$p_{3,j}\,x_{3,j}-$$
$$\frac{(a_2\,b_5-a_4\,b_5+c_5-c_{14}-c_5\,T_1+a_4\,b_5\,T_2+c_{14}\,T_1\,T_2)\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,j}}{(-1+T_1)\,(-1+T_1\,T_2)}-$$
$$\Big(\big(-a_2\,b_3+a_2\,b_5-a_4\,b_5-c_{14}+a_2\,b_3\,T_1-a_4\,b_3\,T_1-a_4\,b_3\,T_2-a_2\,b_5\,T_2+2\,a_4\,b_5\,T_2+c_{14}\,T_2+2\,a_4\,b_3\,T_1\,T_2+$$
$$c_{14}\,T_1\,T_2-a_4\,b_5\,T_2^2-c_{14}\,T_1\,T_2^2\big)\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,j}\Big)\Big/(T_1\,(-1+T_2)\,(-1+T_1\,T_2))+$$
$$\Big(\big(-a_2\,b_3-c_6\,T_1+c_{15}\,T_1-a_4\,b_3\,T_2+a_4\,b_3\,T_1\,T_2+c_6\,T_1\,T_2-c_{15}\,T_1^2\,T_2\big)\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,j}\Big)\Big/$$
$$(T_1\,(-1+T_2)\,(-1+T_1\,T_2))-$$
$$\Big(\big(a_2\,b_3-a_2\,b_3\,T_1-c_{15}\,T_1+c_{15}\,T_1^2+a_4\,b_3\,T_2+a_2\,b_5\,T_2-2\,a_4\,b_3\,T_1\,T_2-a_2\,b_5\,T_1\,T_2+a_4\,b_5\,T_1\,T_2+a_4\,b_3\,T_1^2\,T_2+$$
$$c_{15}\,T_1^2\,T_2-c_{15}\,T_1^3\,T_2+a_4\,b_5\,T_2^2-2\,a_4\,b_5\,T_1\,T_2^2\big)\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,j}\Big)\Big/((-1+T_1)\,T_1\,T_2\,(-1+T_1\,T_2))$$

In[•]:=
```
CF[r42[-1, i, j] /.
  {a4 → 0, b3 → 0, b5 → (T1 - 1) (T2 - 1) (T3 - 1) a2^-1, c4|5|6|13|14|15|49|50|51|52|53|54 → 0}]
```

Out[•]=

$$-\frac{(-1+T_1)\,(-1+T_2)\,p_{1,j}\,p_{2,i}\,x_{1,i}\,x_{2,i}}{T_1^2}+\frac{(-1+T_2)^2\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,i}}{T_2}-$$
$$\frac{(-1+T_1)\,(-1+T_2)\,(-T_1-T_2+T_1\,T_2)\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,i}}{T_1^2\,T_2}-\frac{(-1+T_2)\,p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,i}}{T_1}-$$
$$\frac{(-1+T_1)\,(-1+T_2)\,p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,i}}{T_1}+(-1+T_2)\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,j}-$$
$$\frac{(-1+T_1)\,(-1+T_2)\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,j}}{T_1}+\frac{(-1+T_1)\,(-1+T_2)\,(-1+T_1\,T_2)\,(1+T_1\,T_2)\,p_{3,j}\,x_{3,i}}{T_1^3\,T_2^2}+$$
$$\frac{(-1+T_1)\,(-1+T_2)\,p_{1,j}\,p_{3,i}\,x_{1,i}\,x_{3,i}}{T_1^2}-\frac{(-1+T_2)\,(-1+T_1\,T_2)\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,i}}{T_1\,T_2}+$$
$$\frac{(-1+T_1)\,(-1+T_2)\,(-1+T_1\,T_2)\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,i}}{T_1^2\,T_2}+\frac{(-1+T_2)\,p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,i}}{T_1}-$$
$$\frac{(-1+T_1)\,(-1+T_2)^2\,p_{2,j}\,p_{3,i}\,x_{2,i}\,x_{3,i}}{T_1\,T_2}-\frac{(-1+T_1)\,(-1+T_2)\,(-1+T_1\,T_2)\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,i}}{T_1^2\,T_2}+$$
$$\frac{(-1+T_1)\,(-1+T_2)\,(-1+2\,T_2)\,(-1+T_1\,T_2)\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,i}}{T_1^2\,T_2^2}-$$
$$\frac{(-1+T_1)\,(-1+T_2)\,p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,i}}{T_1}+\frac{(-1+T_1)\,(-1+T_2)\,(-1+T_1\,T_2)\,p_{2,j}\,p_{3,j}\,x_{2,j}\,x_{3,i}}{T_1^2\,T_2}+$$
$$\frac{(-1+T_1)\,(-1+T_2)\,p_{3,j}\,x_{3,j}}{T_1^2\,T_2}+(1-T_2)\,p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,j}+$$
$$\frac{(-1+T_1)\,(-1+T_2)\,p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,j}}{T_1}+\frac{(-1+T_1)\,(-1+T_2)\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,j}}{T_1}$$