

```
In[1]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank\\Rolands_A2"];
Once[<< KnotTheory`];
<< ./Rot.m
<< ./FormalGaussianIntegration.m
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

Loading Rot.m from <http://drorbn.net/AP/Projects/HigherRank> to compute rotation numbers.

Hidden code:

(*General input from A21rev.nb, hard coded!, we set α_4 to z to avoid a bug that mixes up the dummy variable α in an unexplained way : (*)

```
r0[1, i_, j_] := -p3,j x1,i x2,i + p3,j x1,i x2,j;
r0[-1, i_, j_] :=  $\frac{p3,j x1,i x2,i}{T} - \frac{p3,j x1,i x2,j}{T}$ ;
r1[1, i_, j_] := - $\frac{3}{2}$  + c15 p1,i x1,i + c36 p1,j x1,i + c4 p1,i p1,j x1,i2 +
 $\frac{1}{2}$  (2 c4 - T1) (-1 + T1) p1,j x1,i2 -  $\frac{(c15 + c36) p1,j x1,j}{T1}$  - p1,i p1,j x1,i x1,j +
 $\frac{1}{2}$  (1 - 2 c4 + T1) p1,j x1,i x1,j + c56 p2,i x2,i + c69 p2,j x2,i + c28 p1,j p2,i x1,i x2,i -
 $\frac{1}{2}$  ((-c10 c22 - c22 c28 + c10 c22 T1 + c22 T1 - c22 c81 T1 - c22 c86 T1 - c22 c56 T12 - c22 c69 T12 + 2 c10 c22 T2 +
2 c22 c28 T2 - 2 c10 c22 T1 T2 - c15 c22 T1 T2 - 2 c22 T1 T2 - c22 c36 T1 T2 + 2 c22 c81 T1 T2 +
2 c22 c86 T1 T2 - 3 c22 T1 T2 + c15 c22 T1 T2 + 3 c22 c56 T1 T2 + 2 c22 c69 T1 T2 + c22 c81 T1 T2 -
c10 c22 T22 - c22 c28 T22 + c10 c22 T1 T22 + 2 c15 c22 T1 T22 + c22 T1 T22 + 2 c22 c36 T1 T22 - c22 c81 T1 T22 -
c22 c86 T1 T22 + 6 c22 T1 T22 - 2 c15 c22 T1 T22 - 3 c22 c56 T1 T22 - c22 c69 T1 T22 - 2 c22 c81 T1 T22 -
c15 c22 T1 T23 - c22 c36 T1 T23 - 3 c22 T1 T23 + c15 c22 T1 T23 + c22 c56 T1 T23 + c22 c81 T1 T23 + c22 alpha_4 -
c81 alpha_4 - c86 alpha_4 + c22 T1 alpha_4 - c56 T1 alpha_4 - c69 T1 alpha_4 - c10 T2 alpha_4 - c15 T2 alpha_4 - c22 T2 alpha_4 - c28 T2 alpha_4 -
c36 T2 alpha_4 + c81 T2 alpha_4 + c86 T2 alpha_4 - 3 T1 T2 alpha_4 + c10 T1 T2 alpha_4 + c15 T1 T2 alpha_4 - c22 T1 T2 alpha_4 +
2 c56 T1 T2 alpha_4 + c69 T1 T2 alpha_4 + c81 T1 T2 alpha_4 + c10 T22 alpha_4 + c15 T22 alpha_4 + c28 T22 alpha_4 + c36 T22 alpha_4 +
3 T1 T22 alpha_4 - c10 T1 T22 alpha_4 - c15 T1 T22 alpha_4 - c56 T1 T22 alpha_4 - c81 T1 T22 alpha_4 + alpha_42) p1,i p2,j x1,i x2,i) /
 $\frac{1}{(-1 + T1) (-c22 + c22 T2 - T2 alpha_4)} - \frac{-c22 + c22 T2 - T2 alpha_4}{-c22 + c22 T2 - T2 alpha_4}$ 
(-c10 c22 - 2 c22 c28 + c10 c22 T1 + 2 c22 T1 - c22 c81 T1 - c22 c86 T1 - c22 c56 T12 - c22 c69 T12 +
2 c10 c22 T2 + 4 c22 c28 T2 - 2 c10 c22 T1 T2 - c15 c22 T1 T2 - 4 c22 T1 T2 - c22 c36 T1 T2 +
3 c22 c81 T1 T2 + 3 c22 c86 T1 T2 - 3 c22 T1 T2 + c15 c22 T1 T2 + 4 c22 c56 T1 T2 + 3 c22 c69 T1 T2 +
c22 c81 T1 T2 - c10 c22 T22 - 2 c22 c28 T22 + c10 c22 T1 T22 + 3 c15 c22 T1 T22 + 2 c22 T1 T22 +
3 c22 c36 T1 T22 - 2 c22 c81 T1 T22 - 2 c22 c86 T1 T22 + 9 c22 T1 T22 - 3 c15 c22 T1 T22 -
5 c22 c56 T1 T22 - 2 c22 c69 T1 T22 - 3 c22 c81 T1 T22 - 2 c15 c22 T1 T23 - 2 c22 c36 T1 T23 -
6 c22 T1 T23 + 2 c15 c22 T1 T23 + 2 c22 c56 T1 T23 + 2 c22 c81 T1 T23 + c22 alpha_4 - c81 alpha_4 - c86 alpha_4 +
c22 T1 alpha_4 - c56 T1 alpha_4 - c69 T1 alpha_4 - c10 T2 alpha_4 - c15 T2 alpha_4 - c22 T2 alpha_4 - 2 c28 T2 alpha_4 - c36 T2 alpha_4 +
2 c81 T2 alpha_4 + 2 c86 T2 alpha_4 - 3 T1 T2 alpha_4 + c10 T1 T2 alpha_4 + c15 T1 T2 alpha_4 - c22 T1 T2 alpha_4 + 3 c56 T1 T2 alpha_4 +
2 c69 T1 T2 alpha_4 + c81 T1 T2 alpha_4 + c10 T22 alpha_4 + 2 c15 T22 alpha_4 + 2 c28 T22 alpha_4 + 2 c36 T22 alpha_4 + 6 T1 T22 alpha_4 -
```

$$\begin{aligned}
& \left(\left(\mathbf{c}_{10} \mathbf{T}_1 \mathbf{T}_2^2 \alpha_4 - 2 \mathbf{c}_{15} \mathbf{T}_1 \mathbf{T}_2^2 \alpha_4 - 2 \mathbf{c}_{56} \mathbf{T}_1 \mathbf{T}_2^2 \alpha_4 - 2 \mathbf{c}_{81} \mathbf{T}_1 \mathbf{T}_2^2 \alpha_4 + \frac{\alpha_4^2}{4} - \mathbf{T}_2 \alpha_4^2 \right) \mathbf{p}_{1,j} \mathbf{p}_{2,j} \mathbf{x}_{1,i} \mathbf{x}_{2,i} - \right. \\
& \left(\left(\mathbf{c}_{10} \mathbf{c}_{22} + \mathbf{c}_{22}^2 - \mathbf{c}_{22} \mathbf{c}_{81} - \mathbf{c}_{22} \mathbf{c}_{86} - \mathbf{c}_{10} \mathbf{c}_{22} \mathbf{T}_1 - \mathbf{c}_{22}^2 \mathbf{T}_1 - \mathbf{c}_{22} \mathbf{c}_{56} \mathbf{T}_1 - \mathbf{c}_{22} \mathbf{c}_{69} \mathbf{T}_1 + \mathbf{c}_{22} \mathbf{c}_{81} \mathbf{T}_1 + \right. \right. \\
& \left. \left. \mathbf{c}_{22} \mathbf{c}_{86} \mathbf{T}_1 + \mathbf{c}_{22} \mathbf{c}_{56} \mathbf{T}_1^2 + \mathbf{c}_{22} \mathbf{c}_{69} \mathbf{T}_1^2 - \mathbf{c}_{10} \mathbf{c}_{22} \mathbf{T}_2 - \mathbf{c}_{15} \mathbf{c}_{22} \mathbf{T}_2 - \mathbf{c}_{22}^2 \mathbf{T}_2 - \mathbf{c}_{22} \mathbf{c}_{36} \mathbf{T}_2 + \mathbf{c}_{22} \mathbf{c}_{81} \mathbf{T}_2 + \right. \right. \\
& \left. \left. \mathbf{c}_{22} \mathbf{c}_{86} \mathbf{T}_2 - 3 \mathbf{c}_{22} \mathbf{T}_1 \mathbf{T}_2 + \mathbf{c}_{10} \mathbf{c}_{22} \mathbf{T}_1 \mathbf{T}_2 + 2 \mathbf{c}_{15} \mathbf{c}_{22} \mathbf{T}_1 \mathbf{T}_2 + \mathbf{c}_{22}^2 \mathbf{T}_1 \mathbf{T}_2 + \mathbf{c}_{22} \mathbf{c}_{36} \mathbf{T}_1 \mathbf{T}_2 + \right. \right. \\
& \left. \left. 2 \mathbf{c}_{22} \mathbf{c}_{56} \mathbf{T}_1 \mathbf{T}_2 + \mathbf{c}_{22} \mathbf{c}_{69} \mathbf{T}_1 \mathbf{T}_2 - \mathbf{c}_{22} \mathbf{c}_{86} \mathbf{T}_1 \mathbf{T}_2 + 3 \mathbf{c}_{22} \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{c}_{15} \mathbf{c}_{22} \mathbf{T}_1^2 \mathbf{T}_2 - 2 \mathbf{c}_{22} \mathbf{c}_{56} \mathbf{T}_1^2 \mathbf{T}_2 - \right. \right. \\
& \left. \left. \mathbf{c}_{22} \mathbf{c}_{69} \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{c}_{22} \mathbf{c}_{81} \mathbf{T}_1^2 \mathbf{T}_2 + \mathbf{c}_{15} \mathbf{c}_{22} \mathbf{T}_2^2 + \mathbf{c}_{22} \mathbf{c}_{36} \mathbf{T}_2^2 + 3 \mathbf{c}_{22} \mathbf{T}_1 \mathbf{T}_2^2 - 2 \mathbf{c}_{15} \mathbf{c}_{22} \mathbf{T}_1 \mathbf{T}_2^2 - \mathbf{c}_{22} \mathbf{c}_{36} \mathbf{T}_1 \mathbf{T}_2^2 - \right. \right. \\
& \left. \left. \mathbf{c}_{22} \mathbf{c}_{56} \mathbf{T}_1 \mathbf{T}_2^2 - \mathbf{c}_{22} \mathbf{c}_{81} \mathbf{T}_1 \mathbf{T}_2^2 - 3 \mathbf{c}_{22} \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{c}_{15} \mathbf{c}_{22} \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{c}_{22} \mathbf{c}_{56} \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{c}_{22} \mathbf{c}_{81} \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{c}_{81} \alpha_4 + \right. \right. \\
& \left. \left. \mathbf{c}_{86} \alpha_4 - \mathbf{c}_{22} \mathbf{T}_1 \alpha_4 + \mathbf{c}_{56} \mathbf{T}_1 \alpha_4 + \mathbf{c}_{69} \mathbf{T}_1 \alpha_4 + \mathbf{c}_{10} \mathbf{T}_2 \alpha_4 + \mathbf{c}_{15} \mathbf{T}_2 \alpha_4 + \mathbf{c}_{22} \mathbf{T}_2 \alpha_4 + \mathbf{c}_{36} \mathbf{T}_2 \alpha_4 - \right. \right. \\
& \left. \left. \mathbf{c}_{81} \mathbf{T}_2 \alpha_4 - \mathbf{c}_{86} \mathbf{T}_2 \alpha_4 + 3 \mathbf{T}_1 \mathbf{T}_2 \alpha_4 - \mathbf{c}_{10} \mathbf{T}_1 \mathbf{T}_2 \alpha_4 - \mathbf{c}_{15} \mathbf{T}_1 \mathbf{T}_2 \alpha_4 - 2 \mathbf{c}_{56} \mathbf{T}_1 \mathbf{T}_2 \alpha_4 - \mathbf{c}_{69} \mathbf{T}_1 \mathbf{T}_2 \alpha_4 - \right. \right. \\
& \left. \left. \mathbf{c}_{81} \mathbf{T}_1 \mathbf{T}_2 \alpha_4 - \mathbf{c}_{15} \mathbf{T}_2^2 \alpha_4 - \mathbf{c}_{36} \mathbf{T}_2^2 \alpha_4 - 3 \mathbf{T}_1 \mathbf{T}_2^2 \alpha_4 + \mathbf{c}_{15} \mathbf{T}_1 \mathbf{T}_2^2 \alpha_4 + \mathbf{c}_{56} \mathbf{T}_1 \mathbf{T}_2^2 \alpha_4 + \mathbf{c}_{81} \mathbf{T}_1 \mathbf{T}_2^2 \alpha_4 - \alpha_4^2 \right) \mathbf{p}_{1,j} \mathbf{p}_{2,i} \mathbf{x}_{1,j} \mathbf{x}_{2,i} \right) / ((-1 + \mathbf{T}_1) (-\mathbf{c}_{22} + \mathbf{c}_{22} \mathbf{T}_2 - \mathbf{T}_2 \alpha_4)) + \\
& ((-1 + \mathbf{T}_2) (\mathbf{c}_{22} \mathbf{c}_{28} + \mathbf{c}_{22} \mathbf{c}_{81} + \mathbf{c}_{22} \mathbf{c}_{86} - \mathbf{c}_{22}^2 \mathbf{T}_1 + \mathbf{c}_{22} \mathbf{c}_{56} \mathbf{T}_1 + \mathbf{c}_{22} \mathbf{c}_{69} \mathbf{T}_1 + \mathbf{c}_{15} \mathbf{c}_{22} \mathbf{T}_2 - \mathbf{c}_{22} \mathbf{c}_{28} \mathbf{T}_2 + \\
& \mathbf{c}_{22} \mathbf{c}_{36} \mathbf{T}_2 + 3 \mathbf{c}_{22} \mathbf{T}_1 \mathbf{T}_2 - \mathbf{c}_{15} \mathbf{c}_{22} \mathbf{T}_1 \mathbf{T}_2 + \mathbf{c}_{22}^2 \mathbf{T}_1 \mathbf{T}_2 - \mathbf{c}_{22} \mathbf{c}_{56} \mathbf{T}_1 \mathbf{T}_2 - 2 \mathbf{c}_{22} \mathbf{c}_{81} \mathbf{T}_1 \mathbf{T}_2 - \\
& \mathbf{c}_{22} \mathbf{c}_{86} \mathbf{T}_1 \mathbf{T}_2 - \mathbf{c}_{22} \mathbf{c}_{56} \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{c}_{22} \mathbf{c}_{69} \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{c}_{15} \mathbf{c}_{22} \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{c}_{22} \mathbf{c}_{36} \mathbf{T}_1 \mathbf{T}_2^2 - 3 \mathbf{c}_{22} \mathbf{T}_1^2 \mathbf{T}_2^2 + \\
& \mathbf{c}_{15} \mathbf{c}_{22} \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{c}_{22} \mathbf{c}_{56} \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{c}_{22} \mathbf{c}_{81} \mathbf{T}_1^2 \mathbf{T}_2^2 - \mathbf{c}_{22} \alpha_4 + \mathbf{c}_{28} \mathbf{T}_2 \alpha_4) \mathbf{p}_{1,j} \mathbf{p}_{2,i} \mathbf{x}_{1,j} \mathbf{x}_{2,i}) / \\
& ((-1 + \mathbf{T}_1) (-\mathbf{c}_{22} + \mathbf{c}_{22} \mathbf{T}_2 - \mathbf{T}_2 \alpha_4)) + \mathbf{c}_{49} \mathbf{p}_{2,i} \mathbf{p}_{2,j} \mathbf{x}_{2,i}^2 + \frac{1}{2} (2 \mathbf{c}_{49} - \mathbf{T}_2) \\
& (-1 + \mathbf{T}_2) \mathbf{p}_{2,j}^2 \mathbf{x}_{2,i}^2 - \\
& \frac{(\mathbf{c}_{56} + \mathbf{c}_{69}) \mathbf{p}_{2,j} \mathbf{x}_{2,j}}{\mathbf{T}_2} + \mathbf{c}_{10} \mathbf{p}_{1,i} \mathbf{p}_{2,j} \mathbf{x}_{1,i} \mathbf{x}_{2,j} + \\
& \frac{1}{-\mathbf{c}_{22} + \mathbf{c}_{22} \mathbf{T}_2 - \mathbf{T}_2 \alpha_4} \\
& \frac{(\mathbf{c}_{10} \mathbf{c}_{22} + \mathbf{c}_{22} \mathbf{c}_{28} - \mathbf{c}_{10} \mathbf{c}_{22} \mathbf{T}_1 - \mathbf{c}_{22}^2 \mathbf{T}_1 + \mathbf{c}_{22} \mathbf{c}_{81} \mathbf{T}_1 + \mathbf{c}_{22} \mathbf{c}_{86} \mathbf{T}_1 + \mathbf{c}_{22} \mathbf{c}_{56} \mathbf{T}_1^2 + \mathbf{c}_{22} \mathbf{c}_{69} \mathbf{T}_1^2 - \\
& \mathbf{c}_{10} \mathbf{c}_{22} \mathbf{T}_2 - \mathbf{c}_{22} \mathbf{c}_{28} \mathbf{T}_2 + \mathbf{c}_{10} \mathbf{c}_{22} \mathbf{T}_1 \mathbf{T}_2 + \mathbf{c}_{15} \mathbf{c}_{22} \mathbf{T}_1 \mathbf{T}_2 + \mathbf{c}_{22}^2 \mathbf{T}_1 \mathbf{T}_2 + \mathbf{c}_{22} \mathbf{c}_{36} \mathbf{T}_1 \mathbf{T}_2 - \mathbf{c}_{22} \mathbf{c}_{81} \mathbf{T}_1 \mathbf{T}_2 - \\
& \mathbf{c}_{22} \mathbf{c}_{86} \mathbf{T}_1 \mathbf{T}_2 + 3 \mathbf{c}_{22} \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{c}_{15} \mathbf{c}_{22} \mathbf{T}_1^2 \mathbf{T}_2 - 2 \mathbf{c}_{22} \mathbf{c}_{56} \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{c}_{22} \mathbf{c}_{69} \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{c}_{22} \mathbf{c}_{81} \mathbf{T}_1^2 \mathbf{T}_2 - \\
& \mathbf{c}_{15} \mathbf{c}_{22} \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{c}_{22} \mathbf{c}_{36} \mathbf{T}_1 \mathbf{T}_2^2 - 3 \mathbf{c}_{22} \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{c}_{15} \mathbf{c}_{22} \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{c}_{22} \mathbf{c}_{56} \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{c}_{22} \mathbf{c}_{81} \mathbf{T}_1^2 \mathbf{T}_2^2 + \\
& \mathbf{c}_{81} \alpha_4 + \mathbf{c}_{86} \alpha_4 - \mathbf{c}_{22} \mathbf{T}_1 \alpha_4 + \mathbf{c}_{56} \mathbf{T}_1 \alpha_4 + \mathbf{c}_{69} \mathbf{T}_1 \alpha_4 + \mathbf{c}_{10} \mathbf{T}_2 \alpha_4 + \mathbf{c}_{15} \mathbf{T}_2 \alpha_4 + \mathbf{c}_{28} \mathbf{T}_2 \alpha_4 + \\
& \mathbf{c}_{36} \mathbf{T}_2 \alpha_4 + 3 \mathbf{T}_1 \mathbf{T}_2 \alpha_4 - \mathbf{c}_{10} \mathbf{T}_1 \mathbf{T}_2 \alpha_4 - \mathbf{c}_{15} \mathbf{T}_1 \mathbf{T}_2 \alpha_4 - \mathbf{c}_{56} \mathbf{T}_1 \mathbf{T}_2 \alpha_4 - \mathbf{c}_{81} \mathbf{T}_1 \mathbf{T}_2 \alpha_4 - \alpha_4^2) \\
& \mathbf{p}_{1,j} \mathbf{p}_{2,j} \mathbf{x}_{1,i} \mathbf{x}_{2,j} - \mathbf{p}_{2,i} \mathbf{p}_{2,j} \mathbf{x}_{2,i} \mathbf{x}_{2,j} + \frac{1}{2} (1 - 2 \mathbf{c}_{49} + \mathbf{T}_2) \mathbf{p}_{2,j}^2 \mathbf{x}_{2,i} \mathbf{x}_{2,j} + \\
& \frac{1}{\mathbf{B} (\mathbf{c}_{22} - \mathbf{c}_{22} \mathbf{T}_2 + \mathbf{T}_2 \alpha_4)} (\mathbf{c}_{81} + \mathbf{c}_{86} - \mathbf{c}_{22} \mathbf{T}_1 + \mathbf{c}_{56} \mathbf{T}_1 + \mathbf{c}_{69} \mathbf{T}_1 + \mathbf{c}_{15} \mathbf{T}_2 + \mathbf{c}_{36} \mathbf{T}_2 + \\
& 3 \mathbf{T}_1 \mathbf{T}_2 - \mathbf{c}_{15} \mathbf{T}_1 \mathbf{T}_2 + \mathbf{c}_{22} \mathbf{T}_1 \mathbf{T}_2 - \mathbf{c}_{56} \mathbf{T}_1 \mathbf{T}_2 - 2 \mathbf{c}_{81} \mathbf{T}_1 \mathbf{T}_2 - \mathbf{c}_{86} \mathbf{T}_1 \mathbf{T}_2 - \mathbf{c}_{56} \mathbf{T}_1^2 \mathbf{T}_2 - \\
& \mathbf{c}_{69} \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{c}_{15} \mathbf{T}_1 \mathbf{T}_2^2 - \mathbf{c}_{36} \mathbf{T}_1 \mathbf{T}_2^2 - 3 \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{c}_{15} \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{c}_{56} \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{c}_{81} \mathbf{T}_1^2 \mathbf{T}_2^2 - \alpha_4) \\
& \frac{\alpha_4 \mathbf{p}_{1,j} \mathbf{p}_{2,i} \mathbf{x}_{3,i} + \frac{\alpha_4 \mathbf{p}_{1,i} \mathbf{p}_{2,j} \mathbf{x}_{3,i}}{\mathbf{B}} - \frac{1}{\mathbf{B} (\mathbf{c}_{22} - \mathbf{c}_{22} \mathbf{T}_2 + \mathbf{T}_2 \alpha_4)} \alpha_4}{\alpha_4} \\
& (\mathbf{c}_{22} + \mathbf{c}_{81} + \mathbf{c}_{86} - \mathbf{c}_{22} \mathbf{T}_1 + \mathbf{c}_{56} \mathbf{T}_1 + \mathbf{c}_{69} \mathbf{T}_1 + \mathbf{c}_{15} \mathbf{T}_2 - \mathbf{c}_{22} \mathbf{T}_2 + \mathbf{c}_{36} \mathbf{T}_2 + 3 \mathbf{T}_1 \mathbf{T}_2 - \mathbf{c}_{15} \mathbf{T}_1 \mathbf{T}_2 + \\
& \mathbf{c}_{22} \mathbf{T}_1 \mathbf{T}_2 - \mathbf{c}_{56} \mathbf{T}_1 \mathbf{T}_2 - 2 \mathbf{c}_{81} \mathbf{T}_1 \mathbf{T}_2 - \mathbf{c}_{86} \mathbf{T}_1 \mathbf{T}_2 - \mathbf{c}_{56} \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{c}_{69} \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{c}_{15} \mathbf{T}_1 \mathbf{T}_2^2 - \\
& \mathbf{c}_{36} \mathbf{T}_1 \mathbf{T}_2^2 - 3 \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{c}_{15} \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{c}_{56} \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{c}_{81} \mathbf{T}_1^2 \mathbf{T}_2^2 - \alpha_4 + \mathbf{T}_2 \alpha_4) \mathbf{p}_{1,j} \mathbf{p}_{2,j} \mathbf{x}_{3,i} + \\
& \mathbf{c}_{81} \mathbf{p}_{3,i} \mathbf{x}_{3,i} + \mathbf{c}_{86} \mathbf{p}_{3,j} \mathbf{x}_{3,i} + \mathbf{c}_{32} \mathbf{p}_{1,j} \mathbf{p}_{3,i} \mathbf{x}_{1,i} \mathbf{x}_{3,i} + \mathbf{c}_{13} \mathbf{p}_{1,i} \mathbf{p}_{3,j} \mathbf{x}_{1,i} \mathbf{x}_{3,i} + \\
& \frac{1}{-\mathbf{c}_{22} + \mathbf{c}_{22} \mathbf{T}_2 - \mathbf{T}_2 \alpha_4} \\
& (\mathbf{c}_{13} \mathbf{c}_{22} + \mathbf{c}_{22} \mathbf{c}_{32} - \mathbf{c}_{13} \mathbf{c}_{22} \mathbf{T}_1 + \mathbf{c}_{22}^2 \mathbf{T}_1 - \mathbf{c}_{22} \mathbf{c}_{81} \mathbf{T}_1 - \mathbf{c}_{22} \mathbf{c}_{86} \mathbf{T}_1 - \mathbf{c}_{22} \mathbf{c}_{56} \mathbf{T}_1^2 - \mathbf{c}_{22} \mathbf{c}_{69} \mathbf{T}_1^2 -
\end{aligned}$$

$$\begin{aligned}
& \left(\mathbf{c}_{86} T_1 T_2^2 \alpha_4 + 2 T_1^2 T_2^2 \alpha_4 - \mathbf{c}_{15} T_1^2 T_2^2 \alpha_4 - 2 \mathbf{c}_{56} T_1^2 T_2^2 \alpha_4 - \mathbf{c}_{69} T_1^2 T_2^2 \alpha_4 - \mathbf{c}_{81} T_1^2 T_2^2 \alpha_4 - \right. \\
& \left. \mathbf{c}_{15} T_1 T_2^3 \alpha_4 - \mathbf{c}_{36} T_1 T_2^3 \alpha_4 - 3 T_1^2 T_2^3 \alpha_4 + \mathbf{c}_{15} T_1^2 T_2^3 \alpha_4 + \mathbf{c}_{56} T_1^2 T_2^3 \alpha_4 + \mathbf{c}_{81} T_1^2 T_2^3 \alpha_4 - T_1 T_2 \alpha_4^2 \right) \\
& \frac{\mathbf{p}_{3,j}^2 x_{3,i}}{(2 (-1 + T_1) (-\mathbf{c}_{22} + \mathbf{c}_{22} T_2 - T_2 \alpha_4))} - \\
& \frac{1}{T_1 T_2} \left(-\mathbf{c}_{56} T_1 - \mathbf{c}_{69} T_1 - \mathbf{c}_{15} T_2 - \mathbf{c}_{36} T_2 - 3 T_1 T_2 + \mathbf{c}_{15} T_1 T_2 + \mathbf{c}_{56} T_1 T_2 + \mathbf{c}_{81} T_1 T_2 \right) \mathbf{p}_{3,j} x_{3,i} + \\
& \left((-\mathbf{c}_{13} \mathbf{c}_{22} + \mathbf{c}_{22} \mathbf{c}_{32} + \mathbf{c}_{13} \mathbf{c}_{22} T_1 - \mathbf{c}_{22} \mathbf{c}_{81} T_1 - \mathbf{c}_{22} \mathbf{c}_{86} T_1 + \mathbf{c}_{22}^2 T_1^2 - \mathbf{c}_{22} \mathbf{c}_{56} T_1^2 - \mathbf{c}_{22} \mathbf{c}_{69} T_1^2 + \right. \\
& \mathbf{c}_{13} \mathbf{c}_{22} T_2 - \mathbf{c}_{22} \mathbf{c}_{32} T_2 - \mathbf{c}_{13} \mathbf{c}_{22} T_1 T_2 - \mathbf{c}_{15} \mathbf{c}_{22} T_1 T_2 - \mathbf{c}_{22} \mathbf{c}_{32} T_1 T_2 - \mathbf{c}_{22} \mathbf{c}_{36} T_1 T_2 + \\
& \mathbf{c}_{22} \mathbf{c}_{81} T_1 T_2 + \mathbf{c}_{22} \mathbf{c}_{86} T_1 T_2 - 3 \mathbf{c}_{22} T_1^2 T_2 + \mathbf{c}_{15} \mathbf{c}_{22} T_1^2 T_2 - 2 \mathbf{c}_{22}^2 T_1^2 T_2 + 2 \mathbf{c}_{22} \mathbf{c}_{56} T_1^2 T_2 + \\
& \mathbf{c}_{22} \mathbf{c}_{69} T_1^2 T_2 + 2 \mathbf{c}_{22} \mathbf{c}_{81} T_1^2 T_2 + \mathbf{c}_{22} \mathbf{c}_{86} T_1^2 T_2 + \mathbf{c}_{22} \mathbf{c}_{56} T_1^3 T_2 + \mathbf{c}_{22} \mathbf{c}_{69} T_1^3 T_2 + \mathbf{c}_{15} \mathbf{c}_{22} T_1 T_2^2 + \\
& \mathbf{c}_{22} \mathbf{c}_{32} T_1 T_2^2 + \mathbf{c}_{22} \mathbf{c}_{36} T_1 T_2^2 + 3 \mathbf{c}_{22} T_1^2 T_2^2 + \mathbf{c}_{22}^2 T_1^2 T_2^2 + \mathbf{c}_{22} \mathbf{c}_{36} T_1^2 T_2^2 - \mathbf{c}_{22} \mathbf{c}_{56} T_1^2 T_2^2 - \\
& 2 \mathbf{c}_{22} \mathbf{c}_{81} T_1^2 T_2^2 - \mathbf{c}_{22} \mathbf{c}_{86} T_1^2 T_2^2 + 3 \mathbf{c}_{22} T_1^3 T_2^2 - \mathbf{c}_{15} \mathbf{c}_{22} T_1^3 T_2^2 - 2 \mathbf{c}_{22} \mathbf{c}_{56} T_1^3 T_2^2 - \mathbf{c}_{22} \mathbf{c}_{69} T_1^3 T_2^2 - \\
& \mathbf{c}_{22} \mathbf{c}_{81} T_1^3 T_2^2 - \mathbf{c}_{15} \mathbf{c}_{22} T_1^2 T_3^2 - \mathbf{c}_{22} \mathbf{c}_{36} T_1^2 T_3^2 - 3 \mathbf{c}_{22} T_1^3 T_2^3 + \mathbf{c}_{15} \mathbf{c}_{22} T_1^3 T_2^3 + \mathbf{c}_{22} \mathbf{c}_{56} T_1^3 T_2^3 + \\
& \mathbf{c}_{22} \mathbf{c}_{81} T_1^3 T_2^3 - \mathbf{c}_{81} \alpha_4 - \mathbf{c}_{86} \alpha_4 + 2 \mathbf{c}_{22} T_1 \alpha_4 - \mathbf{c}_{56} T_1 \alpha_4 - \mathbf{c}_{69} T_1 \alpha_4 - \mathbf{c}_{13} T_2 \alpha_4 - \mathbf{c}_{15} T_2 \alpha_4 + \\
& \mathbf{c}_{32} T_2 \alpha_4 - \mathbf{c}_{36} T_2 \alpha_4 - 3 T_1 T_2 \alpha_4 + \mathbf{c}_{13} T_1 T_2 \alpha_4 + \mathbf{c}_{15} T_1 T_2 \alpha_4 - 2 \mathbf{c}_{22} T_1 T_2 \alpha_4 + \\
& \mathbf{c}_{56} T_1 T_2 \alpha_4 + 2 \mathbf{c}_{81} T_1 T_2 \alpha_4 + \mathbf{c}_{86} T_1 T_2 \alpha_4 + \mathbf{c}_{56} T_1^2 T_2 \alpha_4 + \mathbf{c}_{69} T_1^2 T_2 \alpha_4 + \mathbf{c}_{15} T_1 T_2^2 \alpha_4 - \\
& \mathbf{c}_{32} T_1 T_2^2 \alpha_4 + \mathbf{c}_{36} T_1 T_2^2 \alpha_4 + 3 T_1^2 T_2^2 \alpha_4 - \mathbf{c}_{15} T_1^2 T_2^2 \alpha_4 - \mathbf{c}_{56} T_1^2 T_2^2 \alpha_4 - \mathbf{c}_{81} T_1^2 T_2^2 \alpha_4 + \alpha_4^2 \Big) \\
& \frac{\mathbf{p}_{1,i} \mathbf{p}_{3,j} x_{1,i} x_{3,j}}{((-1 + T_1) (-1 + T_1 T_2) (-\mathbf{c}_{22} + \mathbf{c}_{22} T_2 - T_2 \alpha_4))} - \\
& \frac{(-\mathbf{c}_{13} + \mathbf{c}_{13} T_1 - \mathbf{c}_{22} T_1 + \mathbf{c}_{22} T_1^2 + T_1 \alpha_4) \mathbf{p}_{1,j} \mathbf{p}_{3,j} x_{1,i} x_{3,j}}{-1 + T_1 T_2} - \\
& \frac{(-\mathbf{c}_{22} - \mathbf{c}_{54} + \mathbf{c}_{65} + \mathbf{c}_{22} T_2 + \mathbf{c}_{54} T_2 - \mathbf{c}_{65} T_1 T_2 - T_2 \alpha_4) \mathbf{p}_{2,i} \mathbf{p}_{3,j} x_{2,i} x_{3,j}}{(-1 + T_2) (-1 + T_1 T_2)} - \\
& \left((-1 + T_2) \left(\mathbf{c}_{22} \mathbf{c}_{54} - \mathbf{c}_{22} \mathbf{c}_{54} T_1 - \mathbf{c}_{22} \mathbf{c}_{54} T_2 - \mathbf{c}_{22} \mathbf{c}_{81} T_2 - \mathbf{c}_{22} \mathbf{c}_{86} T_2 + \mathbf{c}_{22}^2 T_1 T_2 + \right. \right. \\
& \mathbf{c}_{22} \mathbf{c}_{54} T_1 T_2 - \mathbf{c}_{22} \mathbf{c}_{56} T_1 T_2 - \mathbf{c}_{22} \mathbf{c}_{69} T_1 T_2 + \mathbf{c}_{22} \mathbf{c}_{81} T_1 T_2 + \mathbf{c}_{22} \mathbf{c}_{86} T_1 T_2 - \mathbf{c}_{22}^2 T_1^2 T_2 + \\
& \mathbf{c}_{22} \mathbf{c}_{56} T_1^2 T_2 + \mathbf{c}_{22} \mathbf{c}_{69} T_1^2 T_2 - \mathbf{c}_{15} \mathbf{c}_{22} T_2^2 - \mathbf{c}_{22} \mathbf{c}_{36} T_2^2 - 3 \mathbf{c}_{22} T_1 T_2^2 + 2 \mathbf{c}_{15} \mathbf{c}_{22} T_1 T_2^2 - \\
& \mathbf{c}_{22}^2 T_1 T_2^2 + \mathbf{c}_{22} \mathbf{c}_{36} T_1 T_2^2 + \mathbf{c}_{22} \mathbf{c}_{56} T_1 T_2^2 + 2 \mathbf{c}_{22} \mathbf{c}_{81} T_1 T_2^2 + \mathbf{c}_{22} \mathbf{c}_{86} T_1 T_2^2 + 3 \mathbf{c}_{22} T_1^2 T_2^2 - \\
& \mathbf{c}_{15} \mathbf{c}_{22} T_1^2 T_2^2 + \mathbf{c}_{22}^2 T_1^2 T_2^2 + \mathbf{c}_{22} \mathbf{c}_{69} T_1^2 T_2^2 - 2 \mathbf{c}_{22} \mathbf{c}_{81} T_1^2 T_2^2 - \mathbf{c}_{22} \mathbf{c}_{86} T_1^2 T_2^2 - \mathbf{c}_{22} \mathbf{c}_{56} T_1^3 T_2^2 - \\
& \mathbf{c}_{22} \mathbf{c}_{69} T_1^3 T_2^2 + \mathbf{c}_{15} \mathbf{c}_{22} T_1 T_3^2 + \mathbf{c}_{22} \mathbf{c}_{36} T_1 T_3^2 + 3 \mathbf{c}_{22} T_1^2 T_3^2 - 2 \mathbf{c}_{15} \mathbf{c}_{22} T_1^2 T_3^2 - \mathbf{c}_{22} \mathbf{c}_{36} T_1^2 T_3^2 - \\
& \mathbf{c}_{22} \mathbf{c}_{56} T_1^2 T_3^2 - \mathbf{c}_{22} \mathbf{c}_{81} T_1^2 T_3^2 - 3 \mathbf{c}_{22} T_1^3 T_2^3 + \mathbf{c}_{15} \mathbf{c}_{22} T_1^3 T_2^3 + \mathbf{c}_{22} \mathbf{c}_{56} T_1^3 T_2^3 + \mathbf{c}_{22} \mathbf{c}_{81} T_1^3 T_2^3 + \\
& \mathbf{c}_{22} T_2 \alpha_4 + \mathbf{c}_{54} T_2 \alpha_4 + \mathbf{c}_{81} T_2 \alpha_4 + \mathbf{c}_{86} T_2 \alpha_4 - 2 \mathbf{c}_{22} T_1 T_2 \alpha_4 - \mathbf{c}_{54} T_1 T_2 \alpha_4 + \mathbf{c}_{56} T_1 T_2 \alpha_4 + \\
& \mathbf{c}_{69} T_1 T_2 \alpha_4 + \mathbf{c}_{15} T_2 \alpha_4 + \mathbf{c}_{36} T_2 \alpha_4 + 3 T_1 T_2 \alpha_4 - \mathbf{c}_{15} T_1 T_2^2 \alpha_4 + \mathbf{c}_{22} T_1 T_2^2 \alpha_4 - \\
& \mathbf{c}_{56} T_1 T_2^2 \alpha_4 - 2 \mathbf{c}_{81} T_1 T_2^2 \alpha_4 - \mathbf{c}_{86} T_1 T_2^2 \alpha_4 - \mathbf{c}_{56} T_1^2 T_2 \alpha_4 - \mathbf{c}_{69} T_1^2 T_2 \alpha_4 - \mathbf{c}_{15} T_1 T_2^3 \alpha_4 - \\
& \mathbf{c}_{36} T_1 T_2^3 \alpha_4 - 3 T_1^2 T_2^3 \alpha_4 + \mathbf{c}_{15} T_1^2 T_2^3 \alpha_4 + \mathbf{c}_{56} T_1^2 T_2^3 \alpha_4 + \mathbf{c}_{81} T_1^2 T_2^3 \alpha_4 - T_2 \alpha_4^2 \Big) \\
& \frac{\mathbf{p}_{2,j} \mathbf{p}_{3,j} x_{2,i} x_{3,j}}{((-1 + T_1) (-1 + T_1 T_2) (-\mathbf{c}_{22} + \mathbf{c}_{22} T_2 - T_2 \alpha_4))} - \\
& \left(\left(\mathbf{c}_{22} + \mathbf{c}_{22}^2 - \mathbf{c}_{22} \mathbf{c}_{81} - \mathbf{c}_{22} \mathbf{c}_{86} - \mathbf{c}_{22} T_1 - \mathbf{c}_{22}^2 T_1 - \mathbf{c}_{22} \mathbf{c}_{56} T_1 - \mathbf{c}_{22} \mathbf{c}_{69} T_1 + \mathbf{c}_{22} \mathbf{c}_{81} T_1 + \mathbf{c}_{22} \mathbf{c}_{86} T_1 + \right. \right. \\
& \mathbf{c}_{22} \mathbf{c}_{56} T_1^2 + \mathbf{c}_{22} \mathbf{c}_{69} T_1^2 - \mathbf{c}_{22} T_2 - \mathbf{c}_{15} \mathbf{c}_{22} T_2 - \mathbf{c}_{22}^2 T_2 - \mathbf{c}_{22} \mathbf{c}_{36} T_2 + \mathbf{c}_{22} \mathbf{c}_{81} T_2 + \mathbf{c}_{22} \mathbf{c}_{86} T_2 - \\
& 2 \mathbf{c}_{22} T_1 T_2 + 2 \mathbf{c}_{15} \mathbf{c}_{22} T_1 T_2 + \mathbf{c}_{22}^2 T_1 T_2 + \mathbf{c}_{22} \mathbf{c}_{36} T_1 T_2 + 2 \mathbf{c}_{22} \mathbf{c}_{56} T_1 T_2 + \mathbf{c}_{22} \mathbf{c}_{69} T_1 T_2 - \\
& \mathbf{c}_{22} \mathbf{c}_{86} T_1 T_2 + 3 \mathbf{c}_{22} T_1^2 T_2 - \mathbf{c}_{15} \mathbf{c}_{22} T_1^2 T_2 - 2 \mathbf{c}_{22} \mathbf{c}_{56} T_1^2 T_2 - \mathbf{c}_{22} \mathbf{c}_{69} T_1^2 T_2 - \mathbf{c}_{22} \mathbf{c}_{81} T_1^2 T_2 + \\
& \mathbf{c}_{15} \mathbf{c}_{22} T_2^2 + \mathbf{c}_{22} \mathbf{c}_{36} T_2^2 + 3 \mathbf{c}_{22} T_1 T_2^2 - 2 \mathbf{c}_{15} \mathbf{c}_{22} T_1 T_2^2 - \mathbf{c}_{22} \mathbf{c}_{36} T_1 T_2^2 - \mathbf{c}_{22} \mathbf{c}_{56} T_1 T_2^2 - \\
& \mathbf{c}_{22} \mathbf{c}_{81} T_1 T_2^2 - 3 \mathbf{c}_{22} T_1^2 T_2^2 + \mathbf{c}_{15} \mathbf{c}_{22} T_1^2 T_2^2 + \mathbf{c}_{22} \mathbf{c}_{56} T_1^2 T_2^2 + \mathbf{c}_{22} \mathbf{c}_{81} T_1^2 T_2^2 + \mathbf{c}_{81} \alpha_4 + \mathbf{c}_{86} \alpha_4 - \\
& \mathbf{c}_{22} T_1 \alpha_4 + \mathbf{c}_{56} T_1 \alpha_4 + \mathbf{c}_{69} T_1 \alpha_4 + \mathbf{T}_2 \alpha_4 + \mathbf{c}_{15} T_2 \alpha_4 + \mathbf{c}_{22} T_2 \alpha_4 + \mathbf{c}_{36} T_2 \alpha_4 - \mathbf{c}_{81} T_2 \alpha_4 - \\
& \mathbf{c}_{86} T_2 \alpha_4 + 2 T_1 T_2 \alpha_4 - \mathbf{c}_{15} T_1 T_2 \alpha_4 - 2 \mathbf{c}_{56} T_1 T_2 \alpha_4 - \mathbf{c}_{69} T_1 T_2 \alpha_4 - \mathbf{c}_{81} T_1 T_2 \alpha_4 - \\
& \mathbf{c}_{15} T_2 \alpha_4 - \mathbf{c}_{36} T_2 \alpha_4 - 3 T_1 T_2 \alpha_4 + \mathbf{c}_{15} T_1 T_2 \alpha_4 + \mathbf{c}_{56} T_1 T_2 \alpha_4 + \mathbf{c}_{81} T_1 T_2 \alpha_4 - \mathbf{T}_2 \alpha_4^2 \Big) \\
& \frac{\mathbf{p}_{3,i} \mathbf{p}_{3,j} x_{3,i} x_{3,j}}{((-1 + T_1) (-\mathbf{c}_{22} + \mathbf{c}_{22} T_2 - T_2 \alpha_4))} - \\
& \left(\left(\mathbf{c}_{22} + \mathbf{c}_{22}^2 - 2 \mathbf{c}_{22} \mathbf{c}_{78} - \mathbf{c}_{22} \mathbf{c}_{81} - \mathbf{c}_{22} \mathbf{c}_{86} - \mathbf{c}_{22}^2 T_1 - \mathbf{c}_{22} \mathbf{c}_{56} T_1 - \mathbf{c}_{22} \mathbf{c}_{69} T_1 + 2 \mathbf{c}_{22} \mathbf{c}_{78} T_1 + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(p_{1,i} p_{2,j} x_{1,i} x_{2,i} \right) / ((-1 + T_1) T_2 (-c_{22} + c_{22} T_2 - T_2 \alpha_4)) - \\
& \left((-c_{10} c_{22} - c_{22}^2 - 2 c_{22} c_{28} + c_{10} c_{22} T_1 + 2 c_{22}^2 T_1 - 2 c_{22} c_{81} T_1 - 2 c_{22} c_{86} T_1 - 2 c_{22} c_{56} T_1^2 - \right. \\
& \quad 2 c_{22} c_{69} T_1^2 + 2 c_{10} c_{22} T_2 + 2 c_{22}^2 T_2 + 4 c_{22} c_{28} T_2 - c_{22} c_{81} T_2 - c_{22} c_{86} T_2 - 2 c_{10} c_{22} T_1 T_2 - \\
& \quad 2 c_{15} c_{22} T_1 T_2 - 4 c_{22}^2 T_1 T_2 - 2 c_{22} c_{36} T_1 T_2 - c_{22} c_{56} T_1 T_2 - c_{22} c_{69} T_1 T_2 + 4 c_{22} c_{81} T_1 T_2 + \\
& \quad 4 c_{22} c_{86} T_1 T_2 - 6 c_{22} T_1^2 T_2 + 2 c_{15} c_{22} T_1^2 T_2 + 6 c_{22} c_{56} T_1^2 T_2 + 4 c_{22} c_{69} T_1^2 T_2 + 2 c_{22} c_{81} T_1^2 T_2 - \\
& \quad c_{10} c_{22} T_2^2 - c_{15} c_{22} T_2^2 - c_{22}^2 T_2^2 - c_{22} c_{28} T_2^2 - c_{22} c_{36} T_2^2 + c_{22} c_{81} T_2^2 + c_{22} c_{86} T_2^2 - \\
& \quad 3 c_{22} T_1 T_2^2 + c_{10} c_{22} T_1 T_2^2 + 5 c_{15} c_{22} T_1 T_2^2 + 2 c_{22}^2 T_1 T_2^2 + 4 c_{22} c_{36} T_1 T_2^2 + 2 c_{22} c_{56} T_1 T_2^2 + \\
& \quad c_{22} c_{69} T_1 T_2^2 - c_{22} c_{81} T_1 T_2^2 - 2 c_{22} c_{86} T_1 T_2^2 + 12 c_{22} T_1^2 T_2^2 - 4 c_{15} c_{22} T_1^2 T_2^2 - 6 c_{22} c_{56} T_1^2 T_2^2 - \\
& \quad 2 c_{22} c_{69} T_1^2 T_2^2 - 4 c_{22} c_{81} T_1^2 T_2^2 + c_{15} c_{22} T_2^3 + c_{22} c_{36} T_2^3 + 3 c_{22} T_1 T_2^3 - 3 c_{15} c_{22} T_1 T_2^3 - \\
& \quad 2 c_{22} c_{36} T_1 T_2^3 - c_{22} c_{56} T_1 T_2^3 - c_{22} c_{81} T_1 T_2^3 - 6 c_{22} T_1^2 T_2^3 + 2 c_{15} c_{22} T_1^2 T_2^3 + 2 c_{22} c_{56} T_1^2 T_2^3 + \\
& \quad 2 c_{22} c_{81} T_1^2 T_2^3 + c_{22} \alpha_4 - 2 c_{81} \alpha_4 - 2 c_{86} \alpha_4 + 2 c_{22} T_1 \alpha_4 - 2 c_{56} T_1 \alpha_4 - 2 c_{69} T_1 \alpha_4 - c_{10} T_2 \alpha_4 - \\
& \quad 2 c_{15} T_2 \alpha_4 - 2 c_{22} T_2 \alpha_4 - 2 c_{28} T_2 \alpha_4 - 2 c_{36} T_2 \alpha_4 + 2 c_{81} T_2 \alpha_4 + 2 c_{86} T_2 \alpha_4 - 6 T_1 T_2 \alpha_4 + \\
& \quad c_{10} T_1 T_2 \alpha_4 + 2 c_{15} T_1 T_2 \alpha_4 - 2 c_{22} T_1 T_2 \alpha_4 + 4 c_{56} T_1 T_2 \alpha_4 + 2 c_{69} T_1 T_2 \alpha_4 + 2 c_{81} T_1 T_2 \alpha_4 + \\
& \quad c_{10} T_2^2 \alpha_4 + 2 c_{15} T_2^2 \alpha_4 + c_{22} T_2^2 \alpha_4 + 2 c_{28} T_2^2 \alpha_4 + 2 c_{36} T_2^2 \alpha_4 - c_{81} T_2^2 \alpha_4 - c_{86} T_2^2 \alpha_4 + \\
& \quad 6 T_1 T_2^2 \alpha_4 - c_{10} T_1 T_2^2 \alpha_4 - 2 c_{15} T_1 T_2^2 \alpha_4 - 3 c_{56} T_1 T_2^2 \alpha_4 - c_{69} T_1 T_2^2 \alpha_4 - 2 c_{81} T_1 T_2^2 \alpha_4 - \\
& \quad c_{15} T_2^3 \alpha_4 - c_{36} T_2^3 \alpha_4 - 3 T_1 T_2^3 \alpha_4 + c_{15} T_1 T_2^3 \alpha_4 + c_{56} T_1 T_2^3 \alpha_4 + c_{81} T_1 T_2^3 \alpha_4 + 2 \alpha_4^2 - T_2 \alpha_4^2) \\
& \left(p_{1,j} p_{2,j} x_{1,j} x_{2,j} \right) / ((T_1 T_2 (-c_{22} + c_{22} T_2 - T_2 \alpha_4)) + \\
& \left((c_{10} c_{22} + c_{22}^2 - c_{22} c_{81} - c_{22} c_{86} - c_{10} c_{22} T_1 - c_{22}^2 T_1 - c_{22} c_{56} T_1 - c_{22} c_{69} T_1 + c_{22} c_{81} T_1 + \right. \\
& \quad c_{22} c_{86} T_1 + c_{22} c_{56} T_1^2 + c_{22} c_{69} T_1^2 - c_{10} c_{22} T_2 - c_{15} c_{22} T_2 - c_{22}^2 T_2 - c_{22} c_{36} T_2 + c_{22} c_{81} T_2 + \\
& \quad c_{22} c_{86} T_2 - 3 c_{22} T_1 T_2 + c_{10} c_{22} T_1 T_2 + 2 c_{15} c_{22} T_1 T_2 + c_{22}^2 T_1 T_2 + c_{22} c_{36} T_1 T_2 + \\
& \quad 2 c_{22} c_{56} T_1 T_2 + c_{22} c_{69} T_1 T_2 - c_{22} c_{86} T_1 T_2 + 3 c_{22} T_1^2 T_2 - c_{15} c_{22} T_1^2 T_2 - 2 c_{22} c_{56} T_1^2 T_2 - \\
& \quad c_{22} c_{69} T_1^2 T_2 - c_{22} c_{81} T_1^2 T_2 + c_{15} c_{22} T_2^2 + c_{22} c_{36} T_2^2 + 3 c_{22} T_1 T_2^2 - 2 c_{15} c_{22} T_1 T_2^2 - c_{22} c_{36} T_1 T_2^2 - \\
& \quad c_{22} c_{56} T_1 T_2^2 - c_{22} c_{81} T_1 T_2^2 - 3 c_{22} T_1^2 T_2^2 + c_{15} c_{22} T_1^2 T_2^2 + c_{22} c_{56} T_1^2 T_2^2 + c_{22} c_{81} T_1^2 T_2^2 + c_{81} \alpha_4 + \\
& \quad c_{86} \alpha_4 - c_{22} T_1 \alpha_4 + c_{56} T_1 \alpha_4 + c_{69} T_1 \alpha_4 + c_{10} T_2 \alpha_4 + c_{15} T_2 \alpha_4 + c_{22} T_2 \alpha_4 + c_{36} T_2 \alpha_4 - \\
& \quad c_{81} T_2 \alpha_4 - c_{86} T_2 \alpha_4 + 3 T_1 T_2 \alpha_4 - c_{10} T_1 T_2 \alpha_4 - c_{15} T_1 T_2 \alpha_4 - 2 c_{56} T_1 T_2 \alpha_4 - c_{69} T_1 T_2 \alpha_4 - \\
& \quad c_{81} T_1 T_2 \alpha_4 - c_{15} T_2^2 \alpha_4 - c_{36} T_2^2 \alpha_4 - 3 T_1 T_2^2 \alpha_4 + c_{15} T_1 T_2^2 \alpha_4 + c_{56} T_1 T_2^2 \alpha_4 + c_{81} T_1 T_2^2 \alpha_4 - \alpha_4^2) \\
& \left(p_{1,j} p_{2,j} x_{1,j} x_{2,j} \right) / (((-1 + T_1) (-c_{22} + c_{22} T_2 - T_2 \alpha_4)) - \\
& ((-1 + T_2) (c_{10} c_{22} + c_{22}^2 + c_{22} c_{28} - c_{10} c_{22} T_1 - 2 c_{22}^2 T_1 + c_{22} c_{81} T_1 + c_{22} c_{86} T_1 + c_{22} c_{56} T_1^2 + \\
& \quad c_{22} c_{69} T_1^2 - c_{10} c_{22} T_2 - c_{22}^2 T_2 - c_{22} c_{28} T_2 + c_{22} c_{81} T_2 + c_{22} c_{86} T_2 + c_{10} c_{22} T_1 T_2 + c_{15} c_{22} T_1 T_2 + \\
& \quad 2 c_{22}^2 T_1 T_2 + c_{22} c_{36} T_1 T_2 + c_{22} c_{56} T_1 T_2 + c_{22} c_{69} T_1 T_2 - 2 c_{22} c_{81} T_1 T_2 - 2 c_{22} c_{86} T_1 T_2 + \\
& \quad 3 c_{22} T_1^2 T_2 - c_{15} c_{22} T_1^2 T_2 - 3 c_{22} c_{56} T_1^2 T_2 - 2 c_{22} c_{69} T_1^2 T_2 - c_{22} c_{81} T_1^2 T_2 + c_{15} c_{22} T_2^2 + \\
& \quad c_{22} c_{36} T_2^2 + 3 c_{22} T_1 T_2^2 - 3 c_{15} c_{22} T_1 T_2^2 - 2 c_{22} c_{36} T_1 T_2^2 - c_{22} c_{56} T_1 T_2^2 - c_{22} c_{81} T_1 T_2^2 - \\
& \quad 6 c_{22} T_1^2 T_2^2 + 2 c_{15} c_{22} T_1^2 T_2^2 + 2 c_{22} c_{56} T_1^2 T_2^2 + 2 c_{22} c_{81} T_1^2 T_2^2 - c_{22} \alpha_4 + c_{81} \alpha_4 + c_{86} \alpha_4 - \\
& \quad c_{22} T_1 \alpha_4 + c_{56} T_1 \alpha_4 + c_{69} T_1 \alpha_4 + c_{10} T_2 \alpha_4 + c_{15} T_2 \alpha_4 + c_{22} T_2 \alpha_4 + c_{28} T_2 \alpha_4 + c_{36} T_2 \alpha_4 - \\
& \quad c_{81} T_2 \alpha_4 - c_{86} T_2 \alpha_4 + 3 T_1 T_2 \alpha_4 - c_{10} T_1 T_2 \alpha_4 - c_{15} T_1 T_2 \alpha_4 - 2 c_{56} T_1 T_2 \alpha_4 - c_{69} T_1 T_2 \alpha_4 - \\
& \quad c_{81} T_1 T_2 \alpha_4 - c_{15} T_2^2 \alpha_4 - c_{36} T_2^2 \alpha_4 - 3 T_1 T_2^2 \alpha_4 + c_{15} T_1 T_2^2 \alpha_4 + c_{56} T_1 T_2^2 \alpha_4 + c_{81} T_1 T_2^2 \alpha_4 - \alpha_4^2) \\
& \left(p_{1,j} p_{2,j} x_{1,j} x_{2,j} \right) / (((-1 + T_1) T_2 (-c_{22} + c_{22} T_2 - T_2 \alpha_4)) + \\
& \frac{(-1 - c_{49} + T_2) p_{2,i} p_{2,j} x_{2,i}^2}{T_2} - \frac{(-1 + T_2) (-1 - 2 c_{49} + 2 T_2) p_{2,j}^2 x_{2,i}^2}{2 T_2^2} + \\
& \frac{(c_{56} + c_{69}) p_{2,j} x_{2,j}}{T_2} - \\
& \frac{c_{10} p_{1,i} p_{2,j} x_{1,i} x_{2,j} - 1}{T_1 (-c_{22} + c_{22} T_2 - T_2 \alpha_4)}
\end{aligned}$$

$$\begin{aligned}
& \left(c_{22} c_{28} - c_{22}^2 T_1 + c_{22} c_{81} T_1 + c_{22} c_{86} T_1 + c_{22} c_{56} T_1^2 + c_{22} c_{69} T_1^2 - c_{22} c_{28} T_2 + \right. \\
& \quad c_{15} c_{22} T_1 T_2 + c_{22}^2 T_1 T_2 + c_{22} c_{36} T_1 T_2 - c_{22} c_{81} T_1 T_2 - c_{22} c_{86} T_1 T_2 + 3 c_{22} T_1^2 T_2 - \\
& \quad c_{15} c_{22} T_1^2 T_2 - 2 c_{22} c_{56} T_1^2 T_2 - c_{22} c_{69} T_1^2 T_2 - c_{22} c_{81} T_1^2 T_2 - c_{15} c_{22} T_1 T_2^2 - \\
& \quad c_{22} c_{36} T_1 T_2^2 - 3 c_{22} T_1^2 T_2 + c_{15} c_{22} T_1^2 T_2 + c_{22} c_{56} T_1^2 T_2 + c_{22} c_{81} T_1^2 T_2 + c_{81} \alpha_4 + \\
& \quad c_{86} \alpha_4 - c_{22} T_1 \alpha_4 + c_{56} T_1 \alpha_4 + c_{69} T_1 \alpha_4 + c_{15} T_2 \alpha_4 + c_{28} T_2 \alpha_4 + c_{36} T_2 \alpha_4 + \\
& \quad \left. 3 T_1 T_2 \alpha_4 - c_{15} T_1 T_2 \alpha_4 - c_{56} T_1 T_2 \alpha_4 - c_{81} T_1 T_2 \alpha_4 - \alpha_4^2 \right) p_{1,j} p_{2,j} x_{1,i} x_{2,j} + \\
& p_{2,i} p_{2,j} x_{2,i} x_{2,j} - \frac{(-1 - 2 c_{49} + 3 T_2) p_{2,j}^2 x_{2,i} x_{2,j}}{2 T_2} - \frac{1}{B T_1 (c_{22} - c_{22} T_2 + T_2 \alpha_4)} \\
& \left(c_{81} + c_{86} - c_{22} T_1 + c_{56} T_1 + c_{69} T_1 + c_{15} T_2 + c_{36} T_2 + 3 T_1 T_2 - c_{15} T_1 T_2 + c_{22} T_1 T_2 - \right. \\
& \quad c_{56} T_1 T_2 - 2 c_{81} T_1 T_2 - c_{86} T_1 T_2 - c_{56} T_1^2 T_2 - c_{69} T_1^2 T_2 - c_{15} T_1^2 T_2 - c_{36} T_1 T_2^2 - \\
& \quad \left. 3 T_1^2 T_2 + c_{15} T_1^2 T_2 + c_{56} T_1^2 T_2 + c_{81} T_1^2 T_2 - \alpha_4 \right) \alpha_4 p_{1,j} p_{2,i} x_{3,i} - \frac{\alpha_4 p_{1,i} p_{2,j} x_{3,i}}{B T_2} + \\
& (\alpha_4 (c_{22} T_1 + c_{81} T_2 + c_{86} T_2 - 2 c_{22} T_1 T_2 + c_{56} T_1 T_2 + c_{69} T_1 T_2 + c_{15} T_2^2 + c_{36} T_2^2 + 3 T_1 T_2^2 - \\
& \quad c_{15} T_1 T_2^2 + c_{22} T_1 T_2^2 - c_{56} T_1 T_2^2 - 2 c_{81} T_1 T_2^2 - c_{86} T_1 T_2^2 - c_{56} T_1^2 T_2^2 - c_{69} T_1^2 T_2^2 - \\
& \quad c_{15} T_1 T_2^3 - c_{36} T_1 T_2^3 - 3 T_1^2 T_2^3 + c_{15} T_1^2 T_2^3 + c_{56} T_1^2 T_2^3 + c_{81} T_1^2 T_2^3 - T_2 \alpha_4 + T_1 T_2 \alpha_4) \\
& \quad p_{1,j} p_{2,j} x_{3,i}) / (B T_1 T_2 (c_{22} - c_{22} T_2 + T_2 \alpha_4)) - c_{81} p_{3,i} x_{3,i} + \\
& ((-c_{22} c_{56} T_1 - c_{22} c_{69} T_1 - c_{15} c_{22} T_2 - c_{22} c_{36} T_2 - 3 c_{22} T_1 T_2 + c_{15} c_{22} T_1 T_2 + 2 c_{22} c_{56} T_1 T_2 + \\
& \quad c_{22} c_{69} T_1 T_2 + c_{22} c_{81} T_1 T_2 + c_{22}^2 T_1^2 T_2 + c_{15} c_{22} T_2^2 + c_{22} c_{36} T_2^2 + 3 c_{22} T_1 T_2^2 - c_{15} c_{22} T_1 T_2^2 - \\
& \quad c_{22} c_{56} T_1 T_2^2 - c_{22} c_{81} T_1 T_2^2 - 2 c_{22}^2 T_1^2 T_2 + c_{22} c_{86} T_1^2 T_2 + c_{22} c_{56} T_1^2 T_2^2 + c_{22} c_{69} T_1^2 T_2^2 + \\
& \quad c_{15} c_{22} T_1^2 T_2^3 + c_{22}^2 T_1^2 T_2^3 + c_{22} c_{36} T_1^2 T_2^3 - c_{22} c_{86} T_1^2 T_2^3 + 3 c_{22} T_1^3 T_2^3 - c_{15} c_{22} T_1^3 T_2^3 - \\
& \quad 2 c_{22} c_{56} T_1^3 T_2^3 - c_{22} c_{69} T_1^3 T_2^3 - c_{22} c_{81} T_1^3 T_2^3 - c_{15} c_{22} T_1^2 T_2^4 - c_{22} c_{36} T_1^2 T_2^4 - 3 c_{22} T_1^3 T_2^4 + \\
& \quad c_{15} c_{22} T_1^3 T_2^4 + c_{22} c_{56} T_1^3 T_2^4 + c_{22} c_{81} T_1^3 T_2^4 - c_{56} T_1 T_2 \alpha_4 - c_{69} T_1 T_2 \alpha_4 - c_{15} T_2 \alpha_4 - c_{36} T_2 \alpha_4 - \\
& \quad 3 T_1 T_2 \alpha_4 + c_{15} T_1 T_2 \alpha_4 + c_{56} T_1 T_2 \alpha_4 + 2 c_{81} T_1 T_2 \alpha_4 + c_{86} T_1 T_2 \alpha_4 + c_{56} T_1^2 T_2 \alpha_4 + c_{69} T_1^2 T_2 \alpha_4 + \\
& \quad c_{15} T_1 T_2^3 \alpha_4 + c_{36} T_1 T_2^3 \alpha_4 + 3 T_1^2 T_2^3 \alpha_4 - c_{15} T_1^2 T_2^3 \alpha_4 - c_{56} T_1^2 T_2^3 \alpha_4 - 2 c_{81} T_1^2 T_2^3 \alpha_4 - T_1 T_2 \alpha_4^2) \\
& \quad p_{3,j} x_{3,i}) / (T_1^2 T_2^2 (-c_{22} + c_{22} T_2 - T_2 \alpha_4)) - \frac{1}{T_1 (-1 + T_1 T_2)} \\
& (-c_{13} + c_{22} - c_{81} - c_{86} + c_{13} T_1 - c_{22} T_1 - c_{56} T_1 - c_{69} T_1 + c_{22} T_1^2 - c_{15} T_2 - c_{36} T_2 - 3 T_1 T_2 + \\
& \quad c_{15} T_1 T_2 - c_{22} T_1 T_2 + c_{56} T_1 T_2 + 2 c_{81} T_1 T_2 + c_{86} T_1 T_2 + c_{56} T_1^2 T_2 + c_{69} T_1^2 T_2 + c_{15} T_1 T_2^2 + \\
& \quad c_{36} T_1 T_2^2 + 3 T_1^2 T_2^2 - c_{15} T_1^2 T_2^2 - c_{56} T_1^2 T_2^2 - c_{81} T_1^2 T_2^2 + T_1 \alpha_4) p_{1,j} p_{3,i} x_{1,i} x_{3,i} - \\
& ((-c_{22}^2 + c_{22} c_{32} + 2 c_{22}^2 T_1 - c_{22} c_{81} T_1 - c_{22} c_{86} T_1 - c_{22} c_{56} T_1^2 - c_{22} c_{69} T_1^2 + c_{22}^2 T_2 - \\
& \quad c_{22} c_{32} T_2 - c_{15} c_{22} T_1 T_2 - 2 c_{22}^2 T_1 T_2 - c_{22} c_{32} T_1 T_2 - c_{22} c_{36} T_1 T_2 + c_{22} c_{81} T_1 T_2 + \\
& \quad c_{22} c_{86} T_1 T_2 - 3 c_{22} T_1^2 T_2 + c_{15} c_{22} T_1^2 T_2 - c_{22}^2 T_1^2 T_2 + 2 c_{22} c_{56} T_1^2 T_2 + c_{22} c_{69} T_1^2 T_2 + \\
& \quad 2 c_{22} c_{81} T_1^2 T_2 + c_{22} c_{86} T_1^2 T_2 + c_{22} c_{56} T_1^3 T_2 + c_{22} c_{69} T_1^3 T_2 + c_{15} c_{22} T_1 T_2^2 + c_{22} c_{32} T_1 T_2^2 + \\
& \quad c_{22} c_{36} T_1 T_2^2 + 3 c_{22} T_1^2 T_2^2 + c_{22}^2 T_1^2 T_2^2 + c_{22} c_{36} T_1^2 T_2^2 - c_{22} c_{56} T_1^2 T_2^2 - 2 c_{22} c_{81} T_1^2 T_2^2 - \\
& \quad c_{22} c_{86} T_1^2 T_2^2 + 3 c_{22} T_1^3 T_2^2 - c_{15} c_{22} T_1^3 T_2^2 - 2 c_{22} c_{56} T_1^3 T_2^2 - c_{22} c_{69} T_1^3 T_2^2 - c_{22} c_{81} T_1^3 T_2^2 - \\
& \quad c_{15} c_{22} T_1^2 T_3^2 - c_{22} c_{36} T_1^2 T_3^2 - 3 c_{22} T_1^3 T_2^3 + c_{15} c_{22} T_1^3 T_2^3 + c_{22} c_{56} T_1^3 T_2^3 + c_{22} c_{81} T_1^3 T_2^3 + \\
& \quad c_{22} \alpha_4 - c_{81} \alpha_4 - c_{86} \alpha_4 + c_{22} T_1 \alpha_4 - c_{56} T_1 \alpha_4 - c_{69} T_1 \alpha_4 - c_{15} T_2 \alpha_4 - 2 c_{22} T_2 \alpha_4 + \\
& \quad c_{32} T_2 \alpha_4 - c_{36} T_2 \alpha_4 - 3 T_1 T_2 \alpha_4 + c_{15} T_1 T_2 \alpha_4 + c_{22} T_1 T_2 \alpha_4 + c_{56} T_1 T_2 \alpha_4 + 2 c_{81} T_1 T_2 \alpha_4 + \\
& \quad c_{86} T_1 T_2 \alpha_4 - c_{22} T_1^2 T_2 \alpha_4 + c_{56} T_1^2 T_2 \alpha_4 + c_{69} T_1^2 T_2 \alpha_4 + c_{15} T_1 T_2^2 \alpha_4 - c_{32} T_1 T_2^2 \alpha_4 + \\
& \quad c_{36} T_1 T_2^2 \alpha_4 + 3 T_1^2 T_2^2 \alpha_4 - c_{15} T_1^2 T_2^2 \alpha_4 - c_{56} T_1^2 T_2^2 \alpha_4 - c_{81} T_1^2 T_2^2 \alpha_4 + \alpha_4^2 + T_2 \alpha_4^2 - T_1 T_2 \alpha_4^2) \\
& \quad p_{1,i} p_{3,j} x_{1,i} x_{3,i}) / ((-1 + T_1) T_1 T_2 (-c_{22} + c_{22} T_2 - T_2 \alpha_4)) + \\
& ((c_{13} c_{22} - c_{22}^2 + c_{22} c_{32} - c_{13} c_{22} T_1 + 3 c_{22}^2 T_1 - c_{22} c_{81} T_1 - c_{22} c_{86} T_1 - c_{22}^2 T_1^2 - c_{22} c_{56} T_1^2 - \\
& \quad c_{22} c_{69} T_1^2 - c_{13} c_{22} T_2 + c_{22}^2 T_2 - c_{22} c_{32} T_2 + c_{13} c_{22} T_1 T_2 - c_{15} c_{22} T_1 T_2 - 3 c_{22} T_1 T_2 + \\
& \quad c_{22} c_{32} T_1 T_2 - c_{22} c_{36} T_1 T_2 + c_{22} c_{81} T_1 T_2 + c_{22} c_{86} T_1 T_2 - 3 c_{22} T_1^2 T_2 + c_{15} c_{22} T_1^2 T_2 +)
\end{aligned}$$

$$\begin{aligned}
& \frac{2 c_{22} c_{56} T_1^2 T_2 + c_{22} c_{69} T_1^2 T_2 + 2 c_{22} c_{81} T_1^2 T_2 + c_{22} c_{86} T_1^2 T_2 + c_{22} c_{56} T_1^3 T_2 + c_{22} c_{69} T_1^3 T_2 +}{c_{15} c_{22} T_1 T_2^2 + c_{22} c_{32} T_1 T_2^2 + c_{22} c_{36} T_1 T_2^2 + 3 c_{22} T_1 T_2^2 + c_{22} T_1 T_2^2 + c_{22} c_{36} T_1 T_2^2 -} \\
& \frac{c_{22} c_{56} T_1 T_2^2 - 2 c_{22} c_{81} T_1 T_2^2 - c_{22} c_{86} T_1 T_2^2 + 3 c_{22} T_1 T_2^3 - c_{15} c_{22} T_1 T_2^3 - 2 c_{22} c_{56} T_1 T_2^3 -}{c_{22} c_{69} T_1 T_2^3 - c_{22} c_{81} T_1 T_2^3 - c_{15} c_{22} T_1 T_2^3 - c_{22} c_{36} T_1 T_2^3 - 3 c_{22} T_1 T_2^3 + c_{15} c_{22} T_1 T_2^3 +} \\
& \frac{c_{22} c_{56} T_1 T_2^3 + c_{22} c_{81} T_1 T_2^3 + c_{22} \alpha_4 - c_{81} \alpha_4 - c_{86} \alpha_4 - c_{56} T_1 \alpha_4 - c_{69} T_1 \alpha_4 + c_{13} T_2 \alpha_4 -}{c_{15} T_2 \alpha_4 - 2 c_{22} T_2 \alpha_4 + c_{32} T_2 \alpha_4 - c_{36} T_2 \alpha_4 - 3 T_1 T_2 \alpha_4 - c_{13} T_1 T_2 \alpha_4 + c_{15} T_1 T_2 \alpha_4 +} \\
& \frac{3 c_{22} T_1 T_2 \alpha_4 + c_{56} T_1 T_2 \alpha_4 + 2 c_{81} T_1 T_2 \alpha_4 + c_{86} T_1 T_2 \alpha_4 - 2 c_{22} T_1 T_2 \alpha_4 + c_{56} T_1 T_2 \alpha_4 +}{c_{69} T_1 T_2 \alpha_4 + c_{15} T_1 T_2 \alpha_4 - c_{32} T_1 T_2 \alpha_4 + c_{36} T_1 T_2 \alpha_4 + 3 T_1 T_2 \alpha_4 - c_{15} T_1 T_2 \alpha_4 - c_{56} T_1 T_2 \alpha_4 -} \\
& \frac{c_{81} T_1 T_2 \alpha_4 + \alpha_4^2 + T_2 \alpha_4^2 - 2 T_1 T_2 \alpha_4^2) p_{1,j} p_{3,j} x_{1,i} x_{3,i}}{\left(T_1^2 T_2 (-c_{22} + c_{22} T_2 - T_2 \alpha_4) \right) -} \\
& \frac{1}{(-1 + T_1) (-1 + T_1 T_2)} \left(-c_{13} + c_{22} + c_{32} - c_{81} - c_{86} + c_{13} T_1 - c_{22} T_1 - c_{56} T_1 - c_{69} T_1 + \right. \\
& \left. c_{22} T_1^2 - c_{15} T_2 - c_{36} T_2 - 3 T_1 T_2 + c_{15} T_1 T_2 - c_{22} T_1 T_2 - c_{32} T_1 T_2 + c_{56} T_1 T_2 + \right. \\
& \left. 2 c_{81} T_1 T_2 + c_{86} T_1 T_2 + c_{56} T_1 T_2 + c_{69} T_1 T_2 + c_{15} T_1 T_2^2 + c_{36} T_1 T_2^2 + 3 T_1 T_2^2 - \right. \\
& \left. c_{15} T_1 T_2^2 - c_{56} T_1 T_2^2 - c_{81} T_1 T_2^2 + T_1 \alpha_4 \right) p_{1,j} p_{3,i} x_{1,j} x_{3,i} - \frac{c_{22} p_{1,i} p_{3,j} x_{1,j} x_{3,i}}{T_2} + \\
& \left(\left(c_{13} c_{22} - c_{22}^2 - c_{13} c_{22} T_1 + 3 c_{22}^2 T_1 - 2 c_{22}^2 T_1^2 - c_{13} c_{22} T_2 + c_{22}^2 T_2 + c_{13} c_{22} T_1 T_2 - 3 c_{22}^2 T_1 T_2 + \right. \right. \\
& \left. \left. 2 c_{22}^2 T_1 T_2 + c_{22} \alpha_4 - c_{22} T_1 \alpha_4 + c_{13} T_2 \alpha_4 - 2 c_{22} T_2 \alpha_4 + c_{81} T_2 \alpha_4 + c_{86} T_2 \alpha_4 - c_{13} T_1 T_2 \alpha_4 + \right. \right. \\
& \left. \left. 3 c_{22} T_1 T_2 \alpha_4 + c_{56} T_1 T_2 \alpha_4 + c_{69} T_1 T_2 \alpha_4 - 2 c_{22} T_1^2 T_2 \alpha_4 + c_{15} T_2^2 \alpha_4 + c_{36} T_2^2 \alpha_4 + 3 T_1 T_2^2 \alpha_4 - \right. \right. \\
& \left. \left. c_{15} T_1 T_2^2 \alpha_4 + c_{22} T_1 T_2^2 \alpha_4 - c_{56} T_1 T_2^2 \alpha_4 - 2 c_{81} T_1 T_2^2 \alpha_4 - c_{86} T_1 T_2^2 \alpha_4 - c_{56} T_1 T_2^2 \alpha_4 - c_{69} T_1 T_2^2 \alpha_4 - \right. \right. \\
& \left. \left. \alpha_4 - c_{15} T_1 T_2^3 \alpha_4 - c_{36} T_1 T_2^3 \alpha_4 - 3 T_1 T_2^3 \alpha_4 + c_{15} T_1 T_2^3 \alpha_4 + c_{56} T_1 T_2^3 \alpha_4 + c_{81} T_1 T_2^3 \alpha_4 - T_1 T_2 \alpha_4^2 \right) \right. \\
& \left. p_{1,j} p_{3,j} x_{1,j} x_{3,i} \right) / \left((-1 + T_1) T_1 T_2 (-c_{22} + c_{22} T_2 - T_2 \alpha_4) \right) - \\
& \left(\left(-c_{22} c_{54} - c_{22} c_{81} - c_{22} c_{86} + c_{22} c_{54} T_1 - c_{22} c_{56} T_1 - c_{22} c_{69} T_1 + c_{22} c_{81} T_1 + c_{22} c_{86} T_1 + \right. \right. \\
& \left. \left. c_{22} c_{56} T_1^2 + c_{22} c_{69} T_1^2 - c_{15} c_{22} T_2 - c_{22} c_{36} T_2 + 2 c_{22} c_{54} T_2 + 2 c_{22} c_{81} T_2 + 2 c_{22} c_{86} T_2 - \right. \right. \\
& \left. \left. 3 c_{22} T_1 T_2 + 2 c_{15} c_{22} T_1 T_2 - c_{22}^2 T_1 T_2 + c_{22} c_{36} T_1 T_2 - 2 c_{22} c_{54} T_1 T_2 + 3 c_{22} c_{56} T_1 T_2 + \right. \right. \\
& \left. \left. 2 c_{22} c_{69} T_1 T_2 - c_{22} c_{86} T_1 T_2 + 3 c_{22} T_1 T_2 - c_{15} c_{22} T_1 T_2 + c_{22}^2 T_1 T_2 - 2 c_{22} c_{56} T_1 T_2 - \right. \right. \\
& \left. \left. c_{22} c_{69} T_1 T_2 - 2 c_{22} c_{81} T_1 T_2 - c_{22} c_{86} T_1 T_2 - c_{22} c_{56} T_1 T_2 - c_{22} c_{69} T_1 T_2 + 2 c_{15} c_{22} T_2^2 + \right. \right. \\
& \left. \left. 2 c_{22} c_{36} T_2^2 - c_{22} c_{54} T_2^2 - c_{22} c_{81} T_2^2 - c_{22} c_{86} T_2^2 + 6 c_{22} T_1 T_2^2 - 3 c_{15} c_{22} T_1 T_2^2 + 2 c_{22}^2 T_1 T_2^2 - \right. \right. \\
& \left. \left. c_{22} c_{36} T_1 T_2^2 + c_{22} c_{54} T_1 T_2^2 - 3 c_{22} c_{56} T_1 T_2^2 - c_{22} c_{69} T_1 T_2^2 - 3 c_{22} c_{81} T_1 T_2^2 - c_{22} c_{86} T_1 T_2^2 - \right. \right. \\
& \left. \left. 3 c_{22} T_1 T_2^2 - 2 c_{22}^2 T_1 T_2^2 - c_{22} c_{36} T_1 T_2^2 - c_{22} c_{69} T_1 T_2^2 + 3 c_{22} c_{81} T_1 T_2^2 + 2 c_{22} c_{86} T_1 T_2^2 - \right. \right. \\
& \left. \left. 3 c_{22} T_1^3 T_2 + c_{15} c_{22} T_1^3 T_2 + 3 c_{22} c_{56} T_1^3 T_2 + 2 c_{22} c_{69} T_1^3 T_2 + c_{22} c_{81} T_1^3 T_2 - c_{15} c_{22} T_2^3 - \right. \right. \\
& \left. \left. c_{22} c_{36} T_2^3 - 3 c_{22} T_1 T_2^3 - c_{22} c_{56} T_1 T_2^3 - c_{22} c_{69} T_1 T_2^3 + 2 c_{22} c_{81} T_1 T_2^3 + \right. \right. \\
& \left. \left. c_{22} c_{86} T_1 T_2^3 - 3 c_{22} T_1^2 T_2^3 + 3 c_{15} c_{22} T_1^2 T_2^3 + c_{22}^2 T_1^2 T_2^3 + 2 c_{22} c_{36} T_1^2 T_2^3 + 2 c_{22} c_{56} T_1^2 T_2^3 + \right. \right. \\
& \left. \left. c_{22} c_{69} T_1^2 T_2^3 - c_{22} c_{86} T_1^2 T_2^3 + 6 c_{22} T_1^3 T_2^3 - 2 c_{15} c_{22} T_1^3 T_2^3 - 3 c_{22} c_{56} T_1^3 T_2^3 - c_{22} c_{69} T_1^3 T_2^3 - \right. \right. \\
& \left. \left. 2 c_{22} c_{81} T_1^3 T_2^3 + c_{15} c_{22} T_1^2 T_2^4 + c_{22} c_{36} T_1^2 T_2^4 + 3 c_{22} T_1^2 T_2^4 - 2 c_{15} c_{22} T_1^2 T_2^4 - c_{22} c_{36} T_1^2 T_2^4 - \right. \right. \\
& \left. \left. c_{22} c_{56} T_1^2 T_2^4 - c_{22} c_{81} T_1^2 T_2^4 - 3 c_{22} T_1^3 T_2^4 + c_{15} c_{22} T_1^3 T_2^4 + c_{22} c_{56} T_1^3 T_2^4 + c_{22} c_{81} T_1^3 T_2^4 + \right. \right. \\
& \left. \left. c_{81} \alpha_4 + c_{86} \alpha_4 - c_{22} T_1 \alpha_4 + c_{56} T_1 \alpha_4 + c_{69} T_1 \alpha_4 + c_{15} T_2 \alpha_4 - c_{22} T_2 \alpha_4 + c_{36} T_2 \alpha_4 - \right. \right. \\
& \left. \left. c_{54} T_2 \alpha_4 - 2 c_{81} T_2 \alpha_4 - 2 c_{86} T_2 \alpha_4 + 3 T_1 T_2 \alpha_4 - c_{15} T_1 T_2 \alpha_4 + 3 c_{22} T_1 T_2 \alpha_4 + c_{54} T_1 T_2 \alpha_4 - \right. \right. \\
& \left. \left. 3 c_{56} T_1 T_2 \alpha_4 - 2 c_{69} T_1 T_2 \alpha_4 - 2 c_{81} T_1 T_2 \alpha_4 - c_{86} T_1 T_2 \alpha_4 + c_{22} T_1 T_2 \alpha_4 - c_{56} T_1 T_2 \alpha_4 - \right. \right. \\
& \left. \left. c_{69} T_1 T_2 \alpha_4 - 2 c_{15} T_2^2 \alpha_4 + c_{22} T_2^2 \alpha_4 - 2 c_{36} T_2^2 \alpha_4 + c_{54} T_2^2 \alpha_4 + c_{81} T_2^2 \alpha_4 + c_{86} T_2^2 \alpha_4 - \right. \right. \\
& \left. \left. 6 T_1 T_2^2 \alpha_4 + c_{15} T_1 T_2^2 \alpha_4 - 3 c_{22} T_1 T_2^2 \alpha_4 - c_{36} T_1 T_2^2 \alpha_4 - c_{54} T_1 T_2^2 \alpha_4 + 3 c_{56} T_1 T_2^2 \alpha_4 + \right. \right. \\
& \left. \left. c_{69} T_1 T_2^2 \alpha_4 + 4 c_{81} T_1 T_2^2 \alpha_4 + 2 c_{86} T_1 T_2^2 \alpha_4 - 3 T_1 T_2^2 \alpha_4 + c_{15} T_1 T_2^2 \alpha_4 - c_{22} T_1 T_2^2 \alpha_4 + \right. \right. \\
& \left. \left. 3 c_{56} T_1 T_2^2 \alpha_4 + 2 c_{69} T_1 T_2^2 \alpha_4 + c_{81} T_1 T_2^2 \alpha_4 + c_{15} T_1 T_2^3 \alpha_4 + c_{36} T_1 T_2^3 \alpha_4 + 3 T_1 T_2^3 \alpha_4 + c_{15} T_1 T_2^3 \alpha_4 + \right. \right. \\
& \left. \left. c_{22} T_1 T_2^3 \alpha_4 + 2 c_{36} T_1 T_2^3 \alpha_4 - c_{56} T_1 T_2^3 \alpha_4 - 2 c_{81} T_1 T_2^3 \alpha_4 - c_{86} T_1 T_2^3 \alpha_4 + 6 T_1 T_2^3 \alpha_4 - \right. \right. \\
& \left. \left. 2 c_{15} T_1 T_2^3 \alpha_4 - 3 c_{56} T_1 T_2^3 \alpha_4 - c_{69} T_1 T_2^3 \alpha_4 - 2 c_{81} T_1 T_2^3 \alpha_4 - c_{15} T_1 T_2^4 \alpha_4 - c_{36} T_1 T_2^4 \alpha_4 - \right. \right. \\
& \left. \left. 3 T_1 T_2^4 \alpha_4 + c_{15} T_1 T_2^4 \alpha_4 + c_{56} T_1 T_2^4 \alpha_4 + c_{81} T_1 T_2^4 \alpha_4 - \alpha_4^2 + T_2 \alpha_4^2 + T_1 T_2 \alpha_4^2 - T_2 \alpha_4^2 \right) \right.
\end{aligned}$$

$$\begin{aligned}
& \left(p_{2,j} p_{3,i} x_{2,i} x_{3,i} \right) / ((-1 + T_1) T_2 (-1 + T_1 T_2) (-c_{22} + c_{22} T_2 - T_2 \alpha_4)) - \\
& \left((c_{22}^2 - c_{22} c_{65} - c_{22} c_{81} - c_{22} c_{86} - c_{22} c_{56} T_1 + c_{22} c_{65} T_1 - c_{22} c_{69} T_1 + c_{22} c_{81} T_1 + c_{22} c_{86} T_1 - c_{22}^2 T_1^2 + \right. \\
& \quad c_{22} c_{56} T_1^2 + c_{22} c_{69} T_1^2 - c_{15} c_{22} T_2 - 2 c_{22}^2 T_2 - c_{22} c_{36} T_2 + c_{22} c_{65} T_2 + 2 c_{22} c_{81} T_2 + 2 c_{22} c_{86} T_2 - \\
& \quad 3 c_{22} T_1 T_2 + 2 c_{15} c_{22} T_1 T_2 - c_{22}^2 T_1 T_2 + c_{22} c_{36} T_1 T_2 + 3 c_{22} c_{56} T_1 T_2 + 2 c_{22} c_{69} T_1 T_2 - \\
& \quad c_{22} c_{86} T_1 T_2 + 3 c_{22} T_1^2 T_2 - c_{15} c_{22} T_1^2 T_2 + 3 c_{22}^2 T_1^2 T_2 - 2 c_{22} c_{56} T_1^2 T_2 - c_{22} c_{65} T_1^2 T_2 - \\
& \quad c_{22} c_{69} T_1^2 T_2 - 2 c_{22} c_{81} T_1^2 T_2 - c_{22} c_{86} T_1^2 T_2 - c_{22} c_{56} T_1^3 T_2 - c_{22} c_{69} T_1^3 T_2 + 2 c_{15} c_{22} T_1^2 T_2 + c_{22}^2 T_1^2 T_2 + \\
& \quad 2 c_{22} c_{36} T_1^2 T_2 - c_{22} c_{81} T_1^2 T_2 - c_{22} c_{86} T_1^2 T_2 + 6 c_{22} T_1 T_2^2 - 3 c_{15} c_{22} T_1 T_2^2 + 2 c_{22}^2 T_1 T_2^2 - c_{22} c_{36} T_1 T_2^2 - \\
& \quad 3 c_{22} c_{56} T_1 T_2^2 - c_{22} c_{65} T_1 T_2^2 - c_{22} c_{69} T_1 T_2^2 - 3 c_{22} c_{81} T_1 T_2^2 - c_{22} c_{86} T_1 T_2^2 - 3 c_{22} T_1^2 T_2^2 - \\
& \quad 3 c_{22}^2 T_1^2 T_2^2 - c_{22} c_{36} T_1^2 T_2^2 + c_{22} c_{65} T_1^2 T_2^2 - c_{22} c_{69} T_1^2 T_2^2 + 3 c_{22} c_{81} T_1^2 T_2^2 + 2 c_{22} c_{86} T_1^2 T_2^2 - \\
& \quad 3 c_{22} T_1^3 T_2^2 + 3 c_{15} c_{22} T_1^3 T_2^2 + 3 c_{22} c_{56} T_1^3 T_2^2 + 2 c_{22} c_{69} T_1^3 T_2^2 + c_{22} c_{81} T_1^3 T_2^2 - c_{15} c_{22} T_1^3 T_2^2 - \\
& \quad c_{22} c_{36} T_1^3 T_2^2 - 3 c_{22} T_1^3 T_2^2 - c_{22}^2 T_1^3 T_2^2 - c_{22} c_{36} T_1^3 T_2^2 + 2 c_{22} c_{56} T_1^3 T_2^2 + 2 c_{22} c_{81} T_1^3 T_2^2 - \\
& \quad 3 c_{22} T_1^2 T_2^3 + 3 c_{15} c_{22} T_1^2 T_2^3 + 3 c_{22} c_{56} T_1^2 T_2^3 + 2 c_{22} c_{69} T_1^2 T_2^3 + 2 c_{22} c_{81} T_1^2 T_2^3 - 2 c_{22} c_{86} T_1^2 T_2^3 + \\
& \quad c_{22} c_{86} T_1^2 T_2^3 + 6 c_{22} c_{22} T_1^3 T_2^3 - 2 c_{15} c_{22} T_1^3 T_2^3 - 3 c_{22} c_{56} T_1^3 T_2^3 - c_{22} c_{69} T_1^3 T_2^3 - 2 c_{22} c_{81} T_1^3 T_2^3 + \\
& \quad c_{15} c_{22} T_1 T_2^4 + c_{22} c_{36} T_1 T_2^4 + 3 c_{22} T_1^2 T_2^4 - 2 c_{15} c_{22} T_1^2 T_2^4 - c_{22} c_{36} T_1^2 T_2^4 - c_{22} c_{56} T_1^2 T_2^4 - \\
& \quad c_{22} c_{81} T_1^2 T_2^4 - 3 c_{22} T_1^3 T_2^4 + c_{15} c_{22} T_1^3 T_2^4 + c_{22} c_{56} T_1^3 T_2^4 + c_{22} c_{81} T_1^3 T_2^4 + c_{22} \alpha_4 + c_{81} \alpha_4 + c_{86} \alpha_4 - \\
& \quad 2 c_{22} T_1 \alpha_4 + c_{56} T_1 \alpha_4 + c_{69} T_1 \alpha_4 + c_{15} T_2 \alpha_4 + c_{36} T_2 \alpha_4 - c_{65} T_2 \alpha_4 - 2 c_{81} T_2 \alpha_4 - 2 c_{86} T_2 \alpha_4 + \\
& \quad 3 T_1 T_2 \alpha_4 - c_{15} T_1 T_2 \alpha_4 + 3 c_{22} T_1 T_2 \alpha_4 - 3 c_{56} T_1 T_2 \alpha_4 + c_{65} T_1 T_2 \alpha_4 - 2 c_{69} T_1 T_2 \alpha_4 - \\
& \quad 2 c_{81} T_1 T_2 \alpha_4 - c_{86} T_1 T_2 \alpha_4 - c_{56} T_1^2 T_2 \alpha_4 - c_{69} T_1^2 T_2 \alpha_4 - 2 c_{15} T_2^2 \alpha_4 - c_{22} T_2^2 \alpha_4 - 2 c_{36} T_2^2 \alpha_4 + \\
& \quad c_{81} T_2^2 \alpha_4 + c_{86} T_2^2 \alpha_4 - 6 T_1 T_2^2 \alpha_4 + c_{15} T_1 T_2^2 \alpha_4 - 2 c_{22} T_1 T_2^2 \alpha_4 - c_{36} T_1 T_2^2 \alpha_4 + 3 c_{56} T_1 T_2^2 \alpha_4 + \\
& \quad c_{65} T_1 T_2^2 \alpha_4 + c_{69} T_1 T_2^2 \alpha_4 + 4 c_{81} T_1 T_2^2 \alpha_4 + 2 c_{86} T_1 T_2^2 \alpha_4 - 3 T_1^2 T_2^2 \alpha_4 + c_{15} T_2^2 \alpha_4 + \\
& \quad 3 c_{56} T_1^2 T_2^2 \alpha_4 - c_{65} T_1^2 T_2^2 \alpha_4 + 2 c_{69} T_1^2 T_2^2 \alpha_4 + c_{81} T_1^2 T_2^2 \alpha_4 + c_{15} T_2^3 \alpha_4 + c_{36} T_2^3 \alpha_4 + 3 T_1 T_2^3 \alpha_4 + \\
& \quad c_{15} T_1 T_2^3 \alpha_4 + c_{22} T_1 T_2^3 \alpha_4 + 2 c_{36} T_1 T_2^3 \alpha_4 - c_{56} T_1 T_2^3 \alpha_4 - 2 c_{81} T_1 T_2^3 \alpha_4 - c_{86} T_1 T_2^3 \alpha_4 + \\
& \quad 6 T_1^2 T_2^3 \alpha_4 - 2 c_{15} T_1^2 T_2^3 \alpha_4 - 3 c_{56} T_1^2 T_2^3 \alpha_4 - c_{69} T_1^2 T_2^3 \alpha_4 - 2 c_{81} T_1^2 T_2^3 \alpha_4 - c_{15} T_1 T_2^4 \alpha_4 - \\
& \quad c_{36} T_1 T_2^4 \alpha_4 - 3 T_1^2 T_2^4 \alpha_4 + c_{15} T_1^2 T_2^4 \alpha_4 + c_{56} T_1^2 T_2^4 \alpha_4 + c_{81} T_1^2 T_2^4 \alpha_4 - \alpha_4^2 + 2 T_2 \alpha_4^2 - T_1 T_2^2 \alpha_4^2) \\
& \left(p_{2,i} p_{3,j} x_{2,i} x_{3,i} \right) / ((-1 + T_1) T_1 (-1 + T_2) T_2 (-c_{22} + c_{22} T_2 - T_2 \alpha_4)) + \\
& \left((c_{22}^2 - c_{22} c_{54} - c_{22} c_{65} - c_{22} c_{81} - c_{22} c_{86} + c_{22} c_{54} T_1 - c_{22} c_{56} T_1 + c_{22} c_{65} T_1 - c_{22} c_{69} T_1 + \right. \\
& \quad c_{22} c_{81} T_1 + c_{22} c_{86} T_1 - c_{22}^2 T_1^2 + c_{22} c_{56} T_1^2 + c_{22} c_{69} T_1^2 - c_{15} c_{22} T_2 - 2 c_{22}^2 T_2 - c_{22} c_{36} T_2 + \\
& \quad 2 c_{22} c_{54} T_2 + c_{22} c_{65} T_2 + 3 c_{22} c_{81} T_2 + 3 c_{22} c_{86} T_2 - 3 c_{22} T_1 T_2 + 2 c_{15} c_{22} T_1 T_2 - 2 c_{22}^2 T_1 T_2 + \\
& \quad c_{22} c_{36} T_1 T_2 - 2 c_{22} c_{54} T_1 T_2 + 4 c_{22} c_{56} T_1 T_2 + 3 c_{22} c_{69} T_1 T_2 - c_{22} c_{81} T_1 T_2 - 2 c_{22} c_{86} T_1 T_2 + \\
& \quad 3 c_{22} T_1^2 T_2 - c_{15} c_{22} T_1^2 T_2 + 4 c_{22}^2 T_1^2 T_2 - 3 c_{22} c_{56} T_1^2 T_2 - c_{22} c_{65} T_1^2 T_2 - 2 c_{22} c_{69} T_1^2 T_2 - \\
& \quad 2 c_{22} c_{81} T_1^2 T_2 - c_{22} c_{86} T_1^2 T_2 - c_{22} c_{56} T_1^3 T_2 - c_{22} c_{69} T_1^3 T_2 + 3 c_{15} c_{22} T_2^2 + c_{22}^2 T_2^2 + 3 c_{22} c_{36} T_2^2 - \\
& \quad c_{22} c_{54} T_2^2 - 2 c_{22} c_{81} T_2^2 - 2 c_{22} c_{86} T_2^2 + 9 c_{22} T_1 T_2^2 - 5 c_{15} c_{22} T_1 T_2^2 + 4 c_{22}^2 T_1 T_2^2 - 2 c_{22} c_{36} T_1 T_2^2 + \\
& \quad c_{22} c_{54} T_1 T_2^2 - 5 c_{22} c_{56} T_1 T_2^2 - c_{22} c_{65} T_1 T_2^2 - 2 c_{22} c_{69} T_1 T_2^2 - 4 c_{22} c_{81} T_1 T_2^2 - c_{22} c_{86} T_1 T_2^2 - \\
& \quad 6 c_{22} T_1^2 T_2^2 + c_{15} c_{22} T_1^2 T_2^2 - 5 c_{22}^2 T_1^2 T_2^2 - c_{22} c_{36} T_1^2 T_2^2 + c_{22} c_{56} T_1^2 T_2^2 + c_{22} c_{65} T_1^2 T_2^2 - \\
& \quad c_{22} c_{69} T_1^2 T_2^2 + 5 c_{22} c_{81} T_1^2 T_2^2 + 3 c_{22} c_{86} T_1^2 T_2^2 - 3 c_{22} T_1^3 T_2^2 + c_{15} c_{22} T_1^3 T_2^2 + 4 c_{22} c_{56} T_1^3 T_2^2 + \\
& \quad 3 c_{22} c_{69} T_1^3 T_2^2 + c_{22} c_{81} T_1^3 T_2^2 - 2 c_{15} c_{22} T_1^3 T_2^2 - 2 c_{22} c_{36} T_1^3 T_2^2 - 6 c_{22} T_1^4 T_2^2 + c_{15} c_{22} T_1^4 T_2^2 - \\
& \quad 2 c_{22}^2 T_1^4 T_2^2 + c_{22} c_{36} T_1^4 T_2^2 + 2 c_{22} c_{56} T_1^4 T_2^2 + 4 c_{22} c_{81} T_1^4 T_2^2 + 2 c_{22} c_{86} T_1^4 T_2^2 - 3 c_{22} T_1^5 T_2^2 + \\
& \quad 4 c_{15} c_{22} T_1^2 T_3^2 + 2 c_{22}^2 T_1^2 T_3^2 + 3 c_{22} c_{36} T_1^2 T_3^2 + 3 c_{22} c_{56} T_1^2 T_3^2 + 2 c_{22} c_{69} T_1^2 T_3^2 - c_{22} c_{81} T_1^2 T_3^2 - \\
& \quad 2 c_{22} c_{86} T_1^2 T_3^2 + 9 c_{22} T_1^3 T_3^2 - 3 c_{15} c_{22} T_1^3 T_3^2 - 5 c_{22} c_{56} T_1^3 T_3^2 - 2 c_{22} c_{69} T_1^3 T_3^2 - 3 c_{22} c_{81} T_1^3 T_3^2 + \\
& \quad 2 c_{15} c_{22} T_1 T_4^2 + 2 c_{22} c_{36} T_1 T_4^2 + 6 c_{22} T_1^2 T_4^2 - 4 c_{15} c_{22} T_1^2 T_4^2 - 2 c_{22} c_{36} T_1^2 T_4^2 - 2 c_{22} c_{56} T_1^2 T_4^2 - \\
& \quad 2 c_{22} c_{81} T_1 T_4^2 - 6 c_{22} T_1^3 T_4^2 + 2 c_{15} c_{22} T_1^3 T_4^2 + 2 c_{22} c_{56} T_1^3 T_4^2 + 2 c_{22} c_{81} T_1^3 T_4^2 + c_{22} \alpha_4 + c_{81} \alpha_4 + \\
& \quad c_{86} \alpha_4 - 2 c_{22} T_1 \alpha_4 + c_{56} T_1 \alpha_4 + c_{69} T_1 \alpha_4 + c_{15} T_2 \alpha_4 - c_{22} T_2 \alpha_4 + c_{36} T_2 \alpha_4 - c_{54} T_2 \alpha_4 - \\
& \quad c_{65} T_2 \alpha_4 - 3 c_{81} T_2 \alpha_4 - 3 c_{86} T_2 \alpha_4 + 3 T_1 T_2 \alpha_4 - c_{15} T_1 T_2 \alpha_4 + 5 c_{22} T_1 T_2 \alpha_4 + c_{54} T_1 T_2 \alpha_4 - \\
& \quad 4 c_{56} T_1 T_2 \alpha_4 + c_{65} T_1 T_2 \alpha_4 - 3 c_{69} T_1 T_2 \alpha_4 - 2 c_{81} T_1 T_2 \alpha_4 - c_{86} T_1 T_2 \alpha_4 - c_{56} T_1^2 T_2 \alpha_4 - \\
& \quad c_{69} T_1^2 T_2 \alpha_4 - 3 c_{15} T_2^2 \alpha_4 - 3 c_{36} T_2^2 \alpha_4 + c_{54} T_2^2 \alpha_4 + 2 c_{81} T_2^2 \alpha_4 + 2 c_{86} T_2^2 \alpha_4 - 9 T_1 T_2^2 \alpha_4 + \\
& \quad 2 c_{15} T_1 T_2^2 \alpha_4 - 5 c_{22} T_1 T_2^2 \alpha_4 - c_{36} T_1 T_2^2 \alpha_4 - c_{54} T_1 T_2^2 \alpha_4 + 5 c_{56} T_1 T_2^2 \alpha_4 + c_{65} T_1 T_2^2 \alpha_4 +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{T_1 T_2} (-c_{56} T_1 - c_{69} T_1 - c_{15} T_2 - c_{36} T_2 - 3 T_1 T_2 + c_{15} T_1 T_2 + c_{56} T_1 T_2 + c_{81} T_1 T_2) p_{3,j} x_{3,j} - \\
& \left((-c_{13} c_{22} + c_{22} c_{32} + c_{13} c_{22} T_1 - c_{22} c_{81} T_1 - c_{22} c_{86} T_1 + c_{22}^2 T_1^2 - c_{22} c_{56} T_1^2 - c_{22} c_{69} T_1^2 + c_{13} c_{22} T_2 - \right. \\
& \quad c_{22} c_{32} T_2 - c_{13} c_{22} T_1 T_2 - c_{15} c_{22} T_1 T_2 - c_{22} c_{32} T_1 T_2 - c_{22} c_{36} T_1 T_2 + c_{22} c_{81} T_1 T_2 + \\
& \quad c_{22} c_{86} T_1 T_2 - 3 c_{22} T_1^2 T_2 + c_{15} c_{22} T_1^2 T_2 - 2 c_{22}^2 T_1^2 T_2 + 2 c_{22} c_{56} T_1^2 T_2 + c_{22} c_{69} T_1^2 T_2 + \\
& \quad 2 c_{22} c_{81} T_1^2 T_2 + c_{22} c_{86} T_1^2 T_2 + c_{22} c_{56} T_1^3 T_2 + c_{22} c_{69} T_1^3 T_2 + c_{15} c_{22} T_1 T_2^2 + c_{22} c_{32} T_1 T_2^2 + \\
& \quad c_{22} c_{36} T_1 T_2^2 + 3 c_{22} T_1^2 T_2^2 + c_{22}^2 T_1^2 T_2^2 + c_{22} c_{36} T_1^2 T_2^2 - c_{22} c_{56} T_1^2 T_2^2 - 2 c_{22} c_{81} T_1^2 T_2^2 - \\
& \quad c_{22} c_{86} T_1^2 T_2^2 + 3 c_{22} T_1^3 T_2^2 - c_{15} c_{22} T_1^3 T_2^2 - 2 c_{22} c_{56} T_1^3 T_2^2 - c_{22} c_{69} T_1^3 T_2^2 - c_{22} c_{81} T_1^3 T_2^2 - \\
& \quad c_{15} c_{22} T_1^2 T_3^2 - c_{22} c_{36} T_1^2 T_3^2 - 3 c_{22} T_1^3 T_3^2 + c_{15} c_{22} T_1^3 T_3^2 + c_{22} c_{56} T_1^3 T_3^2 + c_{22} c_{81} T_1^3 T_3^2 - \\
& \quad c_{81} \alpha_4 - c_{86} \alpha_4 + 2 c_{22} T_1 \alpha_4 - c_{56} T_1 \alpha_4 - c_{69} T_1 \alpha_4 - c_{13} T_2 \alpha_4 - c_{15} T_2 \alpha_4 + c_{32} T_2 \alpha_4 - \\
& \quad c_{36} T_2 \alpha_4 - 3 T_1 T_2 \alpha_4 + c_{13} T_1 T_2 \alpha_4 + c_{15} T_1 T_2 \alpha_4 - 2 c_{22} T_1 T_2 \alpha_4 + c_{56} T_1 T_2 \alpha_4 + \\
& \quad 2 c_{81} T_1 T_2 \alpha_4 + c_{86} T_1 T_2 \alpha_4 + c_{56} T_1^2 T_2 \alpha_4 + c_{69} T_1^2 T_2 \alpha_4 + c_{15} T_1 T_2^2 \alpha_4 - c_{32} T_1 T_2^2 \alpha_4 + \\
& \quad c_{36} T_1 T_2^2 \alpha_4 + 3 T_1^2 T_2 \alpha_4 - c_{15} T_1^2 T_2 \alpha_4 - c_{56} T_1^2 T_2 \alpha_4 - c_{81} T_1^2 T_2 \alpha_4 + \alpha_4^2) p_{1,i} p_{3,j} x_{1,i} x_{3,j}) / \\
& \quad \frac{1}{((-1 + T_1) (-1 + T_1 T_2) (-c_{22} + c_{22} T_2 - T_2 \alpha_4)) + \frac{1}{T_1 (-c_{22} + c_{22} T_2 - T_2 \alpha_4)}} \\
& \quad \left(-c_{22} c_{32} - c_{22}^2 T_1 + c_{22} c_{81} T_1 + c_{22} c_{86} T_1 + c_{22} c_{56} T_1^2 + c_{22} c_{69} T_1^2 + \right. \\
& \quad c_{22} c_{32} T_2 + c_{15} c_{22} T_1 T_2 + c_{22}^2 T_1 T_2 + c_{22} c_{36} T_1 T_2 - c_{22} c_{81} T_1 T_2 - \\
& \quad c_{22} c_{86} T_1 T_2 + 3 c_{22} T_1^2 T_2 - c_{15} c_{22} T_1^2 T_2 - 2 c_{22} c_{56} T_1^2 T_2 - \\
& \quad c_{22} c_{69} T_1^2 T_2 - c_{22} c_{81} T_1^2 T_2 - c_{15} c_{22} T_1 T_2^2 - c_{22} c_{36} T_1 T_2^2 - 3 c_{22} T_1 T_2^2 + \\
& \quad c_{15} c_{22} T_1^2 T_2^2 + c_{22} c_{56} T_1^2 T_2^2 + c_{22} c_{81} T_1^2 T_2^2 + c_{81} \alpha_4 + c_{86} \alpha_4 - c_{22} T_1 \alpha_4 + \\
& \quad c_{56} T_1 \alpha_4 + c_{69} T_1 \alpha_4 + c_{15} T_2 \alpha_4 - c_{32} T_2 \alpha_4 + c_{36} T_2 \alpha_4 + 3 T_1 T_2 \alpha_4 - \\
& \quad c_{15} T_1 T_2 \alpha_4 - c_{56} T_1 T_2 \alpha_4 - c_{81} T_1 T_2 \alpha_4 - \alpha_4^2) p_{1,j} p_{3,j} x_{1,i} x_{3,j} + \\
& \quad \left. (-c_{22} - c_{54} + c_{65} + c_{22} T_2 + c_{54} T_2 - c_{65} T_1 T_2 - T_2 \alpha_4) p_{2,i} p_{3,j} x_{2,i} x_{3,j} \right) / \\
& \quad (-1 + T_2) (-1 + T_1 T_2) \\
& \quad \left((-c_{22}^2 + c_{22} c_{65} + c_{22}^2 T_1 - c_{22} c_{65} T_1 + 2 c_{22}^2 T_2 - c_{22} c_{65} T_2 - c_{22} c_{81} T_2 - c_{22} c_{86} T_2 - 2 c_{22}^2 T_1 T_2 - \right. \\
& \quad c_{22} c_{56} T_1 T_2 + c_{22} c_{65} T_1 T_2 - c_{22} c_{69} T_1 T_2 + c_{22} c_{81} T_1 T_2 + c_{22} c_{86} T_1 T_2 + c_{22} c_{56} T_1^2 T_2 + \\
& \quad c_{22} c_{69} T_1^2 T_2 - c_{15} c_{22} T_2^2 - c_{22}^2 T_2^2 - c_{22} c_{36} T_2^2 + c_{22} c_{81} T_2^2 + c_{22} c_{86} T_2^2 - 3 c_{22} T_1 T_2^2 + \\
& \quad 2 c_{15} c_{22} T_1 T_2^2 + c_{22}^2 T_1 T_2^2 + c_{22} c_{36} T_1 T_2^2 + 2 c_{22} c_{56} T_1 T_2^2 + c_{22} c_{69} T_1 T_2^2 - c_{22} c_{86} T_1 T_2^2 + \\
& \quad 3 c_{22} T_1^2 T_2^2 - c_{15} c_{22} T_1^2 T_2^2 - 2 c_{22} c_{56} T_1^2 T_2^2 - c_{22} c_{69} T_1^2 T_2^2 - c_{22} c_{81} T_1^2 T_2^2 + c_{15} c_{22} T_2^3 + \\
& \quad c_{22} c_{36} T_2^3 + 3 c_{22} T_1 T_2^3 - 2 c_{15} c_{22} T_1 T_2^3 - c_{22} c_{36} T_1 T_2^3 - c_{22} c_{56} T_1 T_2^3 - c_{22} c_{81} T_1 T_2^3 - \\
& \quad 3 c_{22} T_1^2 T_2^3 + c_{15} c_{22} T_1^2 T_2^3 + c_{22} c_{56} T_1^2 T_2^3 + c_{22} c_{81} T_1^2 T_2^3 - c_{22} T_2 \alpha_4 + c_{65} T_2 \alpha_4 + c_{81} T_2 \alpha_4 + \\
& \quad c_{86} T_2 \alpha_4 + c_{56} T_1 T_2 \alpha_4 - c_{65} T_1 T_2 \alpha_4 + c_{69} T_1 T_2 \alpha_4 + c_{15} T_2^2 \alpha_4 + c_{22} T_2^2 \alpha_4 + c_{36} T_2^2 \alpha_4 - \\
& \quad c_{81} T_2^2 \alpha_4 - c_{86} T_2^2 \alpha_4 + 3 T_1 T_2^2 \alpha_4 - c_{15} T_1 T_2^2 \alpha_4 - 2 c_{56} T_1 T_2^2 \alpha_4 - c_{69} T_1 T_2^2 \alpha_4 - c_{81} T_1 T_2^2 \alpha_4 - \\
& \quad c_{15} T_2^3 \alpha_4 - c_{36} T_2^3 \alpha_4 - 3 T_1 T_2^3 \alpha_4 + c_{15} T_1 T_2^3 \alpha_4 + c_{56} T_1 T_2^3 \alpha_4 + c_{81} T_1 T_2^3 \alpha_4 - T_2 \alpha_4^2) \\
& \quad p_{2,j} p_{3,j} x_{2,i} x_{3,j}) / ((-1 + T_1) T_2 (-c_{22} + c_{22} T_2 - T_2 \alpha_4)) + \\
& \quad \left((c_{22} + c_{22}^2 - c_{22} c_{81} - c_{22} c_{86} - c_{22} T_1 - c_{22}^2 T_1 - c_{22} c_{56} T_1 - c_{22} c_{69} T_1 + c_{22} c_{81} T_1 + c_{22} c_{86} T_1 + \right. \\
& \quad c_{22} c_{56} T_1^2 + c_{22} c_{69} T_1^2 - c_{22} T_2 - c_{15} c_{22} T_2 - c_{22}^2 T_2 - c_{22} c_{36} T_2 + c_{22} c_{81} T_2 + c_{22} c_{86} T_2 - \\
& \quad 2 c_{22} T_1 T_2 + 2 c_{15} c_{22} T_1 T_2 + c_{22}^2 T_1 T_2 + c_{22} c_{36} T_1 T_2 + 2 c_{22} c_{56} T_1 T_2 + c_{22} c_{69} T_1 T_2 - \\
& \quad c_{22} c_{86} T_1 T_2 + 3 c_{22} T_1^2 T_2 - c_{15} c_{22} T_1^2 T_2 - 2 c_{22} c_{56} T_1^2 T_2 - c_{22} c_{69} T_1^2 T_2 - c_{22} c_{81} T_1^2 T_2 + \\
& \quad c_{15} c_{22} T_2^2 + c_{22} c_{36} T_2^2 + 3 c_{22} T_1 T_2^2 - 2 c_{15} c_{22} T_1 T_2^2 - c_{22} c_{36} T_1 T_2^2 - c_{22} c_{56} T_1 T_2^2 - \\
& \quad c_{22} c_{81} T_1 T_2^2 - 3 c_{22} T_1^2 T_2^2 + c_{15} c_{22} T_1^2 T_2^2 + c_{22} c_{56} T_1^2 T_2^2 + c_{22} c_{81} T_1^2 T_2^2 + c_{81} \alpha_4 + c_{86} \alpha_4 - \\
& \quad c_{22} T_1 \alpha_4 + c_{56} T_1 \alpha_4 + c_{69} T_1 \alpha_4 + T_2 \alpha_4 + c_{15} T_2 \alpha_4 + c_{22} T_2 \alpha_4 + c_{36} T_2 \alpha_4 - c_{81} T_2 \alpha_4 - \\
& \quad c_{86} T_2 \alpha_4 + 2 T_1 T_2 \alpha_4 - c_{15} T_1 T_2 \alpha_4 - 2 c_{56} T_1 T_2 \alpha_4 - c_{69} T_1 T_2 \alpha_4 - c_{81} T_1 T_2 \alpha_4 - \\
& \quad c_{15} T_2^2 \alpha_4 - c_{36} T_2^2 \alpha_4 - 3 T_1 T_2^2 \alpha_4 + c_{15} T_1 T_2^2 \alpha_4 + c_{56} T_1 T_2^2 \alpha_4 + c_{81} T_1 T_2^2 \alpha_4 - \alpha_4^2)
\end{aligned}$$

$$\begin{aligned}
& \left(p_{3,i} p_{3,j} x_{3,i} x_{3,j} \right) / ((-1 + T_1) (-c_{22} + c_{22} T_2 - T_2 \alpha_4)) - \\
& \left((-c_{22} - c_{22}^2 - 2 c_{22} c_{78} + c_{22} c_{81} + c_{22} c_{86} + c_{22} T_1 + c_{22}^2 T_1 + c_{22} c_{56} T_1 + c_{22} c_{69} T_1 + \right. \\
& 2 c_{22} c_{78} T_1 - c_{22} c_{81} T_1 - c_{22} c_{86} T_1 - c_{22} c_{56} T_1^2 - c_{22} c_{69} T_1^2 + c_{22} T_2 + c_{15} c_{22} T_2 + \\
& c_{22}^2 T_2 + c_{22} c_{36} T_2 + 2 c_{22} c_{78} T_2 - c_{22} c_{81} T_2 - c_{22} c_{86} T_2 + 5 c_{22} T_1 T_2 - 2 c_{15} c_{22} T_1 T_2 + \\
& 2 c_{22}^2 T_1 T_2 - c_{22} c_{36} T_1 T_2 - 2 c_{22} c_{56} T_1 T_2 - c_{22} c_{69} T_1 T_2 - 2 c_{22} c_{78} T_1 T_2 - 3 c_{22} c_{81} T_1 T_2 - \\
& 2 c_{22} c_{86} T_1 T_2 - 6 c_{22} T_1^2 T_2 + c_{15} c_{22} T_1^2 T_2 - 3 c_{22}^2 T_1^2 T_2 - c_{22} c_{56} T_1^2 T_2 - 2 c_{22} c_{69} T_1^2 T_2 + \\
& 4 c_{22} c_{81} T_1^2 T_2 + 3 c_{22} c_{86} T_1^2 T_2 + 3 c_{22} c_{56} T_1^3 T_2 + 3 c_{22} c_{69} T_1^3 T_2 - c_{15} c_{22} T_1^2 T_2 - c_{22} c_{36} T_1^2 T_2 - \\
& 6 c_{22} T_1 T_2^2 - c_{15} c_{22} T_1 T_2^2 - 3 c_{22}^2 T_1 T_2^2 - 2 c_{22} c_{36} T_1 T_2^2 + c_{22} c_{56} T_1 T_2^2 + 4 c_{22} c_{81} T_1 T_2^2 + \\
& 3 c_{22} c_{86} T_1 T_2^2 - 3 c_{22} T_1^2 T_2^2 + 5 c_{15} c_{22} T_1^2 T_2^2 + 3 c_{22}^2 T_1^2 T_2^2 + 3 c_{22} c_{36} T_1^2 T_2^2 + 5 c_{22} c_{56} T_1^2 T_2^2 + \\
& 3 c_{22} c_{69} T_1^2 T_2^2 - c_{22} c_{81} T_1^2 T_2^2 - 3 c_{22} c_{86} T_1^2 T_2^2 + 9 c_{22} T_1^3 T_2^2 - 3 c_{15} c_{22} T_1^3 T_2^2 - 6 c_{22} c_{56} T_1^3 T_2^2 - \\
& 3 c_{22} c_{69} T_1^3 T_2^2 - 3 c_{22} c_{81} T_1^3 T_2^2 + 3 c_{15} c_{22} T_1^3 T_2^2 + 3 c_{22} c_{36} T_1^3 T_2^2 + 9 c_{22} T_1^2 T_2^3 - 6 c_{15} c_{22} T_1^2 T_2^3 - \\
& 3 c_{22} c_{36} T_1^2 T_2^3 - 3 c_{22} c_{56} T_1^2 T_2^3 - 3 c_{22} c_{81} T_1^2 T_2^3 - 9 c_{22} T_1^3 T_2^3 + 3 c_{15} c_{22} T_1^3 T_2^3 + 3 c_{22} c_{56} T_1^3 T_2^3 + \\
& 3 c_{22} c_{81} T_1^3 T_2^3 - c_{81} \alpha_4 - c_{86} \alpha_4 + c_{22} T_1 \alpha_4 - c_{56} T_1 \alpha_4 - c_{69} T_1 \alpha_4 - T_2 \alpha_4 - c_{15} T_2 \alpha_4 - \\
& c_{22} T_2 \alpha_4 - c_{36} T_2 \alpha_4 - 2 c_{78} T_2 \alpha_4 + c_{81} T_2 \alpha_4 + c_{86} T_2 \alpha_4 - 2 T_1 T_2 \alpha_4 + c_{15} T_1 T_2 \alpha_4 + \\
& 2 c_{56} T_1 T_2 \alpha_4 + c_{69} T_1 T_2 \alpha_4 + 2 c_{78} T_1 T_2 \alpha_4 + 4 c_{81} T_1 T_2 \alpha_4 + 3 c_{86} T_1 T_2 \alpha_4 - 3 c_{22} T_1^2 T_2 \alpha_4 + \\
& 3 c_{56} T_1^2 T_2 \alpha_4 + 3 c_{69} T_1^2 T_2 \alpha_4 + c_{15} T_1^2 T_2 \alpha_4 + c_{36} T_1^2 T_2 \alpha_4 + 6 T_1 T_2^2 \alpha_4 + 2 c_{15} T_1 T_2^2 \alpha_4 + \\
& 3 c_{22} T_1 T_2^2 \alpha_4 + 3 c_{36} T_1 T_2^2 \alpha_4 - c_{56} T_1 T_2^2 \alpha_4 - 4 c_{81} T_1 T_2^2 \alpha_4 - 3 c_{86} T_1 T_2^2 \alpha_4 + 6 T_1^2 T_2^2 \alpha_4 - \\
& 3 c_{15} T_1^2 T_2^2 \alpha_4 - 6 c_{56} T_1^2 T_2^2 \alpha_4 - 3 c_{69} T_1^2 T_2^2 \alpha_4 - 3 c_{81} T_1^2 T_2^2 \alpha_4 - 3 c_{15} T_1 T_2^3 \alpha_4 - 3 c_{36} T_1 T_2^3 \alpha_4 - \\
& 9 T_1^2 T_2^3 \alpha_4 + 3 c_{15} T_1^2 T_2^3 \alpha_4 + 3 c_{56} T_1^2 T_2^3 \alpha_4 + 3 c_{81} T_1^2 T_2^3 \alpha_4 + \alpha_4^2 - 3 T_1 T_2 \alpha_4^2) p_{3,j} x_{3,i} x_{3,j} \Big) / \\
& (2 (-1 + T_1) T_1 T_2 (-c_{22} + c_{22} T_2 - T_2 \alpha_4)) /. \alpha_4 \rightarrow z
\end{aligned}$$

```
In[1]:= {p*, x*, π*, ξ*} = {π, ξ, p, x}; (u_i_) := (u*) i;
```

```
In[2]:= Zip{}[ε_] := ε;
Zip{ξ_,ξs___}[ε_] := (Collect[ε // Zip{ξs}, ξ] /. f_. ξ^d_ → (D[f, {ξ*, d}])) /. ξ* → 0
```

```
In[3]:= px2g[ε_] := Module[{ps, xs, Q},
  ps = Union[Cases[ε, p__], ∞];
  xs = Union[Cases[ε, x__], ∞];
  Q = Sum[p0^x0^g_{p0[[2]],x0[[2]]}, p0[[3]], x0[[3]], {p0, ps}, {x0, xs}];
  Expand[Zip[ps ∪ xs][ε e^Q] /. g_{α_, β_, i_, j_} → If[α == β, g_{α, i, j}, 0]]
]
```

The simplifying substitutions:

```
In[4]:= EasySubs = {c4|10|13|15|22|28|32|36|49|54|56|65|69|69|78|81|86 → 0, B → 1};
(* {z → 0, c15 → 0, c22 → 0, c36 → 0, c56 → 0, c69 → 0, c81 → 0, c86 → 0} *);
```

```
In[1]:= R1[1, i_, j_] := Evaluate[
  Together[(px2g[r1[1, i, j]] + (Coefficient[r1[1, i, j] /. t : (x | p) :> λ t, λ^3] /.
    x3,α_ p1,β_ p2,γ_ :> yα,β,γ)) /. EasySubs] /. z → 0]
R1[-1, i_, j_] := Evaluate[
  Together[(px2g[r1[-1, i, j]] + (Coefficient[r1[-1, i, j] /. t : (x | p) :> λ t, λ^3] /.
    x3,α_ p1,β_ p2,γ_ :> yα,β,γ)) /. EasySubs] /. z → 0]
```

```
In[2]:= Θ[1, i_, j_, α_, β_, γ_] :=
  Evaluate[rθ[1, i, j] /. {p3,j_ :> g3,α,j, x1,i_ :> g1,i,β, x2,i_ :> g2,i,γ} /. EasySubs /. z → 0];
(* The Θ graph with light (pxx) vertex at (1,i,j) and
unspecified heavy (xpp) vertex *)
Θ[-1, i_, j_, α_, β_, γ_] := Evaluate[
  rθ[-1, i, j] /. {p3,j_ :> g3,α,j, x1,i_ :> g1,i,β, x2,i_ :> g2,i,γ} /. EasySubs /. z → 0];
(* The Θ graph with light (pxx) vertex at (-1,i,j) and
unspecified heavy (xpp) vertex *)
```

```
In[3]:= T3 = T1 T2;
CF[ε_] := Factor@Together[ε];
λ[K_] := Module[{Cs, ϕ, n, A, s, i, j, k, Δ, G, gEval, Y, yEval, c, λ1},
  {Cs, ϕ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :> (A[[{i, j}], {i + 1, j + 1}] += {{-T^s T^s - 1}, {0, -1}})];
  Δ = T^{(-Total[ϕ] - Total[Cs[[All, 1]])/2} Det[A];
  G = Inverse[A];
  gEval[ε_] := CF[ε] /.
    {g1,α_,β_ :> (G[[α, β]] /. T → T1),
     g2,α_,β_ :> (G[[α, β]] /. T → T2), g3,α_,β_ :> (G[[α, β]] /. T → T3)};
  Y[α_, β_, γ_] :=
    Y[α, β, γ] = Sum[{s, i, j} = c; (* The expectation value of x3,α p1,β p2,γ *)
      Θ[s, i, j, α, β, γ],
      {c, Cs}]];
  yEval[ε_] := ε /. Y[α, β, γ] :> Y[α, β, γ];
  λ1 = Sum[Cs[[k]] - Sum[Cs[[k]] (g1,k,k + g2,k,k + g3,k,k - 3/2)];
  {Δ, (Δ^2 /. T → T1) (Δ^2 /. T → T2) (Δ^2 /. T → T3) λ1} // yEval // gEval
  ];
];
```

```
In[]:= ygs = Flatten@Join[Table[{ga,b,c, {a, 1, 3}, {b, {i, j}}, {c, {i, j}}}],
Table[{ya,b,c, {a, {i, j}}, {b, {i, j}}, {c, {i, j}}}]
ygCF[&_] := Total@CoefficientRules[&, ygs] /.
{(u_ → x_) → (Factor@Together[x] × Product[ygs[[w]]^u[[w]], {w, 1, Length[ygs]}])}
```

Out[]= {g_{1,i,i}, g_{1,i,j}, g_{1,j,i}, g_{1,j,j}, g_{2,i,i}, g_{2,i,j}, g_{2,j,i}, g_{2,j,j}, g_{3,i,i}, g_{3,i,j}, g_{3,j,i}, g_{3,j,j}, y_{i,i,i}, y_{i,i,j}, y_{i,j,i}, y_{i,j,j}, y_{j,i,i}, y_{j,i,j}, y_{j,j,i}, y_{j,j,j}}

```
In[]:= ygCF[R1[1, i, j]]
```

Out[]=
$$\begin{aligned} & -\frac{3}{2} - g_{1,i,j} g_{1,j,i} - (-1 + T_1) T_1 g_{1,j,i}^2 - g_{1,i,i} g_{1,j,j} + (1 + T_1) g_{1,j,i} g_{1,j,j} - \frac{3 T_1 (-1 + T_2) g_{1,j,j} g_{2,i,i}}{-1 + T_1} + \\ & \frac{3 T_1 (-1 + T_2) g_{1,i,i} g_{2,j,i}}{-1 + T_1} + 3 T_1 (-1 + 2 T_2) g_{1,j,i} g_{2,j,i} - g_{2,i,j} g_{2,j,i} - (-1 + T_2) T_2 g_{2,j,i}^2 - \\ & 3 T_1 g_{1,j,i} g_{2,j,j} - g_{2,i,i} g_{2,j,j} + (1 + T_2) g_{2,j,i} g_{2,j,j} + \frac{3 T_1 T_2 g_{1,j,j} g_{3,i,i}}{-1 + T_1} + \\ & \frac{3 T_1 (-1 + T_2) g_{2,j,j} g_{3,i,i}}{-1 + T_1} - 3 T_1 (-1 + T_1 T_2) g_{1,j,i} g_{3,j,i} + \frac{3 T_1 (-1 + T_1 T_2) g_{2,i,j} g_{3,j,i}}{-1 + T_1} + \\ & 3 T_1 (-1 + T_2) T_2 (-1 + T_1 T_2) g_{2,j,i} g_{3,j,i} - \frac{3 T_1 (-1 + T_1 T_2) g_{2,j,j} g_{3,j,i}}{-1 + T_1} - \\ & \frac{(-1 - 2 T_1 + 3 T_1 T_2) g_{3,i,j} g_{3,j,i}}{-1 + T_1} - \frac{T_1 T_2 (-1 + T_1 T_2) (-1 - 2 T_1 + 3 T_1 T_2) g_{3,j,i}^2}{-1 + T_1} + 3 g_{3,j,j} - \\ & \frac{3 T_1 g_{1,i,i} g_{3,j,j}}{-1 + T_1} - \frac{3 T_1 (-1 + T_2) T_2 g_{2,j,i} g_{3,j,j}}{-1 + T_1} - \frac{(-1 - 2 T_1 + 3 T_1 T_2) g_{3,i,i} g_{3,j,j}}{-1 + T_1} + \\ & \frac{(1 + T_1 T_2) (-1 - 2 T_1 + 3 T_1 T_2) g_{3,j,i} g_{3,j,j}}{-1 + T_1} - \frac{3 T_1 (-1 + T_1 T_2) y_{i,j,i}}{-1 + T_1} + \frac{3 T_1 (-1 + T_1 T_2) y_{i,j,j}}{-1 + T_1} \end{aligned}$$

In[=]:= **ygCF[R1[-1, i, j]]**

Out[=]=

$$\begin{aligned}
 & \frac{3}{2} + \frac{2(-1 + T_1) g_{1,i,i} g_{1,j,i}}{T_1} + g_{1,i,j} g_{1,j,i} - \frac{(-1 + T_1)(-1 + 2T_1) g_{1,j,i}^2}{T_1^2} + g_{1,i,i} g_{1,j,j} - \\
 & \frac{(-1 + 3T_1) g_{1,j,i} g_{1,j,j}}{T_1} + 3(-1 + T_2) g_{1,j,i} g_{2,i,i} + \frac{3T_1(-1 + T_2) g_{1,j,j} g_{2,i,i}}{-1 + T_1} - \\
 & \frac{3T_1(-1 + T_2) g_{1,i,i} g_{2,j,i}}{(-1 + T_1) T_2} - \frac{3(2 - 2T_2 + T_2^2) g_{1,j,i} g_{2,j,i}}{T_2} - \frac{3T_1(-1 + T_2)^2 g_{1,j,j} g_{2,j,i}}{(-1 + T_1) T_2} + \\
 & \frac{2(-1 + T_2) g_{2,i,i} g_{2,j,i}}{T_2} + g_{2,i,j} g_{2,j,i} - \frac{(-1 + T_2)(-1 + 2T_2) g_{2,j,i}^2}{T_2^2} + 3g_{1,j,i} g_{2,j,j} + g_{2,i,i} g_{2,j,j} - \\
 & \frac{(-1 + 3T_2) g_{2,j,i} g_{2,j,j}}{T_2} - 3T_2 g_{1,j,i} g_{3,i,i} - \frac{3T_1 T_2 g_{1,j,j} g_{3,i,i}}{-1 + T_1} - \frac{3T_1(-1 + T_2)^2 g_{2,j,i} g_{3,i,i}}{(-1 + T_1) T_2} - \\
 & \frac{3T_1(-1 + T_2) g_{2,j,j} g_{3,i,i}}{-1 + T_1} - \frac{3(-1 + T_1 T_2) g_{3,j,i}}{T_1 T_2} + \frac{3(-1 + T_1 T_2) g_{1,i,i} g_{3,j,i}}{(-1 + T_1) T_2} - \\
 & \frac{3(-1 + T_1 T_2) g_{1,j,i} g_{3,j,i}}{T_1 T_2} + \frac{3(-1 + T_1 T_2) g_{1,j,j} g_{3,j,i}}{-1 + T_1} - \frac{3(-1 + T_2)(-1 + T_1 T_2) g_{2,i,i} g_{3,j,i}}{(-1 + T_1) T_2} - \\
 & \frac{3(-1 + T_1 T_2) g_{2,i,j} g_{3,j,i}}{-1 + T_1} + \frac{3(-1 + T_2)(-1 + 2T_2)(-1 + T_1 T_2) g_{2,j,i} g_{3,j,i}}{(-1 + T_1) T_2^2} + \\
 & \frac{3(-1 + 2T_2)(-1 + T_1 T_2) g_{2,j,j} g_{3,j,i}}{(-1 + T_1) T_2} + \frac{2(-1 + T_1 T_2)(-1 - 2T_1 + 3T_1 T_2) g_{3,i,i} g_{3,j,i}}{(-1 + T_1) T_1 T_2} + \\
 & \frac{(-1 - 2T_1 + 3T_1 T_2) g_{3,i,j} g_{3,j,i}}{-1 + T_1} - \frac{(-1 + T_1 T_2)(-1 + 2T_1 T_2)(-1 - 2T_1 + 3T_1 T_2) g_{3,j,i}^2}{(-1 + T_1) T_1^2 T_2^2} - 3g_{3,j,j} + \\
 & \frac{3T_1 g_{1,i,i} g_{3,j,j}}{-1 + T_1} - 3g_{1,j,i} g_{3,j,j} + \frac{3T_1(-1 + T_2) g_{2,j,i} g_{3,j,j}}{-1 + T_1} + \frac{(-1 - 2T_1 + 3T_1 T_2) g_{3,i,i} g_{3,j,j}}{-1 + T_1} - \\
 & \frac{(-1 + 3T_1 T_2)(-1 - 2T_1 + 3T_1 T_2) g_{3,j,i} g_{3,j,j}}{(-1 + T_1) T_1 T_2} + \frac{3(-1 + T_1 T_2) y_{i,j,i}}{-1 + T_1} - \frac{3(-1 + T_1 T_2) y_{i,j,j}}{-1 + T_1}
 \end{aligned}$$

In[1]:= **Timing**[λ [Knot[3, 1]]]

Out[1]=

$$\left\{ 0.296875, \left\{ \frac{1 - T + T^2}{T}, - \frac{1}{T_1^5 T_2^5} (-6 + 15 T_1 - 21 T_1^2 + 15 T_1^3 - 6 T_1^4 + 27 T_2 - 54 T_1 T_2 + 55 T_1^2 T_2 - 8 T_1^3 T_2 - 18 T_1^4 T_2 + 19 T_1^5 T_2 - 57 T_2^2 + 58 T_1 T_2^2 + 6 T_1^2 T_2^2 - 114 T_1^3 T_2^2 + 68 T_1^4 T_2^2 - 6 T_1^5 T_2^2 - 43 T_1^6 T_2^2 + 78 T_2^3 - 2 T_1 T_2^3 - 84 T_1^2 T_2^3 + 139 T_1^3 T_2^3 + 84 T_1^4 T_2^3 - 108 T_1^5 T_2^3 + 85 T_1^6 T_2^3 + 55 T_1^7 T_2^3 - 63 T_2^4 - 120 T_1 T_2^4 + 140 T_1^2 T_2^4 - 75 T_1^3 T_2^4 - 284 T_1^4 T_2^4 + 117 T_1^5 T_2^4 - 54 T_1^6 T_2^4 - 116 T_1^7 T_2^4 - 59 T_1^8 T_2^4 + 33 T_2^5 + 148 T_1 T_2^5 + 36 T_1^2 T_2^5 - 177 T_1^3 T_2^5 + 417 T_1^4 T_2^5 + 90 T_1^5 T_2^5 - 94 T_1^6 T_2^5 + 197 T_1^7 T_2^5 + 114 T_1^8 T_2^5 + 44 T_1^9 T_2^5 - 6 T_2^6 - 108 T_1 T_2^6 - 136 T_1^2 T_2^6 + 118 T_1^3 T_2^6 - 132 T_1^4 T_2^6 - 361 T_1^5 T_2^6 + 22 T_1^6 T_2^6 - 21 T_1^7 T_2^6 - 246 T_1^8 T_2^6 - 69 T_1^9 T_2^6 - 24 T_1^{10} T_2^6 + 24 T_1 T_2^7 + 162 T_1^2 T_2^7 - 20 T_1^3 T_2^7 - 50 T_1^4 T_2^7 + 326 T_1^5 T_2^7 + 102 T_1^6 T_2^7 - 8 T_1^7 T_2^7 + 140 T_1^8 T_2^7 + 148 T_1^9 T_2^7 + 33 T_1^{10} T_2^7 + 9 T_1^{11} T_2^7 - 45 T_1^2 T_2^8 - 117 T_1^3 T_2^8 + 115 T_1^4 T_2^8 - 99 T_1^5 T_2^8 - 258 T_1^6 T_2^8 + 107 T_1^7 T_2^8 - 114 T_1^8 T_2^8 - 97 T_1^9 T_2^8 + 48 T_1^{10} T_2^8 - 18 T_1^{11} T_2^8 + 48 T_1^3 T_2^9 + 18 T_1^4 T_2^9 - 61 T_1^5 T_2^9 + 132 T_1^6 T_2^9 + 52 T_1^7 T_2^9 - 64 T_1^8 T_2^9 + 114 T_1^9 T_2^9 + 3 T_1^{10} T_2^9 + 27 T_1^{11} T_2^9 - 30 T_1^4 T_2^{10} + 36 T_1^5 T_2^{10} - 33 T_1^6 T_2^{10} - 27 T_1^7 T_2^{10} + 21 T_1^8 T_2^{10} - 36 T_1^9 T_2^{10} + 6 T_1^{10} T_2^{10} - 18 T_1^{11} T_2^{10} + 9 T_1^5 T_2^{11} - 24 T_1^6 T_2^{11} + 45 T_1^7 T_2^{11} - 48 T_1^8 T_2^{11} + 42 T_1^9 T_2^{11} - 21 T_1^{10} T_2^{11} + 9 T_1^{11} T_2^{11}) \right\} \right\}$$

In[2]:= **λ**[Knot[4, 1]]

Out[2]=

$$\left\{ - \frac{1 - 3 T + T^2}{T}, - \frac{1}{T_1^4 T_2^4} (1 - 3 T_1 + T_1^2) (-1 + T_1 T_2) (1 - 3 T_2 + T_2^2) (1 - 3 T_1 T_2 + T_1^2 T_2^2) (-9 T_1 + 11 T_1^2 - 3 T_2 + 27 T_1 T_2 - 15 T_1^2 T_2 - 19 T_1^3 T_2 + 2 T_2^2 - 42 T_1 T_2^2 + 51 T_1^2 T_2^2 + 24 T_1^3 T_2^2 + 24 T_1^4 T_2^2 - 27 T_1^5 T_2^2 + 6 T_1^6 T_2^2 + 32 T_1 T_2^3 - 27 T_2^4 - 54 T_1^3 T_2^3 - 78 T_1^4 T_2^3 + 30 T_1^5 T_2^3 + 9 T_1^6 T_2^3 - 3 T_1^7 T_2^3 - 15 T_1 T_2^4 - 15 T_1^2 T_2^4 + 96 T_1^3 T_2^4 - 9 T_1^4 T_2^4 + 144 T_1^5 T_2^4 - 60 T_1^6 T_2^4 + 3 T_1^7 T_2^4 + 3 T_1 T_2^5 + 21 T_1^2 T_2^5 - 75 T_1^3 T_2^5 + 63 T_1^4 T_2^5 - 135 T_1^5 T_2^5 + 9 T_1^6 T_2^5 + 12 T_1^7 T_2^5 - 6 T_1^2 T_2^6 + 18 T_1^3 T_2^6 - 18 T_1^4 T_2^6 + 21 T_1^5 T_2^6 + 27 T_1^6 T_2^6 - 12 T_1^7 T_2^6 + 3 T_1^5 T_2^7 - 9 T_1^6 T_2^7 + 3 T_1^7 T_2^7) \right\}$$

In[3]:= **Conway** = λ [Knot[11, NonAlternating, 34]]

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[=]=

$$\left\{ 1, - \frac{1}{T_1^9 T_2^9} \left(9 - 51 T_1 + 153 T_1^2 - 309 T_1^3 + 441 T_1^4 - 462 T_1^5 + 360 T_1^6 - 201 T_1^7 + 72 T_1^8 - 12 T_1^9 - 57 T_2 + 291 T_1 T_2 - 771 T_1^2 T_2 + 1299 T_1^3 T_2 - 1434 T_1^4 T_2 + 924 T_1^5 T_2 - 144 T_1^6 T_2 - 420 T_1^7 T_2 + 507 T_1^8 T_2 - 276 T_1^9 T_2 + 60 T_1^{10} T_2 + 156 T_1^2 - 684 T_1 T_2^2 + 1485 T_1^2 T_2^2 - 1743 T_1^3 T_2^2 + 747 T_1^4 T_2^2 + 1365 T_1^5 T_2^2 - 2571 T_1^6 T_2^2 + 2301 T_1^7 T_2^2 - 960 T_1^8 T_2^2 - 222 T_1^9 T_2^2 + 465 T_1^{10} T_2^2 - 150 T_1^{11} T_2^2 - 264 T_1^3 + 933 T_1 T_2^3 - 1404 T_1^2 T_2^3 - 12 T_1^3 T_2^3 + 3522 T_1^4 T_2^3 - 7788 T_1^5 T_2^3 + 7833 T_1^6 T_2^3 - 5031 T_1^7 T_2^3 + 1341 T_1^8 T_2^3 + 582 T_1^9 T_2^3 - 138 T_1^{10} T_2^3 - 567 T_1^{11} T_2^3 + 270 T_1^{12} T_2^3 + 315 T_1^4 - 759 T_1 T_2^4 + 4770 T_1^3 T_2^4 - 12 576 T_1^4 T_2^4 + 20 028 T_1^5 T_2^4 - 18 207 T_1^6 T_2^4 + 12 219 T_1^7 T_2^4 - 5040 T_1^8 T_2^4 + 1137 T_1^9 T_2^4 - 1335 T_1^{10} T_2^4 + 1008 T_1^{11} T_2^4 + 390 T_1^{12} T_2^4 - 372 T_1^{13} T_2^4 - 264 T_1^{14} T_2^4 + 159 T_1 T_2^5 + 2154 T_1^2 T_2^5 - 10 290 T_1^3 T_2^5 + 21 117 T_1^4 T_2^5 - 28 803 T_1^5 T_2^5 + 22 350 T_1^6 T_2^5 - 12 285 T_1^7 T_2^5 + 2262 T_1^8 T_2^5 + 2031 T_1^9 T_2^5 - 123 T_1^{10} T_2^5 + 915 T_1^{11} T_2^5 - 1800 T_1^{12} T_2^5 + 93 T_1^{13} T_2^5 + 396 T_1^{14} T_2^5 + 168 T_1^{15} T_2^5 + 345 T_1^6 T_2^6 - 3204 T_1^7 T_2^6 + 11 898 T_1^8 T_2^6 - 20 901 T_1^9 T_2^6 + 24 255 T_1^{10} T_2^6 - 15 004 T_1^{11} T_2^6 + 7644 T_1^{12} T_2^6 - 2994 T_1^{13} T_2^6 + 1532 T_1^9 T_2^6 - 3489 T_1^{10} T_2^6 - 1017 T_1^{11} T_2^6 + 1218 T_1^{12} T_2^6 + 1512 T_1^{13} T_2^6 - 567 T_1^{14} T_2^6 - 330 T_1^{15} T_2^6 - 66 T_2^7 - 624 T_1 T_2^7 + 2955 T_1^2 T_2^7 - 8895 T_1^3 T_2^7 + 11 448 T_1^4 T_2^7 - 8934 T_1^5 T_2^7 + 3828 T_1^6 T_2^7 - 5686 T_1^7 T_2^7 + 7299 T_1^8 T_2^7 - 97 T_1^9 T_2^7 - 4620 T_1^{10} T_2^7 + 10 611 T_1^{11} T_2^7 - 3600 T_1^{12} T_2^7 - 1611 T_1^{13} T_2^7 - 591 T_1^{14} T_2^7 + 759 T_1^{15} T_2^7 + 216 T_1^{16} T_2^7 + 12 T_1^8 + 423 T_1 T_2^8 - 882 T_1^2 T_2^8 + 1665 T_1^3 T_2^8 + 1560 T_1^4 T_2^8 - 2571 T_1^5 T_2^8 - 5670 T_1^6 T_2^8 + 13 596 T_1^7 T_2^8 - 8836 T_1^8 T_2^8 - 15 166 T_1^9 T_2^8 + 21 465 T_1^{10} T_2^8 - 16 239 T_1^{11} T_2^8 - 1218 T_1^{12} T_2^8 + 4203 T_1^{13} T_2^8 + 1071 T_1^{14} T_2^8 - 288 T_1^{15} T_2^8 - 645 T_1^{16} T_2^8 - 108 T_1^{17} T_2^8 - 132 T_1 T_2^9 - 321 T_1^2 T_2^9 + 396 T_1^3 T_2^9 - 27 T_1^4 T_2^9 - 7542 T_1^5 T_2^9 + 20 291 T_1^6 T_2^9 - 12 505 T_1^7 T_2^9 - 12 103 T_1^8 T_2^9 + 43 137 T_1^9 T_2^9 - 31 216 T_1^{10} T_2^9 + 9521 T_1^{11} T_2^9 + 7538 T_1^{12} T_2^9 - 2373 T_1^{13} T_2^9 - 3573 T_1^{14} T_2^9 + 81 T_1^{15} T_2^9 + 570 T_1^{16} T_2^9 + 408 T_1^{17} T_2^9 + 36 T_1^{18} T_2^9 + 12 T_1 T_2^{10} + 300 T_1^2 T_2^{10} + 339 T_1^3 T_2^{10} - 2082 T_1^4 T_2^{10} + 5733 T_1^5 T_2^{10} - 3579 T_1^6 T_2^{10} - 21 882 T_1^7 T_2^{10} + 44 004 T_1^8 T_2^{10} - 50 950 T_1^9 T_2^{10} + 15 629 T_1^{10} T_2^{10} + 4635 T_1^{11} T_2^{10} - 7188 T_1^{12} T_2^{10} - 3372 T_1^{13} T_2^{10} + 5145 T_1^{14} T_2^{10} + 624 T_1^{15} T_2^{10} - 153 T_1^{16} T_2^{10} - 600 T_1^{17} T_2^{10} - 165 T_1^{18} T_2^{10} - 6 T_1^{19} T_2^{10} - 48 T_1^2 T_2^{11} - 555 T_1^3 T_2^{11} + 903 T_1^4 T_2^{11} + 360 T_1^5 T_2^{11} - 6642 T_1^6 T_2^{11} + 21 174 T_1^7 T_2^{11} - 18 987 T_1^8 T_2^{11} + 5408 T_1^9 T_2^{11} + 21 123 T_1^{10} T_2^{11} - 15 157 T_1^{11} T_2^{11} + 819 T_1^{12} T_2^{11} + 8913 T_1^{13} T_2^{11} - 5970 T_1^{14} T_2^{11} - 33 T_1^{15} T_2^{11} - 840 T_1^6 T_2^{11} + 522 T_1^7 T_2^{11} + 327 T_1^8 T_2^{11} + 33 T_1^9 T_2^{11} + 126 T_1^10 T_2^{11} + 462 T_1^11 T_2^{11} - 1911 T_1^5 T_2^{12} + 3459 T_1^6 T_2^{12} - 4797 T_1^7 T_2^{12} - 3657 T_1^8 T_2^{12} + 9677 T_1^9 T_2^{12} - 10 521 T_1^{10} T_2^{12} - 9603 T_1^{11} T_2^{12} + 19 073 T_1^{12} T_2^{12} - 16 617 T_1^{13} T_2^{12} + 7233 T_1^{14} T_2^{12} - 861 T_1^{15} T_2^{12} + 939 T_1^{16} T_2^{12} + 66 T_1^{17} T_2^{12} - 414 T_1^{18} T_2^{12} - 84 T_1^{19} T_2^{12} - 198 T_1^4 T_2^{13} - 18 T_1^5 T_2^{13} + 1311 T_1^6 T_2^{13} - 2238 T_1^7 T_2^{13} + 4503 T_1^8 T_2^{13} - 2235 T_1^9 T_2^{13} - 4104 T_1^{10} T_2^{13} + 14 784 T_1^{11} T_2^{13} - 12 411 T_1^{12} T_2^{13} + 6183 T_1^{13} T_2^{13} - 417 T_1^{14} T_2^{13} - 1182 T_1^{15} T_2^{13} + 96 T_1^{16} T_2^{13} - 516 T_1^{17} T_2^{13} + 258 T_1^{18} T_2^{13} + 144 T_1^{19} T_2^{13} + 210 T_1^{20} T_2^{14} - 324 T_1^{21} T_2^{14} - 795 T_1^{22} T_2^{14} + 1746 T_1^{23} T_2^{14} - 3378 T_1^{24} T_2^{14} + 5745 T_1^{10} T_2^{14} - 7365 T_1^{11} T_2^{14} + 3612 T_1^{12} T_2^{14} - 1653 T_1^{13} T_2^{14} + 228 T_1^{14} T_2^{14} - 219 T_1^{15} T_2^{14} + 261 T_1^{16} T_2^{14} + 243 T_1^{17} T_2^{14} + 33 T_1^{18} T_2^{14} - 171 T_1^{19} T_2^{14} - 168 T_1^{20} T_2^{14} + 507 T_1^{21} T_2^{14} + 30 T_1^{22} T_2^{14} - 792 T_1^{23} T_2^{14} + 648 T_1^{24} T_2^{14} - 417 T_1^{11} T_2^{15} + 882 T_1^{12} T_2^{15} - 54 T_1^{13} T_2^{15} + 198 T_1^{14} T_2^{15} - 102 T_1^{15} T_2^{15} + 195 T_1^{16} T_2^{15} - 147 T_1^{17} T_2^{15} - 171 T_1^{18} T_2^{15} + 147 T_1^{19} T_2^{15} + 96 T_1^{20} T_2^{15} - 426 T_1^{21} T_2^{15} + 384 T_1^{22} T_2^{15} + 348 T_1^{23} T_2^{15} - 921 T_1^{24} T_2^{15} + 993 T_1^{25} T_2^{15} - 870 T_1^{13} T_2^{16} + 123 T_1^{14} T_2^{16} + 93 T_1^{15} T_2^{16} - 87 T_1^{16} T_2^{16} - 111 T_1^{17} T_2^{16} + 225 T_1^{18} T_2^{16} - 102 T_1^{19} T_2^{16} - 36 T_1^{20} T_2^{16} + 225 T_1^{21} T_2^{17} - 441 T_1^{22} T_2^{17} + 354 T_1^{23} T_2^{17} - 12 T_1^{24} T_2^{17} - 339 T_1^{25} T_2^{17} + 537 T_1^{26} T_2^{17} - 318 T_1^{27} T_2^{17} + 36 T_1^{28} T_2^{17} + 153 T_1^{29} T_2^{17} - 153 T_1^{30} T_2^{17} + 51 T_1^{31} T_2^{17} + 6 T_1^{32} T_2^{18} - 54 T_1^{33} T_2^{18} + 159 T_1^{34} T_2^{18} - 249 T_1^{35} T_2^{18} + 249 T_1^{36} T_2^{18} - 144 T_1^{37} T_2^{18} - 18 T_1^{38} T_2^{18} + 117 T_1^{39} T_2^{18} - 126 T_1^{40} T_2^{18} + 72 T_1^{41} T_2^{18} - 18 T_1^{42} T_2^{18} + 3 T_1^{43} T_2^{19} - 15 T_1^{44} T_2^{19} + 36 T_1^{45} T_2^{19} - 57 T_1^{46} T_2^{19} + 66 T_1^{47} T_2^{19} - 57 T_1^{48} T_2^{19} + 36 T_1^{49} T_2^{19} - 15 T_1^{50} T_2^{19} + 3 T_1^{51} T_2^{19} \right) \}$$

In[=]:= $\text{KT} = \lambda[\text{Knot}[11, \text{NonAlternating}, 42]]$

Out[=]=

$$\left\{ 1, -\frac{1}{T_1^9 T_2^9} \left(-3 + 18 T_1 - 48 T_1^2 + 87 T_1^3 - 102 T_1^4 + 81 T_1^5 - 36 T_1^6 + 6 T_1^7 + 18 T_2 - 96 T_1 T_2 + 231 T_1^2 T_2 - 381 T_1^3 T_2 + 366 T_1^4 T_2 - 195 T_1^5 T_2 - 33 T_1^6 T_2 + 78 T_1^7 T_2 - 3 T_1^8 T_2 - 6 T_1^9 T_2 - 54 T_2^2 + 252 T_1 T_2^2 - 495 T_1^2 T_2^2 + 567 T_1^3 T_2^2 + 15 T_1^4 T_2^2 - 885 T_1^5 T_2^2 + 1425 T_1^6 T_2^2 - 897 T_1^7 T_2^2 + 132 T_1^8 T_2^2 - 84 T_1^9 T_2^2 + 39 T_1^{10} T_2^2 + 102 T_2^3 - 375 T_1 T_2^3 + 381 T_1^2 T_2^3 + 348 T_1^3 T_2^3 - 2172 T_1^4 T_2^3 + 3453 T_1^5 T_2^3 - 2607 T_1^6 T_2^3 - 342 T_1^7 T_2^3 + 1974 T_1^8 T_2^3 - 444 T_1^9 T_2^3 + 213 T_1^{10} T_2^3 - 111 T_1^{11} T_2^3 - 126 T_1^{12} T_2^3 + 279 T_1 T_2^{13} + 384 T_1^2 T_2^{14} - 1896 T_1^3 T_2^{14} + 3219 T_1^4 T_2^{14} - 1500 T_1^5 T_2^{14} - 4059 T_1^6 T_2^{14} + 8565 T_1^7 T_2^{14} - 7044 T_1^8 T_2^{14} - 294 T_1^9 T_2^{14} - 315 T_1^{10} T_2^{14} + 18 T_1^{11} T_2^{14} + 174 T_1^{12} T_2^{14} + 108 T_2^5 - 18 T_1 T_2^5 - 1137 T_1^2 T_2^5 + 2055 T_1^3 T_2^5 + 18 T_1^4 T_2^5 - 6708 T_1^5 T_2^5 + 15 567 T_1^6 T_2^5 - 14 997 T_1^7 T_2^5 + 6471 T_1^8 T_2^5 + 6219 T_1^9 T_2^5 + 1248 T_1^{10} T_2^5 + 642 T_1^{11} T_2^5 - 585 T_1^{12} T_2^5 - 165 T_1^{13} T_2^5 - 54 T_2^6 - 258 T_1 T_2^6 + 1335 T_1^2 T_2^6 - 345 T_1^3 T_2^6 - 5988 T_1^4 T_2^6 + 15 207 T_1^5 T_2^6 - 22 213 T_1^6 T_2^6 + 12 054 T_1^7 T_2^6 - 1311 T_1^8 T_2^6 - 9616 T_1^9 T_2^6 - 8133 T_1^{10} T_2^6 - 2379 T_1^{11} T_2^6 + 315 T_1^{12} T_2^6 + 1017 T_1^{13} T_2^6 + 81 T_1^{14} T_2^6 + 12 T_2^7 + 249 T_1 T_2^7 - 429 T_1^2 T_2^7 - 2703 T_1^3 T_2^7 + 10 386 T_1^4 T_2^7 - 15 783 T_1^5 T_2^7 + 16 422 T_1^6 T_2^7 - 232 T_1^7 T_2^7 - 456 T_1^8 T_2^7 + 2957 T_1^9 T_2^7 + 18 456 T_1^{10} T_2^7 + 6513 T_1^{11} T_2^7 + 3432 T_1^{12} T_2^7 - 2679 T_1^{13} T_2^7 - 699 T_1^{14} T_2^7 - 12 T_1^{15} T_2^7 - 90 T_1 T_2^8 - 255 T_1^2 T_2^8 + 2799 T_1^3 T_2^8 - 6048 T_1^4 T_2^8 + 3615 T_1^5 T_2^8 - 717 T_1^6 T_2^8 - 9549 T_1^7 T_2^8 - 9067 T_1^8 T_2^8 + 11 528 T_1^9 T_2^8 - 24 393 T_1^{10} T_2^8 - 8349 T_1^{11} T_2^8 - 12 579 T_1^{12} T_2^8 + 2106 T_1^{13} T_2^8 + 2868 T_1^{14} T_2^8 + 117 T_1^{15} T_2^8 + 6 T_1 T_2^9 + 216 T_1^2 T_2^9 - 1017 T_1^3 T_2^9 + 663 T_1^4 T_2^9 + 2703 T_1^5 T_2^9 + 107 T_1^6 T_2^9 - 2413 T_1^7 T_2^9 + 29 681 T_1^8 T_2^9 - 22 935 T_1^9 T_2^9 + 23 513 T_1^{10} T_2^9 + 3131 T_1^{11} T_2^9 + 17 276 T_1^{12} T_2^9 + 7584 T_1^{13} T_2^9 - 6894 T_1^{14} T_2^9 - 561 T_1^{15} T_2^9 - 12 T_1^2 T_2^{10} + 579 T_1^3 T_2^{10} - 879 T_1^4 T_2^{10} - 4593 T_1^5 T_2^{10} + 8424 T_1^6 T_2^{10} - 26 310 T_1^7 T_2^{10} + 12 545 T_1^8 T_2^{10} - 14 005 T_1^9 T_2^{10} - 3189 T_1^{11} T_2^{10} + 117 T_1^{12} T_2^{10} - 26 763 T_1^{13} T_2^{10} + 9870 T_1^{14} T_2^{10} + 1656 T_1^{15} T_2^{10} - 6 T_1 T_2^{11} - 54 T_1^3 T_2^{11} + 192 T_1^4 T_2^{11} - 651 T_1^5 T_2^{11} + 2322 T_1^6 T_2^{11} - 2628 T_1^7 T_2^{11} + 13 011 T_1^8 T_2^{11} - 5122 T_1^9 T_2^{11} + 5505 T_1^{10} T_2^{11} + 18 968 T_1^{11} T_2^{11} - 33 633 T_1^{12} T_2^{11} + 39 975 T_1^{13} T_2^{11} - 7218 T_1^{14} T_2^{11} - 3216 T_1^{15} T_2^{11} + 33 T_1^3 T_2^{12} + 45 T_1^4 T_2^{12} - 537 T_1^5 T_2^{12} + 771 T_1^6 T_2^{12} + 1347 T_1^7 T_2^{12} - 9336 T_1^8 T_2^{12} + 2219 T_1^9 T_2^{12} + 12 168 T_1^{10} T_2^{12} - 43 833 T_1^{11} T_2^{12} + 52 331 T_1^{12} T_2^{12} - 31 965 T_1^{13} T_2^{12} - 726 T_1^{14} T_2^{12} + 4227 T_1^{15} T_2^{12} - 72 T_1^4 T_2^{13} + 147 T_1^5 T_2^{13} + 675 T_1^6 T_2^{13} - 3276 T_1^7 T_2^{13} + 4440 T_1^8 T_2^{13} + 7593 T_1^9 T_2^{13} - 26 265 T_1^{10} T_2^{13} + 43 053 T_1^{11} T_2^{13} - 33 975 T_1^{12} T_2^{13} + 8577 T_1^{13} T_2^{13} + 7326 T_1^{14} T_2^{13} - 3753 T_1^{15} T_2^{13} + 87 T_1^5 T_2^{14} - 447 T_1^6 T_2^{14} + 555 T_1^7 T_2^{14} + 2031 T_1^8 T_2^{14} - 9648 T_1^9 T_2^{14} + 16 401 T_1^{10} T_2^{14} - 16 125 T_1^{11} T_2^{14} + 4986 T_1^{12} T_2^{14} + 6123 T_1^{13} T_2^{14} - 6993 T_1^{14} T_2^{14} + 2121 T_1^{15} T_2^{14} - 51 T_1^6 T_2^{15} + 399 T_1^7 T_2^{15} - 1389 T_1^8 T_2^{15} + 2568 T_1^9 T_2^{15} - 2016 T_1^{10} T_2^{15} - 849 T_1^{11} T_2^{15} + 4476 T_1^{12} T_2^{15} - 5373 T_1^{13} T_2^{15} + 3006 T_1^{14} T_2^{15} - 696 T_1^{15} T_2^{15} + 9 T_1^7 T_2^{16} - 84 T_1^8 T_2^{16} + 366 T_1^9 T_2^{16} - 945 T_1^{10} T_2^{16} + 1551 T_1^{11} T_2^{16} - 1731 T_1^{12} T_2^{16} + 1263 T_1^{13} T_2^{16} - 531 T_1^{14} T_2^{16} + 105 T_1^{15} T_2^{16} \right) \}$$

In[=]:= **Simplify**[KT == Conway]

Out[=]=

$$\left\{ \theta, \frac{1}{T_1^9 T_2^9} \left(3 (4 - 25 T_2 + 70 T_2^2 - 122 T_2^3 + 147 T_2^4 - 124 T_2^5 + 74 T_2^6 - 26 T_2^7 + 4 T_2^8 + T_2^{10}) (-2 + 11 T_2 - 28 T_2^2 + 48 T_2^3 - 57 T_2^4 + 49 T_2^5 - 34 T_2^6 + 17 T_2^7 - 6 T_2^8 + T_2^9) + T_1^{18} T_2^9 (12 - 55 T_2 + 109 T_2^2 - 138 T_2^3 + 86 T_2^4 + 11 T_2^5 - 57 T_2^6 + 75 T_2^7 - 51 T_2^8 + 24 T_2^9 - 5 T_2^{10}) + T_1 (-23 + 129 T_2 - 312 T_2^2 + 436 T_2^3 - 346 T_2^4 + 59 T_2^5 + 201 T_2^6 - 291 T_2^7 + 171 T_2^8 - 46 T_2^9 + 4 T_2^{10}) + T_1^{17} T_2^8 (-36 + 136 T_2 - 200 T_2^2 + 174 T_2^3 + 22 T_2^4 - 172 T_2^5 + 81 T_2^6 - 49 T_2^7 - 37 T_2^8 + 51 T_2^9 - 42 T_2^{10} + 12 T_2^{11}) - T_1^2 (-67 + 334 T_2 - 660 T_2^2 + 595 T_2^3 + 128 T_2^4 - 1097 T_2^5 + 1513 T_2^6 - 1128 T_2^7 + 209 T_2^8 + 179 T_2^9 - 104 T_2^{10} + 14 T_2^{11}) + T_1^{16} T_2^7 (72 - 215 T_2 + 190 T_2^2 - 51 T_2^3 - 280 T_2^4 + 313 T_2^5 + 32 T_2^6 + 87 T_2^7 + 65 T_2^8 - 29 T_2^9 + 12 T_2^{10} + 39 T_2^{11} - 19 T_2^{12}) + T_1^3 (-132 + 560 T_2 - 770 T_2^2 - 120 T_2^3 + 2222 T_2^4 - 4115 T_2^5 + 4081 T_2^6 - 2064 T_2^7 - 378 T_2^8 + 471 T_2^9 + 113 T_2^{10} - 167 T_2^{11} + 31 T_2^{12}) + T_1^4 (181 - 600 T_2 + 244 T_2^2 + 1898 T_2^3 - 5265 T_2^4 + 7033 T_2^5 - 4971 T_2^6 + 354 T_2^7 + 2536 T_2^8 - 230 T_2^9 - 887 T_2^{10} + 237 T_2^{11} + 139 T_2^{12} - 42 T_2^{13}) + T_1^{15} T_2^6 (-110 + 257 T_2 - 135 T_2^2 + 214 T_2^3 - 344 T_2^4 + 1061 T_2^5 - 1696 T_2^6 + 857 T_2^7 - 780 T_2^8 + 198 T_2^9 - 4 T_2^{10} - 106 T_2^{11} - 6 T_2^{12} + 22 T_2^{13}) + T_1^{14} T_2^5 (132 - 216 T_2 + 36 T_2^2 - 599 T_2^3 + 1107 T_2^4 - 1575 T_2^5 + 416 T_2^6 + 2653 T_2^7 - 2581 T_2^8 + 2407 T_2^9 - 936 T_2^{10} + 218 T_2^{11} + 179 T_2^{12} - 48 T_2^{13} - 19 T_2^{14}) + T_1^5 (-181 + 373 T_2 + 750 T_2^2 - 3747 T_2^3 + 7176 T_2^4 - 7365 T_2^5 + 3016 T_2^6 + 2283 T_2^7 - 2062 T_2^8 - 3415 T_2^9 + 2204 T_2^{10} + 337 T_2^{11} - 458 T_2^{12} - 55 T_2^{13} + 41 T_2^{14}) + T_1^6 (132 - 37 T_2 - 1332 T_2^2 + 3480 T_2^3 - 4716 T_2^4 + 2261 T_2^5 + 2403 T_2^6 - 4198 T_2^7 - 1651 T_2^8 + 6728 T_2^9 + 338 T_2^{10} - 2988 T_2^{11} + 896 T_2^{12} + 212 T_2^{13} + 41 T_2^{14} - 39 T_2^{15}) + T_1^{13} T_2^4 (-124 + 86 T_2 + 165 T_2^2 + 356 T_2^3 + 699 T_2^4 - 3319 T_2^5 + 7797 T_2^6 - 10354 T_2^7 + 5116 T_2^8 - 798 T_2^9 - 2592 T_2^{10} + 1773 T_2^{11} - 711 T_2^{12} - 113 T_2^{13} + 83 T_2^{14} + 12 T_2^{15}) - T_1^{12} T_2^3 (-90 - 72 T_2 + 405 T_2^2 - 301 T_2^3 + 2344 T_2^4 - 3787 T_2^5 + 3246 T_2^6 + 2435 T_2^7 - 11484 T_2^8 + 11086 T_2^9 - 7188 T_2^{10} + 458 T_2^{11} + 1198 T_2^{12} - 908 T_2^{13} + 4 T_2^{14} + 83 T_2^{15} + 5 T_2^{16}) + T_1^7 (-69 - 166 T_2 + 1066 T_2^2 - 1563 T_2^3 + 1218 T_2^4 + 904 T_2^5 - 1470 T_2^6 - 1818 T_2^7 + 7715 T_2^8 - 3364 T_2^9 - 10102 T_2^{10} + 7934 T_2^{11} - 2048 T_2^{12} + 346 T_2^{13} - 450 T_2^{14} + 36 T_2^{15} + 29 T_2^{16}) + T_1^{10} T_2^2 (20 + 142 T_2 - 117 T_2^2 - 340 T_2^3 - 457 T_2^4 + 1548 T_2^5 - 7692 T_2^6 + 15286 T_2^7 - 18243 T_2^8 + 9878 T_2^9 + 5206 T_2^{10} - 7563 T_2^{11} + 7387 T_2^{12} - 3552 T_2^{13} + 888 T_2^{14} + 431 T_2^{15} - 147 T_2^{16} - 18 T_2^{17}) + T_1^8 (24 + 170 T_2 - 364 T_2^2 - 211 T_2^3 + 668 T_2^4 - 1403 T_2^5 - 561 T_2^6 + 2585 T_2^7 + 77 T_2^8 - 13928 T_2^9 + 23438 T_2^{10} - 10666 T_2^{11} + 1893 T_2^{12} + 21 T_2^{13} - 95 T_2^{14} + 473 T_2^{15} - 114 T_2^{16} - 12 T_2^{17}) + T_1^{11} T_2^2 (-50 - 152 T_2 + 330 T_2^2 + 91 T_2^3 + 454 T_2^4 + 1366 T_2^5 - 2630 T_2^6 + 2130 T_2^7 + 2608 T_2^8 - 11375 T_2^9 + 11410 T_2^{10} - 9423 T_2^{11} + 2920 T_2^{12} + 144 T_2^{13} - 824 T_2^{14} + 118 T_2^{15} + 53 T_2^{16} + T_2^{17}) + T_1^9 (-4 - 90 T_2 - 46 T_2^2 + 342 T_2^3 + 477 T_2^4 - 1396 T_2^5 + 3716 T_2^6 - 1018 T_2^7 - 8898 T_2^8 + 22024 T_2^9 - 21165 T_2^{10} + 3510 T_2^{11} + 2486 T_2^{12} - 3276 T_2^{13} + 2090 T_2^{14} - 1120 T_2^{15} + 6 T_2^{16} + 75 T_2^{17} + 2 T_2^{18})) \right\} == \{0, 0\}$$

In[¹]:= **Simplify**[KT == Conway /. T₁ → 1]

Out[¹]=

$$\left\{ 0, 3 \left(-12958 - \frac{1}{T_2^9} + \frac{58}{T_2^7} - \frac{381}{T_2^6} + \frac{1391}{T_2^5} - \frac{3602}{T_2^4} + \frac{7118}{T_2^3} - \frac{11014}{T_2^2} + \frac{13462}{T_2} + 9668 T_2 - 5365 T_2^2 + 1942 T_2^3 - 170 T_2^4 - 306 T_2^5 + 227 T_2^6 - 86 T_2^7 + 19 T_2^8 - 2 T_2^9 \right) \right\} == \{0, 0\}$$

In[²]:= **Simplify**[KT == Conway /. T₂ → 1]

Out[²]=

$$\left\{ 0, 3 \left(-2285 + \frac{2}{T_1^9} - \frac{18}{T_1^8} + \frac{84}{T_1^7} - \frac{268}{T_1^6} + \frac{627}{T_1^5} - \frac{1103}{T_1^4} + \frac{1530}{T_1^3} - \frac{1802}{T_1^2} + \frac{1995}{T_1} + 2657 T_1 - 2829 T_1^2 + 2566 T_1^3 - 1924 T_1^4 + 1174 T_1^5 - 572 T_1^6 + 216 T_1^7 - 60 T_1^8 + 11 T_1^9 - T_1^{10} \right) \right\} == \{0, 0\}$$

In[[#]]:= **K31** = $\lambda[\text{Knot}[3, 1]]$

K31 = $\lambda[\text{Mirror}@\text{Knot}[3, 1]]$

Out[[#]]=

$$\left\{ \frac{1 - T + T^2}{T}, \right.$$

$$- \frac{1}{T_1^5 T_2^5} (-6 + 15 T_1 - 21 T_1^2 + 15 T_1^3 - 6 T_1^4 + 27 T_2 - 54 T_1 T_2 + 55 T_1^2 T_2 - 8 T_1^3 T_2 - 18 T_1^4 T_2 + 19 T_1^5 T_2 - 57 T_2^2 +$$

$$58 T_1 T_2^2 + 6 T_1^2 T_2^2 - 114 T_1^3 T_2^2 + 68 T_1^4 T_2^2 - 6 T_1^5 T_2^2 - 43 T_1^6 T_2^2 + 78 T_1^3 T_2^3 - 2 T_1 T_2^3 - 84 T_1^2 T_2^3 + 139 T_1^3 T_2^3 +$$

$$84 T_1^4 T_2^3 - 108 T_1^5 T_2^3 + 85 T_1^6 T_2^3 + 55 T_1^7 T_2^3 - 63 T_2^4 - 120 T_1 T_2^4 + 140 T_1^2 T_2^4 - 75 T_1^3 T_2^4 - 284 T_1^4 T_2^4 +$$

$$117 T_1^5 T_2^4 - 54 T_1^6 T_2^4 - 116 T_1^7 T_2^4 - 59 T_1^8 T_2^4 + 33 T_2^5 + 148 T_1 T_2^5 + 36 T_1^2 T_2^5 - 177 T_1^3 T_2^5 + 417 T_1^4 T_2^5 +$$

$$90 T_1^5 T_2^5 - 94 T_1^6 T_2^5 + 197 T_1^7 T_2^5 + 114 T_1^8 T_2^5 + 44 T_1^9 T_2^5 - 6 T_2^6 - 108 T_1 T_2^6 - 136 T_1^2 T_2^6 + 118 T_1^3 T_2^6 -$$

$$132 T_1^4 T_2^6 - 361 T_1^5 T_2^6 + 22 T_1^6 T_2^6 - 21 T_1^7 T_2^6 - 246 T_1^8 T_2^6 - 69 T_1^9 T_2^6 - 24 T_1^10 T_2^6 + 24 T_1 T_2^7 + 162 T_1^2 T_2^7 -$$

$$20 T_1^3 T_2^7 - 50 T_1^4 T_2^7 + 326 T_1^5 T_2^7 + 102 T_1^6 T_2^7 - 8 T_1^7 T_2^7 + 140 T_1^8 T_2^7 + 148 T_1^9 T_2^7 + 33 T_1^10 T_2^7 + 9 T_1^{11} T_2^7 -$$

$$45 T_1^2 T_2^8 - 117 T_1^3 T_2^8 + 115 T_1^4 T_2^8 - 99 T_1^5 T_2^8 - 258 T_1^6 T_2^8 + 107 T_1^7 T_2^8 - 114 T_1^8 T_2^8 - 97 T_1^9 T_2^8 - 48 T_1^{10} T_2^8 -$$

$$18 T_1^{11} T_2^8 + 48 T_1^3 T_2^9 + 18 T_1^4 T_2^9 - 61 T_1^5 T_2^9 + 132 T_1^6 T_2^9 + 52 T_1^7 T_2^9 - 64 T_1^8 T_2^9 + 114 T_1^9 T_2^9 + 3 T_1^{10} T_2^9 +$$

$$27 T_1^{11} T_2^9 - 30 T_1^4 T_2^{10} + 36 T_1^5 T_2^{10} - 33 T_1^6 T_2^{10} - 27 T_1^7 T_2^{10} + 21 T_1^8 T_2^{10} - 36 T_1^9 T_2^{10} + 6 T_1^{10} T_2^{10} -$$

$$18 T_1^{11} T_2^{10} + 9 T_1^5 T_2^{11} - 24 T_1^6 T_2^{11} + 45 T_1^7 T_2^{11} - 48 T_1^8 T_2^{11} + 42 T_1^9 T_2^{11} - 21 T_1^{10} T_2^{11} + 9 T_1^{11} T_2^{11}) \}$$

Out[[#]]=

$$\left\{ \frac{1 - T + T^2}{T}, \right.$$

$$- \frac{1}{T_1^4 T_2^4} (3 - 5 T_1 + 5 T_1^2 - 3 T_1^3 + T_1^4 - 5 T_2 + 7 T_1^2 T_2 - 18 T_1^3 T_2 + 18 T_1^4 T_2 - 16 T_1^5 T_2 + 6 T_1^6 T_2 - 3 T_1^7 T_2 + 5 T_2^2 +$$

$$7 T_1 T_2^2 - 7 T_1^2 T_2^2 + 24 T_1^3 T_2^2 - 17 T_1^4 T_2^2 + 32 T_1^5 T_2^2 - 7 T_1^6 T_2^2 + 9 T_1^7 T_2^2 + 6 T_1^8 T_2^2 - 3 T_1^3 T_2^3 - 12 T_1 T_2^3 -$$

$$37 T_1^3 T_2^3 + 7 T_1^4 T_2^3 - 18 T_1^5 T_2^3 - 67 T_1^6 T_2^3 + 37 T_1^7 T_2^3 - 42 T_1^8 T_2^3 - 3 T_1^9 T_2^3 + T_1^4 + 9 T_1 T_2^4 + 13 T_1^2 T_2^4 +$$

$$40 T_1^3 T_2^4 + 33 T_1^4 T_2^4 + 3 T_1^5 T_2^4 + 87 T_1^6 T_2^4 + 45 T_1^7 T_2^4 - 22 T_1^8 T_2^4 + 60 T_1^9 T_2^4 - 3 T_1^{10} T_2^4 - 4 T_1 T_2^5 -$$

$$13 T_1^2 T_2^5 - 42 T_1^3 T_2^5 - 78 T_1^4 T_2^5 + 11 T_1^5 T_2^5 - 117 T_1^6 T_2^5 - 74 T_1^7 T_2^5 - 27 T_1^8 T_2^5 - 24 T_1^9 T_2^5 - 45 T_1^{10} T_2^5 +$$

$$6 T_1^{11} T_2^5 + 8 T_1^2 T_2^6 + 26 T_1^3 T_2^6 + 96 T_1^4 T_2^6 + 33 T_1^5 T_2^6 + 47 T_1^6 T_2^6 + 165 T_1^7 T_2^6 + 32 T_1^8 T_2^6 + 30 T_1^9 T_2^6 +$$

$$57 T_1^{10} T_2^6 + 12 T_1^{11} T_2^6 - 3 T_1^{12} T_2^6 - 14 T_1^3 T_2^7 - 63 T_1^4 T_2^7 - 71 T_1^5 T_2^7 - 36 T_1^6 T_2^7 - 87 T_1^7 T_2^7 - 151 T_1^8 T_2^7 +$$

$$27 T_1^9 T_2^7 - 75 T_1^{10} T_2^7 - 27 T_1^{11} T_2^7 + 29 T_1^4 T_2^8 + 57 T_1^5 T_2^8 + 41 T_1^6 T_2^8 + 29 T_1^7 T_2^8 + 126 T_1^8 T_2^8 + 21 T_1^9 T_2^8 +$$

$$27 T_1^{10} T_2^8 + 42 T_1^{11} T_2^8 + 6 T_1^{12} T_2^8 - 33 T_1^5 T_2^9 - 30 T_1^6 T_2^9 - 69 T_1^8 T_2^9 - 30 T_1^9 T_2^9 - 15 T_1^{10} T_2^9 - 21 T_1^{11} T_2^9 -$$

$$15 T_1^{12} T_2^9 + 27 T_1^6 T_2^{10} - 9 T_1^7 T_2^{10} + 18 T_1^8 T_2^{10} + 24 T_1^9 T_2^{10} + 9 T_1^{10} T_2^{10} + 15 T_1^{12} T_2^{10} - 12 T_1^7 T_2^{11} + 12 T_1^8 T_2^{11} -$$

$$12 T_1^9 T_2^{11} - 9 T_1^{10} T_2^{11} + 9 T_1^{11} T_2^{11} - 9 T_1^{12} T_2^{11} + 3 T_1^8 T_2^{12} - 6 T_1^9 T_2^{12} + 9 T_1^{10} T_2^{12} - 6 T_1^{11} T_2^{12} + 3 T_1^{12} T_2^{12}) \}$$

In[[#]]:= **K41** = $\lambda[\text{Knot}[4, 1]]$

Out[[#]]=

$$\left\{ - \frac{1 - 3 T + T^2}{T}, \right.$$

$$- \frac{1}{T_1^4 T_2^4} (1 - 3 T_1 + T_1^2) (-1 + T_1 T_2) (1 - 3 T_2 + T_2^2) (1 - 3 T_1 T_2 + T_1^2 T_2^2) (-9 T_1 + 11 T_1^2 - 3 T_2 + 27 T_1 T_2 -$$

$$15 T_1^2 T_2 - 19 T_1^3 T_2 + 2 T_2^2 - 42 T_1 T_2^2 + 51 T_1^2 T_2^2 + 24 T_1^3 T_2^2 + 24 T_1^4 T_2^2 - 27 T_1^5 T_2^2 + 6 T_1^6 T_2^2 + 32 T_1 T_2^3 -$$

$$27 T_1^2 T_2^3 - 54 T_1^3 T_2^3 - 78 T_1^4 T_2^3 + 30 T_1^5 T_2^3 + 9 T_1^6 T_2^3 - 3 T_1^7 T_2^3 - 15 T_1 T_2^4 - 15 T_1^2 T_2^4 + 96 T_1^3 T_2^4 - 9 T_1^4 T_2^4 +$$

$$144 T_1^5 T_2^4 - 60 T_1^6 T_2^4 + 3 T_1^7 T_2^4 + 3 T_1 T_2^5 + 21 T_1^2 T_2^5 - 75 T_1^3 T_2^5 + 63 T_1^4 T_2^5 - 135 T_1^5 T_2^5 + 9 T_1^6 T_2^5 +$$

$$12 T_1^7 T_2^5 - 6 T_1^2 T_2^6 + 18 T_1^3 T_2^6 - 18 T_1^4 T_2^6 + 21 T_1^5 T_2^6 + 27 T_1^6 T_2^6 - 12 T_1^7 T_2^6 + 3 T_1^5 T_2^7 - 9 T_1^6 T_2^7 + 3 T_1^7 T_2^7) \}$$