(Alt) In[ ]:=

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank\\DunfieldKnots"];
Once[<< KnotTheory`];
<< ../Rot.m
T3 = T1 T2;
```

```
Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at http://katlas.org/wiki/KnotTheory.
```

Loading Rot.m from http://drorbn.net/AP/Projects/HigherRank to compute rotation numbers.

(Alt) In[ ]:=

```
CCF[ℰ_] := ExpandDenominator@ExpandNumerator@Together[ℰ];
CCF[ℰ_] := Factor[ℰ];
CF[ℰ_List] := CF /@ ℰ;
CF[ℰ_] := Module[{vs = Cases[ℰ, (x | p | π | g)__, ∞] ⋃ {x, p, ϵ}, ps, c},
   Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) :> CCF[c] (Times @@ vs^ps)]];
```

Data

(from Talks/Beijing-2407/theta.nb)

(Alt) In[ ]:=

```
R1[1, i_, j_] = CF[
   1/2 - T3 g1ji g2ji - g3ii + g2jj g3ii + T1 (T3 - 1) g1ji g3ji +
    T2 (T3 - 1) g2ji g3ji - T2 g2ji g3jj + (g1jj g2ii + (T3 - 1) g1jj g2ji -
       T1 g1ii g2jj - g1jj g3ii - T1 (T3 - 1) g1jj g3ji + T1 g1ii g3jj) / (T1 - 1)];
```

(Alt) In[ ]:=

```
R1[-1, i_, j_] = CF[
  -1/2 - T1^-1 g1ji g2ii - (1 - T1^-1 - T2^-1) g1ji g2ji - g1jj g2ji - g1ji g2jj + g3ii +
   T1^-1 g1ji g3ii - (1 - T2^-1) g2ji g3ii - g2jj g3ii + (1 - T3^-1) g1ji g3ji - (1 - T3^-1) g2ii g3ji +
   (2 - T2^-1) (1 - T3^-1) g2ji g3ji + (1 - T3^-1) g2jj g3ji + g1ji g3jj + g2ji g3jj + (T1 (1 - T2^-1) g1ii g2ji -
      g1jj g2ii + T1 g1ii g2jj + g1jj g3ii - T2^-1 (T3 - 1) g1ii g3ji - T1 g1ii g3jj) / (T1 - 1)]
```

(Alt) Out[ ]=

$$-\frac{1}{2}-\frac{g_{1,j,i}\,g_{2,i,i}}{T_1}-\frac{g_{1,j,j}\,g_{2,i,i}}{-1+T_1}+\frac{T_1\,(-1+T_2)\,g_{1,i,i}\,g_{2,j,i}}{(-1+T_1)\,T_2}-$$
$$\frac{(-T_1-T_2+T_1\,T_2)\,g_{1,j,i}\,g_{2,j,i}}{T_1\,T_2}-g_{1,j,j}\,g_{2,j,i}+\frac{T_1\,g_{1,i,i}\,g_{2,j,j}}{-1+T_1}-g_{1,j,i}\,g_{2,j,j}+g_{3,i,i}+$$
$$\frac{g_{1,j,i}\,g_{3,i,i}}{T_1}+\frac{g_{1,j,j}\,g_{3,i,i}}{-1+T_1}-\frac{(-1+T_2)\,g_{2,j,i}\,g_{3,i,i}}{T_2}-g_{2,j,j}\,g_{3,i,i}-\frac{(-1+T_1\,T_2)\,g_{1,i,i}\,g_{3,j,i}}{(-1+T_1)\,T_2}+$$
$$\frac{(-1+T_1\,T_2)\,g_{1,j,i}\,g_{3,j,i}}{T_1\,T_2}-\frac{(-1+T_1\,T_2)\,g_{2,i,i}\,g_{3,j,i}}{T_1\,T_2}+\frac{(-1+2\,T_2)\,(-1+T_1\,T_2)\,g_{2,j,i}\,g_{3,j,i}}{T_1\,T_2^2}+$$
$$\frac{(-1+T_1\,T_2)\,g_{2,j,j}\,g_{3,j,i}}{T_1\,T_2}-\frac{T_1\,g_{1,i,i}\,g_{3,j,j}}{-1+T_1}+g_{1,j,i}\,g_{3,j,j}+g_{2,j,i}\,g_{3,j,j}$$

(Alt) In[ ]:=

```
θ[{1, i0_, j0_}, {1, i1_, j1_}] =
  -T1 (T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 + (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 +
   T1 (T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 - (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;
```

(Alt) In[ ]:=

```
θ[{1, i0_, j0_}, {-1, i1_, j1_}] =
  (T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 - T1^-1 (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 -
   (T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 + T1^-1 (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;
```

(Alt) In[ ]:=

```
θ[{-1, i0_, j0_}, {1, i1_, j1_}] = CF[
   T1^-1 T2^-1 (T3 - 1) (g1,j1,i0 g2,i1,i0 g3,j0,i1 -
      T1 g1,j1,j0 g2,i1,i0 g3,j0,i1 - g1,j1,i0 g2,j1,i0 g3,j0,i1 + T1 g1,j1,j0 g2,j1,i0 g3,j0,i1)];
```

(Alt) In[ ]:=

```
θ[{-1, i0_, j0_}, {-1, i1_, j1_}] = CF[
   (1 - T3^-1) (-T1^-1 g1,j1,i0 g2,i1,i0 g3,j0,i1 +
      g1,j1,j0 g2,i1,i0 g3,j0,i1 + T1^-1 g1,j1,i0 g2,j1,i0 g3,j0,i1 - g1,j1,j0 g2,j1,i0 g3,j0,i1)];
```

(Alt) In[ ]:=

```
Γ1[φ_, k_] = -φ / 2 + φ g3,k,k;
```

The Programs

(Alt) In[ ]:=

```
θ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ν, α, β, gEval, c, z},
   {Cs, φ} = Rot[K]; n = Length[Cs];
   A = IdentityMatrix[2 n + 1];
   Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += {{-T^s, T^s - 1}, {0, -1}})];
   Δ = T^(-Total[φ] - Total[Cs[[All, 1]]])/2 Det[A];
   G = Inverse[A]; gEval[ℰ_] := Factor[ℰ /. g_ν_,α_,β_ :> (G[[α, β]] /. T → T_ν)];
   z = gEval[Sum[Sum[θ[Cs[[k1]], Cs[[k2]]], {k2, 1, n}], {k1, 1, n}]];
   z += gEval[Sum[R1 @@ Cs[[k]], {k, 1, n}]];
   z += gEval[Sum[Γ1[φ[[k]], k], {k, 1, 2 n}]];
   {Δ, (Δ /. T → T1) (Δ /. T → T2) (Δ /. T → T3) z} // Factor];
```

(Alt) In[ ]:=

```
PolyPlot[0] = Graphics[{}];
PolyPlot[p_] := Module[{crs, m1, m2, maxc, minc, s, hex},
   crs = CoefficientRules[T1^(m1=-Exponent[p,T1,Min]) T2^(m2=-Exponent[p,T2,Min]) p, {T1, T2}];
   maxc = N@Log@Max@Abs[Last /@ crs];
   minc = N@Log@Min@Select[Abs[Last /@ crs], # > 0 &];
   If[minc == maxc, s[_] = 0, s[c_] := s[c] = (maxc - Log@c) / (maxc - minc)];
   hex = Table[{Cos[α], Sin[α]} / Cos[2 π / 12] / 2, {α, 2 π / 12, 2 π, 2 π / 6}];
   Graphics[crs /. ({x1_, x2_} → c_) :> {
       If[c == 0, White, Lighter[If[c > 0, Red, Blue], 0.88 s[Abs@c]]],
       Polygon[({{1, -1/2}, {0, Sqrt[3]/2}}.{x1 + m1, x2 + m2} + #) & /@ hex]}]];
PolyPlot[{Δ_, Θ_}] := PolyPlot[Θ]
```

Testing

In[ ]:=

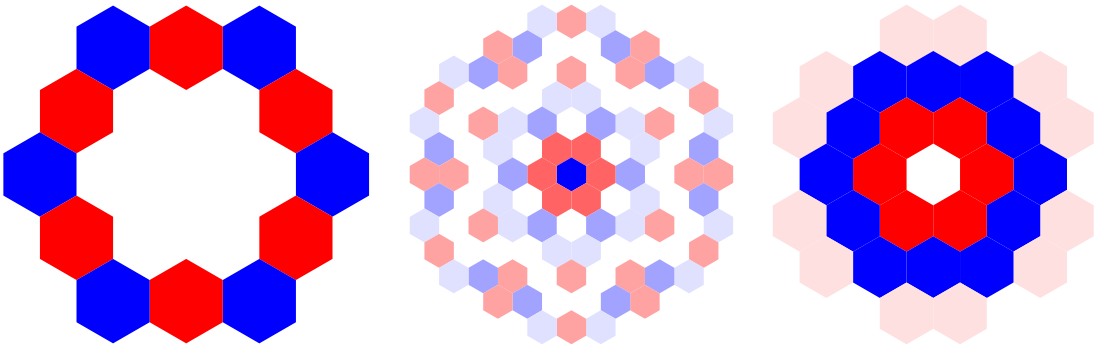
```
GraphicsRow[PolyPlot[Θ[Knot[#]][[2]]] &
  /@ {"3_1", "K11n34", "K11n42"}]
```

KnotTheory: Loading precomputed data in PD4Knots`.

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

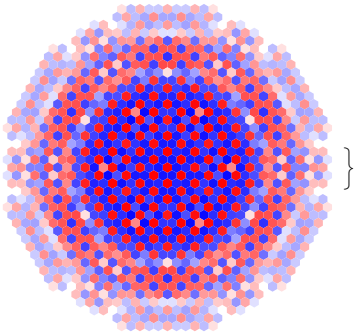
KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[ ]=



(Alt) In[•]:=
```
AbsoluteTiming@
 PolyPlot[θ[EPD[X14,1, X̄2,29, X3,40, X43,4, X̄26,5, X6,95, X96,7, X13,8, X̄9,28, X10,41, X42,11, X̄27,12,
     X30,15, X̄16,61, X̄17,72, X̄18,83, X19,34, X̄89,20, X̄21,92, X̄79,22, X̄68,23, X̄57,24, X̄25,56, X62,31,
     X73,32, X84,33, X̄50,35, X36,81, X37,70, X38,59, X̄39,54, X44,55, X58,45, X69,46, X80,47, X48,91,
     X90,49, X51,82, X52,71, X53,60, X̄63,74, X̄64,85, X̄76,65, X̄87,66, X̄67,94, X̄75,86, X̄88,77, X̄78,93]][[2]]]
```

(Alt) Out[•]=

{13.2989,  }

Run

(Alt) In[•]:=
```
DunfieldKnots =
  ReadList["../../../People/Dunfield/nmd_random_knots"] /. k_Integer :> k + 1;
DK[n_] := DunfieldKnots[[n - 2]];
```

(Alt) In[•]:=
```
DKString[n_] := StringDrop[ToString[1000 + n], 1]
```

In[•]:=
```
DKString[76]
```

Out[•]=
```
076
```

(Alt) In[•]:=
```
Do[
 If[FileExistsQ[from = "D" <> ToString[n] <> ".m"],
  RenameFile[from, "D" <> DKString[n] <> ".m"]];
 If[FileExistsQ[from = "PP" <> ToString[n] <> ".png"],
  RenameFile[from, "PP" <> DKString[n] <> ".png"]],
 {n, 3, 99}]
```

(Alt) In[•]:=

```
Clear[at, pp];
Monitor[
 Do[
  If[(n = k) > 1000, Abort[]];
  If[Not@FileExistsQ["D" <> DKString[n] <> ".m"],
   Put[
    ({at, th} = AbsoluteTiming[θ[DK[n]]]) /. {T1 → T1, T2 → T2},
    "D" <> DKString[n] <> ".m"];
   Export["PP" <> DKString[n] <> ".png", pp = PolyPlot[th[[2]]]];
   mon = {n, at, pp}
  ],
  {k, 3, 1000, 1}
 ],
 mon ~Join~ {n}
]
```

(Alt) In[•]:=

```
AbsoluteTiming[th = θ[DK[100]];]
PolyPlot[th]
```

(Alt) Out[•]=

```
{335.969, Null}
```

(Alt) Out[•]=