

```
In[1]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank\\Data"] ;
Once[<< KnotTheory`];
<< ./Rot.m
T3 = T1 T2;
```

C:\drorbn\AcademicPensieve\Projects\KnotTheory\KnotTheory

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

SetDelayed: Tag Diff in Diff[K_PD, rut_, ag_, n_, m_] is Protected.

>Loading Rot.m from <http://drorbn.net/AP/Projects/HigherRank> to compute rotation numbers.

```
In[2]:= CCF[<*>] := ExpandDenominator@ExpandNumerator@Together[<*>];
CCF[<*>] := Factor[<*>];
CF[<*>List] := CF /@ <*>;
CF[<*>] := Module[{vs = Cases[<*>, (x | p | \[Pi] | g) __, \[Infinity]] \[Union] {x, p, e}, ps, c},
Total[CoefficientRules[Expand[<*>], vs] /. (ps_ \[Rule] c_) \[Rule] CCF[c] (Times @@ vs^ps)] ]];
```

```
In[3]:= R1[1, i_, j_] = CF[
1/2 - T3 g1ji g2ji - g3ii + g2jj g3ii + T1 (T3 - 1) g1ji g3ji +
T2 (T3 - 1) g2ji g3ji - T2 g2ji g3jj + (g1jj g2ii + (T3 - 1) g1jj g2ji -
T1 g1ii g2jj - g1jj g3ii - T1 (T3 - 1) g1jj g3ji + T1 g1ii g3jj) / (T1 - 1)];
```

```
In[4]:= R1[-1, i_, j_] = CF[
-1/2 - T1^-1 g1ji g2ii - (1 - T1^-1 - T2^-1) g1ji g2ji - g1jj g2ji - g1ji g2jj + g3ii +
T1^-1 g1ji g3ii - (1 - T2^-1) g2ji g3ii - g2jj g3ii + (1 - T3^-1) g1ji g3ji - (1 - T3^-1) g2ii g3ji +
(2 - T2^-1) (1 - T3^-1) g2ji g3ji + (1 - T3^-1) g2jj g3ji + g1ji g3jj + g2ji g3jj + (T1 (1 - T2^-1) g1ii g2ji -
g1jj g2ii + T1 g1ii g2jj + g1jj g3ii - T2^-1 (T3 - 1) g1ii g3ji - T1 g1ii g3jj) / (T1 - 1)];
```

```
In[5]:= \[Theta][{1, i0_, j0_}, {1, i1_, j1_}] =
-T1 (T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 + (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 +
T1 (T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 - (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;
```

```
In[6]:= \[Theta][{1, i0_, j0_}, {-1, i1_, j1_}] =
(T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 - T1^-1 (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 -
(T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 + T1^-1 (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;
```

```
In[7]:= \[Theta][{-1, i0_, j0_}, {1, i1_, j1_}] = CF[
T1^-1 T2^-1 (T3 - 1) (g1,j1,i0 g2,i1,i0 g3,j0,i1 -
T1 g1,j1,j0 g2,i1,i0 g3,j0,i1 - g1,j1,i0 g2,j1,i0 g3,j0,i1 + T1 g1,j1,j0 g2,j1,i0 g3,j0,i1)];
```

```
In[1]:= Θ[{-1, iθ_, jθ_}, {-1, i1_, j1_}] = CF[
  (1 - T3-1) (-T1-1 g1,j1,iθ g2,i1,iθ g3,jθ,i1 +
   g1,j1,jθ g2,i1,iθ g3,jθ,i1 + T1-1 g1,j1,iθ g2,j1,iθ g3,jθ,i1 - g1,j1,jθ g2,j1,iθ g3,jθ,i1)];
```

```
In[2]:= Γ1[φ_, k_] = -φ / 2 + φ g3,k,k;
```

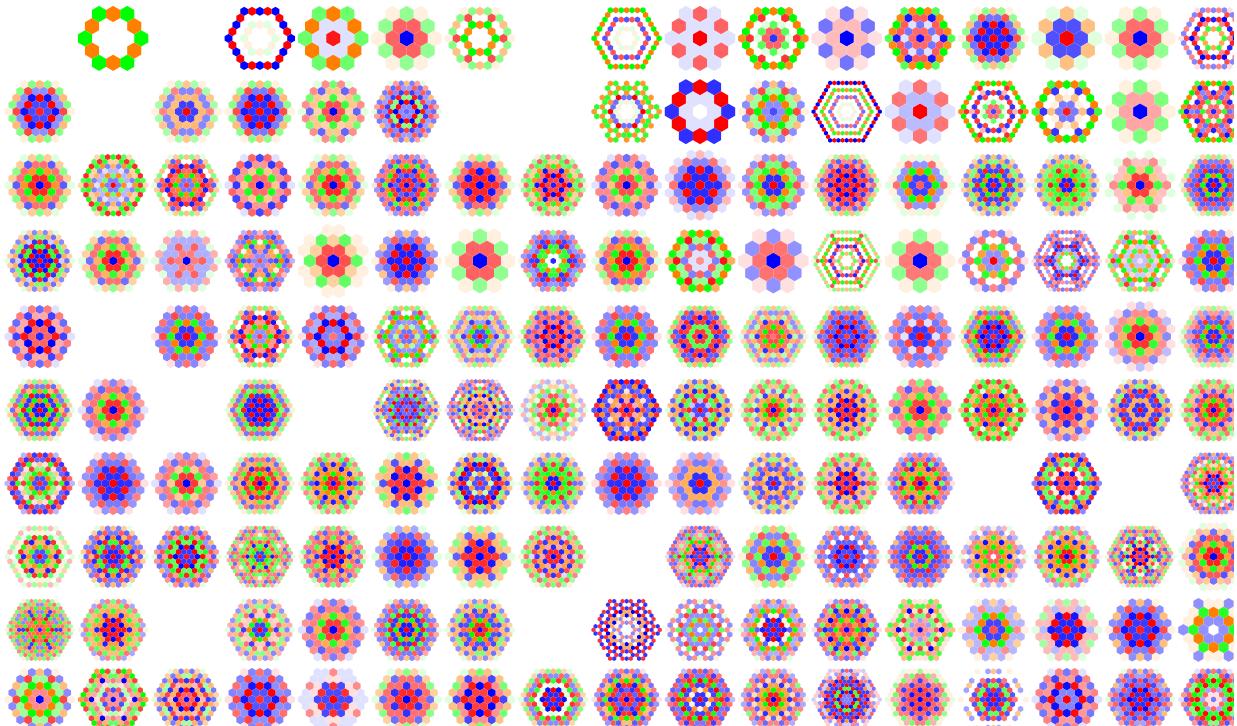
```
In[3]:= Θ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, v, α, β, gEval, c, z},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :> (A[[{i, j}], {i + 1, j + 1}] += {{-Ts Ts - 1}, {0, -1}})];
  Δ = T(-Total[φ] - Total[Cs[[All, 1]]]) / 2 Det[A];
  G = Inverse[A]; gEval[ε_] := Factor[ε /. gv,α,β :> (G[[α, β]] /. T → Tv)];
  z = gEval[Sum[Sum[Θ[Cs[[k1]], Cs[[k2]]], {k1, 1, n}], {k2, 1, n}];
  z += gEval[Sum[R1 @@ Cs[[k]], {k, 1, n}]];
  z += gEval[Sum[T1[φ[[k]], k], {k, 1, n}]];
  {Δ, (Δ /. T → T1) (Δ /. T → T2) (Δ /. T → T3) z} // Factor];
```

```
In[4]:= PolyPlot[θ] = Graphics[{}];
PolyPlot[p_] := Module[{crs, m1, m2, maxc, minc, s, hex},
  crs = CoefficientRules[T1m1 = -Exponent[p, T1, Min] T2m2 = -Exponent[p, T2, Min] p, {T1, T2}];
  maxc = N@Log@Max@Abs[Last /@ crs];
  minc = N@Log@Min@Select[Abs[Last /@ crs], # > 0 &];
  If[minc == maxc, s[_] = 0, s[c_] := s[c] = (maxc - Log@c) / (maxc - minc)];
  hex = Table[{Cos[α], Sin[α]} / Cos[2 π / 12] / 2, {α, 2 π / 12, 2 π, 2 π / 6}];
  Graphics[crs /. ({x1_, x2_} → c_) :> {
    If[c == 0, White, Lighter[Which[
      c > 0 ∧ OddQ[c], Orange,
      c > 0 ∧ EvenQ[c], Red,
      c < 0 ∧ OddQ[c], Green,
      c < 0 ∧ EvenQ[c], Blue
    ], 0.88 s[Abs@c]]],
    Polygon[{{(1 - 1/2), 0}, {(1 + 1/2), 0}, {(0, Sqrt[3]/2)}, {(0, -Sqrt[3]/2)}] & /@ hex]
  }]];
  PolyPlot[{Δ_, θ_}] := PolyPlot[θ]
```

```
In[=]:= tab250 = {0} ~Join~ Table[θ[K] [[2]], {K, AllKnots[{3, 10}]}];
g250 = GraphicsGrid[Partition[PolyPlot /@ tab250, 25], Spacings -> 0]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[=]=



```
In[=]:= Table[n -> NumberOfKnots[n], {n, 3, 15}]
```

Out[=]=

```
{3 -> 1, 4 -> 1, 5 -> 2, 6 -> 3, 7 -> 7, 8 -> 21, 9 -> 49,
10 -> 165, 11 -> 552, 12 -> 2176, 13 -> 9988, 14 -> 46972, 15 -> 253293}
```

In[=]:= $\Theta[\text{Knot}[15, \text{Alternating}, 20902]]$

KnotTheory: Loading precomputed data in KnotTheory/15A.dts.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[=]=

$$\left\{ -\frac{6 - 35 T + 91 T^2 - 125 T^3 + 91 T^4 - 35 T^5 + 6 T^6}{T^3}, \right. \\ -\frac{1}{T_1^6 T_2^6} (45 - 319 T_1 + 878 T_1^2 - 1219 T_1^3 + 878 T_1^4 - 319 T_1^5 + 45 T_1^6 - 319 T_2 + 1870 T_1 T_2 - 3824 T_1^2 T_2 + 2356 T_1^3 T_2 + 2356 T_1^4 T_2 - 3824 T_1^5 T_2 + 1870 T_1^6 T_2 - 319 T_1^7 T_2 + 878 T_1^8 T_2 - 3824 T_1 T_2^2 + 2825 T_1^2 T_2^2 + 10702 T_1^3 T_2^2 - 22185 T_1^4 T_2^2 + 10702 T_1^5 T_2^2 + 2825 T_1^6 T_2^2 - 3824 T_1^7 T_2^2 + 878 T_1^8 T_2^2 - 1219 T_2^3 + 2356 T_1 T_2^3 + 10702 T_1^2 T_2^3 - 37286 T_1^3 T_2^3 + 27271 T_1^4 T_2^3 + 27271 T_1^5 T_2^3 - 37286 T_1^6 T_2^3 + 10702 T_1^7 T_2^3 + 2356 T_1^8 T_2^3 - 1219 T_1^9 T_2^3 + 878 T_1^10 T_2^3 + 2356 T_1^11 T_2^3 - 22185 T_1^12 T_2^3 + 27271 T_1^13 T_2^3 + 45442 T_1^14 T_2^3 - 116098 T_1^15 T_2^3 + 45442 T_1^16 T_2^3 + 27271 T_1^17 T_2^3 - 22185 T_1^18 T_2^3 + 2356 T_1^19 T_2^3 + 878 T_1^20 T_2^3 - 3824 T_1^21 T_2^3 + 89308 T_1^22 T_2^3 + 89308 T_1^23 T_2^3 - 116098 T_1^24 T_2^3 + 89308 T_1^25 T_2^3 + 89308 T_1^26 T_2^3 - 116098 T_1^27 T_2^3 + 27271 T_1^28 T_2^3 + 10702 T_1^29 T_2^3 - 37286 T_1^30 T_2^3 + 45442 T_1^31 T_2^3 + 89308 T_1^32 T_2^3 - 220980 T_1^33 T_2^3 + 89308 T_1^34 T_2^3 + 45442 T_1^35 T_2^3 - 2825 T_1^36 T_2^3 - 37286 T_1^37 T_2^3 + 45442 T_1^38 T_2^3 + 89308 T_1^39 T_2^3 - 220980 T_1^40 T_2^3 + 89308 T_1^41 T_2^3 + 45442 T_1^42 T_2^3 - 37286 T_1^43 T_2^3 + 2825 T_1^44 T_2^3 + 1870 T_1^45 T_2^3 + 45 T_1^46 T_2^3 - 319 T_1 T_2^7 - 3824 T_1^2 T_2^7 + 10702 T_1^3 T_2^7 + 37286 T_1^4 T_2^7 - 116098 T_1^5 T_2^7 + 89308 T_1^6 T_2^7 + 89308 T_1^7 T_2^7 - 116098 T_1^8 T_2^7 + 27271 T_1^9 T_2^7 + 10702 T_1^10 T_2^7 - 3824 T_1^11 T_2^7 + 319 T_1^12 T_2^7 + 878 T_1^13 T_2^7 + 2356 T_1^14 T_2^7 - 22185 T_1^15 T_2^7 + 27271 T_1^16 T_2^7 + 45442 T_1^17 T_2^7 - 116098 T_1^18 T_2^7 + 45442 T_1^19 T_2^7 + 27271 T_1^20 T_2^7 - 22185 T_1^21 T_2^7 + 2356 T_1^22 T_2^7 + 878 T_1^23 T_2^7 - 1219 T_1^24 T_2^7 + 878 T_1^25 T_2^7 - 319 T_1^26 T_2^7 + 45 T_1^27 T_2^7) \Big\}$$

In[=]:= $\Theta[\text{Knot}[15, \text{Alternating}, 20903]]$

Out[=]=

$$\left\{ \frac{(2 - 3 T + 2 T^2) (1 - 6 T + 16 T^2 - 23 T^3 + 25 T^4 - 23 T^5 + 16 T^6 - 6 T^7 + T^8)}{T^5}, \right. \\ -\frac{1}{T_1^{10} T_2^{10}} (25 - 186 T_1 + 639 T_1^2 - 1291 T_1^3 + 1827 T_1^4 - 2015 T_1^5 + 1827 T_1^6 - 1291 T_1^7 + 639 T_1^8 - 186 T_1^9 + 25 T_1^{10} - 186 T_2 + 1200 T_1 T_2 - 3382 T_1^2 T_2 + 4879 T_1^3 T_2 - 4021 T_1^4 T_2 + 1414 T_1^5 T_2 + 1414 T_1^6 T_2 - 4021 T_1^7 T_2 + 4879 T_1^8 T_2 - 3382 T_1^9 T_2 + 1200 T_1^{10} T_2 - 186 T_1^{11} T_2 + 639 T_2^2 - 3382 T_1 T_2^2 + 6708 T_1^2 T_2^2 - 2333 T_1^3 T_2^2 - 8819 T_1^4 T_2^2 + 17455 T_1^5 T_2^2 - 19176 T_1^6 T_2^2 + 17455 T_1^7 T_2^2 - 8819 T_1^8 T_2^2 - 2333 T_1^9 T_2^2 + 6708 T_1^{10} T_2^2 - 3382 T_1^{11} T_2^2 + 639 T_1^{12} T_2^2 - 1291 T_1^{13} T_2^2 + 4879 T_1^{14} T_2^2 - 2333 T_1^{15} T_2^2 - 21460 T_1^{16} T_2^2 + 41279 T_1^{17} T_2^2 - 33366 T_1^{18} T_2^2 + 9255 T_1^{19} T_2^2 + 9255 T_1^{20} T_2^2 - 33366 T_1^{21} T_2^2 + 41279 T_1^{22} T_2^2 - 21460 T_1^{23} T_2^2 - 2333 T_1^{24} T_2^2 + 4879 T_1^{25} T_2^2 - 1291 T_1^{26} T_2^2 + 1827 T_1^{27} T_2^2 - 4021 T_1^{28} T_2^2 - 8819 T_1^{29} T_2^2 + 41279 T_1^{30} T_2^2 - 18452 T_1^{31} T_2^2 - 56189 T_1^{32} T_2^2 + 109308 T_1^{33} T_2^2 - 110634 T_1^{34} T_2^2 + 109308 T_1^{35} T_2^2 - 56189 T_1^{36} T_2^2 - 18452 T_1^{37} T_2^2 + 41279 T_1^{38} T_2^2 - 8819 T_1^{39} T_2^2 - 4021 T_1^{40} T_2^2 + 1827 T_1^{41} T_2^2 - 2015 T_1^{42} T_2^2 + 1414 T_1^{43} T_2^2 + 17455 T_1^{44} T_2^2 - 33366 T_1^{45} T_2^2 + 56189 T_1^{46} T_2^2 + 140578 T_1^{47} T_2^2 - 110869 T_1^{48} T_2^2 + 19830 T_1^{49} T_2^2 + 19830 T_1^{50} T_2^2 - 110869 T_1^{51} T_2^2 + 140578 T_1^{52} T_2^2 - 56189 T_1^{53} T_2^2 - 33366 T_1^{54} T_2^2 + 17455 T_1^{55} T_2^2 + 1414 T_1^{56} T_2^2 - 2015 T_1^{57} T_2^2 + 1827 T_1^{58} T_2^2 + 1414 T_1^{59} T_2^2 - 19176 T_1^{60} T_2^2 + 9255 T_1^{61} T_2^2 + 109308 T_1^{62} T_2^2 - 110869 T_1^{63} T_2^2 - 18442 T_1^{64} T_2^2 + 103490 T_1^{65} T_2^2 -$$

$$\begin{aligned}
& 64 \cdot 338 T_1^8 T_2^6 + 103 \cdot 490 T_1^9 T_2^6 - 18 \cdot 442 T_1^{10} T_2^6 - 110 \cdot 869 T_1^{11} T_2^6 + 109 \cdot 308 T_1^{12} T_2^6 + 9255 T_1^{13} T_2^6 - \\
& 19 \cdot 176 T_1^{14} T_2^6 + 1414 T_1^{15} T_2^6 + 1827 T_1^{16} T_2^6 - 1291 T_1^7 T_2 - 4021 T_1 T_2^7 + 17455 T_1^2 T_2^7 + 9255 T_1^3 T_2^7 - \\
& 110 \cdot 634 T_1^4 T_2^7 + 19 \cdot 830 T_1^5 T_2^7 + 103 \cdot 490 T_1^6 T_2^7 - 81 \cdot 860 T_1^7 T_2^7 - 23 \cdot 747 T_1^8 T_2^7 - 23 \cdot 747 T_1^9 T_2^7 - 81 \cdot 860 T_1^{10} T_2^7 + \\
& 103 \cdot 490 T_1^{11} T_2^7 + 19 \cdot 830 T_1^{12} T_2^7 - 110 \cdot 634 T_1^{13} T_2^7 + 9255 T_1^{14} T_2^7 + 17455 T_1^{15} T_2^7 - 4021 T_1^{16} T_2^7 - \\
& 1291 T_1^{17} T_2^7 + 639 T_1^8 + 4879 T_1 T_2^8 - 8819 T_1^2 T_2^8 - 33 \cdot 366 T_1^3 T_2^8 + 109 \cdot 308 T_1^4 T_2^8 + 19 \cdot 830 T_1^5 T_2^8 - \\
& 64 \cdot 338 T_1^6 T_2^8 - 23 \cdot 747 T_1^7 T_2^8 + 87 \cdot 820 T_1^8 T_2^8 + 11 \cdot 778 T_1^9 T_2^8 + 87 \cdot 820 T_1^{10} T_2^8 - 23 \cdot 747 T_1^{11} T_2^8 - 64 \cdot 338 T_1^{12} T_2^8 + \\
& 19 \cdot 830 T_1^{13} T_2^8 + 109 \cdot 308 T_1^{14} T_2^8 - 33 \cdot 366 T_1^{15} T_2^8 - 8819 T_1^{16} T_2^8 + 4879 T_1^{17} T_2^8 + 639 T_1^{18} T_2^8 - 186 T_2^9 - \\
& 3382 T_1 T_2^9 - 2333 T_1^2 T_2^9 + 41 \cdot 279 T_1^3 T_2^9 - 56 \cdot 189 T_1^4 T_2^9 - 110 \cdot 869 T_1^5 T_2^9 + 103 \cdot 490 T_1^6 T_2^9 - 23 \cdot 747 T_1^7 T_2^9 + \\
& 11 \cdot 778 T_1^8 T_2^9 - 77 \cdot 420 T_1^9 T_2^9 - 77 \cdot 420 T_1^{10} T_2^9 + 11 \cdot 778 T_1^{11} T_2^9 - 23 \cdot 747 T_1^{12} T_2^9 + 103 \cdot 490 T_1^{13} T_2^9 - \\
& 110 \cdot 869 T_1^{14} T_2^9 - 56 \cdot 189 T_1^{15} T_2^9 + 41 \cdot 279 T_1^{16} T_2^9 - 2333 T_1^{17} T_2^9 - 3382 T_1^{18} T_2^9 - 186 T_1^{19} T_2^9 + 25 T_2^{10} + \\
& 1200 T_1 T_2^{10} + 6708 T_1^2 T_2^{10} - 21 \cdot 460 T_1^3 T_2^{10} - 18 \cdot 452 T_1^4 T_2^{10} + 140 \cdot 578 T_1^5 T_2^{10} - 18 \cdot 442 T_1^6 T_2^{10} - 81 \cdot 860 T_1^7 T_2^{10} + \\
& 87 \cdot 820 T_1^8 T_2^{10} - 77 \cdot 420 T_1^9 T_2^{10} + 212 \cdot 052 T_1^{10} T_2^{10} - 77 \cdot 420 T_1^{11} T_2^{10} + 87 \cdot 820 T_1^{12} T_2^{10} - 81 \cdot 860 T_1^{13} T_2^{10} + \\
& 18 \cdot 442 T_1^{14} T_2^{10} + 140 \cdot 578 T_1^{15} T_2^{10} - 18 \cdot 452 T_1^{16} T_2^{10} - 21 \cdot 460 T_1^{17} T_2^{10} + 6708 T_1^{18} T_2^{10} + 1200 T_1^{19} T_2^{10} + \\
& 25 T_1^{20} T_2^{10} - 186 T_1 T_2^{11} - 3382 T_1^2 T_2^{11} - 2333 T_1^3 T_2^{11} + 41 \cdot 279 T_1^4 T_2^{11} - 56 \cdot 189 T_1^5 T_2^{11} - 110 \cdot 869 T_1^6 T_2^{11} + \\
& 103 \cdot 490 T_1^7 T_2^{11} - 23 \cdot 747 T_1^8 T_2^{11} + 11 \cdot 778 T_1^9 T_2^{11} - 77 \cdot 420 T_1^{10} T_2^{11} - 77 \cdot 420 T_1^{11} T_2^{11} + 11 \cdot 778 T_1^{12} T_2^{11} - \\
& 23 \cdot 747 T_1^{13} T_2^{11} + 103 \cdot 490 T_1^{14} T_2^{11} - 110 \cdot 869 T_1^{15} T_2^{11} - 56 \cdot 189 T_1^{16} T_2^{11} + 41 \cdot 279 T_1^{17} T_2^{11} - 2333 T_1^{18} T_2^{11} - \\
& 3382 T_1^{19} T_2^{11} - 186 T_1^{20} T_2^{11} + 639 T_1^2 T_2^{12} + 4879 T_1^3 T_2^{12} - 8819 T_1^4 T_2^{12} - 33 \cdot 366 T_1^5 T_2^{12} + 109 \cdot 308 T_1^6 T_2^{12} + \\
& 19 \cdot 830 T_1^7 T_2^{12} - 64 \cdot 338 T_1^8 T_2^{12} - 23 \cdot 747 T_1^9 T_2^{12} + 87 \cdot 820 T_1^{10} T_2^{12} + 11 \cdot 778 T_1^{11} T_2^{12} + 87 \cdot 820 T_1^{12} T_2^{12} - \\
& 23 \cdot 747 T_1^{13} T_2^{12} - 64 \cdot 338 T_1^{14} T_2^{12} + 19 \cdot 830 T_1^{15} T_2^{12} + 109 \cdot 308 T_1^{16} T_2^{12} - 33 \cdot 366 T_1^{17} T_2^{12} - 8819 T_1^{18} T_2^{12} + \\
& 4879 T_1^{19} T_2^{12} + 639 T_1^{20} T_2^{12} - 1291 T_1^3 T_2^{13} - 4021 T_1^4 T_2^{13} + 17455 T_1^5 T_2^{13} + 9255 T_1^6 T_2^{13} - 110 \cdot 634 T_1^7 T_2^{13} + \\
& 19 \cdot 830 T_1^8 T_2^{13} + 103 \cdot 490 T_1^9 T_2^{13} - 81 \cdot 860 T_1^{10} T_2^{13} - 23 \cdot 747 T_1^{11} T_2^{13} - 23 \cdot 747 T_1^{12} T_2^{13} - 81 \cdot 860 T_1^{13} T_2^{13} + \\
& 103 \cdot 490 T_1^{14} T_2^{13} + 19 \cdot 830 T_1^{15} T_2^{13} - 110 \cdot 634 T_1^{16} T_2^{13} + 9255 T_1^{17} T_2^{13} + 17455 T_1^{18} T_2^{13} - 4021 T_1^{19} T_2^{13} - \\
& 1291 T_1^{20} T_2^{13} + 1827 T_1^4 T_2^{14} + 1414 T_1^5 T_2^{14} - 19 \cdot 176 T_1^6 T_2^{14} + 9255 T_1^7 T_2^{14} + 109 \cdot 308 T_1^8 T_2^{14} - 110 \cdot 869 T_1^9 T_2^{14} - \\
& 18 \cdot 442 T_1^{10} T_2^{14} + 103 \cdot 490 T_1^{11} T_2^{14} - 64 \cdot 338 T_1^{12} T_2^{14} + 103 \cdot 490 T_1^{13} T_2^{14} - 18 \cdot 442 T_1^{14} T_2^{14} - 110 \cdot 869 T_1^{15} T_2^{14} + \\
& 109 \cdot 308 T_1^{16} T_2^{14} + 9255 T_1^{17} T_2^{14} - 19 \cdot 176 T_1^{18} T_2^{14} + 1414 T_1^{19} T_2^{14} + 1827 T_1^{20} T_2^{14} - 2015 T_1^5 T_2^{15} + 1414 T_1^6 T_2^{15} + \\
& 17455 T_1^7 T_2^{15} - 33 \cdot 366 T_1^8 T_2^{15} - 56 \cdot 189 T_1^9 T_2^{15} + 140 \cdot 578 T_1^{10} T_2^{15} - 110 \cdot 869 T_1^{11} T_2^{15} + 19 \cdot 830 T_1^{12} T_2^{15} + \\
& 19 \cdot 830 T_1^{13} T_2^{15} - 110 \cdot 869 T_1^{14} T_2^{15} + 140 \cdot 578 T_1^{15} T_2^{15} - 56 \cdot 189 T_1^{16} T_2^{15} - 33 \cdot 366 T_1^{17} T_2^{15} + 17455 T_1^{18} T_2^{15} + \\
& 1414 T_1^{19} T_2^{15} - 2015 T_1^{20} T_2^{15} + 1827 T_1^6 T_2^{16} - 4021 T_1^7 T_2^{16} - 8819 T_1^8 T_2^{16} + 41 \cdot 279 T_1^9 T_2^{16} - 18 \cdot 452 T_1^{10} T_2^{16} - \\
& 56 \cdot 189 T_1^{11} T_2^{16} + 109 \cdot 308 T_1^{12} T_2^{16} - 110 \cdot 634 T_1^{13} T_2^{16} + 109 \cdot 308 T_1^{14} T_2^{16} - 56 \cdot 189 T_1^{15} T_2^{16} - 18 \cdot 452 T_1^{16} T_2^{16} + \\
& 41 \cdot 279 T_1^{17} T_2^{16} - 8819 T_1^{18} T_2^{16} - 4021 T_1^{19} T_2^{16} + 1827 T_1^{20} T_2^{16} - 1291 T_1^7 T_2^{17} + 4879 T_1^8 T_2^{17} - 2333 T_1^9 T_2^{17} - \\
& 21 \cdot 460 T_1^{10} T_2^{17} + 41 \cdot 279 T_1^{11} T_2^{17} - 33 \cdot 366 T_1^{12} T_2^{17} + 9255 T_1^{13} T_2^{17} + 9255 T_1^{14} T_2^{17} - 33 \cdot 366 T_1^{15} T_2^{17} + \\
& 41 \cdot 279 T_1^{16} T_2^{17} - 21 \cdot 460 T_1^{17} T_2^{17} - 2333 T_1^{18} T_2^{17} + 4879 T_1^{19} T_2^{17} - 1291 T_1^{20} T_2^{17} + 639 T_1^8 T_2^{18} - 3382 T_1^9 T_2^{18} + \\
& 6708 T_1^{10} T_2^{18} - 2333 T_1^{11} T_2^{18} - 8819 T_1^{12} T_2^{18} + 17455 T_1^{13} T_2^{18} - 19 \cdot 176 T_1^{14} T_2^{18} + 17455 T_1^{15} T_2^{18} - \\
& 8819 T_1^{16} T_2^{18} - 2333 T_1^{17} T_2^{18} + 6708 T_1^{18} T_2^{18} - 3382 T_1^{19} T_2^{18} + 639 T_1^{20} T_2^{18} - 186 T_1^9 T_2^{19} + 1200 T_1^{10} T_2^{19} - \\
& 3382 T_1^{11} T_2^{19} + 4879 T_1^{12} T_2^{19} - 4021 T_1^{13} T_2^{19} + 1414 T_1^{14} T_2^{19} + 1414 T_1^{15} T_2^{19} - 4021 T_1^{16} T_2^{19} + 4879 T_1^{17} T_2^{19} - \\
& 3382 T_1^{18} T_2^{19} + 1200 T_1^{19} T_2^{19} - 186 T_1^{20} T_2^{19} + 25 T_1^{10} T_2^{20} - 186 T_1^{11} T_2^{20} + 639 T_1^{12} T_2^{20} - 1291 T_1^{13} T_2^{20} + \\
& 1827 T_1^{14} T_2^{20} - 2015 T_1^{15} T_2^{20} + 1827 T_1^{16} T_2^{20} - 1291 T_1^{17} T_2^{20} + 639 T_1^{18} T_2^{20} - 186 T_1^{19} T_2^{20} + 25 T_1^{20} T_2^{20}) \}
\end{aligned}$$

In[1]:= Θ[Knot[15, Alternating, 20904]]

Out[*n*] =

$$\left\{ \frac{4 - 31 T + 109 T^2 - 217 T^3 + 271 T^4 - 217 T^5 + 109 T^6 - 31 T^7 + 4 T^8}{T^4}, \right.$$

$$\frac{1}{T_1^8 T_2^8} \left(144 - 1104 T_1 + 3840 T_1^2 - 7580 T_1^3 + 9440 T_1^4 - 7580 T_1^5 + 3840 T_1^6 - 1104 T_1^7 + 144 T_1^8 - 1104 T_2 + \right.$$

$$7372 T_1 T_2 - 21032 T_1^2 T_2 + 28778 T_1^3 T_2 - 14318 T_1^4 T_2 - 14318 T_1^5 T_2 + 28778 T_1^6 T_2 - 21032 T_1^7 T_2 +$$

$$7372 T_1^8 T_2 - 1104 T_1^9 T_2 + 3840 T_1^2 - 21032 T_1 T_2^2 + 41413 T_1^2 T_2^2 - 5211 T_1^3 T_2^2 - 93916 T_1^4 T_2^2 +$$

$$154213 T_1^5 T_2^2 - 93916 T_1^6 T_2^2 - 5211 T_1^7 T_2^2 + 41413 T_1^8 T_2^2 - 21032 T_1^9 T_2^2 + 3840 T_1^{10} T_2^2 - 7580 T_1^3 +$$

$$28778 T_1 T_2^3 - 5211 T_1^2 T_2^3 - 172230 T_1^3 T_2^3 + 334651 T_1^4 T_2^3 - 188351 T_1^5 T_2^3 - 188351 T_1^6 T_2^3 +$$

$$334651 T_1^7 T_2^3 - 172230 T_1^8 T_2^3 - 5211 T_1^9 T_2^3 + 28778 T_1^{10} T_2^3 - 7580 T_1^{11} T_2^3 + 9440 T_1^4 T_2^4 - 14318 T_1 T_2^4 -$$

$$93916 T_1^2 T_2^4 + 334651 T_1^3 T_2^4 - 154881 T_1^4 T_2^4 - 611596 T_1^5 T_2^4 + 1123849 T_1^6 T_2^4 - 611596 T_1^7 T_2^4 -$$

$$154881 T_1^8 T_2^4 + 334651 T_1^9 T_2^4 - 93916 T_1^{10} T_2^4 - 14318 T_1^{11} T_2^4 + 9440 T_1^{12} T_2^4 - 7580 T_1^5 - 14318 T_1 T_2^5 +$$

$$154213 T_1^2 T_2^5 - 188351 T_1^3 T_2^5 - 611596 T_1^4 T_2^5 + 1498262 T_1^5 T_2^5 - 903551 T_1^6 T_2^5 - 903551 T_1^7 T_2^5 +$$

$$1498262 T_1^8 T_2^5 - 611596 T_1^9 T_2^5 - 188351 T_1^{10} T_2^5 + 154213 T_1^{11} T_2^5 - 14318 T_1^{12} T_2^5 - 7580 T_1^{13} T_2^5 +$$

$$3840 T_1^6 + 28778 T_1 T_2^6 - 93916 T_1^2 T_2^6 - 188351 T_1^3 T_2^6 + 1123849 T_1^4 T_2^6 - 903551 T_1^5 T_2^6 - 1249936 T_1^6 T_2^6 +$$

$$2819416 T_1^7 T_2^6 - 1249936 T_1^8 T_2^6 - 903551 T_1^9 T_2^6 + 1123849 T_1^{10} T_2^6 - 188351 T_1^{11} T_2^6 - 93916 T_1^{12} T_2^6 +$$

$$28778 T_1^{13} T_2^6 + 3840 T_1^{14} T_2^6 - 1104 T_1^{15} T_2^6 - 21032 T_1 T_2^7 - 5211 T_1^2 T_2^7 + 334651 T_1^3 T_2^7 - 611596 T_1^4 T_2^7 -$$

$$903551 T_1^5 T_2^7 + 2819416 T_1^6 T_2^7 - 1794860 T_1^7 T_2^7 - 1794860 T_1^8 T_2^7 + 2819416 T_1^9 T_2^7 - 903551 T_1^{10} T_2^7 -$$

$$611596 T_1^{11} T_2^7 + 334651 T_1^{12} T_2^7 - 5211 T_1^{13} T_2^7 - 21032 T_1^{14} T_2^7 - 1104 T_1^{15} T_2^7 + 144 T_2^8 + 7372 T_1 T_2^8 +$$

$$41413 T_1^2 T_2^8 - 172230 T_1^3 T_2^8 - 154881 T_1^4 T_2^8 + 1498262 T_1^5 T_2^8 - 1249936 T_1^6 T_2^8 - 1794860 T_1^7 T_2^8 +$$

$$4059468 T_1^8 T_2^8 - 1794860 T_1^9 T_2^8 - 1249936 T_1^{10} T_2^8 + 1498262 T_1^{11} T_2^8 - 154881 T_1^{12} T_2^8 - 172230 T_1^{13} T_2^8 +$$

$$41413 T_1^{14} T_2^8 + 7372 T_1^{15} T_2^8 + 144 T_1^{16} T_2^8 - 1104 T_1 T_2^9 - 21032 T_1^2 T_2^9 - 5211 T_1^3 T_2^9 + 334651 T_1^4 T_2^9 -$$

$$611596 T_1^5 T_2^9 - 903551 T_1^6 T_2^9 + 2819416 T_1^7 T_2^9 - 1794860 T_1^8 T_2^9 - 1794860 T_1^9 T_2^9 + 2819416 T_1^{10} T_2^9 -$$

$$903551 T_1^{11} T_2^9 - 611596 T_1^{12} T_2^9 + 334651 T_1^{13} T_2^9 - 5211 T_1^{14} T_2^9 - 21032 T_1^{15} T_2^9 - 1104 T_1^{16} T_2^9 +$$

$$3840 T_1^2 T_2^{10} + 28778 T_1^3 T_2^{10} - 93916 T_1^4 T_2^{10} - 188351 T_1^5 T_2^{10} + 1123849 T_1^6 T_2^{10} - 903551 T_1^7 T_2^{10} -$$

$$1249936 T_1^8 T_2^{10} + 2819416 T_1^9 T_2^{10} - 1249936 T_1^{10} T_2^{10} - 903551 T_1^{11} T_2^{10} + 1123849 T_1^{12} T_2^{10} -$$

$$188351 T_1^{13} T_2^{10} - 93916 T_1^{14} T_2^{10} + 28778 T_1^{15} T_2^{10} + 3840 T_1^{16} T_2^{10} - 7580 T_1^3 T_2^{11} - 14318 T_1^4 T_2^{11} +$$

$$154213 T_1^5 T_2^{11} - 188351 T_1^6 T_2^{11} - 611596 T_1^7 T_2^{11} + 1498262 T_1^8 T_2^{11} - 903551 T_1^9 T_2^{11} - 903551 T_1^{10} T_2^{11} +$$

$$1498262 T_1^{11} T_2^{11} - 611596 T_1^{12} T_2^{11} - 188351 T_1^{13} T_2^{11} + 154213 T_1^{14} T_2^{11} - 14318 T_1^{15} T_2^{11} - 7580 T_1^{16} T_2^{11} +$$

$$9440 T_1^4 T_2^{12} - 14318 T_1^5 T_2^{12} - 93916 T_1^6 T_2^{12} + 334651 T_1^7 T_2^{12} - 154881 T_1^8 T_2^{12} - 611596 T_1^9 T_2^{12} +$$

$$1123849 T_1^{10} T_2^{12} - 611596 T_1^{11} T_2^{12} - 154881 T_1^{12} T_2^{12} + 334651 T_1^{13} T_2^{12} - 93916 T_1^{14} T_2^{12} - 14318 T_1^{15} T_2^{12} +$$

$$9440 T_1^{16} T_2^{12} - 7580 T_1^5 T_2^{13} + 28778 T_1^6 T_2^{13} - 5211 T_1^7 T_2^{13} - 172230 T_1^8 T_2^{13} + 334651 T_1^9 T_2^{13} -$$

$$188351 T_1^{10} T_2^{13} - 188351 T_1^{11} T_2^{13} + 334651 T_1^{12} T_2^{13} - 172230 T_1^{13} T_2^{13} - 5211 T_1^{14} T_2^{13} + 28778 T_1^{15} T_2^{13} -$$

$$7580 T_1^{16} T_2^{13} + 3840 T_1^6 T_2^{14} - 21032 T_1^7 T_2^{14} + 41413 T_1^8 T_2^{14} - 5211 T_1^9 T_2^{14} - 93916 T_1^{10} T_2^{14} +$$

$$154213 T_1^{11} T_2^{14} - 93916 T_1^{12} T_2^{14} - 5211 T_1^{13} T_2^{14} + 41413 T_1^{14} T_2^{14} - 21032 T_1^{15} T_2^{14} + 3840 T_1^{16} T_2^{14} -$$

$$1104 T_1^7 T_2^{15} + 7372 T_1^8 T_2^{15} - 21032 T_1^9 T_2^{15} + 28778 T_1^{10} T_2^{15} - 14318 T_1^{11} T_2^{15} - 14318 T_1^{12} T_2^{15} +$$

$$28778 T_1^{13} T_2^{15} - 21032 T_1^{14} T_2^{15} + 7372 T_1^{15} T_2^{15} - 1104 T_1^{16} T_2^{15} + 144 T_1^8 T_2^{16} - 1104 T_1^9 T_2^{16} +$$

$$3840 T_1^{10} T_2^{16} - 7580 T_1^{11} T_2^{16} + 9440 T_1^{12} T_2^{16} - 7580 T_1^{13} T_2^{16} + 3840 T_1^{14} T_2^{16} - 1104 T_1^{15} T_2^{16} + 144 T_1^{16} T_2^{16} \right)$$

In[*n*] = Θ[Knot[15, Alternating, 20905]]Out[*n*] =

$$\left\{ \frac{1 - 10 T + 44 T^2 - 118 T^3 + 212 T^4 - 257 T^5 + 212 T^6 - 118 T^7 + 44 T^8 - 10 T^9 + T^{10}}{T^5}, \right.$$

$$\frac{1}{T_1^{10} T_2^{10}} \left(1 - 10 T_1 + 44 T_1^2 - 118 T_1^3 + 212 T_1^4 - 257 T_1^5 + 212 T_1^6 - 118 T_1^7 + 44 T_1^8 - 10 T_1^9 + T_1^{10} - 10 T_2 + \right.$$

$$90 T_1 T_2 - 340 T_1^2 T_2 + 740 T_1^3 T_2 - 940 T_1^4 T_2 + 450 T_1^5 T_2 + 450 T_1^6 T_2 - 940 T_1^7 T_2 + 740 T_1^8 T_2 -$$

$$\begin{aligned}
& 340 T_1^9 T_2 + 90 T_1^{10} T_2 - 10 T_1^{11} T_2 + 44 T_2^2 - 340 T_1 T_2^2 + 985 T_1^2 T_2^2 - 1269 T_1^3 T_2^2 - 401 T_1^4 T_2^2 + 4411 T_1^5 T_2^2 - \\
& 6677 T_1^6 T_2^2 + 4411 T_1^7 T_2^2 - 401 T_1^8 T_2^2 - 1269 T_1^9 T_2^2 + 985 T_1^{10} T_2^2 - 340 T_1^{11} T_2^2 + 44 T_1^{12} T_2^2 - 118 T_2^3 + \\
& 740 T_1 T_2^3 - 1269 T_1^2 T_2^3 - 914 T_1^3 T_2^3 + 7987 T_1^4 T_2^3 - 15079 T_1^5 T_2^3 + 8131 T_1^6 T_2^3 + 8131 T_1^7 T_2^3 - 15079 T_1^8 T_2^3 + \\
& 7987 T_1^9 T_2^3 - 914 T_1^{10} T_2^3 - 1269 T_1^{11} T_2^3 + 740 T_1^{12} T_2^3 - 118 T_1^{13} T_2^3 + 212 T_2^4 - 940 T_1 T_2^4 - 401 T_1^2 T_2^4 + \\
& 7987 T_1^3 T_2^4 - 18980 T_1^4 T_2^4 + 15896 T_1^5 T_2^4 + 19541 T_1^6 T_2^4 - 42394 T_1^7 T_2^4 + 19541 T_1^8 T_2^4 + 15896 T_1^9 T_2^4 - \\
& 18980 T_1^{10} T_2^4 + 7987 T_1^{11} T_2^4 - 401 T_1^{12} T_2^4 - 940 T_1^{13} T_2^4 + 212 T_1^{14} T_2^4 - 257 T_2^5 + 450 T_1 T_2^5 + 4411 T_1^2 T_2^5 - \\
& 15079 T_1^3 T_2^5 + 15896 T_1^4 T_2^5 + 17848 T_1^5 T_2^5 - 76792 T_1^6 T_2^5 + 46980 T_1^7 T_2^5 + 46980 T_1^8 T_2^5 - 76792 T_1^9 T_2^5 + \\
& 17848 T_1^{10} T_2^5 + 15896 T_1^{11} T_2^5 - 15079 T_1^{12} T_2^5 + 4411 T_1^{13} T_2^5 + 450 T_1^{14} T_2^5 - 257 T_1^{15} T_2^5 + 212 T_2^6 + \\
& 450 T_1 T_2^6 - 6677 T_1^2 T_2^6 + 8131 T_1^3 T_2^6 + 19541 T_1^4 T_2^6 - 76792 T_1^5 T_2^6 + 92385 T_1^6 T_2^6 + 67072 T_1^7 T_2^6 - \\
& 176705 T_1^8 T_2^6 + 67072 T_1^9 T_2^6 + 92385 T_1^{10} T_2^6 - 76792 T_1^{11} T_2^6 + 19541 T_1^{12} T_2^6 + 8131 T_1^{13} T_2^6 - 6677 T_1^{14} T_2^6 + \\
& 450 T_1^{15} T_2^6 + 212 T_1^{16} T_2^6 - 118 T_1^{17} T_2^6 - 940 T_1 T_2^7 + 4411 T_1^2 T_2^7 + 8131 T_1^3 T_2^7 - 42394 T_1^4 T_2^7 + 46980 T_1^5 T_2^7 + \\
& 67072 T_1^6 T_2^7 - 296378 T_1^7 T_2^7 + 181777 T_1^8 T_2^7 + 181777 T_1^9 T_2^7 - 296378 T_1^{10} T_2^7 + 67072 T_1^{11} T_2^7 + \\
& 46980 T_1^{12} T_2^7 - 42394 T_1^{13} T_2^7 + 8131 T_1^{14} T_2^7 + 4411 T_1^{15} T_2^7 - 940 T_1^{16} T_2^7 - 118 T_1^{17} T_2^7 + 44 T_2^8 + \\
& 740 T_1 T_2^8 - 401 T_1^2 T_2^8 - 15079 T_1^3 T_2^8 + 19541 T_1^4 T_2^8 + 46980 T_1^5 T_2^8 - 176705 T_1^6 T_2^8 + 181777 T_1^7 T_2^8 + \\
& 320104 T_1^8 T_2^8 - 652662 T_1^9 T_2^8 + 320104 T_1^{10} T_2^8 + 181777 T_1^{11} T_2^8 - 176705 T_1^{12} T_2^8 + 46980 T_1^{13} T_2^8 + \\
& 19541 T_1^{14} T_2^8 - 15079 T_1^{15} T_2^8 - 401 T_1^{16} T_2^8 + 740 T_1^{17} T_2^8 + 44 T_1^{18} T_2^8 - 10 T_2^9 - 340 T_1 T_2^9 - 1269 T_1^2 T_2^9 + \\
& 7987 T_1^3 T_2^9 + 15896 T_1^4 T_2^9 - 76792 T_1^5 T_2^9 + 67072 T_1^6 T_2^9 + 181777 T_1^7 T_2^9 - 652662 T_1^8 T_2^9 + 391094 T_1^9 T_2^9 + \\
& 391094 T_1^{10} T_2^9 - 652662 T_1^{11} T_2^9 + 181777 T_1^{12} T_2^9 + 67072 T_1^{13} T_2^9 - 76792 T_1^{14} T_2^9 + 15896 T_1^{15} T_2^9 + \\
& 7987 T_1^{16} T_2^9 - 1269 T_1^{17} T_2^9 - 340 T_1^{18} T_2^9 - 10 T_1^{19} T_2^9 + T_2^{10} + 90 T_1 T_2^{10} + 985 T_1^2 T_2^{10} - 914 T_1^3 T_2^{10} - \\
& 18980 T_1^4 T_2^{10} + 17848 T_1^5 T_2^{10} + 92385 T_1^6 T_2^{10} - 296378 T_1^7 T_2^{10} + 320104 T_1^8 T_2^{10} + 391094 T_1^9 T_2^{10} - \\
& 864744 T_1^{10} T_2^{10} + 391094 T_1^{11} T_2^{10} + 320104 T_1^{12} T_2^{10} - 296378 T_1^{13} T_2^{10} + 92385 T_1^{14} T_2^{10} + 17848 T_1^{15} T_2^{10} - \\
& 18980 T_1^{16} T_2^{10} - 914 T_1^{17} T_2^{10} + 985 T_1^{18} T_2^{10} + 90 T_1^{19} T_2^{10} + T_1^{20} - 10 T_1 T_2^{11} - 340 T_1^2 T_2^{11} - 1269 T_1^3 T_2^{11} + \\
& 7987 T_1^4 T_2^{11} + 15896 T_1^5 T_2^{11} - 76792 T_1^6 T_2^{11} + 67072 T_1^7 T_2^{11} + 181777 T_1^8 T_2^{11} - 652662 T_1^9 T_2^{11} + \\
& 391094 T_1^{10} T_2^{11} + 391094 T_1^{11} T_2^{11} - 652662 T_1^{12} T_2^{11} + 181777 T_1^{13} T_2^{11} + 67072 T_1^{14} T_2^{11} - 76792 T_1^{15} T_2^{11} + \\
& 15896 T_1^{16} T_2^{11} + 7987 T_1^{17} T_2^{11} - 1269 T_1^{18} T_2^{11} - 340 T_1^{19} T_2^{11} - 10 T_1^{20} T_2^{11} + 44 T_1^2 T_2^{12} + 740 T_1^3 T_2^{12} - \\
& 401 T_1^4 T_2^{12} - 15079 T_1^5 T_2^{12} + 19541 T_1^6 T_2^{12} + 46980 T_1^7 T_2^{12} - 176705 T_1^8 T_2^{12} + 181777 T_1^9 T_2^{12} + \\
& 320104 T_1^{10} T_2^{12} - 652662 T_1^{11} T_2^{12} + 320104 T_1^{12} T_2^{12} + 181777 T_1^{13} T_2^{12} - 176705 T_1^{14} T_2^{12} + 46980 T_1^{15} T_2^{12} + \\
& 19541 T_1^{16} T_2^{12} - 15079 T_1^{17} T_2^{12} - 401 T_1^{18} T_2^{12} + 740 T_1^{19} T_2^{12} + 44 T_1^{20} T_2^{12} - 118 T_1^3 T_2^{13} - 940 T_1^4 T_2^{13} + \\
& 4411 T_1^5 T_2^{13} + 8131 T_1^6 T_2^{13} - 42394 T_1^7 T_2^{13} + 46980 T_1^8 T_2^{13} + 67072 T_1^9 T_2^{13} - 296378 T_1^{10} T_2^{13} + \\
& 181777 T_1^{11} T_2^{13} + 181777 T_1^{12} T_2^{13} - 296378 T_1^{13} T_2^{13} + 67072 T_1^{14} T_2^{13} + 46980 T_1^{15} T_2^{13} - 42394 T_1^{16} T_2^{13} + \\
& 8131 T_1^{17} T_2^{13} + 4411 T_1^{18} T_2^{13} - 940 T_1^{19} T_2^{13} - 118 T_1^{20} T_2^{13} + 212 T_1^4 T_2^{14} + 450 T_1^5 T_2^{14} - 6677 T_1^6 T_2^{14} + \\
& 8131 T_1^7 T_2^{14} + 19541 T_1^8 T_2^{14} - 76792 T_1^9 T_2^{14} + 92385 T_1^{10} T_2^{14} + 67072 T_1^{11} T_2^{14} - 176705 T_1^{12} T_2^{14} + \\
& 67072 T_1^{13} T_2^{14} + 92385 T_1^{14} T_2^{14} - 76792 T_1^{15} T_2^{14} + 19541 T_1^{16} T_2^{14} + 8131 T_1^{17} T_2^{14} - 6677 T_1^{18} T_2^{14} + \\
& 450 T_1^{19} T_2^{14} + 212 T_1^{20} T_2^{14} - 257 T_1^5 T_2^{15} + 450 T_1^6 T_2^{15} + 4411 T_1^7 T_2^{15} - 15079 T_1^8 T_2^{15} + 15896 T_1^9 T_2^{15} + \\
& 17848 T_1^{10} T_2^{15} - 76792 T_1^{11} T_2^{15} + 46980 T_1^{12} T_2^{15} + 46980 T_1^{13} T_2^{15} - 76792 T_1^{14} T_2^{15} + 17848 T_1^{15} T_2^{15} + \\
& 15896 T_1^{16} T_2^{15} - 15079 T_1^{17} T_2^{15} + 4411 T_1^{18} T_2^{15} + 450 T_1^{19} T_2^{15} - 257 T_1^{20} T_2^{15} + 212 T_1^6 T_2^{16} - 940 T_1^7 T_2^{16} - \\
& 401 T_1^8 T_2^{16} + 7987 T_1^9 T_2^{16} - 18980 T_1^{10} T_2^{16} + 15896 T_1^{11} T_2^{16} + 19541 T_1^{12} T_2^{16} - 42394 T_1^{13} T_2^{16} + \\
& 19541 T_1^{14} T_2^{16} + 15896 T_1^{15} T_2^{16} - 18980 T_1^{16} T_2^{16} + 7987 T_1^{17} T_2^{16} - 401 T_1^{18} T_2^{16} - 940 T_1^{19} T_2^{16} + \\
& 212 T_1^{20} T_2^{16} - 118 T_1^7 T_2^{17} + 740 T_1^8 T_2^{17} - 1269 T_1^9 T_2^{17} - 914 T_1^{10} T_2^{17} + 7987 T_1^{11} T_2^{17} - 15079 T_1^{12} T_2^{17} + \\
& 8131 T_1^{13} T_2^{17} + 8131 T_1^{14} T_2^{17} - 15079 T_1^{15} T_2^{17} + 7987 T_1^{16} T_2^{17} - 914 T_1^{17} T_2^{17} - 1269 T_1^{18} T_2^{17} + 740 T_1^{19} T_2^{17} - \\
& 118 T_1^{20} T_2^{17} + 44 T_1^8 T_2^{18} - 340 T_1^9 T_2^{18} + 985 T_1^{10} T_2^{18} - 1269 T_1^{11} T_2^{18} - 401 T_1^{12} T_2^{18} + 4411 T_1^{13} T_2^{18} - \\
& 6677 T_1^{14} T_2^{18} + 4411 T_1^{15} T_2^{18} - 401 T_1^{16} T_2^{18} - 1269 T_1^{17} T_2^{18} + 985 T_1^{18} T_2^{18} - 340 T_1^{19} T_2^{18} + 44 T_1^{20} T_2^{18} - \\
& 10 T_1^9 T_2^{19} + 90 T_1^{10} T_2^{19} - 340 T_1^{11} T_2^{19} + 740 T_1^{12} T_2^{19} - 940 T_1^{13} T_2^{19} + 450 T_1^{14} T_2^{19} + 450 T_1^{15} T_2^{19} - \\
& 940 T_1^{16} T_2^{19} + 740 T_1^{17} T_2^{19} - 340 T_1^{18} T_2^{19} + 90 T_1^{19} T_2^{19} - 10 T_1^{20} T_2^{19} + T_1^{21} T_2^{20} - 10 T_1^{11} T_2^{20} + 44 T_1^{12} T_2^{20} - \\
& 118 T_1^{13} T_2^{20} + 212 T_1^{14} T_2^{20} - 257 T_1^{15} T_2^{20} + 212 T_1^{16} T_2^{20} - 118 T_1^{17} T_2^{20} + 44 T_1^{18} T_2^{20} - 10 T_1^{19} T_2^{20} + T_1^{20} T_2^{20} \Big) \Big\}
\end{aligned}$$

```
In[]:= θ[Knot[15, Alternating, 20905]] // LeafCount
Out[]= 3871

In[]:= θ[Knot[15, Alternating, 20905]] // ByteCount
Out[=]
121264

In[]:= (θ[Knot[15, Alternating, 20905]] /. {T1 → T1, T2 → T2}) // ByteCount
Out[=]
75040

In[]:= AbsoluteTiming@Monitor[
  Table[K → (θ[K] /. {T1 → T1, T2 → T2}), {K, AllKnots[{15, 15}]}] >> "theta15-15.m",
  K
]

::: KnotTheory: Loading precomputed data in KnotTheory/15A.dts.
::: KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of
2005.

AbsoluteTiming@
Monitor[Table[K → HOMFLYPT[PD@K] [a, z], {K, AllKnots[{3, 15}]}] >> "HOMFLYPT3-15.m", K]

AbsoluteTiming@
Monitor[Table[K → Kh[PD@K] [q, t], {K, AllKnots[{3, 15}]}] >> "Data/Kh3-15.m", K]
```