

(Alt) In[]:=

```
SetDirectory["C:\\drorbn\\AcademicPensive\\Projects\\HigherRank\\Data"];
Once[<< KnotTheory`];
<< ../Rot.m
T3 = T1 T2;
```

C:\drorbn\AcademicPensive\Projects\KnotTheory\KnotTheory

Loading KnotTheory` version of September 15, 2024, 14:47:22.7604.

Read more at <http://katlas.org/wiki/KnotTheory>. SetDelayed: Tag Diff in Diff[K_PD, rut_, ag_, n_, m_] is Protected.Loading Rot.m from <http://drorbn.net/AP/Projects/HigherRank> to compute rotation numbers.

(Alt) In[]:=

```
CCF[ε_] := ExpandDenominator@ExpandNumerator@Together[ε];
CCF[ε_] := Factor[ε];
CF[ε_List] := CF /@ ε;
CF[ε_] := Module[{vs = Cases[ε, (x | p | π | g)_, ∞] ∪ {x, p, ε}, ps, c},
  Total[CoefficientRules[Expand[ε], vs] /. (ps_ -> c_) => CCF[c] (Times @@ vs^ps)];
```

(Alt) In[]:=

```
R1[1, i_, j_] = CF[
  1 / 2 - T3 g1ji g2ji - g3ii + g2jj g3ii + T1 (T3 - 1) g1ji g3ji +
  T2 (T3 - 1) g2ji g3ji - T2 g2ji g3jj + (g1jj g2ii + (T3 - 1) g1jj g2ji -
  T1 g1ii g2jj - g1jj g3ii - T1 (T3 - 1) g1jj g3ji + T1 g1ii g3jj) / (T1 - 1)];
```

(Alt) In[]:=

```
R1[-1, i_, j_] = CF[
  -1 / 2 - T1^-1 g1ji g2ii - (1 - T1^-1 - T2^-1) g1ji g2ji - g1jj g2ji - g1ji g2jj + g3ii +
  T1^-1 g1ji g3ii - (1 - T2^-1) g2ji g3ii - g2jj g3ii + (1 - T3^-1) g1ji g3ji - (1 - T3^-1) g2ii g3ji +
  (2 - T2^-1) (1 - T3^-1) g2ji g3ji + (1 - T3^-1) g2jj g3ji + g1ji g3jj + g2ji g3jj + (T1 (1 - T2^-1) g1ii g2ji -
  g1jj g2ii + T1 g1ii g2jj + g1jj g3ii - T2^-1 (T3 - 1) g1ii g3ji - T1 g1ii g3jj) / (T1 - 1)];
```

(Alt) In[]:=

```
θ[{1, i0_, j0_}, {1, i1_, j1_}] =
  -T1 (T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 + (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 +
  T1 (T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 - (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;
```

(Alt) In[]:=

```
θ[{1, i0_, j0_}, {-1, i1_, j1_}] =
  (T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 - T1^-1 (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 -
  (T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 + T1^-1 (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;
```

(Alt) In[]:=

$$\theta[\{-1, i\theta, j\theta\}, \{1, i1, j1\}] = \text{CF} \left[\begin{aligned} &T_1^{-1} T_2^{-1} (T_3 - 1) (g_{1,j1,i\theta} g_{2,i1,i\theta} g_{3,j\theta,i1} - \\ &T_1 g_{1,j1,j\theta} g_{2,i1,i\theta} g_{3,j\theta,i1} - g_{1,j1,i\theta} g_{2,j1,i\theta} g_{3,j\theta,i1} + T_1 g_{1,j1,j\theta} g_{2,j1,i\theta} g_{3,j\theta,i1}) \end{aligned} \right];$$

(Alt) In[]:=

$$\theta[\{-1, i\theta, j\theta\}, \{-1, i1, j1\}] = \text{CF} \left[\begin{aligned} &(1 - T_3^{-1}) (-T_1^{-1} g_{1,j1,i\theta} g_{2,i1,i\theta} g_{3,j\theta,i1} + \\ &g_{1,j1,j\theta} g_{2,i1,i\theta} g_{3,j\theta,i1} + T_1^{-1} g_{1,j1,i\theta} g_{2,j1,i\theta} g_{3,j\theta,i1} - g_{1,j1,j\theta} g_{2,j1,i\theta} g_{3,j\theta,i1}) \end{aligned} \right];$$

(Alt) In[]:=

$$\Gamma_1[\varphi, k] = -\varphi / 2 + \varphi g_{3,k,k};$$

(Alt) In[]:=

$$\begin{aligned} \theta[K_] := &\text{Module} \left[\{Cs, \varphi, n, A, s, i, j, k, \Delta, G, v, \alpha, \beta, \text{gEval}, c, z\}, \right. \\ &\{Cs, \varphi\} = \text{Rot}[K]; n = \text{Length}[Cs]; \\ &A = \text{IdentityMatrix}[2n + 1]; \\ &\text{Cases}[Cs, \{s_, i_, j_ \} \Rightarrow \left(A[\{i, j\}, \{i + 1, j + 1\}] += \begin{pmatrix} -T^s & T^s - 1 \\ \theta & -1 \end{pmatrix} \right)]; \\ &\Delta = T^{(-\text{Total}[\varphi] - \text{Total}[Cs[[All, 1]])] / 2} \text{Det}[A]; \\ &G = \text{Inverse}[A]; \text{gEval}[\mathcal{E}_] := \text{Factor}[\mathcal{E} / . g_{v, \alpha, \beta} \Rightarrow (G[\alpha, \beta] / . T \rightarrow T_v)]; \\ &z = \text{gEval} \left[\sum_{k1=1}^n \sum_{k2=1}^n \theta[Cs[[k1], Cs[[k2]]] \right]; \\ &z += \text{gEval} \left[\sum_{k=1}^n R_1 @\text{Cs}[[k]] \right]; \\ &z += \text{gEval} \left[\sum_{k=1}^{2n} \Gamma_1[\varphi[[k], k] \right]; \\ &\{\Delta, (\Delta / . T \rightarrow T_1) (\Delta / . T \rightarrow T_2) (\Delta / . T \rightarrow T_3) z\} // \text{Factor} \left. \right]; \end{aligned}$$

(Alt) In[]:=

```

PolyPlot[0] = Graphics[{}];
PolyPlot[p_] := Module[{crs, m1, m2, maxc, minc, s, hex},
  crs = CoefficientRules[T1^m1 - Exponent[p, T1, Min] T2^m2 - Exponent[p, T2, Min] p, {T1, T2}];
  maxc = N@Log@Max@Abs[Last /@ crs];
  minc = N@Log@Min@Select[Abs[Last /@ crs], # > 0 &];
  If[minc == maxc, s[_] = 0, s[c_] := s[c] = (maxc - Log@c) / (maxc - minc)];
  hex = Table[{Cos[α], Sin[α]} / Cos[2 π / 12] / 2, {α, 2 π / 12, 2 π, 2 π / 6}];
  Graphics[crs /. ({x1_, x2_} -> c_) -> {
    If[c == 0, White, Lighter[Which[
      c > 0 & OddQ[c], Orange,
      c > 0 & EvenQ[c], Red,
      c < 0 & OddQ[c], Green,
      c < 0 & EvenQ[c], Blue
    ], 0.88 s[Abs@c]]],
    Polygon[{{(1 - 1/2), -1/2}, {0, sqrt(3)/2}} . {x1 + m1, x2 + m2} + #] & /@ hex] ]];
  PolyPlot[{a_, e_}] := PolyPlot[0]

```

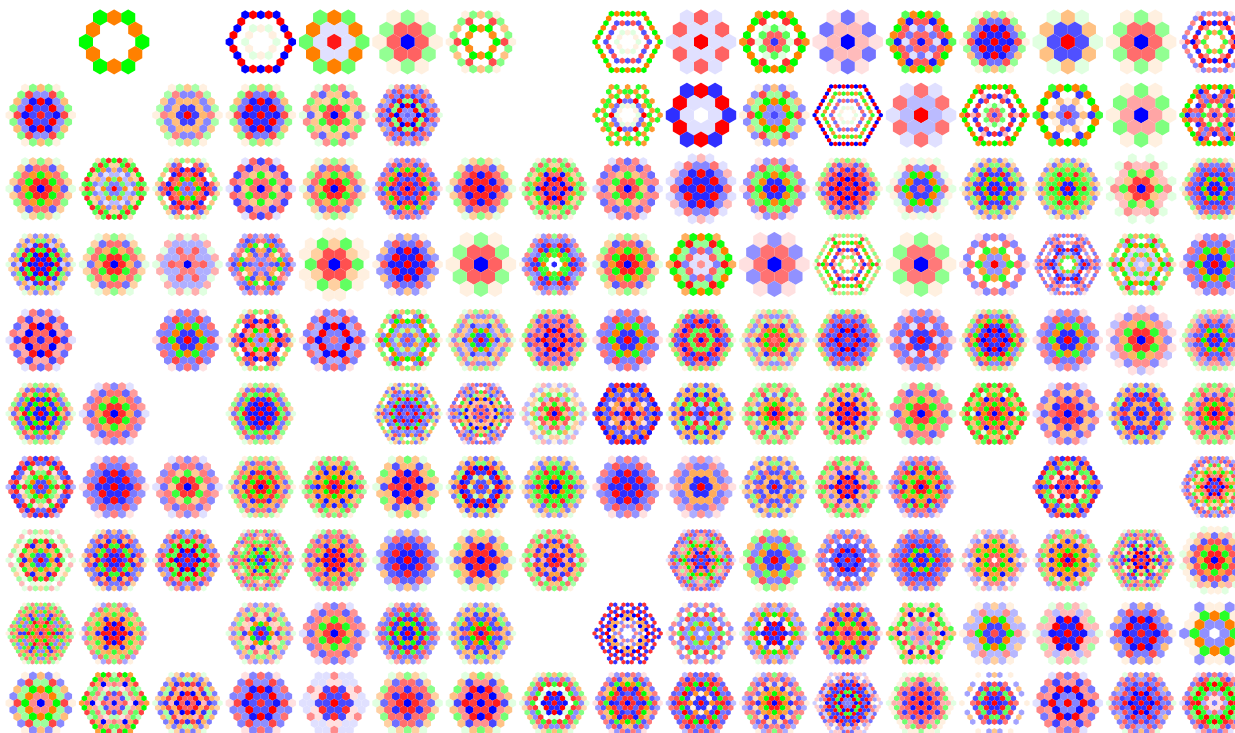
```

In[ ]:= tab250 = {0} ~ Join ~ Table[0[K][[2]], {K, AllKnots[{3, 10}]}];
g250 = GraphicsGrid[Partition[PolyPlot /@ tab250, 25], Spacings -> 0]

```

 KnotTheory: Loading precomputed data in PD4Knots`

Out[]:=




In[*]:= Table[n → NumberOfKnots[n], {n, 3, 15}]

Out[*]=

{3 → 1, 4 → 1, 5 → 2, 6 → 3, 7 → 7, 8 → 21, 9 → 49,
10 → 165, 11 → 552, 12 → 2176, 13 → 9988, 14 → 46 972, 15 → 253 293}

In[*]:= Θ [Knot[15, Alternating, 20902]]

 KnotTheory: Loading precomputed data in KnotTheory/15A.dts.

 KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[*]=

$$\left\{ -\frac{6 - 35 T + 91 T^2 - 125 T^3 + 91 T^4 - 35 T^5 + 6 T^6}{T^3}, \right.$$

$$-\frac{1}{T_1^6 T_2^6} \left(45 - 319 T_1 + 878 T_1^2 - 1219 T_1^3 + 878 T_1^4 - 319 T_1^5 + 45 T_1^6 - 319 T_2 + 1870 T_1 T_2 - 3824 T_1^2 T_2 + \right.$$

$$2356 T_1^3 T_2 + 2356 T_1^4 T_2 - 3824 T_1^5 T_2 + 1870 T_1^6 T_2 - 319 T_1^7 T_2 + 878 T_1^8 T_2 - 3824 T_1 T_2^2 + 2825 T_1^2 T_2^2 +$$

$$10702 T_1^3 T_2^2 - 22185 T_1^4 T_2^2 + 10702 T_1^5 T_2^2 + 2825 T_1^6 T_2^2 - 3824 T_1^7 T_2^2 + 878 T_1^8 T_2^2 - 1219 T_2^3 +$$

$$2356 T_1 T_2^3 + 10702 T_1^2 T_2^3 - 37286 T_1^3 T_2^3 + 27271 T_1^4 T_2^3 + 27271 T_1^5 T_2^3 - 37286 T_1^6 T_2^3 + 10702 T_1^7 T_2^3 +$$

$$2356 T_1^8 T_2^3 - 1219 T_1^9 T_2^3 + 878 T_1^4 T_2^4 + 2356 T_1 T_2^4 - 22185 T_1^2 T_2^4 + 27271 T_1^3 T_2^4 + 45442 T_1^4 T_2^4 -$$

$$116098 T_1^5 T_2^4 + 45442 T_1^6 T_2^4 + 27271 T_1^7 T_2^4 - 22185 T_1^8 T_2^4 + 2356 T_1^9 T_2^4 + 878 T_1^{10} T_2^4 - 319 T_2^5 -$$

$$3824 T_1 T_2^5 + 10702 T_1^2 T_2^5 + 27271 T_1^3 T_2^5 - 116098 T_1^4 T_2^5 + 89308 T_1^5 T_2^5 + 89308 T_1^6 T_2^5 -$$

$$116098 T_1^7 T_2^5 + 27271 T_1^8 T_2^5 + 10702 T_1^9 T_2^5 - 3824 T_1^{10} T_2^5 - 319 T_1^{11} T_2^5 + 45 T_2^6 + 1870 T_1 T_2^6 +$$

$$2825 T_1^2 T_2^6 - 37286 T_1^3 T_2^6 + 45442 T_1^4 T_2^6 + 89308 T_1^5 T_2^6 - 220980 T_1^6 T_2^6 + 89308 T_1^7 T_2^6 + 45442 T_1^8 T_2^6 -$$

$$37286 T_1^9 T_2^6 + 2825 T_1^{10} T_2^6 + 1870 T_1^{11} T_2^6 + 45 T_1^{12} T_2^6 - 319 T_1 T_2^7 - 3824 T_1^2 T_2^7 + 10702 T_1^3 T_2^7 +$$

$$27271 T_1^4 T_2^7 - 116098 T_1^5 T_2^7 + 89308 T_1^6 T_2^7 + 89308 T_1^7 T_2^7 - 116098 T_1^8 T_2^7 + 27271 T_1^9 T_2^7 +$$

$$10702 T_1^{10} T_2^7 - 3824 T_1^{11} T_2^7 - 319 T_1^{12} T_2^7 + 878 T_1^2 T_2^8 + 2356 T_1^3 T_2^8 - 22185 T_1^4 T_2^8 + 27271 T_1^5 T_2^8 +$$

$$45442 T_1^6 T_2^8 - 116098 T_1^7 T_2^8 + 45442 T_1^8 T_2^8 + 27271 T_1^9 T_2^8 - 22185 T_1^{10} T_2^8 + 2356 T_1^{11} T_2^8 +$$

$$878 T_1^{12} T_2^8 - 1219 T_1^3 T_2^9 + 2356 T_1^4 T_2^9 + 10702 T_1^5 T_2^9 - 37286 T_1^6 T_2^9 + 27271 T_1^7 T_2^9 + 27271 T_1^8 T_2^9 -$$

$$37286 T_1^9 T_2^9 + 10702 T_1^{10} T_2^9 + 2356 T_1^{11} T_2^9 - 1219 T_1^{12} T_2^9 + 878 T_1^4 T_2^{10} - 3824 T_1^5 T_2^{10} + 2825 T_1^6 T_2^{10} +$$

$$10702 T_1^7 T_2^{10} - 22185 T_1^8 T_2^{10} + 10702 T_1^9 T_2^{10} + 2825 T_1^{10} T_2^{10} - 3824 T_1^{11} T_2^{10} + 878 T_1^{12} T_2^{10} - 319 T_1^5 T_2^{11} +$$

$$1870 T_1^6 T_2^{11} - 3824 T_1^7 T_2^{11} + 2356 T_1^8 T_2^{11} + 2356 T_1^9 T_2^{11} - 3824 T_1^{10} T_2^{11} + 1870 T_1^{11} T_2^{11} - 319 T_1^{12} T_2^{11} +$$

$$45 T_1^6 T_2^{12} - 319 T_1^7 T_2^{12} + 878 T_1^8 T_2^{12} - 1219 T_1^9 T_2^{12} + 878 T_1^{10} T_2^{12} - 319 T_1^{11} T_2^{12} + 45 T_1^{12} T_2^{12} \left. \right\}$$

In[*]:= Θ [Knot[15, Alternating, 20903]]

Out[*]=

$$\left\{ \frac{(2 - 3 T + 2 T^2) (1 - 6 T + 16 T^2 - 23 T^3 + 25 T^4 - 23 T^5 + 16 T^6 - 6 T^7 + T^8)}{T^5}, \right.$$

$$\frac{1}{T_1^{10} T_2^{10}} \left(25 - 186 T_1 + 639 T_1^2 - 1291 T_1^3 + 1827 T_1^4 - 2015 T_1^5 + 1827 T_1^6 - 1291 T_1^7 + 639 T_1^8 - 186 T_1^9 + 25 T_1^{10} - \right.$$

$$186 T_2 + 1200 T_1 T_2 - 3382 T_1^2 T_2 + 4879 T_1^3 T_2 - 4021 T_1^4 T_2 + 1414 T_1^5 T_2 + 1414 T_1^6 T_2 - 4021 T_1^7 T_2 +$$

$$4879 T_1^8 T_2 - 3382 T_1^9 T_2 + 1200 T_1^{10} T_2 - 186 T_1^{11} T_2 + 639 T_2^2 - 3382 T_1 T_2^2 + 6708 T_1^2 T_2^2 - 2333 T_1^3 T_2^2 -$$

$$8819 T_1^4 T_2^2 + 17455 T_1^5 T_2^2 - 19176 T_1^6 T_2^2 + 17455 T_1^7 T_2^2 - 8819 T_1^8 T_2^2 - 2333 T_1^9 T_2^2 + 6708 T_1^{10} T_2^2 -$$

$$3382 T_1^{11} T_2^2 + 639 T_1^{12} T_2^2 - 1291 T_2^3 + 4879 T_1 T_2^3 - 2333 T_1^2 T_2^3 - 21460 T_1^3 T_2^3 + 41279 T_1^4 T_2^3 - 33366 T_1^5 T_2^3 +$$

$$9255 T_1^6 T_2^3 + 9255 T_1^7 T_2^3 - 33366 T_1^8 T_2^3 + 41279 T_1^9 T_2^3 - 21460 T_1^{10} T_2^3 - 2333 T_1^{11} T_2^3 + 4879 T_1^{12} T_2^3 -$$

$$1291 T_1^{13} T_2^3 + 1827 T_2^4 - 4021 T_1 T_2^4 - 8819 T_1^2 T_2^4 + 41279 T_1^3 T_2^4 - 18452 T_1^4 T_2^4 - 56189 T_1^5 T_2^4 +$$

$$109308 T_1^6 T_2^4 - 110634 T_1^7 T_2^4 + 109308 T_1^8 T_2^4 - 56189 T_1^9 T_2^4 - 18452 T_1^{10} T_2^4 + 41279 T_1^{11} T_2^4 -$$

$$\begin{aligned}
 & 8819 T_1^{12} T_2^4 - 4021 T_1^{13} T_2^4 + 1827 T_1^{14} T_2^4 - 2015 T_2^5 + 1414 T_1 T_2^5 + 17455 T_1^2 T_2^5 - 33366 T_1^3 T_2^5 - \\
 & 56189 T_1^4 T_2^5 + 140578 T_1^5 T_2^5 - 110869 T_1^6 T_2^5 + 19830 T_1^7 T_2^5 + 19830 T_1^8 T_2^5 - 110869 T_1^9 T_2^5 + \\
 & 140578 T_1^{10} T_2^5 - 56189 T_1^{11} T_2^5 - 33366 T_1^{12} T_2^5 + 17455 T_1^{13} T_2^5 + 1414 T_1^{14} T_2^5 - 2015 T_1^{15} T_2^5 + 1827 T_2^6 + \\
 & 1414 T_1 T_2^6 - 19176 T_1^2 T_2^6 + 9255 T_1^3 T_2^6 + 109308 T_1^4 T_2^6 - 110869 T_1^5 T_2^6 - 18442 T_1^6 T_2^6 + 103490 T_1^7 T_2^6 - \\
 & 64338 T_1^8 T_2^6 + 103490 T_1^9 T_2^6 - 18442 T_1^{10} T_2^6 - 110869 T_1^{11} T_2^6 + 109308 T_1^{12} T_2^6 + 9255 T_1^{13} T_2^6 - \\
 & 19176 T_1^{14} T_2^6 + 1414 T_1^{15} T_2^6 + 1827 T_1^{16} T_2^6 - 1291 T_2^7 - 4021 T_1 T_2^7 + 17455 T_1^2 T_2^7 + 9255 T_1^3 T_2^7 - \\
 & 110634 T_1^4 T_2^7 + 19830 T_1^5 T_2^7 + 103490 T_1^6 T_2^7 - 81860 T_1^7 T_2^7 - 23747 T_1^8 T_2^7 - 23747 T_1^9 T_2^7 - 81860 T_1^{10} T_2^7 + \\
 & 103490 T_1^{11} T_2^7 + 19830 T_1^{12} T_2^7 - 110634 T_1^{13} T_2^7 + 9255 T_1^{14} T_2^7 + 17455 T_1^{15} T_2^7 - 4021 T_1^{16} T_2^7 - \\
 & 1291 T_1^{17} T_2^7 + 639 T_2^8 + 4879 T_1 T_2^8 - 8819 T_1^2 T_2^8 - 33366 T_1^3 T_2^8 + 109308 T_1^4 T_2^8 + 19830 T_1^5 T_2^8 - \\
 & 64338 T_1^6 T_2^8 - 23747 T_1^7 T_2^8 + 87820 T_1^8 T_2^8 + 11778 T_1^9 T_2^8 + 87820 T_1^{10} T_2^8 - 23747 T_1^{11} T_2^8 - 64338 T_1^{12} T_2^8 + \\
 & 19830 T_1^{13} T_2^8 + 109308 T_1^{14} T_2^8 - 33366 T_1^{15} T_2^8 - 8819 T_1^{16} T_2^8 + 4879 T_1^{17} T_2^8 + 639 T_1^{18} T_2^8 - 186 T_2^9 - \\
 & 3382 T_1 T_2^9 - 2333 T_1^2 T_2^9 + 41279 T_1^3 T_2^9 - 56189 T_1^4 T_2^9 - 110869 T_1^5 T_2^9 + 103490 T_1^6 T_2^9 - 23747 T_1^7 T_2^9 + \\
 & 11778 T_1^8 T_2^9 - 77420 T_1^9 T_2^9 - 77420 T_1^{10} T_2^9 + 11778 T_1^{11} T_2^9 - 23747 T_1^{12} T_2^9 + 103490 T_1^{13} T_2^9 - \\
 & 110869 T_1^{14} T_2^9 - 56189 T_1^{15} T_2^9 + 41279 T_1^{16} T_2^9 - 2333 T_1^{17} T_2^9 - 3382 T_1^{18} T_2^9 - 186 T_1^{19} T_2^9 + 25 T_2^{10} + \\
 & 1200 T_1 T_2^{10} + 6708 T_1^2 T_2^{10} - 21460 T_1^3 T_2^{10} - 18452 T_1^4 T_2^{10} + 140578 T_1^5 T_2^{10} - 18442 T_1^6 T_2^{10} - 81860 T_1^7 T_2^{10} + \\
 & 87820 T_1^8 T_2^{10} - 77420 T_1^9 T_2^{10} + 212052 T_1^{10} T_2^{10} - 77420 T_1^{11} T_2^{10} + 87820 T_1^{12} T_2^{10} - 81860 T_1^{13} T_2^{10} - \\
 & 18442 T_1^{14} T_2^{10} + 140578 T_1^{15} T_2^{10} - 18452 T_1^{16} T_2^{10} - 21460 T_1^{17} T_2^{10} + 6708 T_1^{18} T_2^{10} + 1200 T_1^{19} T_2^{10} + \\
 & 25 T_1^{20} T_2^{10} - 186 T_1 T_2^{11} - 3382 T_1^2 T_2^{11} - 2333 T_1^3 T_2^{11} + 41279 T_1^4 T_2^{11} - 56189 T_1^5 T_2^{11} - 110869 T_1^6 T_2^{11} + \\
 & 103490 T_1^7 T_2^{11} - 23747 T_1^8 T_2^{11} + 11778 T_1^9 T_2^{11} - 77420 T_1^{10} T_2^{11} - 77420 T_1^{11} T_2^{11} + 11778 T_1^{12} T_2^{11} - \\
 & 23747 T_1^{13} T_2^{11} + 103490 T_1^{14} T_2^{11} - 110869 T_1^{15} T_2^{11} - 56189 T_1^{16} T_2^{11} + 41279 T_1^{17} T_2^{11} - 2333 T_1^{18} T_2^{11} - \\
 & 3382 T_1^{19} T_2^{11} - 186 T_1^{20} T_2^{11} + 639 T_1^2 T_2^{12} + 4879 T_1^3 T_2^{12} - 8819 T_1^4 T_2^{12} - 33366 T_1^5 T_2^{12} + 109308 T_1^6 T_2^{12} + \\
 & 19830 T_1^7 T_2^{12} - 64338 T_1^8 T_2^{12} - 23747 T_1^9 T_2^{12} + 87820 T_1^{10} T_2^{12} + 11778 T_1^{11} T_2^{12} + 87820 T_1^{12} T_2^{12} - \\
 & 23747 T_1^{13} T_2^{12} - 64338 T_1^{14} T_2^{12} + 19830 T_1^{15} T_2^{12} + 109308 T_1^{16} T_2^{12} - 33366 T_1^{17} T_2^{12} - 8819 T_1^{18} T_2^{12} + \\
 & 4879 T_1^{19} T_2^{12} + 639 T_1^{20} T_2^{12} - 1291 T_1^3 T_2^{13} - 4021 T_1^4 T_2^{13} + 17455 T_1^5 T_2^{13} + 9255 T_1^6 T_2^{13} - 110634 T_1^7 T_2^{13} + \\
 & 19830 T_1^8 T_2^{13} + 103490 T_1^9 T_2^{13} - 81860 T_1^{10} T_2^{13} - 23747 T_1^{11} T_2^{13} - 23747 T_1^{12} T_2^{13} - 81860 T_1^{13} T_2^{13} + \\
 & 103490 T_1^{14} T_2^{13} + 19830 T_1^{15} T_2^{13} - 110634 T_1^{16} T_2^{13} + 9255 T_1^{17} T_2^{13} + 17455 T_1^{18} T_2^{13} - 4021 T_1^{19} T_2^{13} - \\
 & 1291 T_1^{20} T_2^{13} + 1827 T_1^4 T_2^{14} + 1414 T_1^5 T_2^{14} - 19176 T_1^6 T_2^{14} + 9255 T_1^7 T_2^{14} + 109308 T_1^8 T_2^{14} - 110869 T_1^9 T_2^{14} - \\
 & 18442 T_1^{10} T_2^{14} + 103490 T_1^{11} T_2^{14} - 64338 T_1^{12} T_2^{14} + 103490 T_1^{13} T_2^{14} - 18442 T_1^{14} T_2^{14} - 110869 T_1^{15} T_2^{14} + \\
 & 109308 T_1^{16} T_2^{14} + 9255 T_1^{17} T_2^{14} - 19176 T_1^{18} T_2^{14} + 1414 T_1^{19} T_2^{14} + 1827 T_1^{20} T_2^{14} - 2015 T_1^5 T_2^{15} + 1414 T_1^6 T_2^{15} + \\
 & 17455 T_1^7 T_2^{15} - 33366 T_1^8 T_2^{15} - 56189 T_1^9 T_2^{15} + 140578 T_1^{10} T_2^{15} - 110869 T_1^{11} T_2^{15} + 19830 T_1^{12} T_2^{15} + \\
 & 19830 T_1^{13} T_2^{15} - 110869 T_1^{14} T_2^{15} + 140578 T_1^{15} T_2^{15} - 56189 T_1^{16} T_2^{15} - 33366 T_1^{17} T_2^{15} + 17455 T_1^{18} T_2^{15} + \\
 & 1414 T_1^{19} T_2^{15} - 2015 T_1^{20} T_2^{15} + 1827 T_1^6 T_2^{16} - 4021 T_1^7 T_2^{16} - 8819 T_1^8 T_2^{16} + 41279 T_1^9 T_2^{16} - 18452 T_1^{10} T_2^{16} - \\
 & 56189 T_1^{11} T_2^{16} + 109308 T_1^{12} T_2^{16} - 110634 T_1^{13} T_2^{16} + 109308 T_1^{14} T_2^{16} - 56189 T_1^{15} T_2^{16} - 18452 T_1^{16} T_2^{16} + \\
 & 41279 T_1^{17} T_2^{16} - 8819 T_1^{18} T_2^{16} - 4021 T_1^{19} T_2^{16} + 1827 T_1^{20} T_2^{16} - 1291 T_1^7 T_2^{17} + 4879 T_1^8 T_2^{17} - 2333 T_1^9 T_2^{17} - \\
 & 21460 T_1^{10} T_2^{17} + 41279 T_1^{11} T_2^{17} - 33366 T_1^{12} T_2^{17} + 9255 T_1^{13} T_2^{17} + 9255 T_1^{14} T_2^{17} - 33366 T_1^{15} T_2^{17} + \\
 & 41279 T_1^{16} T_2^{17} - 21460 T_1^{17} T_2^{17} - 2333 T_1^{18} T_2^{17} + 4879 T_1^{19} T_2^{17} - 1291 T_1^{20} T_2^{17} + 639 T_1^8 T_2^{18} - 3382 T_1^9 T_2^{18} + \\
 & 6708 T_1^{10} T_2^{18} - 2333 T_1^{11} T_2^{18} - 8819 T_1^{12} T_2^{18} + 17455 T_1^{13} T_2^{18} - 19176 T_1^{14} T_2^{18} + 17455 T_1^{15} T_2^{18} - \\
 & 8819 T_1^{16} T_2^{18} - 2333 T_1^{17} T_2^{18} + 6708 T_1^{18} T_2^{18} - 3382 T_1^{19} T_2^{18} + 639 T_1^{20} T_2^{18} - 186 T_1^9 T_2^{19} + 1200 T_1^{10} T_2^{19} - \\
 & 3382 T_1^{11} T_2^{19} + 4879 T_1^{12} T_2^{19} - 4021 T_1^{13} T_2^{19} + 1414 T_1^{14} T_2^{19} + 1414 T_1^{15} T_2^{19} - 4021 T_1^{16} T_2^{19} + 4879 T_1^{17} T_2^{19} - \\
 & 3382 T_1^{18} T_2^{19} + 1200 T_1^{19} T_2^{19} - 186 T_1^{20} T_2^{19} + 25 T_1^{10} T_2^{20} - 186 T_1^{11} T_2^{20} + 639 T_1^{12} T_2^{20} - 1291 T_1^{13} T_2^{20} + \\
 & 1827 T_1^{14} T_2^{20} - 2015 T_1^{15} T_2^{20} + 1827 T_1^{16} T_2^{20} - 1291 T_1^{17} T_2^{20} + 639 T_1^{18} T_2^{20} - 186 T_1^{19} T_2^{20} + 25 T_1^{20} T_2^{20} \}
 \end{aligned}$$

In[*]:= θ [Knot[15, Alternating, 20904]]

Out[]:=

$$\left\{ \frac{4 - 31 T + 109 T^2 - 217 T^3 + 271 T^4 - 217 T^5 + 109 T^6 - 31 T^7 + 4 T^8}{T^4}, \right.$$

$$\frac{1}{T_1^8 T_2^8} \left(144 - 1104 T_1 + 3840 T_1^2 - 7580 T_1^3 + 9440 T_1^4 - 7580 T_1^5 + 3840 T_1^6 - 1104 T_1^7 + 144 T_1^8 - 1104 T_2 + \right.$$

$$7372 T_1 T_2 - 21032 T_1^2 T_2 + 28778 T_1^3 T_2 - 14318 T_1^4 T_2 - 14318 T_1^5 T_2 + 28778 T_1^6 T_2 - 21032 T_1^7 T_2 +$$

$$7372 T_1^8 T_2 - 1104 T_1^9 T_2 + 3840 T_2^2 - 21032 T_1 T_2^2 + 41413 T_1^2 T_2^2 - 5211 T_1^3 T_2^2 - 93916 T_1^4 T_2^2 +$$

$$154213 T_1^5 T_2^2 - 93916 T_1^6 T_2^2 - 5211 T_1^7 T_2^2 + 41413 T_1^8 T_2^2 - 21032 T_1^9 T_2^2 + 3840 T_1^{10} T_2^2 - 7580 T_2^3 +$$

$$28778 T_1 T_2^3 - 5211 T_1^2 T_2^3 - 172230 T_1^3 T_2^3 + 334651 T_1^4 T_2^3 - 188351 T_1^5 T_2^3 - 188351 T_1^6 T_2^3 +$$

$$334651 T_1^7 T_2^3 - 172230 T_1^8 T_2^3 - 5211 T_1^9 T_2^3 + 28778 T_1^{10} T_2^3 - 7580 T_1^{11} T_2^3 + 9440 T_2^4 - 14318 T_1 T_2^4 -$$

$$93916 T_1^2 T_2^4 + 334651 T_1^3 T_2^4 - 154881 T_1^4 T_2^4 - 611596 T_1^5 T_2^4 + 1123849 T_1^6 T_2^4 - 611596 T_1^7 T_2^4 -$$

$$154881 T_1^8 T_2^4 + 334651 T_1^9 T_2^4 - 93916 T_1^{10} T_2^4 - 14318 T_1^{11} T_2^4 + 9440 T_1^{12} T_2^4 - 7580 T_2^5 - 14318 T_1 T_2^5 +$$

$$154213 T_1^2 T_2^5 - 188351 T_1^3 T_2^5 - 611596 T_1^4 T_2^5 + 1498262 T_1^5 T_2^5 - 903551 T_1^6 T_2^5 - 903551 T_1^7 T_2^5 +$$

$$1498262 T_1^8 T_2^5 - 611596 T_1^9 T_2^5 - 188351 T_1^{10} T_2^5 + 154213 T_1^{11} T_2^5 - 14318 T_1^{12} T_2^5 - 7580 T_1^{13} T_2^5 +$$

$$3840 T_2^6 + 28778 T_1 T_2^6 - 93916 T_1^2 T_2^6 - 188351 T_1^3 T_2^6 + 1123849 T_1^4 T_2^6 - 903551 T_1^5 T_2^6 - 1249936 T_1^6 T_2^6 +$$

$$2819416 T_1^7 T_2^6 - 1249936 T_1^8 T_2^6 - 903551 T_1^9 T_2^6 + 1123849 T_1^{10} T_2^6 - 188351 T_1^{11} T_2^6 - 93916 T_1^{12} T_2^6 +$$

$$28778 T_1^{13} T_2^6 + 3840 T_1^{14} T_2^6 - 1104 T_2^7 - 21032 T_1 T_2^7 - 5211 T_1^2 T_2^7 + 334651 T_1^3 T_2^7 - 611596 T_1^4 T_2^7 -$$

$$903551 T_1^5 T_2^7 + 2819416 T_1^6 T_2^7 - 1794860 T_1^7 T_2^7 - 1794860 T_1^8 T_2^7 + 2819416 T_1^9 T_2^7 - 903551 T_1^{10} T_2^7 -$$

$$611596 T_1^{11} T_2^7 + 334651 T_1^{12} T_2^7 - 5211 T_1^{13} T_2^7 - 21032 T_1^{14} T_2^7 - 1104 T_1^{15} T_2^7 + 144 T_2^8 + 7372 T_1 T_2^8 +$$

$$41413 T_1^2 T_2^8 - 172230 T_1^3 T_2^8 - 154881 T_1^4 T_2^8 + 1498262 T_1^5 T_2^8 - 1249936 T_1^6 T_2^8 - 1794860 T_1^7 T_2^8 +$$

$$4059468 T_1^8 T_2^8 - 1794860 T_1^9 T_2^8 - 1249936 T_1^{10} T_2^8 + 1498262 T_1^{11} T_2^8 - 154881 T_1^{12} T_2^8 - 172230 T_1^{13} T_2^8 +$$

$$41413 T_1^{14} T_2^8 + 7372 T_1^{15} T_2^8 + 144 T_1^{16} T_2^8 - 1104 T_1 T_2^9 - 21032 T_1^2 T_2^9 - 5211 T_1^3 T_2^9 + 334651 T_1^4 T_2^9 -$$

$$611596 T_1^5 T_2^9 - 903551 T_1^6 T_2^9 + 2819416 T_1^7 T_2^9 - 1794860 T_1^8 T_2^9 - 1794860 T_1^9 T_2^9 + 2819416 T_1^{10} T_2^9 -$$

$$903551 T_1^{11} T_2^9 - 611596 T_1^{12} T_2^9 + 334651 T_1^{13} T_2^9 - 5211 T_1^{14} T_2^9 - 21032 T_1^{15} T_2^9 - 1104 T_1^{16} T_2^9 +$$

$$3840 T_1^2 T_2^{10} + 28778 T_1^3 T_2^{10} - 93916 T_1^4 T_2^{10} - 188351 T_1^5 T_2^{10} + 1123849 T_1^6 T_2^{10} - 903551 T_1^7 T_2^{10} -$$

$$1249936 T_1^8 T_2^{10} + 2819416 T_1^9 T_2^{10} - 1249936 T_1^{10} T_2^{10} - 903551 T_1^{11} T_2^{10} + 1123849 T_1^{12} T_2^{10} -$$

$$188351 T_1^{13} T_2^{10} - 93916 T_1^{14} T_2^{10} + 28778 T_1^{15} T_2^{10} + 3840 T_1^{16} T_2^{10} - 7580 T_1^3 T_2^{11} - 14318 T_1^4 T_2^{11} +$$

$$154213 T_1^5 T_2^{11} - 188351 T_1^6 T_2^{11} - 611596 T_1^7 T_2^{11} + 1498262 T_1^8 T_2^{11} - 903551 T_1^9 T_2^{11} - 903551 T_1^{10} T_2^{11} +$$

$$1498262 T_1^{11} T_2^{11} - 611596 T_1^{12} T_2^{11} - 188351 T_1^{13} T_2^{11} + 154213 T_1^{14} T_2^{11} - 14318 T_1^{15} T_2^{11} - 7580 T_1^{16} T_2^{11} +$$

$$9440 T_1^4 T_2^{12} - 14318 T_1^5 T_2^{12} - 93916 T_1^6 T_2^{12} + 334651 T_1^7 T_2^{12} - 154881 T_1^8 T_2^{12} - 611596 T_1^9 T_2^{12} +$$

$$1123849 T_1^{10} T_2^{12} - 611596 T_1^{11} T_2^{12} - 154881 T_1^{12} T_2^{12} + 334651 T_1^{13} T_2^{12} - 93916 T_1^{14} T_2^{12} - 14318 T_1^{15} T_2^{12} +$$

$$9440 T_1^{16} T_2^{12} - 7580 T_1^5 T_2^{13} + 28778 T_1^6 T_2^{13} - 5211 T_1^7 T_2^{13} - 172230 T_1^8 T_2^{13} + 334651 T_1^9 T_2^{13} -$$

$$188351 T_1^{10} T_2^{13} - 188351 T_1^{11} T_2^{13} + 334651 T_1^{12} T_2^{13} - 172230 T_1^{13} T_2^{13} - 5211 T_1^{14} T_2^{13} + 28778 T_1^{15} T_2^{13} -$$

$$7580 T_1^{16} T_2^{13} + 3840 T_1^6 T_2^{14} - 21032 T_1^7 T_2^{14} + 41413 T_1^8 T_2^{14} - 5211 T_1^9 T_2^{14} - 93916 T_1^{10} T_2^{14} +$$

$$154213 T_1^{11} T_2^{14} - 93916 T_1^{12} T_2^{14} - 5211 T_1^{13} T_2^{14} + 41413 T_1^{14} T_2^{14} - 21032 T_1^{15} T_2^{14} + 3840 T_1^{16} T_2^{14} -$$

$$1104 T_1^7 T_2^{15} + 7372 T_1^8 T_2^{15} - 21032 T_1^9 T_2^{15} + 28778 T_1^{10} T_2^{15} - 14318 T_1^{11} T_2^{15} - 14318 T_1^{12} T_2^{15} +$$

$$28778 T_1^{13} T_2^{15} - 21032 T_1^{14} T_2^{15} + 7372 T_1^{15} T_2^{15} - 1104 T_1^{16} T_2^{15} + 144 T_1^8 T_2^{16} - 1104 T_1^9 T_2^{16} +$$

$$3840 T_1^{10} T_2^{16} - 7580 T_1^{11} T_2^{16} + 9440 T_1^{12} T_2^{16} - 7580 T_1^{13} T_2^{16} + 3840 T_1^{14} T_2^{16} - 1104 T_1^{15} T_2^{16} + 144 T_1^{16} T_2^{16} \left. \right\}$$

In[]:= θ [Knot[15, Alternating, 20905]]

Out[]:=

$$\left\{ \frac{1 - 10 T + 44 T^2 - 118 T^3 + 212 T^4 - 257 T^5 + 212 T^6 - 118 T^7 + 44 T^8 - 10 T^9 + T^{10}}{T^5}, \right.$$

$$\frac{1}{T_1^{10} T_2^{10}} \left(1 - 10 T_1 + 44 T_1^2 - 118 T_1^3 + 212 T_1^4 - 257 T_1^5 + 212 T_1^6 - 118 T_1^7 + 44 T_1^8 - 10 T_1^9 + T_1^{10} - 10 T_2 + \right.$$

$$90 T_1 T_2 - 340 T_1^2 T_2 + 740 T_1^3 T_2 - 940 T_1^4 T_2 + 450 T_1^5 T_2 + 450 T_1^6 T_2 - 940 T_1^7 T_2 + 740 T_1^8 T_2 -$$

$$\begin{aligned}
 & 340 T_1^9 T_2 + 90 T_1^{10} T_2 - 10 T_1^{11} T_2 + 44 T_2^2 - 340 T_1 T_2^2 + 985 T_1^2 T_2^2 - 1269 T_1^3 T_2^2 - 401 T_1^4 T_2^2 + 4411 T_1^5 T_2^2 - \\
 & 6677 T_1^6 T_2^2 + 4411 T_1^7 T_2^2 - 401 T_1^8 T_2^2 - 1269 T_1^9 T_2^2 + 985 T_1^{10} T_2^2 - 340 T_1^{11} T_2^2 + 44 T_1^{12} T_2^2 - 118 T_2^3 + \\
 & 740 T_1 T_2^3 - 1269 T_1^2 T_2^3 - 914 T_1^3 T_2^3 + 7987 T_1^4 T_2^3 - 15079 T_1^5 T_2^3 + 8131 T_1^6 T_2^3 + 8131 T_1^7 T_2^3 - 15079 T_1^8 T_2^3 + \\
 & 7987 T_1^9 T_2^3 - 914 T_1^{10} T_2^3 - 1269 T_1^{11} T_2^3 + 740 T_1^{12} T_2^3 - 118 T_1^{13} T_2^3 + 212 T_2^4 - 940 T_1 T_2^4 - 401 T_1^2 T_2^4 + \\
 & 7987 T_1^3 T_2^4 - 18980 T_1^4 T_2^4 + 15896 T_1^5 T_2^4 + 19541 T_1^6 T_2^4 - 42394 T_1^7 T_2^4 + 19541 T_1^8 T_2^4 + 15896 T_1^9 T_2^4 - \\
 & 18980 T_1^{10} T_2^4 + 7987 T_1^{11} T_2^4 - 401 T_1^{12} T_2^4 - 940 T_1^{13} T_2^4 + 212 T_1^{14} T_2^4 - 257 T_2^5 + 450 T_1 T_2^5 + 4411 T_1^2 T_2^5 - \\
 & 15079 T_1^3 T_2^5 + 15896 T_1^4 T_2^5 + 17848 T_1^5 T_2^5 - 76792 T_1^6 T_2^5 + 46980 T_1^7 T_2^5 + 46980 T_1^8 T_2^5 - 76792 T_1^9 T_2^5 + \\
 & 17848 T_1^{10} T_2^5 + 15896 T_1^{11} T_2^5 - 15079 T_1^{12} T_2^5 + 4411 T_1^{13} T_2^5 + 450 T_1^{14} T_2^5 - 257 T_1^{15} T_2^5 + 212 T_2^6 + \\
 & 450 T_1 T_2^6 - 6677 T_1^2 T_2^6 + 8131 T_1^3 T_2^6 + 19541 T_1^4 T_2^6 - 76792 T_1^5 T_2^6 + 92385 T_1^6 T_2^6 + 67072 T_1^7 T_2^6 - \\
 & 176705 T_1^8 T_2^6 + 67072 T_1^9 T_2^6 + 92385 T_1^{10} T_2^6 - 76792 T_1^{11} T_2^6 + 19541 T_1^{12} T_2^6 + 8131 T_1^{13} T_2^6 - 6677 T_1^{14} T_2^6 + \\
 & 450 T_1^{15} T_2^6 + 212 T_1^{16} T_2^6 - 118 T_2^7 - 940 T_1 T_2^7 + 4411 T_1^2 T_2^7 + 8131 T_1^3 T_2^7 - 42394 T_1^4 T_2^7 + 46980 T_1^5 T_2^7 + \\
 & 67072 T_1^6 T_2^7 - 296378 T_1^7 T_2^7 + 181777 T_1^8 T_2^7 + 181777 T_1^9 T_2^7 - 296378 T_1^{10} T_2^7 + 67072 T_1^{11} T_2^7 + \\
 & 46980 T_1^{12} T_2^7 - 42394 T_1^{13} T_2^7 + 8131 T_1^{14} T_2^7 + 4411 T_1^{15} T_2^7 - 940 T_1^{16} T_2^7 - 118 T_1^{17} T_2^7 + 44 T_2^8 + \\
 & 740 T_1 T_2^8 - 401 T_1^2 T_2^8 - 15079 T_1^3 T_2^8 + 19541 T_1^4 T_2^8 + 46980 T_1^5 T_2^8 - 176705 T_1^6 T_2^8 + 181777 T_1^7 T_2^8 + \\
 & 320104 T_1^8 T_2^8 - 652662 T_1^9 T_2^8 + 320104 T_1^{10} T_2^8 + 181777 T_1^{11} T_2^8 - 176705 T_1^{12} T_2^8 + 46980 T_1^{13} T_2^8 + \\
 & 19541 T_1^{14} T_2^8 - 15079 T_1^{15} T_2^8 - 401 T_1^{16} T_2^8 + 740 T_1^{17} T_2^8 + 44 T_1^{18} T_2^8 - 10 T_2^9 - 340 T_1 T_2^9 - 1269 T_1^2 T_2^9 + \\
 & 7987 T_1^3 T_2^9 + 15896 T_1^4 T_2^9 - 76792 T_1^5 T_2^9 + 67072 T_1^6 T_2^9 + 181777 T_1^7 T_2^9 - 652662 T_1^8 T_2^9 + 391094 T_1^9 T_2^9 + \\
 & 391094 T_1^{10} T_2^9 - 652662 T_1^{11} T_2^9 + 181777 T_1^{12} T_2^9 + 67072 T_1^{13} T_2^9 - 76792 T_1^{14} T_2^9 + 15896 T_1^{15} T_2^9 + \\
 & 7987 T_1^{16} T_2^9 - 1269 T_1^{17} T_2^9 - 340 T_1^{18} T_2^9 - 10 T_1^{19} T_2^9 + T_2^{10} + 90 T_1 T_2^{10} + 985 T_1^2 T_2^{10} - 914 T_1^3 T_2^{10} - \\
 & 18980 T_1^4 T_2^{10} + 17848 T_1^5 T_2^{10} + 92385 T_1^6 T_2^{10} - 296378 T_1^7 T_2^{10} + 320104 T_1^8 T_2^{10} + 391094 T_1^9 T_2^{10} - \\
 & 864744 T_1^{10} T_2^{10} + 391094 T_1^{11} T_2^{10} + 320104 T_1^{12} T_2^{10} - 296378 T_1^{13} T_2^{10} + 92385 T_1^{14} T_2^{10} + 17848 T_1^{15} T_2^{10} - \\
 & 18980 T_1^{16} T_2^{10} - 914 T_1^{17} T_2^{10} + 985 T_1^{18} T_2^{10} + 90 T_1^{19} T_2^{10} + T_2^{20} - 10 T_1 T_2^{11} - 340 T_1^2 T_2^{11} - 1269 T_1^3 T_2^{11} + \\
 & 7987 T_1^4 T_2^{11} + 15896 T_1^5 T_2^{11} - 76792 T_1^6 T_2^{11} + 67072 T_1^7 T_2^{11} + 181777 T_1^8 T_2^{11} - 652662 T_1^9 T_2^{11} + \\
 & 391094 T_1^{10} T_2^{11} + 391094 T_1^{11} T_2^{11} - 652662 T_1^{12} T_2^{11} + 181777 T_1^{13} T_2^{11} + 67072 T_1^{14} T_2^{11} - 76792 T_1^{15} T_2^{11} + \\
 & 15896 T_1^{16} T_2^{11} + 7987 T_1^{17} T_2^{11} - 1269 T_1^{18} T_2^{11} - 340 T_1^{19} T_2^{11} - 10 T_1^{20} T_2^{11} + 44 T_1^{21} T_2^{11} + 740 T_1^3 T_2^{12} - \\
 & 401 T_1^4 T_2^{12} - 15079 T_1^5 T_2^{12} + 19541 T_1^6 T_2^{12} + 46980 T_1^7 T_2^{12} - 176705 T_1^8 T_2^{12} + 181777 T_1^9 T_2^{12} + \\
 & 320104 T_1^{10} T_2^{12} - 652662 T_1^{11} T_2^{12} + 320104 T_1^{12} T_2^{12} + 181777 T_1^{13} T_2^{12} - 176705 T_1^{14} T_2^{12} + 46980 T_1^{15} T_2^{12} + \\
 & 19541 T_1^{16} T_2^{12} - 15079 T_1^{17} T_2^{12} - 401 T_1^{18} T_2^{12} + 740 T_1^{19} T_2^{12} + 44 T_1^{20} T_2^{12} - 118 T_1^3 T_2^{13} - 940 T_1^4 T_2^{13} + \\
 & 4411 T_1^5 T_2^{13} + 8131 T_1^6 T_2^{13} - 42394 T_1^7 T_2^{13} + 46980 T_1^8 T_2^{13} + 67072 T_1^9 T_2^{13} - 296378 T_1^{10} T_2^{13} + \\
 & 181777 T_1^{11} T_2^{13} + 181777 T_1^{12} T_2^{13} - 296378 T_1^{13} T_2^{13} + 67072 T_1^{14} T_2^{13} + 46980 T_1^{15} T_2^{13} - 42394 T_1^{16} T_2^{13} + \\
 & 8131 T_1^{17} T_2^{13} + 4411 T_1^{18} T_2^{13} - 940 T_1^{19} T_2^{13} - 118 T_1^{20} T_2^{13} + 212 T_1^4 T_2^{14} + 450 T_1^5 T_2^{14} - 6677 T_1^6 T_2^{14} + \\
 & 8131 T_1^7 T_2^{14} + 19541 T_1^8 T_2^{14} - 76792 T_1^9 T_2^{14} + 92385 T_1^{10} T_2^{14} + 67072 T_1^{11} T_2^{14} - 176705 T_1^{12} T_2^{14} + \\
 & 67072 T_1^{13} T_2^{14} + 92385 T_1^{14} T_2^{14} - 76792 T_1^{15} T_2^{14} + 19541 T_1^{16} T_2^{14} + 8131 T_1^{17} T_2^{14} - 6677 T_1^{18} T_2^{14} + \\
 & 450 T_1^{19} T_2^{14} + 212 T_1^{20} T_2^{14} - 257 T_1^5 T_2^{15} + 450 T_1^6 T_2^{15} + 4411 T_1^7 T_2^{15} - 15079 T_1^8 T_2^{15} + 15896 T_1^9 T_2^{15} + \\
 & 17848 T_1^{10} T_2^{15} - 76792 T_1^{11} T_2^{15} + 46980 T_1^{12} T_2^{15} + 46980 T_1^{13} T_2^{15} - 76792 T_1^{14} T_2^{15} + 17848 T_1^{15} T_2^{15} + \\
 & 15896 T_1^{16} T_2^{15} - 15079 T_1^{17} T_2^{15} + 4411 T_1^{18} T_2^{15} + 450 T_1^{19} T_2^{15} - 257 T_1^{20} T_2^{15} + 212 T_1^6 T_2^{16} - 940 T_1^7 T_2^{16} - \\
 & 401 T_1^8 T_2^{16} + 7987 T_1^9 T_2^{16} - 18980 T_1^{10} T_2^{16} + 15896 T_1^{11} T_2^{16} + 19541 T_1^{12} T_2^{16} - 42394 T_1^{13} T_2^{16} + \\
 & 19541 T_1^{14} T_2^{16} + 15896 T_1^{15} T_2^{16} - 18980 T_1^{16} T_2^{16} + 7987 T_1^{17} T_2^{16} - 401 T_1^{18} T_2^{16} - 940 T_1^{19} T_2^{16} + \\
 & 212 T_1^{20} T_2^{16} - 118 T_1^7 T_2^{17} + 740 T_1^8 T_2^{17} - 1269 T_1^9 T_2^{17} - 914 T_1^{10} T_2^{17} + 7987 T_1^{11} T_2^{17} - 15079 T_1^{12} T_2^{17} + \\
 & 8131 T_1^{13} T_2^{17} + 8131 T_1^{14} T_2^{17} - 15079 T_1^{15} T_2^{17} + 7987 T_1^{16} T_2^{17} - 914 T_1^{17} T_2^{17} - 1269 T_1^{18} T_2^{17} + 740 T_1^{19} T_2^{17} - \\
 & 118 T_1^{20} T_2^{17} + 44 T_1^8 T_2^{18} - 340 T_1^9 T_2^{18} + 985 T_1^{10} T_2^{18} - 1269 T_1^{11} T_2^{18} - 401 T_1^{12} T_2^{18} + 4411 T_1^{13} T_2^{18} - \\
 & 6677 T_1^{14} T_2^{18} + 4411 T_1^{15} T_2^{18} - 401 T_1^{16} T_2^{18} - 1269 T_1^{17} T_2^{18} + 985 T_1^{18} T_2^{18} - 340 T_1^{19} T_2^{18} + 44 T_1^{20} T_2^{18} - \\
 & 10 T_1^9 T_2^{19} + 90 T_1^{10} T_2^{19} - 340 T_1^{11} T_2^{19} + 740 T_1^{12} T_2^{19} - 940 T_1^{13} T_2^{19} + 450 T_1^{14} T_2^{19} + 450 T_1^{15} T_2^{19} - \\
 & 940 T_1^{16} T_2^{19} + 740 T_1^{17} T_2^{19} - 340 T_1^{18} T_2^{19} + 90 T_1^{19} T_2^{19} - 10 T_1^{20} T_2^{19} + T_1^{20} T_2^{20} - 10 T_1^{11} T_2^{20} + 44 T_1^{12} T_2^{20} - \\
 & 118 T_1^{13} T_2^{20} + 212 T_1^{14} T_2^{20} - 257 T_1^{15} T_2^{20} + 212 T_1^{16} T_2^{20} - 118 T_1^{17} T_2^{20} + 44 T_1^{18} T_2^{20} - 10 T_1^{19} T_2^{20} + T_1^{20} T_2^{20} \}
 \end{aligned}$$

```

In[*]:=  $\Theta$ [Knot[15, Alternating, 20905]] // LeafCount
Out[*]=
3871

In[*]:=  $\Theta$ [Knot[15, Alternating, 20905]] // ByteCount
Out[*]=
121264

In[*]:= ( $\Theta$ [Knot[15, Alternating, 20905]] /. {T1 → T1, T2 → T2}) // ByteCount
Out[*]=
75040

In[*]:= AbsoluteTiming@Monitor [
  Table[K → ( $\Theta$ [K] /. {T1 → T1, T2 → T2}), {K, AllKnots[{15, 15}}] >> "theta15-15.m",
  K
]
KnotTheory: Loading precomputed data in KnotTheory/15A.dts.
KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of
2005.

AbsoluteTiming@
Monitor[Table[K → HOMFLYPT[PD@K][a, z], {K, AllKnots[{3, 15}}] >> "HOMFLYPT3-15.m", K]
(Alt) In[*]:=
RKh[K_] := Module[{n = 0, kh},
  While[$Failed === (kh = Kh[K]) ∧ (n < 10),
    Kh[K] = .;
    Print["JavaKh failure ", ++n, " at ", K, "."];
  ];
  If[kh === $Failed, kh &, kh]
]

In[*]:= AbsoluteTiming@
Monitor[Table[K → RKh[PD@K][q, t], {K, AllKnots[{3, 10}}] >> "Kh3-10.m", K]
KnotTheory: Loading precomputed data in PD4Knots`.
KnotTheory: The Khovanov homology program JavaKh-v2 is an update of Jeremy Green's program JavaKh-v1, written by Scott
Morrison in 2008 at Microsoft Station Q.

Out[*]=
{75.4257, Null}

In[*]:= Do[Print@Short[K → RKh[PD@K][q, t]], {K, AllKnots[11, Alternating][[1 ;; 250]]}

```

```
In[*]:= AbsoluteTiming@
```

```
Monitor[Table[K → RKh[PD@K][q, t], {K, AllKnots[{3, 11]}]}] >> "Kh3-11.m", K]
```

```
⋯⋯ KnotTheory: Loading precomputed data in PD4Knots`.
```

```
⋯⋯ KnotTheory: The Khovanov homology program JavaKh-v2 is an update of Jeremy Green's program JavaKh-v1, written by Scott Morrison in 2008 at Microsoft Station Q.
```

```
⋯⋯ KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.
```

```
⋯⋯ KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.
```

```
Out[*]=
```

```
{267.324, Null}
```

```
In[*]:= AbsoluteTiming@Monitor[Table[K → RKh[K][q, t], {K, AllKnots[12]}] >> "Kh12.m", K]
```

```
⋯⋯ KnotTheory: Loading precomputed data in KnotTheory/12A.dts.
```

```
⋯⋯ KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.
```

```
⋯⋯ KnotTheory: The Khovanov homology program JavaKh-v2 is an update of Jeremy Green's program JavaKh-v1, written by Scott Morrison in 2008 at Microsoft Station Q.
```

```
⋯⋯ KnotTheory: Loading precomputed data in KnotTheory/12N.dts.
```

```
Out[*]=
```

```
{929.135, Null}
```

```
In[*]:= AbsoluteTiming@Monitor[Table[K → RKh[K][q, t], {K, AllKnots[13]}] >> "Kh13.m", K]
```

```
⋯⋯ KnotTheory: Loading precomputed data in KnotTheory/13A.dts.
```

```
⋯⋯ KnotTheory: Loading precomputed data in KnotTheory/13N.dts.
```

```
Out[*]=
```

```
{4871.25, Null}
```

```
In[*]:= AbsoluteTiming@Monitor[Table[K → RKh[K][q, t], {K, AllKnots[14]}] >> "Kh14.m", K]
```

```
⋯⋯ KnotTheory: Loading precomputed data in KnotTheory/14A.dts.
```

```
⋯⋯ KnotTheory: Loading precomputed data in KnotTheory/14N.dts.
```

```
Out[*]=
```

```
{33709.5, Null}
```

```
(Alt) In[*]:=
```

```
AbsoluteTiming@
```

```
Monitor[Table[K → RKh[K][q, t], {K, AllKnots[15, Alternating]}] >> "Kh15a.m", K]
```

```
⋯⋯ KnotTheory: Loading precomputed data in KnotTheory/15A.dts.
```

```
⋯⋯ KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.
```

```
⋯⋯ KnotTheory: The Khovanov homology program JavaKh-v2 is an update of Jeremy Green's program JavaKh-v1, written by Scott Morrison in 2008 at Microsoft Station Q.
```

```
(Alt) Out[*]=
```

```
{95994.4, Null}
```

(Alt) In[]:=

AbsoluteTiming@

Monitor[Table[K → RKh[K][q, t], {K, AllKnots[15, NonAlternating]}] >> "Kh15n.m", K]

 **KnotTheory**: Loading precomputed data in KnotTheory/15N.dts.

(Alt) Out[]:=

{161.295., Null}