

```

SetDirectory["~/AcademicPensieve/Projects/HigherRank/Data"];
(*SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank\\Data"];*)
<< ../../KnotTheory/KnotTheory/init.m
AppendTo[$Path, KnotTheoryDirectory[] = "../../KnotTheory"];
Print[$Path = Cases[$Path, _String]];
<< ../Rot.m
T3 = T1 T2;

```

SetDirectory: Cannot set current directory to ~/AcademicPensieve/Projects/HigherRank/Data.

C:\drorbn\AcademicPensieve\Projects\KnotTheory\KnotTheory

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

SetDelayed: Tag Diff in Diff[K_PD, rut_, ag_, n_, m_] is Protected.

Loading Rot.m from <http://drorbn.net/AP/Projects/HigherRank> to compute rotation numbers.

```

In[ ]:= CCF[ε_] := ExpandDenominator@ExpandNumerator@Together[ε];
CCF[ε_] := Factor[ε];
CF[ε_List] := CF /@ ε;
CF[ε_] := Module[{vs = Cases[ε, (x | p | π | g)_, ∞] ∪ {x, p, ε}, ps, c},
  Total[CoefficientRules[Expand[ε], vs] /. (ps_ -> c_) => CCF[c] (Times @@ vs^ps)]];

```

Data

(from Talks/Beijing-2407/theta.nb)

```

In[ ]:= R1[1, i_, j_] = CF[
  1 / 2 - T3 g1ji g2ji - g3ii + g2jj g3ii + T1 (T3 - 1) g1ji g3ji +
  T2 (T3 - 1) g2ji g3ji - T2 g2ji g3jj + (g1jj g2ii + (T3 - 1) g1jj g2ji -
  T1 g1ii g2jj - g1jj g3ii - T1 (T3 - 1) g1jj g3ji + T1 g1ii g3jj) / (T1 - 1)];

```

```

In[ ]:= R1[-1, i_, j_] = CF[
  -1 / 2 - T1^-1 g1ji g2ii - (1 - T1^-1 - T2^-1) g1ji g2ji - g1jj g2ji - g1ji g2jj + g3ii +
  T1^-1 g1ji g3ii - (1 - T2^-1) g2ji g3ii - g2jj g3ii + (1 - T3^-1) g1ji g3ji - (1 - T3^-1) g2ii g3ji +
  (2 - T2^-1) (1 - T3^-1) g2ji g3ji + (1 - T3^-1) g2jj g3ji + g1ji g3jj + g2ji g3jj + (T1 (1 - T2^-1) g1ii g2ji -
  g1jj g2ii + T1 g1ii g2jj + g1jj g3ii - T2^-1 (T3 - 1) g1ii g3ji - T1 g1ii g3jj) / (T1 - 1)];

```

```

In[ ]:= θ[{1, i0_, j0_}, {1, i1_, j1_}] =
  -T1 (T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 + (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 +
  T1 (T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 - (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;

```

$$\begin{aligned} \text{In[*]} := & \theta[\{1, i0_, j0_ \}, \{-1, i1_, j1_ \}] = \\ & (T_3 - 1) g_{1,j1,i0} g_{2,i1,i0} g_{3,j0,i1} - T_1^{-1} (T_3 - 1) g_{1,j1,j0} g_{2,i1,i0} g_{3,j0,i1} - \\ & (T_3 - 1) g_{1,j1,i0} g_{2,j1,i0} g_{3,j0,i1} + T_1^{-1} (T_3 - 1) g_{1,j1,j0} g_{2,j1,i0} g_{3,j0,i1}; \end{aligned}$$

$$\begin{aligned} \text{In[*]} := & \theta[\{-1, i0_, j0_ \}, \{1, i1_, j1_ \}] = \text{CF} [\\ & T_1^{-1} T_2^{-1} (T_3 - 1) (g_{1,j1,i0} g_{2,i1,i0} g_{3,j0,i1} - \\ & T_1 g_{1,j1,j0} g_{2,i1,i0} g_{3,j0,i1} - g_{1,j1,i0} g_{2,j1,i0} g_{3,j0,i1} + T_1 g_{1,j1,j0} g_{2,j1,i0} g_{3,j0,i1})]; \end{aligned}$$

$$\begin{aligned} \text{In[*]} := & \theta[\{-1, i0_, j0_ \}, \{-1, i1_, j1_ \}] = \text{CF} [\\ & (1 - T_3^{-1}) (-T_1^{-1} g_{1,j1,i0} g_{2,i1,i0} g_{3,j0,i1} + \\ & g_{1,j1,j0} g_{2,i1,i0} g_{3,j0,i1} + T_1^{-1} g_{1,j1,i0} g_{2,j1,i0} g_{3,j0,i1} - g_{1,j1,j0} g_{2,j1,i0} g_{3,j0,i1})]; \end{aligned}$$

$$\text{In[*]} := T_1[\varphi_, k_] = -\varphi / 2 + \varphi g_{3,k,k};$$

The Programs

```

In[*] := Module[{Cs, phi, n, A, s, i, j, k, Delta, G, v, alpha, beta, gEval, c, z},
  {Cs, phi} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} >=> (A[[{i, j}, {i + 1, j + 1}]] += (
    -T^s T^s - 1
  ))];
  Delta = T^(-Total[phi] - Total[Cs[[All, 1]]) / 2) Det[A];
  G = Inverse[A]; gEval[epsilon_] := Factor[epsilon /. g_{v_, alpha_, beta_} >=> (G[[alpha, beta]] /. T -> T_v)];
  z = gEval[Sum[theta[Cs[[k1]], Cs[[k2]], {k1, n}, {k2, n}]];
  z += gEval@Sum[R_1 @@ Cs[[k]], {k, n}];
  z += gEval@Sum[T_1[phi[[k]], k], {k, 2 n}];
  {Delta, (Delta /. T -> T_1) (Delta /. T -> T_2) (Delta /. T -> T_3) z} // Factor];

```

```

In[*] := theta[Knot[3, 1]]

```

 KnotTheory: Loading precomputed data in PD4Knots`

$$\text{Out[*]} = \left\{ \frac{1 - T + T^2}{T}, -\frac{1 - T_1 + T_1^2 - T_2 - T_1^3 T_2 + T_2^2 + T_1^4 T_2^2 - T_1 T_2^3 - T_1^4 T_2^3 + T_1^2 T_2^4 - T_1^3 T_2^4 + T_1^4 T_2^4}{T_1^2 T_2^2} \right\}$$

The Run

```

In[*] := Table[n -> NumberOfKnots[n], {n, 3, 15}]

```

```

Out[*] = {3 -> 1, 4 -> 1, 5 -> 2, 6 -> 3, 7 -> 7, 8 -> 21, 9 -> 49,
  10 -> 165, 11 -> 552, 12 -> 2176, 13 -> 9988, 14 -> 46972, 15 -> 253293}

```

```
AbsoluteTiming[
  Table[Echo[K] → ( $\Theta$ [K] /. {T1 → T1, T2 → T2}),
    {K, AllKnots[{15, 15}][[200001 ;; 253293]]} >> "theta15(200001-253293).m"
]
```

Out[*]=

```
{0.134328, Null}
```

```
AbsoluteTiming[
  Table[Echo[K] → HOMFLYPT[PD@K][a, z], {K, AllKnots[{3, 15]}}] >> "HOMFLYPT3-15.m"
]
```

```
AbsoluteTiming[
  Table[Echo[K] → Kh[PD@K][q, t], {K, AllKnots[{3, 15]}}] >> "Data/Kh3-15.m"
]
```