In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank"];
Once[<< KnotTheory`];
<< Rot.m
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

Loading Rot.m from http://drorbn.net/AP/Projects/HigherRank to compute rotation numbers.
```

In[•]:=
```
CCF[ℰ_] := ExpandDenominator@ExpandNumerator@Together[ℰ];
CCF[ℰ_] := Factor[ℰ];
CF[ℰ_List] := CF /@ ℰ;
CF[ℰ_] := Module[{vs = Cases[ℰ, (x | p | π | g)__, ∞] ⋃ {x, p, ϵ}, ps, c},
   Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) :> CCF[c] (Times @@ vs^ps)]];
```

In[•]:=
```
{
  {r0,pxx[1, i_, j_], r0,pxx[-1, i_, j_]},
  {r1,ppx[1, i_, j_], r1,ppx[-1, i_, j_]},
  {r1,rest[1, i_, j_], r1,rest[-1, i_, j_]},
  γ1[φ_, k_]
 } = CF[Plus[
   α {1, 1 - T1 T2, 1 - T1 T2, 1 - T1 T2} * Get["px-data.m"],
   β[{{0, 0},
     {0, 0},
     {1 / 2 + T3 x3,i (p3,i+1 - p3,j+1), -1 / 2 - T3^-1 x3,i (p3,i+1 - p3,j+1)},
     φ / 2
    }
  ]]
```

Out[•]=

$$\left\{\left\{\alpha\, p_{3,j}\, x_{1,i}\, x_{2,i} - \frac{\alpha\, p_{3,j}\, x_{1,j}\, x_{2,i}}{T_1},\ -\frac{\alpha\, p_{3,j}\, x_{1,i}\, x_{2,i}}{T_1^2\, T_2} + \frac{\alpha\, p_{3,j}\, x_{1,j}\, x_{2,i}}{T_1\, T_2}\right\},\right.$$
$$\left\{-\alpha\,(-1+T_1\,T_2)\, p_{1,j}\, p_{2,i}\, x_{3,i} + \alpha\,(-1+T_1\,T_2)\, p_{1,j}\, p_{2,j}\, x_{3,i},\right.$$
$$\left.\frac{\alpha\,(-1+T_1\,T_2)\, p_{1,j}\, p_{2,i}\, x_{3,i}}{T_1} - \frac{\alpha\,(-1+T_1\,T_2)\, p_{1,j}\, p_{2,j}\, x_{3,i}}{T_1}\right\},$$
$$\left\{\frac{\beta}{2} - \alpha\, T_2\, p_{1,j}\, p_{2,j}\, x_{1,i}\, x_{2,i} + \frac{\alpha\, p_{1,j}\, p_{2,i}\, x_{1,j}\, x_{2,i}}{(-1+T_1)\, T_1} + \frac{\alpha\,(-1+T_1\,T_2)\, p_{1,j}\, p_{2,j}\, x_{1,j}\, x_{2,i}}{(-1+T_1)\, T_1} -\right.$$
$$\frac{\alpha\, p_{1,i}\, p_{2,j}\, x_{1,i}\, x_{2,j}}{-1+T_1} + \beta\, T_3\, p_{3,1+i}\, x_{3,i} - \frac{\alpha\, p_{3,j}\, x_{3,i}}{T_1} - \beta\, T_3\, p_{3,1+j}\, x_{3,i} + \alpha\,(-1+T_1\,T_2)\, p_{1,j}\, p_{3,j}\, x_{1,i}\, x_{3,i} -$$
$$\frac{\alpha\, p_{1,j}\, p_{3,i}\, x_{1,j}\, x_{3,i}}{(-1+T_1)\, T_1} - \frac{\alpha\,(-1+T_1\,T_2)\, p_{1,j}\, p_{3,j}\, x_{1,j}\, x_{3,i}}{-1+T_1} + \frac{\alpha\, T_2\,(-1+T_1\,T_2)\, p_{2,j}\, p_{3,j}\, x_{2,i}\, x_{3,i}}{T_1} +$$
$$\frac{\alpha\, p_{2,j}\, p_{3,i}\, x_{2,j}\, x_{3,i}}{T_1} + \frac{\alpha\, p_{1,i}\, p_{3,j}\, x_{1,i}\, x_{3,j}}{-1+T_1} - \frac{\alpha\, T_2\, p_{2,j}\, p_{3,j}\, x_{2,i}\, x_{3,j}}{T_1},$$
$$-\frac{\beta}{2} - \frac{\alpha\, p_{1,j}\, p_{2,i}\, x_{1,i}\, x_{2,i}}{T_1^2} + \frac{\alpha\,(-1+T_2)\, p_{1,i}\, p_{2,j}\, x_{1,i}\, x_{2,i}}{(-1+T_1)\, T_2} - \frac{\alpha\,(-T_1-T_2+T_1\,T_2)\, p_{1,j}\, p_{2,j}\, x_{1,i}\, x_{2,i}}{T_1^2\, T_2} -$$
$$\frac{\alpha\, p_{1,j}\, p_{2,i}\, x_{1,j}\, x_{2,i}}{(-1+T_1)\, T_1} - \frac{\alpha\, p_{1,j}\, p_{2,j}\, x_{1,j}\, x_{2,i}}{T_1} + \frac{\alpha\, p_{1,i}\, p_{2,j}\, x_{1,i}\, x_{2,j}}{-1+T_1} - \frac{\alpha\, p_{1,j}\, p_{2,j}\, x_{1,i}\, x_{2,j}}{T_1} -$$
$$\frac{\beta\, p_{3,1+i}\, x_{3,i}}{T_3} + \frac{\alpha\, p_{3,j}\, x_{3,i}}{T_1} + \frac{\beta\, p_{3,1+j}\, x_{3,i}}{T_3} + \frac{\alpha\, p_{1,j}\, p_{3,i}\, x_{1,i}\, x_{3,i}}{T_1^2} - \frac{\alpha\,(-1+T_1\,T_2)\, p_{1,i}\, p_{3,j}\, x_{1,i}\, x_{3,i}}{(-1+T_1)\, T_1\, T_2} +$$
$$\frac{\alpha\,(-1+T_1\,T_2)\, p_{1,j}\, p_{3,j}\, x_{1,i}\, x_{3,i}}{T_1^2\, T_2} + \frac{\alpha\, p_{1,j}\, p_{3,i}\, x_{1,j}\, x_{3,i}}{(-1+T_1)\, T_1} - \frac{\alpha\,(-1+T_2)\, p_{2,j}\, p_{3,i}\, x_{2,i}\, x_{3,i}}{T_1\, T_2} -$$
$$\frac{\alpha\,(-1+T_1\,T_2)\, p_{2,i}\, p_{3,j}\, x_{2,i}\, x_{3,i}}{T_1^2\, T_2} + \frac{\alpha\,(-1+2\,T_2)\,(-1+T_1\,T_2)\, p_{2,j}\, p_{3,j}\, x_{2,i}\, x_{3,i}}{T_1^2\, T_2^2} -$$
$$\frac{\alpha\, p_{2,j}\, p_{3,i}\, x_{2,j}\, x_{3,i}}{T_1} + \frac{\alpha\,(-1+T_1\,T_2)\, p_{2,j}\, p_{3,j}\, x_{2,j}\, x_{3,i}}{T_1^2\, T_2} - \frac{\alpha\, p_{1,i}\, p_{3,j}\, x_{1,i}\, x_{3,j}}{-1+T_1} +$$
$$\left.\frac{\alpha\, p_{1,j}\, p_{3,j}\, x_{1,i}\, x_{3,j}}{T_1} + \frac{\alpha\, p_{2,j}\, p_{3,j}\, x_{2,i}\, x_{3,j}}{T_1}\right\},\ \frac{\beta\,\varphi}{2} + \frac{\alpha\,\varphi\, p_{3,k}\, x_{3,k}}{T_1}\Big\}$$

In[•]:= `r_{1,rest}[1, 4, 5]`

Out[•]=

$$\frac{\beta\,\gamma}{2} - \alpha\, T_2\, p_{1,5}\, p_{2,5}\, x_{1,4}\, x_{2,4} + \frac{\alpha\, p_{1,5}\, p_{2,4}\, x_{1,5}\, x_{2,4}}{(-1+T_1)\, T_1} + \frac{\alpha\,(-1+T_1\,T_2)\, p_{1,5}\, p_{2,5}\, x_{1,5}\, x_{2,4}}{(-1+T_1)\, T_1} -$$
$$\frac{\alpha\, p_{1,4}\, p_{2,5}\, x_{1,4}\, x_{2,5}}{-1+T_1} - \frac{\alpha\, p_{3,5}\, x_{3,4}}{T_1} + \beta\, T_1\, T_2\, p_{3,5}\, x_{3,4} - \beta\, T_1\, T_2\, p_{3,6}\, x_{3,4} +$$
$$\alpha\,(-1+T_1\,T_2)\, p_{1,5}\, p_{3,5}\, x_{1,4}\, x_{3,4} - \frac{\alpha\, p_{1,5}\, p_{3,4}\, x_{1,5}\, x_{3,4}}{(-1+T_1)\, T_1} - \frac{\alpha\,(-1+T_1\,T_2)\, p_{1,5}\, p_{3,5}\, x_{1,5}\, x_{3,4}}{-1+T_1} +$$
$$\frac{\alpha\, T_2\,(-1+T_1\,T_2)\, p_{2,5}\, p_{3,5}\, x_{2,4}\, x_{3,4}}{T_1} + \frac{\alpha\, p_{2,5}\, p_{3,4}\, x_{2,5}\, x_{3,4}}{T_1} + \frac{\alpha\, p_{1,4}\, p_{3,5}\, x_{1,4}\, x_{3,5}}{-1+T_1} - \frac{\alpha\, T_2\, p_{2,5}\, p_{3,5}\, x_{2,4}\, x_{3,5}}{T_1}$$

In[•]:= `{p*, x*, π*, ξ*} = {π, ξ, p, x};  (u_i__)* := (u*)_i;`

```
In[•]:= Zip{}[ℰ_] := ℰ;
        Zip{ξ_,ξs___}[ℰ_] := (Collect[ℰ // Zip{ξs}, ξ] /. f_. ξ^d_. :> (D[f, {ξ*, d}])) /. ξ* → 0
```

```
In[•]:= px2g[ℰ_] := CF@Module[{ps, xs, Q, α, β},
          ps = Union[Cases[ℰ, p__, ∞]]; xs = Union[Cases[ℰ, x__, ∞]];
          Q = Sum[p0* x0* g_{p0[[2]],x0[[2]],p0[[3]],x0[[3]]}, {p0, ps}, {x0, xs}];
          Expand[Zip_{ps∪xs}[ℰ e^Q] /. g_{α_,β_,i_,j_} :> If[α === β, g_{α,i,j}, 0]]
        ]
```

```
In[•]:= px2g[p_{2,j}^2 x_{2,i} x_{2,j}]
```

```
Out[•]=
2 g_{2,j,i} g_{2,j,j}
```

```
In[•]:= R1[1, i_, j_] = px2g[r_{1,rest}[1, i, j]]
```

```
Out[•]=
β/2 + (α g_{1,j,j} g_{2,i,i})/((-1 + T1) T1) - α T2 g_{1,j,i} g_{2,j,i} + (α (-1 + T1 T2) g_{1,j,j} g_{2,j,i})/((-1 + T1) T1) - (α g_{1,i,i} g_{2,j,j})/(-1 + T1) - (α g_{1,j,j} g_{3,i,i})/((-1 + T1) T1) +
(α g_{2,j,j} g_{3,i,i})/T1 + β T3 g_{3,1+i,i} - (α g_{3,j,i})/T1 + α (-1 + T1 T2) g_{1,j,i} g_{3,j,i} - (α (-1 + T1 T2) g_{1,j,j} g_{3,j,i})/(-1 + T1) +
(α T2 (-1 + T1 T2) g_{2,j,i} g_{3,j,i})/T1 + (α g_{1,i,i} g_{3,j,j})/(-1 + T1) - (α T2 g_{2,j,i} g_{3,j,j})/T1 - β T3 g_{3,1+j,i}
```

```
In[•]:= R1[-1, i_, j_] = px2g[r_{1,rest}[-1, i, j]]
```

```
Out[•]=
-β/2 - (α g_{1,j,i} g_{2,i,i})/T1^2 - (α g_{1,j,j} g_{2,i,i})/((-1 + T1) T1) + (α (-1 + T2) g_{1,i,i} g_{2,j,i})/((-1 + T1) T2) - (α (-T1 - T2 + T1 T2) g_{1,j,i} g_{2,j,i})/(T1^2 T2) -
(α g_{1,j,j} g_{2,j,i})/T1 + (α g_{1,i,i} g_{2,j,j})/(-1 + T1) - (α g_{1,j,i} g_{2,j,j})/T1 + (α g_{1,j,i} g_{3,i,i})/T1^2 + (α g_{1,j,j} g_{3,i,i})/((-1 + T1) T1) -
(α (-1 + T2) g_{2,j,i} g_{3,i,i})/(T1 T2) - (α g_{2,j,j} g_{3,i,i})/T1 - (β g_{3,1+i,i})/T3 + (α g_{3,j,i})/T1 - (α (-1 + T1 T2) g_{1,i,i} g_{3,j,i})/((-1 + T1) T1 T2) +
(α (-1 + T1 T2) g_{1,j,i} g_{3,j,i})/(T1^2 T2) - (α (-1 + T1 T2) g_{2,i,i} g_{3,j,i})/(T1^2 T2) + (α (-1 + 2 T2) (-1 + T1 T2) g_{2,j,i} g_{3,j,i})/(T1^2 T2^2) +
(α (-1 + T1 T2) g_{2,j,j} g_{3,j,i})/(T1^2 T2) - (α g_{1,i,i} g_{3,j,j})/(-1 + T1) + (α g_{1,j,i} g_{3,j,j})/T1 + (α g_{2,j,i} g_{3,j,j})/T1 + (β g_{3,1+j,i})/T3
```

```
In[•]:= px2g[r_{0,pxx}[1, i0, j0] r_{1,ppx}[1, i1, j1]]
```

```
Out[•]=
-α^2 (-1 + T1 T2) g_{1,j1,i0} g_{2,i1,i0} g_{3,j0,i1} + (α^2 (-1 + T1 T2) g_{1,j1,j0} g_{2,i1,i0} g_{3,j0,i1})/T1 +
α^2 (-1 + T1 T2) g_{1,j1,i0} g_{2,j1,i0} g_{3,j0,i1} - (α^2 (-1 + T1 T2) g_{1,j1,j0} g_{2,j1,i0} g_{3,j0,i1})/T1
```

In[•]:=
```
θ[{1, i0_, j0_}, {1, i1_, j1_}] = px2g[r0,pxx[1, i0, j0] r1,ppx[1, i1, j1]]
θ[{1, i0_, j0_}, {-1, i1_, j1_}] = px2g[r0,pxx[1, i0, j0] r1,ppx[-1, i1, j1]]
θ[{-1, i0_, j0_}, {1, i1_, j1_}] = px2g[r0,pxx[-1, i0, j0] r1,ppx[1, i1, j1]]
θ[{-1, i0_, j0_}, {-1, i1_, j1_}] = px2g[r0,pxx[-1, i0, j0] r1,ppx[-1, i1, j1]]
```

Out[•]=

$$-\alpha^2\,(-1+T_1\,T_2)\,g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}+\frac{\alpha^2\,(-1+T_1\,T_2)\,g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1}+$$
$$\alpha^2\,(-1+T_1\,T_2)\,g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}-\frac{\alpha^2\,(-1+T_1\,T_2)\,g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1}$$

Out[•]=

$$\frac{\alpha^2\,(-1+T_1\,T_2)\,g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1}-\frac{\alpha^2\,(-1+T_1\,T_2)\,g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1^2}-$$
$$\frac{\alpha^2\,(-1+T_1\,T_2)\,g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1}+\frac{\alpha^2\,(-1+T_1\,T_2)\,g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1^2}$$

Out[•]=

$$\frac{\alpha^2\,(-1+T_1\,T_2)\,g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1^2\,T_2}-\frac{\alpha^2\,(-1+T_1\,T_2)\,g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1\,T_2}-$$
$$\frac{\alpha^2\,(-1+T_1\,T_2)\,g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1^2\,T_2}+\frac{\alpha^2\,(-1+T_1\,T_2)\,g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1\,T_2}$$

Out[•]=

$$-\frac{\alpha^2\,(-1+T_1\,T_2)\,g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1^3\,T_2}+\frac{\alpha^2\,(-1+T_1\,T_2)\,g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1^2\,T_2}+$$
$$\frac{\alpha^2\,(-1+T_1\,T_2)\,g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1^3\,T_2}-\frac{\alpha^2\,(-1+T_1\,T_2)\,g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1^2\,T_2}$$

In[•]:=
```
Γ1[φ_, k_] = px2g[γ1[φ, k]]
```

Out[•]=

$$\frac{\beta\,\varphi}{2}+\frac{\alpha\,\varphi\,g_{3,k,k}}{T_1}$$

In[•]:=
```
T3 = T1 T2;
θ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ν, α, β, gEval, Y, yEval, c, z},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += {{-T^s, T^s - 1}, {0, -1}})];
  Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
  G = Inverse[A]; gEval[ℰ_] := CCF[ℰ /. g_{ν_,α_,β_} :> (G[[α, β]] /. T → T_ν)];
  z = gEval[Sum[Sum[θ[Cs[[k1]], Cs[[k2]]], {k2, n}], {k1, n}]];
  z += gEval[Sum[R1 @@ Cs[[k]], {k, n}]];
  z += gEval[Sum[Γ1[φ[[k]], k], {k, 2 n}]];
  {Δ, (Δ /. T → T1) (Δ /. T → T2) (Δ /. T → T3) z} // CCF
 ];
```

In[•]:=
```
θ_{T1_,T2_}[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, gEval, Y, yEval, c, z = 0},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  temp = PrintTemporary["At work, n=", n];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += {{-T^s, T^s - 1}, {0, -1}})];
  Δ[0] := Δ[0] = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
  G[0] := G[0] = Inverse[A];
  {Δ[1], G[1]} = If[NumberQ@T1,
    {Det[A /. T → T1], Inverse[A /. T → T1]}, {Δ[0], G[0]} /. T → T1 ];
  temp = PrintTemporary@"Done with {Δ[1],G[1]}.";
  {Δ[2], G[2]} = If[NumberQ@T2,
    {Det[A /. T → T2], Inverse[A /. T → T2]}, {Δ[0], G[0]} /. T → T2 ];
  NotebookDelete[temp]; temp = PrintTemporary@"Done with {Δ[2],G[2]}.";
  {Δ[3], G[3]} = If[NumberQ[T1 T2],
    {Det[A /. T → T1 T2], Inverse[A /. T → T1 T2]}, {Δ[0], G[0]} /. T → T1 T2 ];
  NotebookDelete[temp]; temp = PrintTemporary@"Done with {Δ[3],G[3]}.";
  gEval[ℰ_] := CCF[ℰ //. {T1 → T1, T2 → T2, g_{ν_,α_,β_} :> G[ν][[α, β]]}];
  Do[z += gEval[θ[Cs[[k1]], Cs[[k2]]]], {k1, n}, {k2, n}];
  Do[z += gEval[R1 @@ Cs[[k]]], {k, n}];
  Do[z += gEval[Γ1[φ[[k]], k]], {k, 2 n}];
  NotebookDelete[temp];
  {Δ[1], Δ[2], Δ[3], Δ[1] Δ[2] Δ[3] z} // CCF
 ];
```

```
In[•]:= Timing[Factor[θ[Knot[3, 1]] /. {α → 1, β → 0}]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[•]=

$$\left\{0.,\ \left\{\frac{1-T+T^2}{T},\ -\frac{-1+T_1-T_1^2+T_2-T_1^2\,T_2+2\,T_1^3\,T_2-T_2^2-T_1\,T_2^2+T_1^2\,T_2^2-2\,T_1^3\,T_2^2+2\,T_1\,T_2^3-2\,T_1^2\,T_2^3+2\,T_1^3\,T_2^3}{T_1^2\,T_2}\right\}\right\}$$

```
In[•]:= K = Knot[8, 17]; Timing[Factor[θ[K] //. {α → 0, β → 1}]]
```

Out[•]=

$$\left\{0.,\ \left\{-\frac{1-4\,T+8\,T^2-11\,T^3+8\,T^4-4\,T^5+T^6}{T^3},\right.\right.$$
$$\left.\left.\frac{1}{T_1^6\,T_2^6}\left(1-4\,T_1+8\,T_1^2-11\,T_1^3+8\,T_1^4-4\,T_1^5+T_1^6\right)\left(-1+T_1\,T_2\right)\left(1+T_1\,T_2\right)\right.\right.$$
$$\left.\left.\left(1-T_1\,T_2+T_1^2\,T_2^2\right)\left(3-5\,T_1\,T_2+3\,T_1^2\,T_2^2\right)\left(1-4\,T_2+8\,T_2^2-11\,T_2^3+8\,T_2^4-4\,T_2^5+T_2^6\right)\right\}\right\}$$

```
In[•]:= Factor[T D[-(1 - 4 T + 8 T^2 - 11 T^3 + 8 T^4 - 4 T^5 + T^6)/T^3, T]]
```

Out[•]=

$$-\frac{(-1+T)\,(1+T)\left(1-T+T^2\right)\left(3-5\,T+3\,T^2\right)}{T^3}$$

```
In[•]:= θ[K][[1]] /. T → T1
```

Out[•]=

$$-\frac{1-4\,T_1+8\,T_1^2-11\,T_1^3+8\,T_1^4-4\,T_1^5+T_1^6}{T_1^3}$$

```
In[•]:= K = Knot[3, 1]; Timing[Expand[T1 θ[K] //. {α → 1, β → -T1^-1}]]
```

Out[•]=

$$\left\{0.,\ \left\{-T_1+\frac{T_1}{T}+T\,T_1,\ -\frac{1}{T_1^2}-T_1^2-\frac{1}{T_2^2}-\frac{1}{T_1^2\,T_2^2}+\frac{1}{T_1\,T_2^2}+\frac{1}{T_1^2\,T_2}+\frac{T_1}{T_2}+\frac{T_2}{T_1}+T_1^2\,T_2-T_2^2+T_1\,T_2^2-T_1^2\,T_2^2\right\}\right\}$$

In[•]:=
```
K = Knot[8, 19]; Timing[Expand[T1 θ[K] //. {α → 1, β → -T1^-1}]]
```

Out[•]=

$$\Big\{0.015625,\ \Big\{T_1+\frac{T_1}{T^3}-\frac{T_1}{T^2}-T^2\,T_1+T^3\,T_1,\ \frac{3}{T_1^6}-\frac{3}{T_1^4}+\frac{4}{T_1^3}-\frac{1}{T_1^2}-T_1^2+4\,T_1^3-3\,T_1^4+3\,T_1^6+\frac{3}{T_2^6}+\frac{3}{T_1^6\,T_2^6}-\frac{3}{T_1^5\,T_2^6}+\frac{3}{T_1^3\,T_2^6}-\frac{3}{T_1\,T_2^6}-\frac{3}{T_1^6\,T_2^5}+\frac{3}{T_1^4\,T_2^5}-\frac{3}{T_1^3\,T_2^5}-\frac{3}{T_1^2\,T_2^5}+\frac{3}{T_1\,T_2^5}-\frac{3\,T_1}{T_2^5}-\frac{3}{T_2^4}+\frac{3}{T_1^5\,T_2^4}-\frac{3}{T_1^4\,T_2^4}+\frac{3}{T_1^2\,T_2^4}+\frac{3\,T_1}{T_2^4}+\frac{4}{T_2^3}+\frac{3}{T_1^6\,T_2^3}-\frac{3}{T_1^5\,T_2^3}+\frac{4}{T_1^3\,T_2^3}-\frac{2}{T_1^2\,T_2^3}-\frac{2}{T_1\,T_2^3}-\frac{3\,T_1^2}{T_2^3}+\frac{3\,T_1^3}{T_2^3}-\frac{1}{T_2^2}-\frac{3}{T_1^5\,T_2^2}+\frac{3}{T_1^4\,T_2^2}-\frac{2}{T_1^3\,T_2^2}-\frac{1}{T_1^2\,T_2^2}+\frac{1}{T_1\,T_2^2}-\frac{2\,T_1}{T_2^2}+\frac{3\,T_1^2}{T_2^2}-\frac{3\,T_1^3}{T_2^2}-\frac{3}{T_1^6\,T_2}+\frac{3}{T_1^5\,T_2}-\frac{2}{T_1^3\,T_2}+\frac{1}{T_1^2\,T_2}+\frac{T_1}{T_2}-\frac{2\,T_1^2}{T_2}+\frac{3\,T_1^4}{T_2}-\frac{3\,T_1^5}{T_2}-\frac{3\,T_2}{T_1^5}+\frac{3\,T_2}{T_1^4}-\frac{2\,T_2}{T_1^2}+\frac{T_2}{T_1}+T_1^2\,T_2-2\,T_1^3\,T_2+3\,T_1^5\,T_2-3\,T_1^6\,T_2-T_2^2-\frac{3\,T_2^2}{T_1^3}+\frac{3\,T_2^2}{T_1^2}-\frac{2\,T_2^2}{T_1}+T_1\,T_2^2-T_1^2\,T_2^2-2\,T_1^3\,T_2^2+3\,T_1^4\,T_2^2-3\,T_1^5\,T_2^2+4\,T_2^3+\frac{3\,T_2^3}{T_1^3}-\frac{3\,T_2^3}{T_1^2}-2\,T_1\,T_2^3-2\,T_1^2\,T_2^3+4\,T_1^3\,T_2^3-3\,T_1^5\,T_2^3+3\,T_1^6\,T_2^3-3\,T_2^4+\frac{3\,T_2^4}{T_1}+3\,T_1^2\,T_2^4-3\,T_1^4\,T_2^4+3\,T_1^5\,T_2^4-\frac{3\,T_2^5}{T_1}+3\,T_1\,T_2^5-3\,T_1^2\,T_2^5-3\,T_1^3\,T_2^5+3\,T_1^4\,T_2^5-3\,T_1^6\,T_2^5+3\,T_2^6-3\,T_1\,T_2^6+3\,T_1^3\,T_2^6-3\,T_1^5\,T_2^6+3\,T_1^6\,T_2^6\Big\}\Big\}$$

In[•]:=
```
Timing[θ_{T1,T2}[Knot[3, 1]]]
```

Out[•]=

$$\Big\{0.,\ \Big\{\frac{1-T_1+T_1^2}{T_1},\ \frac{1-T_2+T_2^2}{T_2},\ \frac{1-T_1\,T_2+T_1^2\,T_2^2}{T_1\,T_2},\ -\frac{-1+T_1-T_1^2+T_2-T_1^2\,T_2+2\,T_1^3\,T_2-T_2^2-T_1\,T_2^2+T_1^2\,T_2^2-2\,T_1^3\,T_2^2+2\,T_1\,T_2^3-2\,T_1^2\,T_2^3+2\,T_1^3\,T_2^3}{T_1^2\,T_2}\Big\}\Big\}$$

In[•]:=
```
Timing[θ[Knot[4, 1]]]
```

Out[•]=

$$\Big\{0.,\ \Big\{-\frac{1-3\,T+T^2}{T},\ \frac{\left(1-3\,T_1+T_1^2\right)\,(-1+T_1\,T_2)\,(1+T_1\,T_2)\,\left(1-3\,T_2+T_2^2\right)}{T_1^3\,T_2^2}\Big\}\Big\}$$

In[•]:=
```
Timing[θ[Knot["K11n34"]]]
```

Out[•]=

$$\Big\{0.03125,\ \Big\{1,\ -\frac{1}{T_1^7\,T_2^6}\,\Big(T_1^2-2\,T_1^3+T_1^4-2\,T_1\,T_2+2\,T_1^2\,T_2+2\,T_1^5\,T_2-2\,T_1^6\,T_2+T_2^2+2\,T_1\,T_2^2-2\,T_1^2\,T_2^2-2\,T_1^4\,T_2^2-2\,T_1^6\,T_2^2+2\,T_1^7\,T_2^2+T_1^8\,T_2^2-2\,T_1^3\,T_2^3+T_1^4\,T_2^3+T_1^5\,T_2^3-2\,T_1^9\,T_2^3+T_2^4-2\,T_1^2\,T_2^4+T_1^3\,T_2^4+2\,T_1^4\,T_2^4+2\,T_1^6\,T_2^4+T_1^7\,T_2^4-2\,T_1^8\,T_2^4+T_1^{10}\,T_2^4+2\,T_1\,T_2^5+T_1^3\,T_2^5-4\,T_1^5\,T_2^5-4\,T_1^6\,T_2^5+T_1^8\,T_2^5+2\,T_1^{10}\,T_2^5-2\,T_1\,T_2^6-2\,T_1^2\,T_2^6+2\,T_1^4\,T_2^6-4\,T_1^5\,T_2^6+12\,T_1^6\,T_2^6-4\,T_1^7\,T_2^6+2\,T_1^8\,T_2^6-2\,T_1^{10}\,T_2^6-2\,T_1^{11}\,T_2^6+2\,T_1^2\,T_2^7+T_1^4\,T_2^7-4\,T_1^6\,T_2^7-4\,T_1^7\,T_2^7+T_1^9\,T_2^7+2\,T_1^{11}\,T_2^7+T_1^2\,T_2^8-2\,T_1^4\,T_2^8+T_1^5\,T_2^8+2\,T_1^6\,T_2^8+2\,T_1^8\,T_2^8+T_1^9\,T_2^8-2\,T_1^{10}\,T_2^8+T_1^{12}\,T_2^8-2\,T_1^3\,T_2^9+T_1^7\,T_2^9+T_1^8\,T_2^9-2\,T_1^{12}\,T_2^9+T_1^4\,T_2^{10}+2\,T_1^5\,T_2^{10}-2\,T_1^6\,T_2^{10}-2\,T_1^8\,T_2^{10}-2\,T_1^{10}\,T_2^{10}+2\,T_1^{11}\,T_2^{10}+T_1^{12}\,T_2^{10}-2\,T_1^6\,T_2^{11}+2\,T_1^7\,T_2^{11}+2\,T_1^{10}\,T_2^{11}-2\,T_1^{11}\,T_2^{11}+T_1^8\,T_2^{12}-2\,T_1^9\,T_2^{12}+T_1^{10}\,T_2^{12}\Big)\Big\}\Big\}$$

In[•]:=
```
Timing[θ[Knot["K11n42"]]]
```

Out[•]=

$$\left\{0.,\ \left\{1,\ \frac{1}{T_1^4\,T_2^3}\left(T_1+T_1^2+T_2-2\,T_1\,T_2-2\,T_1^2\,T_2-2\,T_1^3\,T_2+T_1^4\,T_2+T_2^2-2\,T_1\,T_2^2+2\,T_1^2\,T_2^2+2\,T_1^3\,T_2^2-2\,T_1^4\,T_2^2+T_1^5\,T_2^2-\right.\right.\right.$$
$$\left.\left.\left.2\,T_1\,T_2^3+2\,T_1^2\,T_2^3+2\,T_1^4\,T_2^3-2\,T_1^5\,T_2^3+T_1\,T_2^4-2\,T_1^2\,T_2^4+2\,T_1^3\,T_2^4+2\,T_1^4\,T_2^4-\right.\right.\right.$$
$$\left.\left.\left.2\,T_1^5\,T_2^4+T_1^6\,T_2^4+T_1^2\,T_2^5-2\,T_1^3\,T_2^5-2\,T_1^4\,T_2^5-2\,T_1^5\,T_2^5+T_1^6\,T_2^5+T_1^4\,T_2^6+T_1^5\,T_2^6\right)\right\}\right\}$$

In[•]:=
```
PD[GST48] = PD[X[1, 15, 2, 14], X[29, 2, 30, 3], X[40, 4, 41, 3],
   X[4, 44, 5, 43], X[5, 26, 6, 27], X[95, 7, 96, 6], X[7, 1, 8, 96], X[8, 14, 9, 13],
   X[28, 9, 29, 10], X[41, 11, 42, 10], X[11, 43, 12, 42], X[12, 27, 13, 28],
   X[15, 31, 16, 30], X[61, 16, 62, 17], X[72, 17, 73, 18], X[83, 18, 84, 19],
   X[34, 20, 35, 19], X[20, 89, 21, 90], X[92, 21, 93, 22], X[22, 79, 23, 80],
   X[23, 68, 24, 69], X[24, 57, 25, 58], X[56, 25, 57, 26], X[31, 63, 32, 62],
   X[32, 74, 33, 73], X[33, 85, 34, 84], X[35, 50, 36, 51], X[81, 37, 82, 36],
   X[70, 38, 71, 37], X[59, 39, 60, 38], X[54, 39, 55, 40], X[55, 45, 56, 44],
   X[45, 59, 46, 58], X[46, 70, 47, 69], X[47, 81, 48, 80], X[91, 49, 92, 48],
   X[49, 91, 50, 90], X[82, 52, 83, 51], X[71, 53, 72, 52], X[60, 54, 61, 53],
   X[74, 63, 75, 64], X[85, 64, 86, 65], X[65, 76, 66, 77], X[66, 87, 67, 88],
   X[94, 67, 95, 68], X[86, 75, 87, 76], X[77, 88, 78, 89], X[93, 78, 94, 79]];
```

In[•]:=
```
AbsoluteTiming[gst48 = θ[GST48]]
```

Out[•]=

$$\left\{11.4869,\ \left\{-\frac{\left(-1+2\,T-T^2-T^3+2\,T^4-T^5+T^8\right)\left(-1+T^3-2\,T^4+T^5+T^6-2\,T^7+T^8\right)}{T^8},\right.\right.$$
$$\frac{1}{T_1^{21}\,T_2^{20}}\left(T_1^5-3\,T_1^6+4\,T_1^7-2\,T_1^8-2\,T_1^9+4\,T_1^{10}-2\,T_1^{11}-2\,T_1^{12}+4\,T_1^{13}-3\,T_1^{14}+T_1^{15}-3\,T_1^5\,T_2+6\,T_1^6\,T_2-3\,T_1^7\,T_2-\right.$$
$$6\,T_1^8\,T_2+12\,T_1^9\,T_2-6\,T_1^{10}\,T_2-6\,T_1^{11}\,T_2+12\,T_1^{12}\,T_2-6\,T_1^{13}\,T_2-3\,T_1^{14}\,T_2+6\,T_1^{15}\,T_2-3\,T_1^{16}\,T_2-T_1^3\,T_2^2+$$
$$3\,T_1^4\,T_2^2-T_1^6\,T_2^2-4\,T_1^7\,T_2^2+9\,T_1^8\,T_2^2-7\,T_1^9\,T_2^2-3\,T_1^{10}\,T_2^2+8\,T_1^{11}\,T_2^2-3\,T_1^{12}\,T_2^2-7\,T_1^{13}\,T_2^2+9\,T_1^{14}\,T_2^2-$$
$$4\,T_1^{15}\,T_2^2-T_1^{16}\,T_2^2+3\,T_1^{18}\,T_2^2-T_1^{19}\,T_2^2-T_1^2\,T_2^3+6\,T_1^3\,T_2^3-10\,T_1^4\,T_2^3+3\,T_1^5\,T_2^3+2\,T_1^6\,T_2^3-3\,T_1^7\,T_2^3+4\,T_1^8\,T_2^3-$$
$$2\,T_1^9\,T_2^3+2\,T_1^{10}\,T_2^3-T_1^{11}\,T_2^3-T_1^{12}\,T_2^3+2\,T_1^{13}\,T_2^3-2\,T_1^{14}\,T_2^3+4\,T_1^{15}\,T_2^3-3\,T_1^{16}\,T_2^3+2\,T_1^{17}\,T_2^3+3\,T_1^{18}\,T_2^3-$$
$$10\,T_1^{19}\,T_2^3+6\,T_1^{20}\,T_2^3-T_1^{21}\,T_2^3+3\,T_1^2\,T_2^4-10\,T_1^3\,T_2^4+3\,T_1^4\,T_2^4+17\,T_1^5\,T_2^4-19\,T_1^6\,T_2^4+10\,T_1^7\,T_2^4-7\,T_1^8\,T_2^4+$$
$$6\,T_1^9\,T_2^4-T_1^{10}\,T_2^4-18\,T_1^{11}\,T_2^4+35\,T_1^{12}\,T_2^4-18\,T_1^{13}\,T_2^4-T_1^{14}\,T_2^4+6\,T_1^{15}\,T_2^4-7\,T_1^{16}\,T_2^4+10\,T_1^{17}\,T_2^4-$$
$$19\,T_1^{18}\,T_2^4+17\,T_1^{19}\,T_2^4+3\,T_1^{20}\,T_2^4-10\,T_1^{21}\,T_2^4+3\,T_1^{22}\,T_2^4+T_2^5-3\,T_1\,T_2^5+3\,T_1^3\,T_2^5+17\,T_1^4\,T_2^5-38\,T_1^5\,T_2^5+$$
$$22\,T_1^6\,T_2^5+7\,T_1^7\,T_2^5-11\,T_1^8\,T_2^5-6\,T_1^9\,T_2^5+14\,T_1^{10}\,T_2^5+11\,T_1^{11}\,T_2^5-31\,T_1^{12}\,T_2^5+9\,T_1^{13}\,T_2^5+T_1^{14}\,T_2^5+8\,T_1^{16}\,T_2^5-$$
$$15\,T_1^{17}\,T_2^5+9\,T_1^{18}\,T_2^5+16\,T_1^{19}\,T_2^5-32\,T_1^{20}\,T_2^5+15\,T_1^{21}\,T_2^5+3\,T_1^{22}\,T_2^5-3\,T_1^{24}\,T_2^5+T_1^{25}\,T_2^5-3\,T_2^6+6\,T_1\,T_2^6-$$
$$T_1^2\,T_2^6+2\,T_1^3\,T_2^6-19\,T_1^4\,T_2^6+22\,T_1^5\,T_2^6+24\,T_1^6\,T_2^6-68\,T_1^7\,T_2^6+43\,T_1^8\,T_2^6+9\,T_1^9\,T_2^6-29\,T_1^{10}\,T_2^6+2\,T_1^{11}\,T_2^6-$$
$$12\,T_1^{12}\,T_2^6+28\,T_1^{13}\,T_2^6-42\,T_1^{14}\,T_2^6+26\,T_1^{15}\,T_2^6-29\,T_1^{16}\,T_2^6-T_1^{17}\,T_2^6+45\,T_1^{18}\,T_2^6-64\,T_1^{19}\,T_2^6+24\,T_1^{20}\,T_2^6+$$
$$18\,T_1^{21}\,T_2^6-17\,T_1^{22}\,T_2^6+2\,T_1^{23}\,T_2^6-T_1^{24}\,T_2^6+6\,T_1^{25}\,T_2^6-3\,T_1^{26}\,T_2^6+4\,T_2^7-3\,T_1\,T_2^7-4\,T_1^2\,T_2^7-3\,T_1^3\,T_2^7+$$
$$10\,T_1^4\,T_2^7+7\,T_1^5\,T_2^7-68\,T_1^6\,T_2^7+74\,T_1^7\,T_2^7+14\,T_1^8\,T_2^7-56\,T_1^9\,T_2^7+14\,T_1^{10}\,T_2^7+55\,T_1^{11}\,T_2^7-23\,T_1^{12}\,T_2^7+$$
$$11\,T_1^{13}\,T_2^7+51\,T_1^{14}\,T_2^7-33\,T_1^{15}\,T_2^7+41\,T_1^{16}\,T_2^7+28\,T_1^{17}\,T_2^7-60\,T_1^{18}\,T_2^7+16\,T_1^{19}\,T_2^7+68\,T_1^{20}\,T_2^7-$$
$$62\,T_1^{21}\,T_2^7+5\,T_1^{22}\,T_2^7+10\,T_1^{23}\,T_2^7-3\,T_1^{24}\,T_2^7-4\,T_1^{25}\,T_2^7-3\,T_1^{26}\,T_2^7+4\,T_1^{27}\,T_2^7-2\,T_2^8-6\,T_1\,T_2^8+9\,T_1^2\,T_2^8+$$
$$4\,T_1^3\,T_2^8-7\,T_1^4\,T_2^8-11\,T_1^5\,T_2^8+43\,T_1^6\,T_2^8+14\,T_1^7\,T_2^8-123\,T_1^8\,T_2^8+133\,T_1^9\,T_2^8-36\,T_1^{10}\,T_2^8-89\,T_1^{11}\,T_2^8+$$
$$136\,T_1^{12}\,T_2^8-127\,T_1^{13}\,T_2^8+31\,T_1^{14}\,T_2^8-31\,T_1^{15}\,T_2^8+16\,T_1^{16}\,T_2^8-33\,T_1^{17}\,T_2^8-28\,T_1^{18}\,T_2^8+109\,T_1^{19}\,T_2^8-$$

$$115\,T_1^{20}\,T_2^8+14\,T_1^{21}\,T_2^8+51\,T_1^{22}\,T_2^8-27\,T_1^{23}\,T_2^8+T_1^{24}\,T_2^8+4\,T_1^{25}\,T_2^8+9\,T_1^{26}\,T_2^8-6\,T_1^{27}\,T_2^8-2\,T_1^{28}\,T_2^8-2\,T_2^9+12\,T_1\,T_2^9-7\,T_1^2\,T_2^9-2\,T_1^3\,T_2^9+6\,T_1^4\,T_2^9-6\,T_1^5\,T_2^9+9\,T_1^6\,T_2^9-56\,T_1^7\,T_2^9+133\,T_1^8\,T_2^9-149\,T_1^9\,T_2^9-10\,T_1^{10}\,T_2^9+224\,T_1^{11}\,T_2^9-314\,T_1^{12}\,T_2^9+67\,T_1^{13}\,T_2^9+111\,T_1^{14}\,T_2^9-124\,T_1^{15}\,T_2^9+38\,T_1^{16}\,T_2^9-49\,T_1^{17}\,T_2^9+50\,T_1^{18}\,T_2^9-38\,T_1^{19}\,T_2^9-47\,T_1^{20}\,T_2^9+95\,T_1^{21}\,T_2^9-68\,T_1^{22}\,T_2^9+8\,T_1^{23}\,T_2^9+32\,T_1^{24}\,T_2^9-19\,T_1^{25}\,T_2^9-2\,T_1^{26}\,T_2^9-7\,T_1^{27}\,T_2^9+12\,T_1^{28}\,T_2^9-2\,T_1^{29}\,T_2^9+4\,T_2^{10}-6\,T_1\,T_2^{10}-3\,T_1^2\,T_2^{10}+2\,T_1^3\,T_2^{10}-T_1^4\,T_2^{10}+14\,T_1^5\,T_2^{10}-29\,T_1^6\,T_2^{10}+14\,T_1^7\,T_2^{10}-36\,T_1^8\,T_2^{10}-10\,T_1^9\,T_2^{10}+240\,T_1^{10}\,T_2^{10}-314\,T_1^{11}\,T_2^{10}+74\,T_1^{12}\,T_2^{10}+431\,T_1^{13}\,T_2^{10}-386\,T_1^{14}\,T_2^{10}+200\,T_1^{15}\,T_2^{10}+34\,T_1^{16}\,T_2^{10}-37\,T_1^{17}\,T_2^{10}+186\,T_1^{18}\,T_2^{10}-186\,T_1^{19}\,T_2^{10}+136\,T_1^{20}\,T_2^{10}-22\,T_1^{21}\,T_2^{10}-12\,T_1^{22}\,T_2^{10}+46\,T_1^{23}\,T_2^{10}-93\,T_1^{24}\,T_2^{10}+30\,T_1^{25}\,T_2^{10}+11\,T_1^{26}\,T_2^{10}+2\,T_1^{27}\,T_2^{10}-3\,T_1^{28}\,T_2^{10}-6\,T_1^{29}\,T_2^{10}+4\,T_1^{30}\,T_2^{10}-2\,T_2^{11}-6\,T_1\,T_2^{11}+8\,T_1^2\,T_2^{11}-T_1^3\,T_2^{11}-18\,T_1^4\,T_2^{11}+11\,T_1^5\,T_2^{11}+2\,T_1^6\,T_2^{11}+55\,T_1^7\,T_2^{11}-89\,T_1^8\,T_2^{11}+224\,T_1^9\,T_2^{11}-314\,T_1^{10}\,T_2^{11}-92\,T_1^{11}\,T_2^{11}+764\,T_1^{12}\,T_2^{11}-899\,T_1^{13}\,T_2^{11}+273\,T_1^{14}\,T_2^{11}+176\,T_1^{15}\,T_2^{11}-382\,T_1^{16}\,T_2^{11}+391\,T_1^{17}\,T_2^{11}-420\,T_1^{18}\,T_2^{11}+75\,T_1^{19}\,T_2^{11}+212\,T_1^{20}\,T_2^{11}-156\,T_1^{21}\,T_2^{11}-46\,T_1^{22}\,T_2^{11}-6\,T_1^{23}\,T_2^{11}+65\,T_1^{24}\,T_2^{11}+76\,T_1^{25}\,T_2^{11}-107\,T_1^{26}\,T_2^{11}+31\,T_1^{27}\,T_2^{11}-T_1^{28}\,T_2^{11}+8\,T_1^{29}\,T_2^{11}-6\,T_1^{30}\,T_2^{11}-2\,T_1^{31}\,T_2^{11}-2\,T_2^{12}+12\,T_1\,T_2^{12}-3\,T_1^2\,T_2^{12}-T_1^3\,T_2^{12}+35\,T_1^4\,T_2^{12}-31\,T_1^5\,T_2^{12}-12\,T_1^6\,T_2^{12}-23\,T_1^7\,T_2^{12}+136\,T_1^8\,T_2^{12}-314\,T_1^9\,T_2^{12}+74\,T_1^{10}\,T_2^{12}+764\,T_1^{11}\,T_2^{12}-1304\,T_1^{12}\,T_2^{12}+293\,T_1^{13}\,T_2^{12}+744\,T_1^{14}\,T_2^{12}-996\,T_1^{15}\,T_2^{12}+616\,T_1^{16}\,T_2^{12}-380\,T_1^{17}\,T_2^{12}-68\,T_1^{18}\,T_2^{12}+589\,T_1^{19}\,T_2^{12}-596\,T_1^{20}\,T_2^{12}-72\,T_1^{21}\,T_2^{12}+294\,T_1^{22}\,T_2^{12}+38\,T_1^{23}\,T_2^{12}-64\,T_1^{24}\,T_2^{12}-123\,T_1^{25}\,T_2^{12}+60\,T_1^{26}\,T_2^{12}+93\,T_1^{27}\,T_2^{12}-69\,T_1^{28}\,T_2^{12}-T_1^{29}\,T_2^{12}-3\,T_1^{30}\,T_2^{12}+12\,T_1^{31}\,T_2^{12}-2\,T_1^{32}\,T_2^{12}+4\,T_2^{13}-6\,T_1\,T_2^{13}-7\,T_1^2\,T_2^{13}+2\,T_1^3\,T_2^{13}-18\,T_1^4\,T_2^{13}+9\,T_1^5\,T_2^{13}+28\,T_1^6\,T_2^{13}+11\,T_1^7\,T_2^{13}-127\,T_1^8\,T_2^{13}+67\,T_1^9\,T_2^{13}+431\,T_1^{10}\,T_2^{13}-899\,T_1^{11}\,T_2^{13}+293\,T_1^{12}\,T_2^{13}+1556\,T_1^{13}\,T_2^{13}-1724\,T_1^{14}\,T_2^{13}+887\,T_1^{15}\,T_2^{13}+223\,T_1^{16}\,T_2^{13}-480\,T_1^{17}\,T_2^{13}+998\,T_1^{18}\,T_2^{13}-905\,T_1^{19}\,T_2^{13}+212\,T_1^{20}\,T_2^{13}+686\,T_1^{21}\,T_2^{13}-294\,T_1^{22}\,T_2^{13}-313\,T_1^{23}\,T_2^{13}+146\,T_1^{24}\,T_2^{13}+24\,T_1^{25}\,T_2^{13}+123\,T_1^{26}\,T_2^{13}-238\,T_1^{27}\,T_2^{13}+65\,T_1^{28}\,T_2^{13}+45\,T_1^{29}\,T_2^{13}+2\,T_1^{30}\,T_2^{13}-7\,T_1^{31}\,T_2^{13}-6\,T_1^{32}\,T_2^{13}+4\,T_1^{33}\,T_2^{13}-3\,T_2^{14}-3\,T_1\,T_2^{14}+9\,T_1^2\,T_2^{14}-2\,T_1^3\,T_2^{14}-T_1^4\,T_2^{14}+T_1^5\,T_2^{14}-42\,T_1^6\,T_2^{14}+51\,T_1^7\,T_2^{14}+31\,T_1^8\,T_2^{14}+111\,T_1^9\,T_2^{14}-386\,T_1^{10}\,T_2^{14}+273\,T_1^{11}\,T_2^{14}+744\,T_1^{12}\,T_2^{14}-1724\,T_1^{13}\,T_2^{14}+705\,T_1^{14}\,T_2^{14}+482\,T_1^{15}\,T_2^{14}-1315\,T_1^{16}\,T_2^{14}+1061\,T_1^{17}\,T_2^{14}-855\,T_1^{18}\,T_2^{14}-140\,T_1^{19}\,T_2^{14}+809\,T_1^{20}\,T_2^{14}-758\,T_1^{21}\,T_2^{14}-370\,T_1^{22}\,T_2^{14}+595\,T_1^{23}\,T_2^{14}+58\,T_1^{24}\,T_2^{14}-229\,T_1^{25}\,T_2^{14}+T_1^{26}\,T_2^{14}+95\,T_1^{27}\,T_2^{14}+124\,T_1^{28}\,T_2^{14}-151\,T_1^{29}\,T_2^{14}+19\,T_1^{30}\,T_2^{14}-2\,T_1^{31}\,T_2^{14}+9\,T_1^{32}\,T_2^{14}-3\,T_1^{33}\,T_2^{14}-3\,T_1^{34}\,T_2^{14}+T_2^{15}+6\,T_1\,T_2^{15}-4\,T_1^2\,T_2^{15}+4\,T_1^3\,T_2^{15}+6\,T_1^4\,T_2^{15}+26\,T_1^6\,T_2^{15}-33\,T_1^7\,T_2^{15}-31\,T_1^8\,T_2^{15}-124\,T_1^9\,T_2^{15}+200\,T_1^{10}\,T_2^{15}+176\,T_1^{11}\,T_2^{15}-996\,T_1^{12}\,T_2^{15}+887\,T_1^{13}\,T_2^{15}+482\,T_1^{14}\,T_2^{15}-1534\,T_1^{15}\,T_2^{15}+1712\,T_1^{16}\,T_2^{15}-619\,T_1^{17}\,T_2^{15}-569\,T_1^{18}\,T_2^{15}+1420\,T_1^{19}\,T_2^{15}-914\,T_1^{20}\,T_2^{15}-229\,T_1^{21}\,T_2^{15}+992\,T_1^{22}\,T_2^{15}-257\,T_1^{23}\,T_2^{15}-598\,T_1^{24}\,T_2^{15}+440\,T_1^{25}\,T_2^{15}-15\,T_1^{26}\,T_2^{15}-50\,T_1^{27}\,T_2^{15}-167\,T_1^{28}\,T_2^{15}+92\,T_1^{29}\,T_2^{15}+74\,T_1^{30}\,T_2^{15}-49\,T_1^{31}\,T_2^{15}+4\,T_1^{32}\,T_2^{15}-4\,T_1^{33}\,T_2^{15}+6\,T_1^{34}\,T_2^{15}+T_1^{35}\,T_2^{15}-3\,T_1\,T_2^{16}-T_1^2\,T_2^{16}-3\,T_1^3\,T_2^{16}-7\,T_1^4\,T_2^{16}+8\,T_1^5\,T_2^{16}-29\,T_1^6\,T_2^{16}+41\,T_1^7\,T_2^{16}+16\,T_1^8\,T_2^{16}+38\,T_1^9\,T_2^{16}+34\,T_1^{10}\,T_2^{16}-382\,T_1^{11}\,T_2^{16}+616\,T_1^{12}\,T_2^{16}+223\,T_1^{13}\,T_2^{16}-1315\,T_1^{14}\,T_2^{16}+1712\,T_1^{15}\,T_2^{16}-720\,T_1^{16}\,T_2^{16}-1180\,T_1^{17}\,T_2^{16}+2146\,T_1^{18}\,T_2^{16}-1310\,T_1^{19}\,T_2^{16}-260\,T_1^{20}\,T_2^{16}+1108\,T_1^{21}\,T_2^{16}-545\,T_1^{22}\,T_2^{16}-555\,T_1^{23}\,T_2^{16}+792\,T_1^{24}\,T_2^{16}-94\,T_1^{25}\,T_2^{16}-350\,T_1^{26}\,T_2^{16}+256\,T_1^{27}\,T_2^{16}-24\,T_1^{28}\,T_2^{16}+109\,T_1^{29}\,T_2^{16}-189\,T_1^{30}\,T_2^{16}+60\,T_1^{31}\,T_2^{16}+17\,T_1^{32}\,T_2^{16}-3\,T_1^{33}\,T_2^{16}-T_1^{34}\,T_2^{16}-3\,T_1^{35}\,T_2^{16}+2\,T_1^3\,T_2^{17}+10\,T_1^4\,T_2^{17}-15\,T_1^5\,T_2^{17}-T_1^6\,T_2^{17}+28\,T_1^7\,T_2^{17}-33\,T_1^8\,T_2^{17}-49\,T_1^9\,T_2^{17}-37\,T_1^{10}\,T_2^{17}+391\,T_1^{11}\,T_2^{17}-380\,T_1^{12}\,T_2^{17}-480\,T_1^{13}\,T_2^{17}+1061\,T_1^{14}\,T_2^{17}-619\,T_1^{15}\,T_2^{17}-1180\,T_1^{16}\,T_2^{17}+2566\,T_1^{17}\,T_2^{17}-1730\,T_1^{18}\,T_2^{17}-591\,T_1^{19}\,T_2^{17}+1520\,T_1^{20}\,T_2^{17}-933\,T_1^{21}\,T_2^{17}-265\,T_1^{22}\,T_2^{17}+476\,T_1^{23}\,T_2^{17}+123\,T_1^{24}\,T_2^{17}-791\,T_1^{25}\,T_2^{17}+681\,T_1^{26}\,T_2^{17}-213\,T_1^{27}\,T_2^{17}-82\,T_1^{28}\,T_2^{17}-8\,T_1^{29}\,T_2^{17}+74\,T_1^{30}\,T_2^{17}+42\,T_1^{31}\,T_2^{17}-59\,T_1^{32}\,T_2^{17}+10\,T_1^{33}\,T_2^{17}+2\,T_1^{34}\,T_2^{17}+3\,T_1^2\,T_2^{18}+3\,T_1^3\,T_2^{18}-19\,T_1^4\,T_2^{18}+9\,T_1^5\,T_2^{18}+45\,T_1^6\,T_2^{18}-60\,T_1^7\,T_2^{18}-28\,T_1^8\,T_2^{18}+50\,T_1^9\,T_2^{18}+186\,T_1^{10}\,T_2^{18}-420\,T_1^{11}\,T_2^{18}-68\,T_1^{12}\,T_2^{18}+998\,T_1^{13}\,T_2^{18}-855\,T_1^{14}\,T_2^{18}-569\,T_1^{15}\,T_2^{18}+2146\,T_1^{16}\,T_2^{18}-1730\,T_1^{17}\,T_2^{18}-492\,T_1^{18}\,T_2^{18}+2218\,T_1^{19}\,T_2^{18}-1372\,T_1^{20}\,T_2^{18}-146\,T_1^{21}\,T_2^{18}+878\,T_1^{22}\,T_2^{18}-163\,T_1^{23}\,T_2^{18}-695\,T_1^{24}\,T_2^{18}+872\,T_1^{25}\,T_2^{18}-162\,T_1^{26}\,T_2^{18}-458\,T_1^{27}\,T_2^{18}+506\,T_1^{28}\,T_2^{18}-208\,T_1^{29}\,T_2^{18}+44\,T_1^{30}\,T_2^{18}-100\,T_1^{31}\,T_2^{18}+79\,T_1^{32}\,T_2^{18}-19\,T_1^{33}\,T_2^{18}-5\,T_1^{34}\,T_2^{18}+3\,T_1^{35}\,T_2^{18}+3\,T_1^{36}\,T_2^{18}-T_1^2\,T_2^{19}-10\,T_1^3\,T_2^{19}+17\,T_1^4\,T_2^{19}+16\,T_1^5\,T_2^{19}-64\,T_1^6\,T_2^{19}+16\,T_1^7\,T_2^{19}+109\,T_1^8\,T_2^{19}-38\,T_1^9\,T_2^{19}-186\,T_1^{10}\,T_2^{19}+75\,T_1^{11}\,T_2^{19}+589\,T_1^{12}\,T_2^{19}-905\,T_1^{13}\,T_2^{19}-140\,T_1^{14}\,T_2^{19}+1420\,T_1^{15}\,T_2^{19}-1310\,T_1^{16}\,T_2^{19}-591\,T_1^{17}\,T_2^{19}+2218\,T_1^{18}\,T_2^{19}-2027\,T_1^{19}\,T_2^{19}+155\,T_1^{20}\,T_2^{19}+1033\,T_1^{21}\,T_2^{19}-840\,T_1^{22}\,T_2^{19}-49\,T_1^{23}\,T_2^{19}+464\,T_1^{24}\,T_2^{19}+37\,T_1^{25}\,T_2^{19}-842\,T_1^{26}\,T_2^{19}+972\,T_1^{27}\,T_2^{19}-$$

$$412\,T_1^{28}\,T_2^{19} - 44\,T_1^{29}\,T_2^{19} + 150\,T_1^{30}\,T_2^{19} - 21\,T_1^{31}\,T_2^{19} - 10\,T_1^{32}\,T_2^{19} - 42\,T_1^{33}\,T_2^{19} + 50\,T_1^{34}\,T_2^{19} - 13\,T_1^{35}\,T_2^{19} - 10\,T_1^{36}\,T_2^{19} - T_1^{37}\,T_2^{19} + 6\,T_1^{3}\,T_2^{20} + 3\,T_1^{4}\,T_2^{20} - 32\,T_1^{5}\,T_2^{20} + 24\,T_1^{6}\,T_2^{20} + 68\,T_1^{7}\,T_2^{20} - 115\,T_1^{8}\,T_2^{20} - 47\,T_1^{9}\,T_2^{20} + 136\,T_1^{10}\,T_2^{20} + 212\,T_1^{11}\,T_2^{20} - 596\,T_1^{12}\,T_2^{20} + 212\,T_1^{13}\,T_2^{20} + 809\,T_1^{14}\,T_2^{20} - 914\,T_1^{15}\,T_2^{20} - 260\,T_1^{16}\,T_2^{20} + 1520\,T_1^{17}\,T_2^{20} - 1372\,T_1^{18}\,T_2^{20} + 155\,T_1^{19}\,T_2^{20} + 1056\,T_1^{20}\,T_2^{20} - 1291\,T_1^{21}\,T_2^{20} + 674\,T_1^{22}\,T_2^{20} - 128\,T_1^{23}\,T_2^{20} - 56\,T_1^{24}\,T_2^{20} - 374\,T_1^{25}\,T_2^{20} + 603\,T_1^{26}\,T_2^{20} - 180\,T_1^{27}\,T_2^{20} - 504\,T_1^{28}\,T_2^{20} + 592\,T_1^{29}\,T_2^{20} - 340\,T_1^{30}\,T_2^{20} + 71\,T_1^{31}\,T_2^{20} - 39\,T_1^{32}\,T_2^{20} + 100\,T_1^{33}\,T_2^{20} - 60\,T_1^{34}\,T_2^{20} - 8\,T_1^{35}\,T_2^{20} + 19\,T_1^{36}\,T_2^{20} + 6\,T_1^{37}\,T_2^{20} - T_1^{3}\,T_2^{21} - 10\,T_1^{4}\,T_2^{21} + 15\,T_1^{5}\,T_2^{21} + 18\,T_1^{6}\,T_2^{21} - 62\,T_1^{7}\,T_2^{21} + 14\,T_1^{8}\,T_2^{21} + 95\,T_1^{9}\,T_2^{21} - 22\,T_1^{10}\,T_2^{21} - 156\,T_1^{11}\,T_2^{21} - 72\,T_1^{12}\,T_2^{21} + 686\,T_1^{13}\,T_2^{21} - 758\,T_1^{14}\,T_2^{21} - 229\,T_1^{15}\,T_2^{21} + 1108\,T_1^{16}\,T_2^{21} - 933\,T_1^{17}\,T_2^{21} - 146\,T_1^{18}\,T_2^{21} + 1033\,T_1^{19}\,T_2^{21} - 1291\,T_1^{20}\,T_2^{21} + 891\,T_1^{21}\,T_2^{21} - 152\,T_1^{22}\,T_2^{21} - 395\,T_1^{23}\,T_2^{21} + 328\,T_1^{24}\,T_2^{21} + 152\,T_1^{25}\,T_2^{21} - 52\,T_1^{26}\,T_2^{21} - 695\,T_1^{27}\,T_2^{21} + 1069\,T_1^{28}\,T_2^{21} - 559\,T_1^{29}\,T_2^{21} - 14\,T_1^{30}\,T_2^{21} + 166\,T_1^{31}\,T_2^{21} - 35\,T_1^{32}\,T_2^{21} - 12\,T_1^{33}\,T_2^{21} - 40\,T_1^{34}\,T_2^{21} + 52\,T_1^{35}\,T_2^{21} - 15\,T_1^{36}\,T_2^{21} - 10\,T_1^{37}\,T_2^{21} - T_1^{38}\,T_2^{21} + 3\,T_1^{4}\,T_2^{22} + 3\,T_1^{5}\,T_2^{22} - 17\,T_1^{6}\,T_2^{22} + 5\,T_1^{7}\,T_2^{22} + 51\,T_1^{8}\,T_2^{22} - 68\,T_1^{9}\,T_2^{22} - 12\,T_1^{10}\,T_2^{22} - 46\,T_1^{11}\,T_2^{22} + 294\,T_1^{12}\,T_2^{22} - 294\,T_1^{13}\,T_2^{22} - 370\,T_1^{14}\,T_2^{22} + 992\,T_1^{15}\,T_2^{22} - 545\,T_1^{16}\,T_2^{22} - 265\,T_1^{17}\,T_2^{22} + 878\,T_1^{18}\,T_2^{22} - 840\,T_1^{19}\,T_2^{22} + 674\,T_1^{20}\,T_2^{22} - 152\,T_1^{21}\,T_2^{22} - 206\,T_1^{22}\,T_2^{22} + 744\,T_1^{23}\,T_2^{22} - 390\,T_1^{24}\,T_2^{22} + 141\,T_1^{25}\,T_2^{22} - 385\,T_1^{26}\,T_2^{22} + 866\,T_1^{27}\,T_2^{22} - 464\,T_1^{28}\,T_2^{22} - 332\,T_1^{29}\,T_2^{22} + 614\,T_1^{30}\,T_2^{22} - 304\,T_1^{31}\,T_2^{22} + 60\,T_1^{32}\,T_2^{22} - 108\,T_1^{33}\,T_2^{22} + 85\,T_1^{34}\,T_2^{22} - 23\,T_1^{35}\,T_2^{22} - 3\,T_1^{36}\,T_2^{22} + 3\,T_1^{37}\,T_2^{22} + 3\,T_1^{38}\,T_2^{22} + 2\,T_1^{6}\,T_2^{23} + 10\,T_1^{7}\,T_2^{23} - 27\,T_1^{8}\,T_2^{23} + 8\,T_1^{9}\,T_2^{23} + 46\,T_1^{10}\,T_2^{23} - 6\,T_1^{11}\,T_2^{23} + 38\,T_1^{12}\,T_2^{23} - 313\,T_1^{13}\,T_2^{23} + 595\,T_1^{14}\,T_2^{23} - 257\,T_1^{15}\,T_2^{23} - 555\,T_1^{16}\,T_2^{23} + 476\,T_1^{17}\,T_2^{23} - 163\,T_1^{18}\,T_2^{23} - 49\,T_1^{19}\,T_2^{23} - 128\,T_1^{20}\,T_2^{23} - 395\,T_1^{21}\,T_2^{23} + 744\,T_1^{22}\,T_2^{23} - 1174\,T_1^{23}\,T_2^{23} + 198\,T_1^{24}\,T_2^{23} + 191\,T_1^{25}\,T_2^{23} - 109\,T_1^{26}\,T_2^{23} + 48\,T_1^{27}\,T_2^{23} - 668\,T_1^{28}\,T_2^{23} + 885\,T_1^{29}\,T_2^{23} - 489\,T_1^{30}\,T_2^{23} + 5\,T_1^{31}\,T_2^{23} + 19\,T_1^{32}\,T_2^{23} + 92\,T_1^{33}\,T_2^{23} + 51\,T_1^{34}\,T_2^{23} - 71\,T_1^{35}\,T_2^{23} + 10\,T_1^{36}\,T_2^{23} + 2\,T_1^{37}\,T_2^{23} - 3\,T_1^{5}\,T_2^{24} - T_1^{6}\,T_2^{24} - 3\,T_1^{7}\,T_2^{24} + T_1^{8}\,T_2^{24} + 32\,T_1^{9}\,T_2^{24} - 93\,T_1^{10}\,T_2^{24} + 65\,T_1^{11}\,T_2^{24} - 64\,T_1^{12}\,T_2^{24} + 146\,T_1^{13}\,T_2^{24} + 58\,T_1^{14}\,T_2^{24} - 598\,T_1^{15}\,T_2^{24} + 792\,T_1^{16}\,T_2^{24} + 123\,T_1^{17}\,T_2^{24} - 695\,T_1^{18}\,T_2^{24} + 464\,T_1^{19}\,T_2^{24} - 56\,T_1^{20}\,T_2^{24} + 328\,T_1^{21}\,T_2^{24} - 390\,T_1^{22}\,T_2^{24} + 198\,T_1^{23}\,T_2^{24} + 404\,T_1^{24}\,T_2^{24} - 140\,T_1^{25}\,T_2^{24} + 75\,T_1^{26}\,T_2^{24} - 655\,T_1^{27}\,T_2^{24} + 968\,T_1^{28}\,T_2^{24} - 310\,T_1^{29}\,T_2^{24} - 326\,T_1^{30}\,T_2^{24} + 364\,T_1^{31}\,T_2^{24} - 104\,T_1^{32}\,T_2^{24} + 133\,T_1^{33}\,T_2^{24} - 253\,T_1^{34}\,T_2^{24} + 84\,T_1^{35}\,T_2^{24} + 25\,T_1^{36}\,T_2^{24} - 3\,T_1^{37}\,T_2^{24} - T_1^{38}\,T_2^{24} - 3\,T_1^{39}\,T_2^{24} + T_1^{5}\,T_2^{25} + 6\,T_1^{6}\,T_2^{25} - 4\,T_1^{7}\,T_2^{25} + 4\,T_1^{8}\,T_2^{25} - 19\,T_1^{9}\,T_2^{25} + 30\,T_1^{10}\,T_2^{25} + 76\,T_1^{11}\,T_2^{25} - 123\,T_1^{12}\,T_2^{25} + 24\,T_1^{13}\,T_2^{25} - 229\,T_1^{14}\,T_2^{25} + 440\,T_1^{15}\,T_2^{25} - 94\,T_1^{16}\,T_2^{25} - 791\,T_1^{17}\,T_2^{25} + 872\,T_1^{18}\,T_2^{25} + 37\,T_1^{19}\,T_2^{25} - 374\,T_1^{20}\,T_2^{25} + 152\,T_1^{21}\,T_2^{25} + 141\,T_1^{22}\,T_2^{25} + 191\,T_1^{23}\,T_2^{25} - 140\,T_1^{24}\,T_2^{25} + 246\,T_1^{25}\,T_2^{25} - 674\,T_1^{26}\,T_2^{25} + 977\,T_1^{27}\,T_2^{25} - 52\,T_1^{28}\,T_2^{25} - 868\,T_1^{29}\,T_2^{25} + 680\,T_1^{30}\,T_2^{25} - 120\,T_1^{31}\,T_2^{25} + 5\,T_1^{32}\,T_2^{25} - 257\,T_1^{33}\,T_2^{25} + 142\,T_1^{34}\,T_2^{25} + 104\,T_1^{35}\,T_2^{25} - 74\,T_1^{36}\,T_2^{25} + 4\,T_1^{37}\,T_2^{25} - 4\,T_1^{38}\,T_2^{25} + 6\,T_1^{39}\,T_2^{25} + T_1^{40}\,T_2^{25} - 3\,T_1^{6}\,T_2^{26} - 3\,T_1^{7}\,T_2^{26} + 9\,T_1^{8}\,T_2^{26} - 2\,T_1^{9}\,T_2^{26} + 11\,T_1^{10}\,T_2^{26} - 107\,T_1^{11}\,T_2^{26} + 60\,T_1^{12}\,T_2^{26} + 123\,T_1^{13}\,T_2^{26} + T_1^{14}\,T_2^{26} - 15\,T_1^{15}\,T_2^{26} - 350\,T_1^{16}\,T_2^{26} + 681\,T_1^{17}\,T_2^{26} - 162\,T_1^{18}\,T_2^{26} - 842\,T_1^{19}\,T_2^{26} + 603\,T_1^{20}\,T_2^{26} - 52\,T_1^{21}\,T_2^{26} - 385\,T_1^{22}\,T_2^{26} - 109\,T_1^{23}\,T_2^{26} + 75\,T_1^{24}\,T_2^{26} - 674\,T_1^{25}\,T_2^{26} + 707\,T_1^{26}\,T_2^{26} + 124\,T_1^{27}\,T_2^{26} - 1276\,T_1^{28}\,T_2^{26} + 1003\,T_1^{29}\,T_2^{26} + 94\,T_1^{30}\,T_2^{26} - 355\,T_1^{31}\,T_2^{26} - 29\,T_1^{32}\,T_2^{26} + 167\,T_1^{33}\,T_2^{26} + 226\,T_1^{34}\,T_2^{26} - 259\,T_1^{35}\,T_2^{26} + 31\,T_1^{36}\,T_2^{26} - 2\,T_1^{37}\,T_2^{26} + 9\,T_1^{38}\,T_2^{26} - 3\,T_1^{39}\,T_2^{26} - 3\,T_1^{40}\,T_2^{26} + 4\,T_1^{7}\,T_2^{27} - 6\,T_1^{8}\,T_2^{27} - 7\,T_1^{9}\,T_2^{27} + 2\,T_1^{10}\,T_2^{27} + 31\,T_1^{11}\,T_2^{27} + 93\,T_1^{12}\,T_2^{27} - 238\,T_1^{13}\,T_2^{27} + 95\,T_1^{14}\,T_2^{27} - 50\,T_1^{15}\,T_2^{27} + 256\,T_1^{16}\,T_2^{27} - 213\,T_1^{17}\,T_2^{27} - 458\,T_1^{18}\,T_2^{27} + 972\,T_1^{19}\,T_2^{27} - 180\,T_1^{20}\,T_2^{27} - 695\,T_1^{21}\,T_2^{27} + 866\,T_1^{22}\,T_2^{27} + 48\,T_1^{23}\,T_2^{27} - 655\,T_1^{24}\,T_2^{27} + 977\,T_1^{25}\,T_2^{27} + 124\,T_1^{26}\,T_2^{27} - 1524\,T_1^{27}\,T_2^{27} + 1365\,T_1^{28}\,T_2^{27} + 147\,T_1^{29}\,T_2^{27} - 957\,T_1^{30}\,T_2^{27} + 335\,T_1^{31}\,T_2^{27} + 101\,T_1^{32}\,T_2^{27} + 207\,T_1^{33}\,T_2^{27} - 504\,T_1^{34}\,T_2^{27} + 149\,T_1^{35}\,T_2^{27} + 94\,T_1^{36}\,T_2^{27} + 2\,T_1^{37}\,T_2^{27} - 7\,T_1^{38}\,T_2^{27} - 6\,T_1^{39}\,T_2^{27} + 4\,T_1^{40}\,T_2^{27} - 2\,T_1^{8}\,T_2^{28} + 12\,T_1^{9}\,T_2^{28} - 3\,T_1^{10}\,T_2^{28} - T_1^{11}\,T_2^{28} - 69\,T_1^{12}\,T_2^{28} + 65\,T_1^{13}\,T_2^{28} + 124\,T_1^{14}\,T_2^{28} - 167\,T_1^{15}\,T_2^{28} - 24\,T_1^{16}\,T_2^{28} - 82\,T_1^{17}\,T_2^{28} + 506\,T_1^{18}\,T_2^{28} - 412\,T_1^{19}\,T_2^{28} - 504\,T_1^{20}\,T_2^{28} + 1069\,T_1^{21}\,T_2^{28} - 464\,T_1^{22}\,T_2^{28} - 668\,T_1^{23}\,T_2^{28} + 968\,T_1^{24}\,T_2^{28} - 52\,T_1^{25}\,T_2^{28} - 1276\,T_1^{26}\,T_2^{28} + 1365\,T_1^{27}\,T_2^{28} + 204\,T_1^{28}\,T_2^{28} - 1248\,T_1^{29}\,T_2^{28} + 726\,T_1^{30}\,T_2^{28} + 270\,T_1^{31}\,T_2^{28} - 224\,T_1^{32}\,T_2^{28} - 267\,T_1^{33}\,T_2^{28} + 196\,T_1^{34}\,T_2^{28} + 189\,T_1^{35}\,T_2^{28} - 173\,T_1^{36}\,T_2^{28} - T_1^{37}\,T_2^{28} - 3\,T_1^{38}\,T_2^{28} + 12\,T_1^{39}\,T_2^{28} - 2\,T_1^{40}\,T_2^{28} - 2\,T_1^{9}\,T_2^{29} - 6\,T_1^{10}\,T_2^{29} + 8\,T_1^{11}\,T_2^{29} - T_1^{12}\,T_2^{29} + 45\,T_1^{13}\,T_2^{29} - 151\,T_1^{14}\,T_2^{29} + 92\,T_1^{15}\,T_2^{29} + 109\,T_1^{16}\,T_2^{29} - 8\,T_1^{17}\,T_2^{29} - 208\,T_1^{18}\,T_2^{29} - 44\,T_1^{19}\,T_2^{29} + 592\,T_1^{20}\,T_2^{29} - 559\,T_1^{21}\,T_2^{29} - 332\,T_1^{22}\,T_2^{29} + 885\,T_1^{23}\,T_2^{29} - 310\,T_1^{24}\,T_2^{29} - 868\,T_1^{25}\,T_2^{29} + 1003\,T_1^{26}\,T_2^{29} + 147\,T_1^{27}\,T_2^{29} - 1248\,T_1^{28}\,T_2^{29} + 896\,T_1^{29}\,T_2^{29} + 114\,T_1^{30}\,T_2^{29} - 478\,T_1^{31}\,T_2^{29} +$$

$$75\,T_1^{32}\,T_2^{29}+119\,T_1^{33}\,T_2^{29}+166\,T_1^{34}\,T_2^{29}-269\,T_1^{35}\,T_2^{29}+94\,T_1^{36}\,T_2^{29}-T_1^{37}\,T_2^{29}+8\,T_1^{38}\,T_2^{29}-6\,T_1^{39}\,T_2^{29}-$$
$$2\,T_1^{40}\,T_2^{29}+4\,T_1^{10}\,T_2^{30}-6\,T_1^{11}\,T_2^{30}-3\,T_1^{12}\,T_2^{30}+2\,T_1^{13}\,T_2^{30}+19\,T_1^{14}\,T_2^{30}+74\,T_1^{15}\,T_2^{30}-189\,T_1^{16}\,T_2^{30}+$$
$$74\,T_1^{17}\,T_2^{30}+44\,T_1^{18}\,T_2^{30}+150\,T_1^{19}\,T_2^{30}-340\,T_1^{20}\,T_2^{30}-14\,T_1^{21}\,T_2^{30}+614\,T_1^{22}\,T_2^{30}-489\,T_1^{23}\,T_2^{30}-$$
$$326\,T_1^{24}\,T_2^{30}+680\,T_1^{25}\,T_2^{30}+94\,T_1^{26}\,T_2^{30}-957\,T_1^{27}\,T_2^{30}+726\,T_1^{28}\,T_2^{30}+114\,T_1^{29}\,T_2^{30}-444\,T_1^{30}\,T_2^{30}+$$
$$138\,T_1^{31}\,T_2^{30}+68\,T_1^{32}\,T_2^{30}+106\,T_1^{33}\,T_2^{30}-253\,T_1^{34}\,T_2^{30}+90\,T_1^{35}\,T_2^{30}+31\,T_1^{36}\,T_2^{30}+2\,T_1^{37}\,T_2^{30}-3\,T_1^{38}\,T_2^{30}-$$
$$6\,T_1^{39}\,T_2^{30}+4\,T_1^{40}\,T_2^{30}-2\,T_1^{11}\,T_2^{31}+12\,T_1^{12}\,T_2^{31}-7\,T_1^{13}\,T_2^{31}-2\,T_1^{14}\,T_2^{31}-49\,T_1^{15}\,T_2^{31}+60\,T_1^{16}\,T_2^{31}+$$
$$42\,T_1^{17}\,T_2^{31}-100\,T_1^{18}\,T_2^{31}-21\,T_1^{19}\,T_2^{31}+71\,T_1^{20}\,T_2^{31}+166\,T_1^{21}\,T_2^{31}-304\,T_1^{22}\,T_2^{31}+5\,T_1^{23}\,T_2^{31}+364\,T_1^{24}\,T_2^{31}-$$
$$120\,T_1^{25}\,T_2^{31}-355\,T_1^{26}\,T_2^{31}+335\,T_1^{27}\,T_2^{31}+270\,T_1^{28}\,T_2^{31}-478\,T_1^{29}\,T_2^{31}+138\,T_1^{30}\,T_2^{31}+173\,T_1^{31}\,T_2^{31}-$$
$$59\,T_1^{32}\,T_2^{31}-112\,T_1^{33}\,T_2^{31}+41\,T_1^{34}\,T_2^{31}+98\,T_1^{35}\,T_2^{31}-74\,T_1^{36}\,T_2^{31}-2\,T_1^{37}\,T_2^{31}-7\,T_1^{38}\,T_2^{31}+12\,T_1^{39}\,T_2^{31}-$$
$$2\,T_1^{40}\,T_2^{31}-2\,T_1^{12}\,T_2^{32}-6\,T_1^{13}\,T_2^{32}+9\,T_1^{14}\,T_2^{32}+4\,T_1^{15}\,T_2^{32}+17\,T_1^{16}\,T_2^{32}-59\,T_1^{17}\,T_2^{32}+79\,T_1^{18}\,T_2^{32}-$$
$$10\,T_1^{19}\,T_2^{32}-39\,T_1^{20}\,T_2^{32}-35\,T_1^{21}\,T_2^{32}+60\,T_1^{22}\,T_2^{32}+19\,T_1^{23}\,T_2^{32}-104\,T_1^{24}\,T_2^{32}+5\,T_1^{25}\,T_2^{32}-29\,T_1^{26}\,T_2^{32}+$$
$$101\,T_1^{27}\,T_2^{32}-224\,T_1^{28}\,T_2^{32}+75\,T_1^{29}\,T_2^{32}+68\,T_1^{30}\,T_2^{32}-59\,T_1^{31}\,T_2^{32}-31\,T_1^{32}\,T_2^{32}-10\,T_1^{33}\,T_2^{32}+87\,T_1^{34}\,T_2^{32}-$$
$$75\,T_1^{35}\,T_2^{32}+25\,T_1^{36}\,T_2^{32}+4\,T_1^{37}\,T_2^{32}+9\,T_1^{38}\,T_2^{32}-6\,T_1^{39}\,T_2^{32}-2\,T_1^{40}\,T_2^{32}+4\,T_1^{13}\,T_2^{33}-3\,T_1^{14}\,T_2^{33}-$$
$$4\,T_1^{15}\,T_2^{33}-3\,T_1^{16}\,T_2^{33}+10\,T_1^{17}\,T_2^{33}-19\,T_1^{18}\,T_2^{33}-42\,T_1^{19}\,T_2^{33}+100\,T_1^{20}\,T_2^{33}-12\,T_1^{21}\,T_2^{33}-108\,T_1^{22}\,T_2^{33}+$$
$$92\,T_1^{23}\,T_2^{33}+133\,T_1^{24}\,T_2^{33}-257\,T_1^{25}\,T_2^{33}+167\,T_1^{26}\,T_2^{33}+207\,T_1^{27}\,T_2^{33}-267\,T_1^{28}\,T_2^{33}+119\,T_1^{29}\,T_2^{33}+$$
$$106\,T_1^{30}\,T_2^{33}-112\,T_1^{31}\,T_2^{33}-10\,T_1^{32}\,T_2^{33}+94\,T_1^{33}\,T_2^{33}-36\,T_1^{34}\,T_2^{33}-21\,T_1^{35}\,T_2^{33}+10\,T_1^{36}\,T_2^{33}-3\,T_1^{37}\,T_2^{33}-$$
$$4\,T_1^{38}\,T_2^{33}-3\,T_1^{39}\,T_2^{33}+4\,T_1^{40}\,T_2^{33}-3\,T_1^{14}\,T_2^{34}+6\,T_1^{15}\,T_2^{34}-T_1^{16}\,T_2^{34}+2\,T_1^{17}\,T_2^{34}-5\,T_1^{18}\,T_2^{34}+50\,T_1^{19}\,T_2^{34}-$$
$$60\,T_1^{20}\,T_2^{34}-40\,T_1^{21}\,T_2^{34}+85\,T_1^{22}\,T_2^{34}+51\,T_1^{23}\,T_2^{34}-253\,T_1^{24}\,T_2^{34}+142\,T_1^{25}\,T_2^{34}+226\,T_1^{26}\,T_2^{34}-$$
$$504\,T_1^{27}\,T_2^{34}+196\,T_1^{28}\,T_2^{34}+166\,T_1^{29}\,T_2^{34}-253\,T_1^{30}\,T_2^{34}+41\,T_1^{31}\,T_2^{34}+87\,T_1^{32}\,T_2^{34}-36\,T_1^{33}\,T_2^{34}-$$
$$60\,T_1^{34}\,T_2^{34}+46\,T_1^{35}\,T_2^{34}-3\,T_1^{36}\,T_2^{34}+2\,T_1^{37}\,T_2^{34}-T_1^{38}\,T_2^{34}+6\,T_1^{39}\,T_2^{34}-3\,T_1^{40}\,T_2^{34}+T_1^{15}\,T_2^{35}-3\,T_1^{16}\,T_2^{35}+$$
$$3\,T_1^{18}\,T_2^{35}-13\,T_1^{19}\,T_2^{35}-8\,T_1^{20}\,T_2^{35}+52\,T_1^{21}\,T_2^{35}-23\,T_1^{22}\,T_2^{35}-71\,T_1^{23}\,T_2^{35}+84\,T_1^{24}\,T_2^{35}+104\,T_1^{25}\,T_2^{35}-$$
$$259\,T_1^{26}\,T_2^{35}+149\,T_1^{27}\,T_2^{35}+189\,T_1^{28}\,T_2^{35}-269\,T_1^{29}\,T_2^{35}+90\,T_1^{30}\,T_2^{35}+98\,T_1^{31}\,T_2^{35}-75\,T_1^{32}\,T_2^{35}-$$
$$21\,T_1^{33}\,T_2^{35}+46\,T_1^{34}\,T_2^{35}-2\,T_1^{35}\,T_2^{35}-15\,T_1^{36}\,T_2^{35}+3\,T_1^{37}\,T_2^{35}-3\,T_1^{39}\,T_2^{35}+T_1^{40}\,T_2^{35}+3\,T_1^{18}\,T_2^{36}-$$
$$10\,T_1^{19}\,T_2^{36}+19\,T_1^{20}\,T_2^{36}-15\,T_1^{21}\,T_2^{36}-3\,T_1^{22}\,T_2^{36}+10\,T_1^{23}\,T_2^{36}+25\,T_1^{24}\,T_2^{36}-74\,T_1^{25}\,T_2^{36}+31\,T_1^{26}\,T_2^{36}+$$
$$94\,T_1^{27}\,T_2^{36}-173\,T_1^{28}\,T_2^{36}+94\,T_1^{29}\,T_2^{36}+31\,T_1^{30}\,T_2^{36}-74\,T_1^{31}\,T_2^{36}+25\,T_1^{32}\,T_2^{36}+10\,T_1^{33}\,T_2^{36}-3\,T_1^{34}\,T_2^{36}-$$
$$15\,T_1^{35}\,T_2^{36}+19\,T_1^{36}\,T_2^{36}-10\,T_1^{37}\,T_2^{36}+3\,T_1^{38}\,T_2^{36}-T_1^{19}\,T_2^{37}+6\,T_1^{20}\,T_2^{37}-10\,T_1^{21}\,T_2^{37}+3\,T_1^{22}\,T_2^{37}+$$
$$2\,T_1^{23}\,T_2^{37}-3\,T_1^{24}\,T_2^{37}+4\,T_1^{25}\,T_2^{37}-2\,T_1^{26}\,T_2^{37}+2\,T_1^{27}\,T_2^{37}-T_1^{28}\,T_2^{37}-T_1^{29}\,T_2^{37}+2\,T_1^{30}\,T_2^{37}-2\,T_1^{31}\,T_2^{37}+$$
$$4\,T_1^{32}\,T_2^{37}-3\,T_1^{33}\,T_2^{37}+2\,T_1^{34}\,T_2^{37}+3\,T_1^{35}\,T_2^{37}-10\,T_1^{36}\,T_2^{37}+6\,T_1^{37}\,T_2^{37}-T_1^{38}\,T_2^{37}-T_1^{21}\,T_2^{38}+3\,T_1^{22}\,T_2^{38}-$$
$$T_1^{24}\,T_2^{38}-4\,T_1^{25}\,T_2^{38}+9\,T_1^{26}\,T_2^{38}-7\,T_1^{27}\,T_2^{38}-3\,T_1^{28}\,T_2^{38}+8\,T_1^{29}\,T_2^{38}-3\,T_1^{30}\,T_2^{38}-7\,T_1^{31}\,T_2^{38}+9\,T_1^{32}\,T_2^{38}-$$
$$4\,T_1^{33}\,T_2^{38}-T_1^{34}\,T_2^{38}+3\,T_1^{36}\,T_2^{38}-T_1^{37}\,T_2^{38}-3\,T_1^{24}\,T_2^{39}+6\,T_1^{25}\,T_2^{39}-3\,T_1^{26}\,T_2^{39}-6\,T_1^{27}\,T_2^{39}+12\,T_1^{28}\,T_2^{39}-$$
$$6\,T_1^{29}\,T_2^{39}-6\,T_1^{30}\,T_2^{39}+12\,T_1^{31}\,T_2^{39}-6\,T_1^{32}\,T_2^{39}-3\,T_1^{33}\,T_2^{39}+6\,T_1^{34}\,T_2^{39}-3\,T_1^{35}\,T_2^{39}+T_1^{25}\,T_2^{40}-3\,T_1^{26}\,T_2^{40}+$$
$$4\,T_1^{27}\,T_2^{40}-2\,T_1^{28}\,T_2^{40}-2\,T_1^{29}\,T_2^{40}+4\,T_1^{30}\,T_2^{40}-2\,T_1^{31}\,T_2^{40}-2\,T_1^{32}\,T_2^{40}+4\,T_1^{33}\,T_2^{40}-3\,T_1^{34}\,T_2^{40}+T_1^{35}\,T_2^{40}\Big)\Big\}\Big\}$$

In[•]:=
```
AbsoluteTiming[Θ_{T1,T2}[GST48];]
```

Out[•]=

{47.8885, Null}

In[•]:=
```
AbsoluteTiming[θ22/7,34/21[GST48]]
```

Out[•]=

$$\left\{0.414346,\ \left\{-\frac{1\,422\,357\,287\,561\,349\,859\,889}{10\,190\,414\,377\,180\,576},\ -\frac{486\,885\,265\,100\,293\,177\,259\,569}{15\,915\,006\,754\,796\,041\,036\,704},\right.\right.$$
$$-\frac{6\,215\,902\,990\,719\,340\,337\,664\,427\,997\,383\,765\,280\,900\,656\,009}{162\,180\,513\,646\,999\,558\,542\,864\,476\,199\,651\,861\,504},$$
$$27\,865\,447\,243\,020\,145\,692\,710\,782\,578\,866\,822\,243\,946\,130\,255\,307\,494\,889\,384\,165\,533\,451\,655\,811\,667\,$$
$$624\,690\,321\,276\,677\,875\,188\,699\,659\ /$$
$$14\,859\,552\,982\,340\,980\,842\,664\,641\,721\,957\,485\,175\,777\,711\,994\,175\,132\,487\,335\,203\,798\,845\,978\,446\,$$
$$\left.\left.417\,306\,385\,907\,712\right\}\right\}$$

In[•]:=
```
DuplicateFreeQ[θ /@ AllKnots[{3, 10}]]
```

Out[•]=

True

In[•]:=
```
DuplicateFreeQ[θ /@ AllKnots[{3, 12}]]
```

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

Out[•]=

False

In[•]:=
```
tab11 = Table[K → θ@K, {K, AllKnots[{3, 11}] }]
```

Out[•]=

$$\left\{\text{Knot}[3,1]\to\left\{\frac{1-T+T^2}{T},\ -\frac{-1+T_1-T_1^2+T_2-T_1^2T_2+2T_1^3T_2-T_2^2-T_1T_2^2+T_1^2T_2^2-2T_1^3T_2^2+2T_1T_2^3-2T_1^2T_2^3+2T_1^3T_2^3}{T_1^2T_2}\right\},\right.$$
$$\text{Knot}[4,1]\to\left\{-\frac{1-3T+T^2}{T},\ \frac{\left(1-3T_1+T_1^2\right)\left(-1+T_1T_2\right)\left(1+T_1T_2\right)\left(1-3T_2+T_2^2\right)}{T_1^3T_2^2}\right\},\ \text{Knot}[5,1]\to\left\{\frac{1-T+T^2-T^3+T^4}{T^2},\ -\frac{-1+\langle\cdots 67\cdots\rangle+4T_1^7T_2^7}{T_1^4T_2^3}\right\},$$
$$\langle\cdots 795\cdots\rangle,\ \text{Knot}[11,\text{NonAlternating},183]\to\left\{\frac{\langle\cdots 1\cdots\rangle}{T^3},\ \langle\cdots 1\cdots\rangle\right\},$$
$$\text{Knot}[11,\text{NonAlternating},184]\to\left\{\frac{\left(1-T+T^2\right)\left(2-7T+11T^2-7T^3+2T^4\right)}{T^3},\ -\frac{-33+\langle\cdots 183\cdots\rangle+15T_1^{12}T_2^{12}}{T_1^7T_2^6}\right\},$$
$$\text{Knot}[11,\text{NonAlternating},185]\to\left\{-\frac{\left(1-3T+T^2\right)\left(1-T+T^2\right)\left(2-3T+2T^2\right)}{T^3},\right.$$
$$\left.\left.\frac{-41+225T_1-490T_1^2+633T_1^3-490T_1^4+225T_1^5-41T_1^6+225T_2-1054T_1T_2+\langle\cdots 156\cdots\rangle+222T_1^{11}T_2^{11}-39T_1^{12}T_2^{11}+7T_1^6T_2^{12}-39T_1^7T_2^{12}+86T_1^8T_2^{12}-111T_1^9T_2^{12}+86T_1^{10}T_2^{12}-39T_1^{11}T_2^{12}+7T_1^{12}T_2^{12}}{T_1^7T_2^6}\right\}\right\}$$

Full expression not available (original memory size: 35.9 MB)

In[•]:=
```
Gather[tab11, Last[#1] === Last[#2] &]
```

Out[•]=

$$\left\{\left\{\text{Knot}[3,1]\to\left\{\frac{1-T+T^2}{T},\ -\frac{-1+T_1-T_1^2+T_2-T_1^2T_2+2T_1^3T_2-T_2^2-T_1T_2^2+T_1^2T_2^2-2T_1^3T_2^2+2T_1T_2^3-2T_1^2T_2^3+2T_1^3T_2^3}{T_1^2T_2}\right\}\right\},\right.$$

$$\left\{\text{Knot}[4,1]\to\left\{-\frac{1-3T+T^2}{T},\ \frac{\left(1-3T_1+T_1^2\right)\left(-1+T_1T_2\right)\left(1+T_1T_2\right)\left(1-3T_2+T_2^2\right)}{T_1^3T_2^2}\right\}\right\},$$

$$\left\{\text{Knot}[5,1]\to\left\{\frac{1-T+T^2-T^3+T^4}{T^2},\ -\frac{-1+\langle\!\langle 67\rangle\!\rangle+\langle\!\langle 1\rangle\!\rangle}{T_1^4T_2^3}\right\}\right\},\ \langle\!\langle 792\rangle\!\rangle,\ \left\{\text{Knot}[11,\text{NonAlternating},183]\to\left\{\langle\!\langle 1\rangle\!\rangle\right\}\right\},$$

$$\left\{\text{Knot}[11,\text{NonAlternating},184]\to\left\{\frac{\left(1-T+T^2\right)\left(2-7T+11T^2-7T^3+2T^4\right)}{T^3},\ -\frac{-33+\langle\!\langle 183\rangle\!\rangle+15T_1^{12}T_2^{12}}{T_1^7T_2^6}\right\}\right\},$$

$$\left\{\text{Knot}[11,\text{NonAlternating},185]\to\left\{-\frac{\left(1-3T+T^2\right)\left(1-T+T^2\right)\left(2-3T+2T^2\right)}{T^3},\right.\right.$$

$$\left.\left.\frac{-41+225T_1-490T_1^2+633T_1^3-490T_1^4+225T_1^5-41T_1^6+225T_2-1054T_1T_2+\langle\!\langle 156\rangle\!\rangle+222T_1^{11}T_2^{11}-39T_1^{12}T_2^{11}+7T_1^6T_2^{12}-39T_1^7T_2^{12}+86T_1^8T_2^{12}-111T_1^9T_2^{12}+86T_1^{10}T_2^{12}-39T_1^{11}T_2^{12}+7T_1^{12}T_2^{12}}{T_1^7T_2^6}\right\}\right\}\right\}$$

Full expression not available (original memory size: 36 MB)

In[•]:=
```
Select[Gather[tab11, Last[#1] === Last[#2] &], Length[#] > 1 &]
```

Out[•]=

$$\left\{\left\{\text{Knot}[11,\text{Alternating},44]\to\right.\right.$$

$$\left\{\frac{\left(1-T+T^2\right)^2\left(1-3T+5T^2-3T^3+T^4\right)}{T^4},\ -\frac{1}{T_1^9T_2^8}\left(1-T_1+T_1^2\right)\left(1-T_2+T_2^2\right)\left(1-T_1T_2+T_1^2T_2^2\right)\right.$$

$$\left(-4+16T_1-36T_1^2+44T_1^3-36T_1^4+16T_1^5-4T_1^6+16T_2-53T_1T_2+100T_1^2T_2-77T_1^3T_2+\right.$$
$$23T_1^4T_2+35T_1^5T_2-28T_1^6T_2+11T_1^7T_2-36T_2^2+100T_1T_2^2-161T_1^2T_2^2+54T_1^3T_2^2+$$
$$39T_1^4T_2^2-107T_1^5T_2^2+23T_1^6T_2^2+8T_1^7T_2^2-13T_1^8T_2^2+44T_2^3-77T_1T_2^3+54T_1^2T_2^3+189T_1^3T_2^3-$$
$$215T_1^4T_2^3+153T_1^5T_2^3+106T_1^6T_2^3-107T_1^7T_2^3+52T_1^8T_2^3-36T_2^4+23T_1T_2^4+39T_1^2T_2^4-$$
$$215T_1^3T_2^4-23T_1^4T_2^4+238T_1^5T_2^4-478T_1^6T_2^4+214T_1^7T_2^4-4T_1^8T_2^4-52T_1^9T_2^4+13T_1^{10}T_2^4+$$
$$16T_2^5+35T_1T_2^5-107T_1^2T_2^5+153T_1^3T_2^5+238T_1^4T_2^5-485T_1^5T_2^5+547T_1^6T_2^5-52T_1^7T_2^5-$$
$$194T_1^8T_2^5+111T_1^9T_2^5-8T_1^{10}T_2^5-11T_1^{11}T_2^5-4T_2^6-28T_1T_2^6+23T_1^2T_2^6+106T_1^3T_2^6-478T_1^4T_2^6+$$
$$547T_1^5T_2^6-192T_1^6T_2^6-419T_1^7T_2^6+498T_1^8T_2^6-126T_1^9T_2^6-23T_1^{10}T_2^6+28T_1^{11}T_2^6+4T_1^{12}T_2^6+$$
$$11T_1T_2^7+8T_1^2T_2^7-107T_1^3T_2^7+214T_1^4T_2^7-52T_1^5T_2^7-419T_1^6T_2^7+613T_1^7T_2^7-342T_1^8T_2^7-$$
$$133T_1^9T_2^7+111T_1^{10}T_2^7-35T_1^{11}T_2^7-16T_1^{12}T_2^7-13T_1^2T_2^8+52T_1^3T_2^8-4T_1^4T_2^8-194T_1^5T_2^8+$$
$$498T_1^6T_2^8-342T_1^7T_2^8+43T_1^8T_2^8+235T_1^9T_2^8-47T_1^{10}T_2^8-23T_1^{11}T_2^8+36T_1^{12}T_2^8-52T_1^4T_2^9+$$
$$111T_1^5T_2^9-126T_1^6T_2^9-133T_1^7T_2^9+235T_1^8T_2^9-209T_1^9T_2^9-50T_1^{10}T_2^9+77T_1^{11}T_2^9-44T_1^{12}T_2^9+$$
$$13T_1^4T_2^{10}-8T_1^5T_2^{10}-23T_1^6T_2^{10}+111T_1^7T_2^{10}-47T_1^8T_2^{10}-50T_1^9T_2^{10}+161T_1^{10}T_2^{10}-100T_1^{11}T_2^{10}+$$
$$36T_1^{12}T_2^{10}-11T_1^5T_2^{11}+28T_1^6T_2^{11}-35T_1^7T_2^{11}-23T_1^8T_2^{11}+77T_1^9T_2^{11}-100T_1^{10}T_2^{11}+53T_1^{11}T_2^{11}-$$
$$\left.\left.16T_1^{12}T_2^{11}+4T_1^6T_2^{12}-16T_1^7T_2^{12}+36T_1^8T_2^{12}-44T_1^9T_2^{12}+36T_1^{10}T_2^{12}-16T_1^{11}T_2^{12}+4T_1^{12}T_2^{12}\right)\right\},$$

$$\text{Knot}[11,\text{Alternating},47]\to\left\{\frac{\left(1-T+T^2\right)^2\left(1-3T+5T^2-3T^3+T^4\right)}{T^4},\right.$$

$$-\frac{1}{T_1^9T_2^8}\left(1-T_1+T_1^2\right)\left(1-T_2+T_2^2\right)\left(1-T_1T_2+T_1^2T_2^2\right)$$

$$\left(-4+16T_1-36T_1^2+44T_1^3-36T_1^4+16T_1^5-4T_1^6+16T_2-53T_1T_2+100T_1^2T_2-77T_1^3T_2+\right.$$
$$23T_1^4T_2+35T_1^5T_2-28T_1^6T_2+11T_1^7T_2-36T_2^2+100T_1T_2^2-161T_1^2T_2^2+54T_1^3T_2^2+$$
$$39T_1^4T_2^2-107T_1^5T_2^2+23T_1^6T_2^2+8T_1^7T_2^2-13T_1^8T_2^2+44T_2^3-77T_1T_2^3+54T_1^2T_2^3+189T_1^3T_2^3-$$
$$215T_1^4T_2^3+153T_1^5T_2^3+106T_1^6T_2^3-107T_1^7T_2^3+52T_1^8T_2^3-36T_2^4+23T_1T_2^4+39T_1^2T_2^4-$$
$$215T_1^3T_2^4-23T_1^4T_2^4+238T_1^5T_2^4-478T_1^6T_2^4+214T_1^7T_2^4-4T_1^8T_2^4-52T_1^9T_2^4+13T_1^{10}T_2^4+$$

$$16\,T_2^5+35\,T_1\,T_2^5-107\,T_1^2\,T_2^5+153\,T_1^3\,T_2^5+238\,T_1^4\,T_2^5-485\,T_1^5\,T_2^5+547\,T_1^6\,T_2^5-52\,T_1^7\,T_2^5-194\,T_1^8\,T_2^5+111\,T_1^9\,T_2^5-8\,T_1^{10}\,T_2^5-11\,T_1^{11}\,T_2^5-4\,T_2^6-28\,T_1\,T_2^6+23\,T_1^2\,T_2^6+106\,T_1^3\,T_2^6-478\,T_1^4\,T_2^6+547\,T_1^5\,T_2^6-192\,T_1^6\,T_2^6-419\,T_1^7\,T_2^6+498\,T_1^8\,T_2^6-126\,T_1^9\,T_2^6-23\,T_1^{10}\,T_2^6+28\,T_1^{11}\,T_2^6+4\,T_1^{12}\,T_2^6+11\,T_1\,T_2^7+8\,T_1^2\,T_2^7-107\,T_1^3\,T_2^7+214\,T_1^4\,T_2^7-52\,T_1^5\,T_2^7-419\,T_1^6\,T_2^7+613\,T_1^7\,T_2^7-342\,T_1^8\,T_2^7-133\,T_1^9\,T_2^7+111\,T_1^{10}\,T_2^7-35\,T_1^{11}\,T_2^7-16\,T_1^{12}\,T_2^7-13\,T_1^2\,T_2^8+52\,T_1^3\,T_2^8-4\,T_1^4\,T_2^8-194\,T_1^5\,T_2^8+498\,T_1^6\,T_2^8-342\,T_1^7\,T_2^8+43\,T_1^8\,T_2^8+235\,T_1^9\,T_2^8-47\,T_1^{10}\,T_2^8-23\,T_1^{11}\,T_2^8+36\,T_1^{12}\,T_2^8-52\,T_1^4\,T_2^9+111\,T_1^5\,T_2^9-126\,T_1^6\,T_2^9-133\,T_1^7\,T_2^9+235\,T_1^8\,T_2^9-209\,T_1^9\,T_2^9-50\,T_1^{10}\,T_2^9+77\,T_1^{11}\,T_2^9-44\,T_1^{12}\,T_2^9+13\,T_1^4\,T_2^{10}-8\,T_1^5\,T_2^{10}-23\,T_1^6\,T_2^{10}+111\,T_1^7\,T_2^{10}-47\,T_1^8\,T_2^{10}-50\,T_1^9\,T_2^{10}+161\,T_1^{10}\,T_2^{10}-100\,T_1^{11}\,T_2^{10}+36\,T_1^{12}\,T_2^{10}-11\,T_1^5\,T_2^{11}+28\,T_1^6\,T_2^{11}-35\,T_1^7\,T_2^{11}-23\,T_1^8\,T_2^{11}+77\,T_1^9\,T_2^{11}-100\,T_1^{10}\,T_2^{11}+53\,T_1^{11}\,T_2^{11}-16\,T_1^{12}\,T_2^{11}+4\,T_1^6\,T_2^{12}-16\,T_1^7\,T_2^{12}+36\,T_1^8\,T_2^{12}-44\,T_1^9\,T_2^{12}+36\,T_1^{10}\,T_2^{12}-16\,T_1^{11}\,T_2^{12}+4\,T_1^{12}\,T_2^{12}\Big)\Big\}\Big\},$$

$$\Big\{\text{Knot}[11,\text{Alternating},57]\to\Big\{-\frac{\left(1-T+T^2\right)^2\left(1-3\,T+3\,T^2-3\,T^3+T^4\right)}{T^4},$$

$$\frac{1}{T_1^9\,T_2^8}\left(1-T_1+T_1^2\right)\left(1-T_2+T_2^2\right)\left(1-T_1\,T_2+T_1^2\,T_2^2\right)\Big(-3+12\,T_1-21\,T_1^2+27\,T_1^3-21\,T_1^4+12\,T_1^5-3\,T_1^6+12\,T_2-41\,T_1\,T_2+56\,T_1^2\,T_2-59\,T_1^3\,T_2+21\,T_1^4\,T_2+T_1^5\,T_2-16\,T_1^6\,T_2+7\,T_1^7\,T_2-21\,T_2^2+56\,T_1\,T_2^2-37\,T_1^2\,T_2^2+5\,T_1^3\,T_2^2+85\,T_1^4\,T_2^2-90\,T_1^5\,T_2^2+77\,T_1^6\,T_2^2-20\,T_1^7\,T_2^2-2\,T_1^8\,T_2^2+27\,T_2^3-59\,T_1\,T_2^3+5\,T_1^2\,T_2^3+39\,T_1^3\,T_2^3-152\,T_1^4\,T_2^3+108\,T_1^5\,T_2^3-52\,T_1^6\,T_2^3-42\,T_1^7\,T_2^3+44\,T_1^8\,T_2^3-9\,T_1^9\,T_2^3-21\,T_2^4+21\,T_1\,T_2^4+85\,T_1^2\,T_2^4-152\,T_1^3\,T_2^4+257\,T_1^4\,T_2^4-118\,T_1^5\,T_2^4-16\,T_1^6\,T_2^4+109\,T_1^7\,T_2^4-52\,T_1^8\,T_2^4-28\,T_1^9\,T_2^4+16\,T_1^{10}\,T_2^4+12\,T_2^5+T_1\,T_2^5-90\,T_1^2\,T_2^5+108\,T_1^3\,T_2^5-118\,T_1^4\,T_2^5-55\,T_1^5\,T_2^5+133\,T_1^6\,T_2^5-68\,T_1^7\,T_2^5-35\,T_1^8\,T_2^5+92\,T_1^9\,T_2^5-4\,T_1^{10}\,T_2^5-15\,T_1^{11}\,T_2^5-3\,T_2^6-16\,T_1\,T_2^6+77\,T_1^2\,T_2^6-52\,T_1^3\,T_2^6-16\,T_1^4\,T_2^6+133\,T_1^5\,T_2^6-156\,T_1^6\,T_2^6-T_1^7\,T_2^6+4\,T_1^8\,T_2^6-12\,T_1^9\,T_2^6-93\,T_1^{10}\,T_2^6+40\,T_1^{11}\,T_2^6+5\,T_1^{12}\,T_2^6+7\,T_1\,T_2^7-20\,T_1^2\,T_2^7-42\,T_1^3\,T_2^7+109\,T_1^4\,T_2^7-68\,T_1^5\,T_2^7-T_1^6\,T_2^7+187\,T_1^7\,T_2^7-18\,T_1^8\,T_2^7-34\,T_1^9\,T_2^7+140\,T_1^{10}\,T_2^7-25\,T_1^{11}\,T_2^7-20\,T_1^{12}\,T_2^7-2\,T_1^2\,T_2^8+44\,T_1^3\,T_2^8-52\,T_1^4\,T_2^8-35\,T_1^5\,T_2^8+4\,T_1^6\,T_2^8-18\,T_1^7\,T_2^8-269\,T_1^8\,T_2^8+226\,T_1^9\,T_2^8-189\,T_1^{10}\,T_2^8-5\,T_1^{11}\,T_2^8+35\,T_1^{12}\,T_2^8-9\,T_1^3\,T_2^9-28\,T_1^4\,T_2^9+92\,T_1^5\,T_2^9-12\,T_1^6\,T_2^9-34\,T_1^7\,T_2^9+226\,T_1^8\,T_2^9-103\,T_1^9\,T_2^9+45\,T_1^{10}\,T_2^9+75\,T_1^{11}\,T_2^9-45\,T_1^{12}\,T_2^9+16\,T_1^4\,T_2^{10}-4\,T_1^5\,T_2^{10}-93\,T_1^6\,T_2^{10}+140\,T_1^7\,T_2^{10}-189\,T_1^8\,T_2^{10}+45\,T_1^9\,T_2^{10}+21\,T_1^{10}\,T_2^{10}-80\,T_1^{11}\,T_2^{10}+35\,T_1^{12}\,T_2^{10}-15\,T_1^5\,T_2^{11}+40\,T_1^6\,T_2^{11}-25\,T_1^7\,T_2^{11}-5\,T_1^8\,T_2^{11}+75\,T_1^9\,T_2^{11}-80\,T_1^{10}\,T_2^{11}+65\,T_1^{11}\,T_2^{11}-20\,T_1^{12}\,T_2^{11}+5\,T_1^6\,T_2^{12}-20\,T_1^7\,T_2^{12}+35\,T_1^8\,T_2^{12}-45\,T_1^9\,T_2^{12}+35\,T_1^{10}\,T_2^{12}-20\,T_1^{11}\,T_2^{12}+5\,T_1^{12}\,T_2^{12}\Big)\Big\},$$

$$\text{Knot}[11,\text{Alternating},231]\to\Big\{-\frac{\left(1-T+T^2\right)^2\left(1-3\,T+3\,T^2-3\,T^3+T^4\right)}{T^4},$$

$$\frac{1}{T_1^9\,T_2^8}\left(1-T_1+T_1^2\right)\left(1-T_2+T_2^2\right)\left(1-T_1\,T_2+T_1^2\,T_2^2\right)\Big(-3+12\,T_1-21\,T_1^2+27\,T_1^3-21\,T_1^4+12\,T_1^5-3\,T_1^6+12\,T_2-41\,T_1\,T_2+56\,T_1^2\,T_2-59\,T_1^3\,T_2+21\,T_1^4\,T_2+T_1^5\,T_2-16\,T_1^6\,T_2+7\,T_1^7\,T_2-21\,T_2^2+56\,T_1\,T_2^2-37\,T_1^2\,T_2^2+5\,T_1^3\,T_2^2+85\,T_1^4\,T_2^2-90\,T_1^5\,T_2^2+77\,T_1^6\,T_2^2-20\,T_1^7\,T_2^2-2\,T_1^8\,T_2^2+27\,T_2^3-59\,T_1\,T_2^3+5\,T_1^2\,T_2^3+39\,T_1^3\,T_2^3-152\,T_1^4\,T_2^3+108\,T_1^5\,T_2^3-52\,T_1^6\,T_2^3-42\,T_1^7\,T_2^3+44\,T_1^8\,T_2^3-9\,T_1^9\,T_2^3-21\,T_2^4+21\,T_1\,T_2^4+85\,T_1^2\,T_2^4-152\,T_1^3\,T_2^4+257\,T_1^4\,T_2^4-118\,T_1^5\,T_2^4-16\,T_1^6\,T_2^4+109\,T_1^7\,T_2^4-52\,T_1^8\,T_2^4-28\,T_1^9\,T_2^4+16\,T_1^{10}\,T_2^4+12\,T_2^5+T_1\,T_2^5-90\,T_1^2\,T_2^5+108\,T_1^3\,T_2^5-118\,T_1^4\,T_2^5-55\,T_1^5\,T_2^5+133\,T_1^6\,T_2^5-68\,T_1^7\,T_2^5-35\,T_1^8\,T_2^5+92\,T_1^9\,T_2^5-4\,T_1^{10}\,T_2^5-15\,T_1^{11}\,T_2^5-3\,T_2^6-16\,T_1\,T_2^6+77\,T_1^2\,T_2^6-52\,T_1^3\,T_2^6-16\,T_1^4\,T_2^6+133\,T_1^5\,T_2^6-156\,T_1^6\,T_2^6-T_1^7\,T_2^6+4\,T_1^8\,T_2^6-12\,T_1^9\,T_2^6-93\,T_1^{10}\,T_2^6+40\,T_1^{11}\,T_2^6+5\,T_1^{12}\,T_2^6+7\,T_1\,T_2^7-20\,T_1^2\,T_2^7-42\,T_1^3\,T_2^7+109\,T_1^4\,T_2^7-68\,T_1^5\,T_2^7-T_1^6\,T_2^7+187\,T_1^7\,T_2^7-18\,T_1^8\,T_2^7-34\,T_1^9\,T_2^7+140\,T_1^{10}\,T_2^7-25\,T_1^{11}\,T_2^7-20\,T_1^{12}\,T_2^7-2\,T_1^2\,T_2^8+44\,T_1^3\,T_2^8-52\,T_1^4\,T_2^8-35\,T_1^5\,T_2^8+4\,T_1^6\,T_2^8-18\,T_1^7\,T_2^8-269\,T_1^8\,T_2^8+226\,T_1^9\,T_2^8-189\,T_1^{10}\,T_2^8-5\,T_1^{11}\,T_2^8+35\,T_1^{12}\,T_2^8-9\,T_1^3\,T_2^9-$$

$$28\,T_1^4\,T_2^9+92\,T_1^5\,T_2^9-12\,T_1^6\,T_2^9-34\,T_1^7\,T_2^9+226\,T_1^8\,T_2^9-103\,T_1^9\,T_2^9+45\,T_1^{10}\,T_2^9+75\,T_1^{11}\,T_2^9-45\,T_1^{12}\,T_2^9+16\,T_1^4\,T_2^{10}-4\,T_1^5\,T_2^{10}-93\,T_1^6\,T_2^{10}+140\,T_1^7\,T_2^{10}-189\,T_1^8\,T_2^{10}+45\,T_1^9\,T_2^{10}+21\,T_1^{10}\,T_2^{10}-80\,T_1^{11}\,T_2^{10}+35\,T_1^{12}\,T_2^{10}-15\,T_1^5\,T_2^{11}+40\,T_1^6\,T_2^{11}-25\,T_1^7\,T_2^{11}-5\,T_1^8\,T_2^{11}+75\,T_1^9\,T_2^{11}-80\,T_1^{10}\,T_2^{11}+65\,T_1^{11}\,T_2^{11}-20\,T_1^{12}\,T_2^{11}+5\,T_1^6\,T_2^{12}-20\,T_1^7\,T_2^{12}+35\,T_1^8\,T_2^{12}-45\,T_1^9\,T_2^{12}+35\,T_1^{10}\,T_2^{12}-20\,T_1^{11}\,T_2^{12}+5\,T_1^{12}\,T_2^{12}\Big)\Big\}\Big\},$$

$$\Big\{\text{Knot}[11,\text{NonAlternating},73]\to\Big\{\frac{\left(1-T+T^2\right)^2}{T^2},$$

$$-\frac{1}{T_1^5\,T_2^4}\,2\left(1-T_1+T_1^2\right)\left(1-T_2+T_2^2\right)\left(1-T_1\,T_2+T_1^2\,T_2^2\right)$$

$$\left(-1+T_1-T_1^2+T_2+2\,T_1^2\,T_2-T_2^2+2\,T_1\,T_2^2-6\,T_1^2\,T_2^2+T_1^4\,T_2^2+2\,T_1^3\,T_2^3-T_1^4\,T_2^3+T_1^2\,T_2^4-T_1^3\,T_2^4+T_1^4\,T_2^4\right)\Big\},$$

$$\text{Knot}[11,\text{NonAlternating},74]\to\Big\{\frac{\left(1-T+T^2\right)^2}{T^2},$$

$$-\frac{1}{T_1^5\,T_2^4}\,2\left(1-T_1+T_1^2\right)\left(1-T_2+T_2^2\right)\left(1-T_1\,T_2+T_1^2\,T_2^2\right)$$

$$\left(-1+T_1-T_1^2+T_2+2\,T_1^2\,T_2-T_2^2+2\,T_1\,T_2^2-6\,T_1^2\,T_2^2+T_1^4\,T_2^2+2\,T_1^3\,T_2^3-T_1^4\,T_2^3+T_1^2\,T_2^4-T_1^3\,T_2^4+T_1^4\,T_2^4\right)\Big\}\Big\}\Big\}$$

In[•]:=
```
tab12 = Table[K → θ@K, {K, AllKnots[{3, 12}] }]
```

In[•]:=
```
dup12 = Map[First, Select[Gather[tab12, Last[#1] === Last[#2] &], Length[#] > 1 &], {2}]
```

Out[•]=
```
{{Knot[10, 106], Knot[12, NonAlternating, 369]},
 {Knot[11, Alternating, 44], Knot[11, Alternating, 47]},
 {Knot[11, Alternating, 57], Knot[11, Alternating, 231]},
 {Knot[11, NonAlternating, 73], Knot[11, NonAlternating, 74]},
 {Knot[12, Alternating, 30], Knot[12, Alternating, 33]},
 {Knot[12, Alternating, 122], Knot[12, Alternating, 182]},
 {Knot[12, Alternating, 164], Knot[12, Alternating, 166]},
 {Knot[12, Alternating, 167], Knot[12, Alternating, 692]},
 {Knot[12, Alternating, 273], Knot[12, Alternating, 890]},
 {Knot[12, Alternating, 341], Knot[12, Alternating, 627]},
 {Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]},
 {Knot[12, Alternating, 458], Knot[12, Alternating, 887]},
 {Knot[12, Alternating, 510], Knot[12, Alternating, 821]},
 {Knot[12, NonAlternating, 56], Knot[12, NonAlternating, 57]},
 {Knot[12, NonAlternating, 60], Knot[12, NonAlternating, 61]},
 {Knot[12, NonAlternating, 62], Knot[12, NonAlternating, 66]},
 {Knot[12, NonAlternating, 144], Knot[12, NonAlternating, 507]},
 {Knot[12, NonAlternating, 313], Knot[12, NonAlternating, 430]}}
```

In[•]:=
```
Length /@ dup12
```

Out[•]=
```
{2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 2, 2, 2, 2, 2, 2, 2}
```

In[•]:=
```
Total[(Length /@ dup12) - 1]
```

Out[•]=
```
19
```

```
In[•]:= Length /@ Select[
          Gather[tab12 /. {T1 → 22 / 7, T2 → 13 / 21}, Last[#1] === Last[#2] &], Length[#] > 1 &]
```

```
Out[•]= {2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 2, 2, 2, 2, 2, 2, 2}
```

```
In[•]:= Put[tab12 /. {T1 → T1, T2 → T2}, "Data12.m"]
```

```
In[•]:= Table[K → θ[K],
         {K, {Knot[6, 1], Knot[8, 8], Knot[8, 9], Knot[8, 20], Knot[9, 27], Knot[9, 41],
           Knot[9, 46], Knot[10, 3], Knot[10, 22], Knot[10, 35], Knot[10, 42], Knot[10, 48],
           Knot[10, 75], Knot[10, 87], Knot[10, 99], Knot[10, 123], Knot[10, 129],
           Knot[10, 137], Knot[10, 140], Knot[10, 153], Knot[10, 155]}}]
```

```
Out[•]= {Knot[6, 1] →
    {-((-2 + T) (-1 + 2 T))/T, 1/(T1^3 T2^2) (-7 + 17 T1 - 7 T1^2 + 17 T2 - 44 T1 T2 + 26 T1^2 T2 - 3 T1^3 T2 - 7 T2^2 + 26 T1 T2^2 -
        24 T1^2 T2^2 - 14 T1^3 T2^2 + 9 T1^4 T2^2 - 3 T1 T2^3 - 14 T1^2 T2^3 + 56 T1^3 T2^3 - 23 T1^4 T2^3 + 9 T1^2 T2^4 - 23 T1^3 T2^4 + 9 T1^4 T2^4)},
  Knot[8, 8] → {(2 - 2 T + T^2) (1 - 2 T + 2 T^2)/T^2,
    -1/(T1^5 T2^4) (-17 + 51 T1 - 77 T1^2 + 51 T1^3 - 17 T1^4 + 51 T2 - 126 T1 T2 + 150 T1^2 T2 - 30 T1^3 T2 - 30 T1^4 T2 +
      27 T1^5 T2 - 77 T2^2 + 150 T1 T2^2 - 117 T1^2 T2^2 - 113 T1^3 T2^2 + 135 T1^4 T2^2 - 66 T1^5 T2^2 - 5 T1^6 T2^2 + 51 T2^3 -
      30 T1 T2^3 - 113 T1^2 T2^3 + 336 T1^3 T2^3 - 186 T1^4 T2^3 - 5 T1^5 T2^3 + 78 T1^6 T2^3 - 21 T1^7 T2^3 - 17 T2^4 - 30 T1 T2^4 +
      135 T1^2 T2^4 - 186 T1^3 T2^4 - 60 T1^4 T2^4 + 222 T1^5 T2^4 - 153 T1^6 T2^4 + 18 T1^7 T2^4 + 15 T1^8 T2^4 + 27 T1 T2^5 -
      66 T1^2 T2^5 - 5 T1^3 T2^5 + 222 T1^4 T2^5 - 300 T1^5 T2^5 + 103 T1^6 T2^5 + 42 T1^7 T2^5 - 45 T1^8 T2^5 - 5 T1^2 T2^6 + 78 T1^3 T2^6 -
      153 T1^4 T2^6 + 103 T1^5 T2^6 + 99 T1^6 T2^6 - 138 T1^7 T2^6 + 67 T1^8 T2^6 - 21 T1^3 T2^7 + 18 T1^4 T2^7 + 42 T1^5 T2^7 -
      138 T1^6 T2^7 + 114 T1^7 T2^7 - 45 T1^8 T2^7 + 15 T1^4 T2^8 - 45 T1^5 T2^8 + 67 T1^6 T2^8 - 45 T1^7 T2^8 + 15 T1^8 T2^8)},
  Knot[8, 9] → {-((-1 + T - 2 T^2 + T^3) (-1 + 2 T - T^2 + T^3))/T^3,
    1/(T1^7 T2^6) (-1 + T1 - 2 T1^2 + T1^3) (-1 + 2 T1 - T1^2 + T1^3) (-1 + T1 T2) (1 + T1 T2)
      (-1 + T2 - 2 T2^2 + T2^3) (-1 + 2 T2 - T2^2 + T2^3) (3 - 6 T1 T2 + 8 T1^2 T2^2 - 6 T1^3 T2^3 + 3 T1^4 T2^4)},
  Knot[8, 20] → {(1 - T + T^2)^2/T^2, -1/(T1^5 T2^4) 2 (-1 + 2 T1 - 3 T1^2 + 2 T1^3 - T1^4 + 2 T2 - 3 T1 T2 + 4 T1^2 T2 -
      T1^3 T2 + T1^5 T2 - 3 T2^2 + 4 T1 T2^2 - 2 T1^2 T2^2 - 4 T1^3 T2^2 + 4 T1^4 T2^2 - 2 T1^5 T2^2 + 2 T2^3 - T1 T2^3 - 4 T1^2 T2^3 + 5 T1^3 T2^3 -
      T1^4 T2^3 - 4 T1^5 T2^3 + 2 T1^6 T2^3 - T1^7 T2^3 - T2^4 + 4 T1^2 T2^4 - T1^3 T2^4 + 3 T1^5 T2^4 + 2 T1^6 T2^4 + T1^8 T2^4 + T1 T2^5 - 2 T1^2 T2^5 -
      4 T1^3 T2^5 + 3 T1^4 T2^5 - 3 T1^5 T2^5 - 4 T1^6 T2^5 + T1^7 T2^5 - 2 T1^8 T2^5 + 2 T1^3 T2^6 + 2 T1^4 T2^6 - 4 T1^5 T2^6 + 8 T1^6 T2^6 - 4 T1^7 T2^6 +
      3 T1^8 T2^6 - T1^3 T2^7 + T1^5 T2^7 - 4 T1^6 T2^7 + 3 T1^7 T2^7 - 2 T1^8 T2^7 + T1^4 T2^8 - 2 T1^5 T2^8 + 3 T1^6 T2^8 - 2 T1^7 T2^8 + T1^8 T2^8)},
  Knot[9, 27] → {-((-1 + 2 T - 3 T^2 + T^3) (-1 + 3 T - 2 T^2 + T^3))/T^3,
    1/(T1^7 T2^6) (-3 + 15 T1 - 33 T1^2 + 45 T1^3 - 33 T1^4 + 15 T1^5 - 3 T1^6 + 15 T2 - 65 T1 T2 + 115 T1^2 T2 - 115 T1^3 T2 +
```

$$15\,T_1^4\,T_2+35\,T_1^5\,T_2-35\,T_1^6\,T_2+10\,T_1^7\,T_2-33\,T_2^2+115\,T_1\,T_2^2-125\,T_1^2\,T_2^2+T_1^3\,T_2^2+265\,T_1^4\,T_2^2-219\,T_1^5\,T_2^2+95\,T_1^6\,T_2^2+5\,T_1^7\,T_2^2-11\,T_1^8\,T_2^2+45\,T_2^3-115\,T_1\,T_2^3+T_1^2\,T_2^3+268\,T_1^3\,T_2^3-554\,T_1^4\,T_2^3+156\,T_1^5\,T_2^3+108\,T_1^6\,T_2^3-164\,T_1^7\,T_2^3+55\,T_1^8\,T_2^3-33\,T_2^4+15\,T_1\,T_2^4+265\,T_1^2\,T_2^4-554\,T_1^3\,T_2^4+518\,T_1^4\,T_2^4+342\,T_1^5\,T_2^4-542\,T_1^6\,T_2^4+286\,T_1^7\,T_2^4-T_1^8\,T_2^4-55\,T_1^9\,T_2^4+11\,T_1^{10}\,T_2^4+15\,T_2^5+35\,T_1\,T_2^5-219\,T_1^2\,T_2^5+156\,T_1^3\,T_2^5+342\,T_1^4\,T_2^5-1095\,T_1^5\,T_2^5+705\,T_1^6\,T_2^5+67\,T_1^7\,T_2^5-294\,T_1^8\,T_2^5+166\,T_1^9\,T_2^5-5\,T_1^{10}\,T_2^5-10\,T_1^{11}\,T_2^5-3\,T_2^6-35\,T_1\,T_2^6+95\,T_1^2\,T_2^6+108\,T_1^3\,T_2^6-542\,T_1^4\,T_2^6+705\,T_1^5\,T_2^6+12\,T_1^6\,T_2^6-745\,T_1^7\,T_2^6+444\,T_1^8\,T_2^6-92\,T_1^9\,T_2^6-97\,T_1^{10}\,T_2^6+35\,T_1^{11}\,T_2^6+3\,T_1^{12}\,T_2^6+10\,T_1\,T_2^7+5\,T_1^2\,T_2^7-164\,T_1^3\,T_2^7+286\,T_1^4\,T_2^7+67\,T_1^5\,T_2^7-745\,T_1^6\,T_2^7+1055\,T_1^7\,T_2^7-208\,T_1^8\,T_2^7-164\,T_1^9\,T_2^7+221\,T_1^{10}\,T_2^7-35\,T_1^{11}\,T_2^7-15\,T_1^{12}\,T_2^7-11\,T_1^2\,T_2^8+55\,T_1^3\,T_2^8-T_1^4\,T_2^8-294\,T_1^5\,T_2^8+444\,T_1^6\,T_2^8-208\,T_1^7\,T_2^8-616\,T_1^8\,T_2^8+546\,T_1^9\,T_2^8-267\,T_1^{10}\,T_2^8-15\,T_1^{11}\,T_2^8+33\,T_1^{12}\,T_2^8-55\,T_1^4\,T_2^9+166\,T_1^5\,T_2^9-92\,T_1^6\,T_2^9-164\,T_1^7\,T_2^9+546\,T_1^8\,T_2^9-252\,T_1^9\,T_2^9+T_1^{10}\,T_2^9+115\,T_1^{11}\,T_2^9-45\,T_1^{12}\,T_2^9+11\,T_1^4\,T_2^{10}-5\,T_1^5\,T_2^{10}-97\,T_1^6\,T_2^{10}+221\,T_1^7\,T_2^{10}-267\,T_1^8\,T_2^{10}+T_1^9\,T_2^{10}+123\,T_1^{10}\,T_2^{10}-115\,T_1^{11}\,T_2^{10}+33\,T_1^{12}\,T_2^{10}-10\,T_1^5\,T_2^{11}+35\,T_1^6\,T_2^{11}-35\,T_1^7\,T_2^{11}-15\,T_1^8\,T_2^{11}+115\,T_1^9\,T_2^{11}-115\,T_1^{10}\,T_2^{11}+65\,T_1^{11}\,T_2^{11}-15\,T_1^{12}\,T_2^{11}+3\,T_1^6\,T_2^{12}-15\,T_1^7\,T_2^{12}+33\,T_1^8\,T_2^{12}-45\,T_1^9\,T_2^{12}+33\,T_1^{10}\,T_2^{12}-15\,T_1^{11}\,T_2^{12}+3\,T_1^{12}\,T_2^{12}\Big)\Big\},$$

$$\text{Knot}[9,41]\to\Big\{\frac{\left(3-3\,T+T^2\right)\left(1-3\,T+3\,T^2\right)}{T^2},$$

$$-\frac{1}{T_1^5\,T_2^4}\Big(-51+201\,T_1-315\,T_1^2+201\,T_1^3-51\,T_1^4+201\,T_2-698\,T_1\,T_2+880\,T_1^2\,T_2-236\,T_1^3\,T_2-158\,T_1^4\,T_2+93\,T_1^5\,T_2-315\,T_2^2+880\,T_1\,T_2^2-519\,T_1^2\,T_2^2-1035\,T_1^3\,T_2^2+1305\,T_1^4\,T_2^2-488\,T_1^5\,T_2^2+27\,T_1^6\,T_2^2+201\,T_2^3-236\,T_1\,T_2^3-1035\,T_1^2\,T_2^3+2964\,T_1^3\,T_2^3-2484\,T_1^4\,T_2^3+333\,T_1^5\,T_2^3+376\,T_1^6\,T_2^3-123\,T_1^7\,T_2^3-51\,T_2^4-158\,T_1\,T_2^4+1305\,T_1^2\,T_2^4-2484\,T_1^3\,T_2^4+1188\,T_1^4\,T_2^4+1692\,T_1^5\,T_2^4-1467\,T_1^6\,T_2^4+274\,T_1^7\,T_2^4+57\,T_1^8\,T_2^4+93\,T_1\,T_2^5-488\,T_1^2\,T_2^5+333\,T_1^3\,T_2^5+1692\,T_1^4\,T_2^5-3756\,T_1^5\,T_2^5+1701\,T_1^6\,T_2^5+124\,T_1^7\,T_2^5-231\,T_1^8\,T_2^5+27\,T_1^2\,T_2^6+376\,T_1^3\,T_2^6-1467\,T_1^4\,T_2^6+1701\,T_1^5\,T_2^6+357\,T_1^6\,T_2^6-992\,T_1^7\,T_2^6+369\,T_1^8\,T_2^6-123\,T_1^3\,T_2^7+274\,T_1^4\,T_2^7+124\,T_1^5\,T_2^7-992\,T_1^6\,T_2^7+814\,T_1^7\,T_2^7-231\,T_1^8\,T_2^7+57\,T_1^4\,T_2^8-231\,T_1^5\,T_2^8+369\,T_1^6\,T_2^8-231\,T_1^7\,T_2^8+57\,T_1^8\,T_2^8\Big)\Big\},$$

$$\text{Knot}[9,46]\to\Big\{-\frac{(-2+T)\,(-1+2\,T)}{T},\frac{1}{T_1^3\,T_2^2}\Big(-5+11\,T_1-5\,T_1^2+11\,T_2-32\,T_1\,T_2+38\,T_1^2\,T_2-9\,T_1^3\,T_2-5\,T_2^2+38\,T_1\,T_2^2-72\,T_1^2\,T_2^2-2\,T_1^3\,T_2^2+11\,T_1^4\,T_2^2-9\,T_1\,T_2^3-2\,T_1^2\,T_2^3+68\,T_1^3\,T_2^3-29\,T_1^4\,T_2^3+11\,T_1^2\,T_2^4-29\,T_1^3\,T_2^4+11\,T_1^4\,T_2^4\Big)\Big\},$$

$$\text{Knot}[10,3]\to\Big\{-\frac{(-3+2\,T)\,(-2+3\,T)}{T},\frac{1}{T_1^3\,T_2^2}\Big(-171+367\,T_1-171\,T_1^2+367\,T_2-888\,T_1\,T_2+594\,T_1^2\,T_2-101\,T_1^3\,T_2-171\,T_2^2+594\,T_1\,T_2^2-420\,T_1^2\,T_2^2-342\,T_1^3\,T_2^2+261\,T_1^4\,T_2^2-101\,T_1\,T_2^3-342\,T_1^2\,T_2^3+1140\,T_1^3\,T_2^3-569\,T_1^4\,T_2^3+261\,T_1^2\,T_2^4-569\,T_1^3\,T_2^4+261\,T_1^4\,T_2^4\Big)\Big\},$$

$$\text{Knot}[10,22]\to\Big\{-\frac{\left(-2+2\,T-2\,T^2+T^3\right)\left(-1+2\,T-2\,T^2+2\,T^3\right)}{T^3},$$

$$\frac{1}{T_1^7\,T_2^6}\Big(-25+75\,T_1-125\,T_1^2+163\,T_1^3-125\,T_1^4+75\,T_1^5-25\,T_1^6+75\,T_2-174\,T_1\,T_2+222\,T_1^2\,T_2-234\,T_1^3\,T_2+42\,T_1^4\,T_2+30\,T_1^5\,T_2-78\,T_1^6\,T_2+51\,T_1^7\,T_2-125\,T_2^2+222\,T_1\,T_2^2-211\,T_1^2\,T_2^2+189\,T_1^3\,T_2^2+141\,T_1^4\,T_2^2-91\,T_1^5\,T_2^2+109\,T_1^6\,T_2^2-18\,T_1^7\,T_2^2-45\,T_1^8\,T_2^2+163\,T_2^3-234\,T_1\,T_2^3+189\,T_1^2\,T_2^3-204\,T_1^3\,T_2^3-184\,T_1^4\,T_2^3-52\,T_1^5\,T_2^3+6\,T_1^6\,T_2^3-111\,T_1^7\,T_2^3+114\,T_1^8\,T_2^3+7\,T_1^9\,T_2^3-125\,T_2^4+42\,T_1\,T_2^4+141\,T_1^2\,T_2^4-184\,T_1^3\,T_2^4+557\,T_1^4\,T_2^4-83\,T_1^5\,T_2^4+105\,T_1^6\,T_2^4+24\,T_1^7\,T_2^4+5\,T_1^8\,T_2^4-126\,T_1^9\,T_2^4+35\,T_1^{10}\,T_2^4+75\,T_2^5+30\,T_1\,T_2^5-91\,T_1^2\,T_2^5-52\,T_1^3\,T_2^5-83\,T_1^4\,T_2^5-416\,T_1^5\,T_2^5+174\,T_1^6\,T_2^5-63\,T_1^7\,T_2^5-32\,T_1^8\,T_2^5+129\,T_1^9\,T_2^5+30\,T_1^{10}\,T_2^5-45\,T_1^{11}\,T_2^5-25\,T_2^6-78\,T_1\,T_2^6+109\,T_1^2\,T_2^6+6\,T_1^3\,T_2^6+105\,T_1^4\,T_2^6+174\,T_1^5\,T_2^6+108\,T_1^6\,T_2^6-$$

$$218\,T_1^7\,T_2^6+73\,T_1^8\,T_2^6-66\,T_1^9\,T_2^6-115\,T_1^{10}\,T_2^6+66\,T_1^{11}\,T_2^6+23\,T_1^{12}\,T_2^6+51\,T_1\,T_2^7-18\,T_1^2\,T_2^7-$$
$$111\,T_1^3\,T_2^7+24\,T_1^4\,T_2^7-63\,T_1^5\,T_2^7-218\,T_1^6\,T_2^7+372\,T_1^7\,T_2^7-43\,T_1^8\,T_2^7+44\,T_1^9\,T_2^7+109\,T_1^{10}\,T_2^7-$$
$$18\,T_1^{11}\,T_2^7-69\,T_1^{12}\,T_2^7-45\,T_1^2\,T_2^8+114\,T_1^3\,T_2^8+5\,T_1^4\,T_2^8-32\,T_1^5\,T_2^8+73\,T_1^6\,T_2^8-43\,T_1^7\,T_2^8-379\,T_1^8\,T_2^8+$$
$$176\,T_1^9\,T_2^8-131\,T_1^{10}\,T_2^8-54\,T_1^{11}\,T_2^8+115\,T_1^{12}\,T_2^8+7\,T_1^3\,T_2^9-126\,T_1^4\,T_2^9+129\,T_1^5\,T_2^9-66\,T_1^6\,T_2^9+$$
$$44\,T_1^7\,T_2^9+176\,T_1^8\,T_2^9+144\,T_1^9\,T_2^9-171\,T_1^{10}\,T_2^9+222\,T_1^{11}\,T_2^9-149\,T_1^{12}\,T_2^9+35\,T_1^4\,T_2^{10}+30\,T_1^5\,T_2^{10}-$$
$$115\,T_1^6\,T_2^{10}+109\,T_1^7\,T_2^{10}-131\,T_1^8\,T_2^{10}-171\,T_1^9\,T_2^{10}+205\,T_1^{10}\,T_2^{10}-210\,T_1^{11}\,T_2^{10}+115\,T_1^{12}\,T_2^{10}-$$
$$45\,T_1^5\,T_2^{11}+66\,T_1^6\,T_2^{11}-18\,T_1^7\,T_2^{11}-54\,T_1^8\,T_2^{11}+222\,T_1^9\,T_2^{11}-210\,T_1^{10}\,T_2^{11}+162\,T_1^{11}\,T_2^{11}-69\,T_1^{12}\,T_2^{11}+$$
$$23\,T_1^6\,T_2^{12}-69\,T_1^7\,T_2^{12}+115\,T_1^8\,T_2^{12}-149\,T_1^9\,T_2^{12}+115\,T_1^{10}\,T_2^{12}-69\,T_1^{11}\,T_2^{12}+23\,T_1^{12}\,T_2^{12}\Big)\Big\},$$

$$\text{Knot}[10,35]\to\Big\{\frac{\left(2-4\,T+T^2\right)\,\left(1-4\,T+2\,T^2\right)}{T^2},$$

$$-\frac{1}{T_1^5\,T_2^4}\,\Big(-17+103\,T_1-181\,T_1^2+103\,T_1^3-17\,T_1^4+103\,T_2-570\,T_1\,T_2+762\,T_1^2\,T_2-30\,T_1^3\,T_2-234\,T_1^4\,T_2+$$
$$55\,T_1^5\,T_2-181\,T_2^2+762\,T_1\,T_2^2+112\,T_1^2\,T_2^2-2442\,T_1^3\,T_2^2+1708\,T_1^4\,T_2^2-246\,T_1^5\,T_2^2-13\,T_1^6\,T_2^2+103\,T_2^3-$$
$$30\,T_1\,T_2^3-2442\,T_1^2\,T_2^3+5136\,T_1^3\,T_2^3-2172\,T_1^4\,T_2^3-426\,T_1^5\,T_2^3+330\,T_1^6\,T_2^3-41\,T_1^7\,T_2^3-17\,T_2^4-234\,T_1\,T_2^4+$$
$$1708\,T_1^2\,T_2^4-2172\,T_1^3\,T_2^4-1392\,T_1^4\,T_2^4+3108\,T_1^5\,T_2^4-1412\,T_1^6\,T_2^4+150\,T_1^7\,T_2^4+15\,T_1^8\,T_2^4+55\,T_1\,T_2^5-$$
$$246\,T_1^2\,T_2^5-426\,T_1^3\,T_2^5+3108\,T_1^4\,T_2^5-4200\,T_1^5\,T_2^5+1590\,T_1^6\,T_2^5+114\,T_1^7\,T_2^5-89\,T_1^8\,T_2^5-13\,T_1^2\,T_2^6+$$
$$330\,T_1^3\,T_2^6-1412\,T_1^4\,T_2^6+1590\,T_1^5\,T_2^6+184\,T_1^6\,T_2^6-678\,T_1^7\,T_2^6+155\,T_1^8\,T_2^6-41\,T_1^3\,T_2^7+150\,T_1^4\,T_2^7+$$
$$114\,T_1^5\,T_2^7-678\,T_1^6\,T_2^7+486\,T_1^7\,T_2^7-89\,T_1^8\,T_2^7+15\,T_1^4\,T_2^8-89\,T_1^5\,T_2^8+155\,T_1^6\,T_2^8-89\,T_1^7\,T_2^8+15\,T_1^8\,T_2^8\Big)\Big\},$$

$$\text{Knot}[10,42]\to\Big\{-\frac{\left(-1+3\,T-4\,T^2+T^3\right)\,\left(-1+4\,T-3\,T^2+T^3\right)}{T^3},$$

$$\frac{1}{T_1^7\,T_2^6}\,\Big(-3+21\,T_1-57\,T_1^2+81\,T_1^3-57\,T_1^4+21\,T_1^5-3\,T_1^6+21\,T_2-133\,T_1\,T_2+301\,T_1^2\,T_2-301\,T_1^3\,T_2+$$
$$21\,T_1^4\,T_2+119\,T_1^5\,T_2-77\,T_1^6\,T_2+14\,T_1^7\,T_2-57\,T_2^2+301\,T_1\,T_2^2-422\,T_1^2\,T_2^2-166\,T_1^3\,T_2^2+1164\,T_1^4\,T_2^2-$$
$$926\,T_1^5\,T_2^2+262\,T_1^6\,T_2^2+35\,T_1^7\,T_2^2-19\,T_1^8\,T_2^2+81\,T_2^3-301\,T_1\,T_2^3-166\,T_1^2\,T_2^3+1864\,T_1^3\,T_2^3-3062\,T_1^4\,T_2^3+$$
$$950\,T_1^5\,T_2^3+674\,T_1^6\,T_2^3-641\,T_1^7\,T_2^3+133\,T_1^8\,T_2^3-57\,T_2^4+21\,T_1\,T_2^4+1164\,T_1^2\,T_2^4-3062\,T_1^3\,T_2^4+$$
$$2392\,T_1^4\,T_2^4+2552\,T_1^5\,T_2^4-3800\,T_1^6\,T_2^4+1690\,T_1^7\,T_2^4-38\,T_1^8\,T_2^4-133\,T_1^9\,T_2^4+19\,T_1^{10}\,T_2^4+21\,T_2^5+$$
$$119\,T_1\,T_2^5-926\,T_1^2\,T_2^5+950\,T_1^3\,T_2^5+2552\,T_1^4\,T_2^5-7805\,T_1^5\,T_2^5+5699\,T_1^6\,T_2^5-279\,T_1^7\,T_2^5-1582\,T_1^8\,T_2^5+$$
$$689\,T_1^9\,T_2^5-35\,T_1^{10}\,T_2^5-14\,T_1^{11}\,T_2^5-3\,T_2^6-77\,T_1\,T_2^6+262\,T_1^2\,T_2^6+674\,T_1^3\,T_2^6-3800\,T_1^4\,T_2^6+5699\,T_1^5\,T_2^6-$$
$$552\,T_1^6\,T_2^6-5255\,T_1^7\,T_2^6+4018\,T_1^8\,T_2^6-818\,T_1^9\,T_2^6-274\,T_1^{10}\,T_2^6+77\,T_1^{11}\,T_2^6+3\,T_1^{12}\,T_2^6+14\,T_1\,T_2^7+$$
$$35\,T_1^2\,T_2^7-641\,T_1^3\,T_2^7+1690\,T_1^4\,T_2^7-279\,T_1^5\,T_2^7-5255\,T_1^6\,T_2^7+8249\,T_1^7\,T_2^7-3110\,T_1^8\,T_2^7-842\,T_1^9\,T_2^7+$$
$$974\,T_1^{10}\,T_2^7-119\,T_1^{11}\,T_2^7-21\,T_1^{12}\,T_2^7-19\,T_1^2\,T_2^8+133\,T_1^3\,T_2^8-38\,T_1^4\,T_2^8-1582\,T_1^5\,T_2^8+4018\,T_1^6\,T_2^8-$$
$$3110\,T_1^7\,T_2^8-2174\,T_1^8\,T_2^8+3170\,T_1^9\,T_2^8-1240\,T_1^{10}\,T_2^8-21\,T_1^{11}\,T_2^8+57\,T_1^{12}\,T_2^8-133\,T_1^4\,T_2^9+689\,T_1^5\,T_2^9-$$
$$818\,T_1^6\,T_2^9-842\,T_1^7\,T_2^9+3170\,T_1^8\,T_2^9-2008\,T_1^9\,T_2^9+214\,T_1^{10}\,T_2^9+301\,T_1^{11}\,T_2^9-81\,T_1^{12}\,T_2^9+19\,T_1^4\,T_2^{10}-$$
$$35\,T_1^5\,T_2^{10}-274\,T_1^6\,T_2^{10}+974\,T_1^7\,T_2^{10}-1240\,T_1^8\,T_2^{10}+214\,T_1^9\,T_2^{10}+410\,T_1^{10}\,T_2^{10}-301\,T_1^{11}\,T_2^{10}+$$
$$57\,T_1^{12}\,T_2^{10}-14\,T_1^5\,T_2^{11}+77\,T_1^6\,T_2^{11}-119\,T_1^7\,T_2^{11}-21\,T_1^8\,T_2^{11}+301\,T_1^9\,T_2^{11}-301\,T_1^{10}\,T_2^{11}+133\,T_1^{11}\,T_2^{11}-$$
$$21\,T_1^{12}\,T_2^{11}+3\,T_1^6\,T_2^{12}-21\,T_1^7\,T_2^{12}+57\,T_1^8\,T_2^{12}-81\,T_1^9\,T_2^{12}+57\,T_1^{10}\,T_2^{12}-21\,T_1^{11}\,T_2^{12}+3\,T_1^{12}\,T_2^{12}\Big)\Big\},$$

$$\text{Knot}[10,48]\to\Big\{\frac{\left(1-T+2\,T^2-2\,T^3+T^4\right)\,\left(1-2\,T+2\,T^2-T^3+T^4\right)}{T^4},$$

$$-\frac{1}{T_1^9\,T_2^8}\,\Big(-4+12\,T_1-24\,T_1^2+36\,T_1^3-44\,T_1^4+36\,T_1^5-24\,T_1^6+12\,T_1^7-4\,T_1^8+12\,T_2-27\,T_1\,T_2+45\,T_1^2\,T_2-$$
$$54\,T_1^3\,T_2+51\,T_1^4\,T_2-9\,T_1^5\,T_2-9\,T_1^6\,T_2+18\,T_1^7\,T_2-15\,T_1^8\,T_2+9\,T_1^9\,T_2-24\,T_2^2+45\,T_1\,T_2^2-74\,T_1^2\,T_2^2+$$
$$88\,T_1^3\,T_2^2-91\,T_1^4\,T_2^2+26\,T_1^5\,T_2^2-31\,T_1^6\,T_2^2+16\,T_1^7\,T_2^2-14\,T_1^8\,T_2^2+9\,T_1^9\,T_2^2-12\,T_1^{10}\,T_2^2+36\,T_2^3-$$
$$54\,T_1\,T_2^3+88\,T_1^2\,T_2^3-97\,T_1^3\,T_2^3+100\,T_1^4\,T_2^3-3\,T_1^5\,T_2^3+42\,T_1^6\,T_2^3-8\,T_1^7\,T_2^3+11\,T_1^8\,T_2^3-2\,T_1^9\,T_2^3+$$

$$9\,T_1^{10}\,T_2^3+9\,T_1^{11}\,T_2^3-44\,T_2^4+51\,T_1\,T_2^4-91\,T_1^2\,T_2^4+100\,T_1^3\,T_2^4-115\,T_1^4\,T_2^4-83\,T_1^6\,T_2^4+6\,T_1^7\,T_2^4+T_1^8\,T_2^4-$$
$$26\,T_1^9\,T_2^4+11\,T_1^{10}\,T_2^4-27\,T_1^{11}\,T_2^4+36\,T_2^5-9\,T_1\,T_2^5+26\,T_1^2\,T_2^5-3\,T_1^3\,T_2^5+110\,T_1^5\,T_2^5+44\,T_1^6\,T_2^5+$$
$$44\,T_1^7\,T_2^5-16\,T_1^8\,T_2^5+9\,T_1^9\,T_2^5+24\,T_1^{10}\,T_2^5-T_1^{11}\,T_2^5+27\,T_1^{12}\,T_2^5-9\,T_1^{13}\,T_2^5-24\,T_2^6-9\,T_1\,T_2^6-31\,T_1^2\,T_2^6+$$
$$42\,T_1^3\,T_2^6-83\,T_1^4\,T_2^6+44\,T_1^5\,T_2^6-212\,T_1^6\,T_2^6+38\,T_1^7\,T_2^6-20\,T_1^8\,T_2^6+8\,T_1^9\,T_2^6+T_1^{10}\,T_2^6-30\,T_1^{11}\,T_2^6-$$
$$7\,T_1^{12}\,T_2^6-9\,T_1^{13}\,T_2^6+12\,T_1^{14}\,T_2^6+12\,T_2^7+18\,T_1\,T_2^7+16\,T_1^2\,T_2^7-8\,T_1^3\,T_2^7+6\,T_1^4\,T_2^7+44\,T_1^5\,T_2^7+38\,T_1^6\,T_2^7+$$
$$158\,T_1^7\,T_2^7-118\,T_1^8\,T_2^7+29\,T_1^9\,T_2^7-28\,T_1^{10}\,T_2^7+9\,T_1^{11}\,T_2^7+28\,T_1^{12}\,T_2^7-2\,T_1^{13}\,T_2^7-9\,T_1^{14}\,T_2^7-9\,T_1^{15}\,T_2^7-$$
$$4\,T_2^8-15\,T_1\,T_2^8-14\,T_1^2\,T_2^8+11\,T_1^3\,T_2^8+T_1^4\,T_2^8-16\,T_1^5\,T_2^8-20\,T_1^6\,T_2^8-118\,T_1^7\,T_2^8+48\,T_1^8\,T_2^8+74\,T_1^9\,T_2^8+$$
$$10\,T_1^{10}\,T_2^8+20\,T_1^{11}\,T_2^8-19\,T_1^{12}\,T_2^8-7\,T_1^{13}\,T_2^8+16\,T_1^{14}\,T_2^8+15\,T_1^{15}\,T_2^8+4\,T_1^{16}\,T_2^8+9\,T_1\,T_2^9+9\,T_1^2\,T_2^9-$$
$$2\,T_1^3\,T_2^9-26\,T_1^4\,T_2^9+9\,T_1^5\,T_2^9+8\,T_1^6\,T_2^9+29\,T_1^7\,T_2^9+74\,T_1^8\,T_2^9-202\,T_1^9\,T_2^9+20\,T_1^{10}\,T_2^9-64\,T_1^{11}\,T_2^9+$$
$$12\,T_1^{12}\,T_2^9+10\,T_1^{13}\,T_2^9-20\,T_1^{14}\,T_2^9-18\,T_1^{15}\,T_2^9-12\,T_1^{16}\,T_2^9-12\,T_1^2\,T_2^{10}+9\,T_1^3\,T_2^{10}+11\,T_1^4\,T_2^{10}+$$
$$24\,T_1^5\,T_2^{10}+T_1^6\,T_2^{10}-28\,T_1^7\,T_2^{10}+10\,T_1^8\,T_2^{10}+20\,T_1^9\,T_2^{10}+202\,T_1^{10}\,T_2^{10}-64\,T_1^{11}\,T_2^{10}+85\,T_1^{12}\,T_2^{10}-$$
$$48\,T_1^{13}\,T_2^{10}+35\,T_1^{14}\,T_2^{10}+9\,T_1^{15}\,T_2^{10}+24\,T_1^{16}\,T_2^{10}+9\,T_1^3\,T_2^{11}-27\,T_1^4\,T_2^{11}-T_1^5\,T_2^{11}-30\,T_1^6\,T_2^{11}+$$
$$9\,T_1^7\,T_2^{11}+20\,T_1^8\,T_2^{11}-64\,T_1^9\,T_2^{11}-64\,T_1^{10}\,T_2^{11}-106\,T_1^{11}\,T_2^{11}+18\,T_1^{12}\,T_2^{11}-3\,T_1^{13}\,T_2^{11}-28\,T_1^{14}\,T_2^{11}+$$
$$9\,T_1^{15}\,T_2^{11}-36\,T_1^{16}\,T_2^{11}+27\,T_1^5\,T_2^{12}-7\,T_1^6\,T_2^{12}+28\,T_1^7\,T_2^{12}-19\,T_1^8\,T_2^{12}+12\,T_1^9\,T_2^{12}+85\,T_1^{10}\,T_2^{12}+$$
$$18\,T_1^{11}\,T_2^{12}+97\,T_1^{12}\,T_2^{12}-98\,T_1^{13}\,T_2^{12}+95\,T_1^{14}\,T_2^{12}-51\,T_1^{15}\,T_2^{12}+44\,T_1^{16}\,T_2^{12}-9\,T_1^5\,T_2^{13}-9\,T_1^6\,T_2^{13}-$$
$$2\,T_1^7\,T_2^{13}-7\,T_1^8\,T_2^{13}+10\,T_1^9\,T_2^{13}-48\,T_1^{10}\,T_2^{13}-3\,T_1^{11}\,T_2^{13}-98\,T_1^{12}\,T_2^{13}+101\,T_1^{13}\,T_2^{13}-92\,T_1^{14}\,T_2^{13}+$$
$$54\,T_1^{15}\,T_2^{13}-36\,T_1^{16}\,T_2^{13}+12\,T_1^6\,T_2^{14}-9\,T_1^7\,T_2^{14}+16\,T_1^8\,T_2^{14}-20\,T_1^9\,T_2^{14}+35\,T_1^{10}\,T_2^{14}-28\,T_1^{11}\,T_2^{14}+$$
$$95\,T_1^{12}\,T_2^{14}-92\,T_1^{13}\,T_2^{14}+76\,T_1^{14}\,T_2^{14}-45\,T_1^{15}\,T_2^{14}+24\,T_1^{16}\,T_2^{14}-9\,T_1^7\,T_2^{15}+15\,T_1^8\,T_2^{15}-18\,T_1^9\,T_2^{15}+$$
$$9\,T_1^{10}\,T_2^{15}+9\,T_1^{11}\,T_2^{15}-51\,T_1^{12}\,T_2^{15}+54\,T_1^{13}\,T_2^{15}-45\,T_1^{14}\,T_2^{15}+27\,T_1^{15}\,T_2^{15}-12\,T_1^{16}\,T_2^{15}+4\,T_1^8\,T_2^{16}-$$
$$12\,T_1^9\,T_2^{16}+24\,T_1^{10}\,T_2^{16}-36\,T_1^{11}\,T_2^{16}+44\,T_1^{12}\,T_2^{16}-36\,T_1^{13}\,T_2^{16}+24\,T_1^{14}\,T_2^{16}-12\,T_1^{15}\,T_2^{16}+4\,T_1^{16}\,T_2^{16}\Big)\Big\},$$

$$\text{Knot}[10,\,75]\to\Big\{-\frac{\left(-1+3\,T-4\,T^2+T^3\right)\,\left(-1+4\,T-3\,T^2+T^3\right)}{T^3},$$

$$\frac{1}{T_1^7\,T_2^6}\,\Big(-3+21\,T_1-57\,T_1^2+81\,T_1^3-57\,T_1^4+21\,T_1^5-3\,T_1^6+21\,T_2-133\,T_1\,T_2+301\,T_1^2\,T_2-301\,T_1^3\,T_2+$$
$$21\,T_1^4\,T_2+119\,T_1^5\,T_2-77\,T_1^6\,T_2+14\,T_1^7\,T_2-57\,T_2^2+301\,T_1\,T_2^2-414\,T_1^2\,T_2^2-198\,T_1^3\,T_2^2+1218\,T_1^4\,T_2^2-$$
$$958\,T_1^5\,T_2^2+270\,T_1^6\,T_2^2+35\,T_1^7\,T_2^2-19\,T_1^8\,T_2^2+81\,T_2^3-301\,T_1\,T_2^3-198\,T_1^2\,T_2^3+1954\,T_1^3\,T_2^3-3148\,T_1^4\,T_2^3+$$
$$864\,T_1^5\,T_2^3+764\,T_1^6\,T_2^3-673\,T_1^7\,T_2^3+133\,T_1^8\,T_2^3-57\,T_2^4+21\,T_1\,T_2^4+1218\,T_1^2\,T_2^4-3148\,T_1^3\,T_2^4+$$
$$2358\,T_1^4\,T_2^4+2906\,T_1^5\,T_2^4-3834\,T_1^6\,T_2^4+1604\,T_1^7\,T_2^4+16\,T_1^8\,T_2^4-133\,T_1^9\,T_2^4+19\,T_1^{10}\,T_2^4+21\,T_2^5+$$
$$119\,T_1\,T_2^5-958\,T_1^2\,T_2^5+864\,T_1^3\,T_2^5+2906\,T_1^4\,T_2^5-8283\,T_1^5\,T_2^5+5221\,T_1^6\,T_2^5+75\,T_1^7\,T_2^5-1668\,T_1^8\,T_2^5+$$
$$657\,T_1^9\,T_2^5-35\,T_1^{10}\,T_2^5-14\,T_1^{11}\,T_2^5-3\,T_2^6-77\,T_1\,T_2^6+270\,T_1^2\,T_2^6+764\,T_1^3\,T_2^6-3834\,T_1^4\,T_2^6+5221\,T_1^5\,T_2^6+$$
$$900\,T_1^6\,T_2^6-5733\,T_1^7\,T_2^6+3984\,T_1^8\,T_2^6-728\,T_1^9\,T_2^6-266\,T_1^{10}\,T_2^6+77\,T_1^{11}\,T_2^6+3\,T_1^{12}\,T_2^6+14\,T_1\,T_2^7+$$
$$35\,T_1^2\,T_2^7-673\,T_1^3\,T_2^7+1604\,T_1^4\,T_2^7+75\,T_1^5\,T_2^7-5733\,T_1^6\,T_2^7+7771\,T_1^7\,T_2^7-2756\,T_1^8\,T_2^7-928\,T_1^9\,T_2^7+$$
$$942\,T_1^{10}\,T_2^7-119\,T_1^{11}\,T_2^7-21\,T_1^{12}\,T_2^7-19\,T_1^2\,T_2^8+133\,T_1^3\,T_2^8+16\,T_1^4\,T_2^8-1668\,T_1^5\,T_2^8+3984\,T_1^6\,T_2^8-$$
$$2756\,T_1^7\,T_2^8-2208\,T_1^8\,T_2^8+3084\,T_1^9\,T_2^8-1186\,T_1^{10}\,T_2^8-21\,T_1^{11}\,T_2^8+57\,T_1^{12}\,T_2^8-133\,T_1^4\,T_2^9+657\,T_1^5\,T_2^9-$$
$$728\,T_1^6\,T_2^9-928\,T_1^7\,T_2^9+3084\,T_1^8\,T_2^9-1918\,T_1^9\,T_2^9+182\,T_1^{10}\,T_2^9+301\,T_1^{11}\,T_2^9-81\,T_1^{12}\,T_2^9+19\,T_1^4\,T_2^{10}-$$
$$35\,T_1^5\,T_2^{10}-266\,T_1^6\,T_2^{10}+942\,T_1^7\,T_2^{10}-1186\,T_1^8\,T_2^{10}+182\,T_1^9\,T_2^{10}+418\,T_1^{10}\,T_2^{10}-301\,T_1^{11}\,T_2^{10}+$$
$$57\,T_1^{12}\,T_2^{10}-14\,T_1^5\,T_2^{11}+77\,T_1^6\,T_2^{11}-119\,T_1^7\,T_2^{11}-21\,T_1^8\,T_2^{11}+301\,T_1^9\,T_2^{11}-301\,T_1^{10}\,T_2^{11}+133\,T_1^{11}\,T_2^{11}-$$
$$21\,T_1^{12}\,T_2^{11}+3\,T_1^6\,T_2^{12}-21\,T_1^7\,T_2^{12}+57\,T_1^8\,T_2^{12}-81\,T_1^9\,T_2^{12}+57\,T_1^{10}\,T_2^{12}-21\,T_1^{11}\,T_2^{12}+3\,T_1^{12}\,T_2^{12}\Big)\Big\},$$

$$\text{Knot}[10,\,87]\to\Big\{-\frac{(-2+T)\,(-1+2\,T)\,\left(1-T+T^2\right)^2}{T^3},$$

$$\frac{1}{T_1^7\,T_2^6}\,\Big(-25+112\,T_1-224\,T_1^2+287\,T_1^3-224\,T_1^4+112\,T_1^5-25\,T_1^6+112\,T_2-426\,T_1\,T_2+664\,T_1^2\,T_2-$$
$$606\,T_1^3\,T_2+132\,T_1^4\,T_2+178\,T_1^5\,T_2-228\,T_1^6\,T_2+76\,T_1^7\,T_2-224\,T_2^2+664\,T_1\,T_2^2-570\,T_1^2\,T_2^2-$$
$$95\,T_1^3\,T_2^2+1152\,T_1^4\,T_2^2-1103\,T_1^5\,T_2^2+582\,T_1^6\,T_2^2+16\,T_1^7\,T_2^2-80\,T_1^8\,T_2^2+287\,T_2^3-606\,T_1\,T_2^3-$$

$$95\,T_1^2\,T_2^3+1138\,T_1^3\,T_2^3-2061\,T_1^4\,T_2^3+855\,T_1^5\,T_2^3+214\,T_1^6\,T_2^3-797\,T_1^7\,T_2^3+312\,T_1^8\,T_2^3+11\,T_1^9\,T_2^3-224\,T_2^4+132\,T_1\,T_2^4+1152\,T_1^2\,T_2^4-2061\,T_1^3\,T_2^4+2114\,T_1^4\,T_2^4+358\,T_1^5\,T_2^4-1378\,T_1^6\,T_2^4+1251\,T_1^7\,T_2^4+18\,T_1^8\,T_2^4-336\,T_1^9\,T_2^4+64\,T_1^{10}\,T_2^4+112\,T_2^5+178\,T_1\,T_2^5-1103\,T_1^2\,T_2^5+855\,T_1^3\,T_2^5+358\,T_1^4\,T_2^5-2758\,T_1^5\,T_2^5+2084\,T_1^6\,T_2^5-290\,T_1^7\,T_2^5-1269\,T_1^8\,T_2^5+823\,T_1^9\,T_2^5+16\,T_1^{10}\,T_2^5-68\,T_1^{11}\,T_2^5-25\,T_2^6-228\,T_1\,T_2^6+582\,T_1^2\,T_2^6+214\,T_1^3\,T_2^6-1378\,T_1^4\,T_2^6+2084\,T_1^5\,T_2^6+72\,T_1^6\,T_2^6-2020\,T_1^7\,T_2^6+1970\,T_1^8\,T_2^6-338\,T_1^9\,T_2^6-606\,T_1^{10}\,T_2^6+204\,T_1^{11}\,T_2^6+23\,T_1^{12}\,T_2^6+76\,T_1\,T_2^7+16\,T_1^2\,T_2^7-797\,T_1^3\,T_2^7+1251\,T_1^4\,T_2^7-290\,T_1^5\,T_2^7-2020\,T_1^6\,T_2^7+2822\,T_1^7\,T_2^7-938\,T_1^8\,T_2^7-873\,T_1^9\,T_2^7+1129\,T_1^{10}\,T_2^7-146\,T_1^{11}\,T_2^7-104\,T_1^{12}\,T_2^7-80\,T_1^2\,T_2^8+312\,T_1^3\,T_2^8+18\,T_1^4\,T_2^8-1269\,T_1^5\,T_2^8+1970\,T_1^6\,T_2^8-938\,T_1^7\,T_2^8-1522\,T_1^8\,T_2^8+2043\,T_1^9\,T_2^8-1116\,T_1^{10}\,T_2^8-156\,T_1^{11}\,T_2^8+208\,T_1^{12}\,T_2^8+11\,T_1^3\,T_2^9-336\,T_1^4\,T_2^9+823\,T_1^5\,T_2^9-338\,T_1^6\,T_2^9-873\,T_1^7\,T_2^9+2043\,T_1^8\,T_2^9-1262\,T_1^9\,T_2^9+121\,T_1^{10}\,T_2^9+582\,T_1^{11}\,T_2^9-265\,T_1^{12}\,T_2^9+64\,T_1^4\,T_2^{10}+16\,T_1^5\,T_2^{10}-606\,T_1^6\,T_2^{10}+1129\,T_1^7\,T_2^{10}-1116\,T_1^8\,T_2^{10}+121\,T_1^9\,T_2^{10}+546\,T_1^{10}\,T_2^{10}-632\,T_1^{11}\,T_2^{10}+208\,T_1^{12}\,T_2^{10}-68\,T_1^5\,T_2^{11}+204\,T_1^6\,T_2^{11}-146\,T_1^7\,T_2^{11}-156\,T_1^8\,T_2^{11}+582\,T_1^9\,T_2^{11}-632\,T_1^{10}\,T_2^{11}+402\,T_1^{11}\,T_2^{11}-104\,T_1^{12}\,T_2^{11}+23\,T_1^6\,T_2^{12}-104\,T_1^7\,T_2^{12}+208\,T_1^8\,T_2^{12}-265\,T_1^9\,T_2^{12}+208\,T_1^{10}\,T_2^{12}-104\,T_1^{11}\,T_2^{12}+23\,T_1^{12}\,T_2^{12}\Big)\Big\},\ \text{Knot}[10,\,99]\to$$

$$\left\{\frac{\left(1-T+T^2\right)^4}{T^4},\ -\frac{4\left(1-T_1+T_1^2\right)^4\left(-1+T_1\,T_2\right)\left(1+T_1\,T_2\right)\left(1-T_2+T_2^2\right)^4\left(1-T_1\,T_2+T_1^2\,T_2^2\right)^3}{T_1^9\,T_2^8}\right\},$$

$$\text{Knot}[10,\,123]\to$$

$$\left\{\frac{\left(1-3\,T+3\,T^2-3\,T^3+T^4\right)^2}{T^4},\right.$$

$$-\frac{1}{T_1^9\,T_2^8}\,2\left(1-3\,T_1+3\,T_1^2-3\,T_1^3+T_1^4\right)^2\left(-1+T_1\,T_2\right)\left(1+T_1\,T_2\right)\left(2-3\,T_1\,T_2+2\,T_1^2\,T_2^2\right)$$

$$\left.\left(1-3\,T_2+3\,T_2^2-3\,T_2^3+T_2^4\right)^2\left(1-3\,T_1\,T_2+3\,T_1^2\,T_2^2-3\,T_1^3\,T_2^3+T_1^4\,T_2^4\right)\right\},$$

$$\text{Knot}[10,\,129]\to\left\{\frac{\left(2-2\,T+T^2\right)\left(1-2\,T+2\,T^2\right)}{T^2},\right.$$

$$-\frac{1}{T_1^5\,T_2^4}\left(-17+50\,T_1-75\,T_1^2+50\,T_1^3-17\,T_1^4+50\,T_2-124\,T_1\,T_2+146\,T_1^2\,T_2-34\,T_1^3\,T_2-28\,T_1^4\,T_2+26\,T_1^5\,T_2-75\,T_2^2+146\,T_1\,T_2^2-77\,T_1^2\,T_2^2-151\,T_1^3\,T_2^2+175\,T_1^4\,T_2^2-70\,T_1^5\,T_2^2-3\,T_1^6\,T_2^2+50\,T_2^3-34\,T_1\,T_2^3-151\,T_1^2\,T_2^3+332\,T_1^3\,T_2^3-190\,T_1^4\,T_2^3-43\,T_1^5\,T_2^3+74\,T_1^6\,T_2^3-22\,T_1^7\,T_2^3-17\,T_2^4-28\,T_1\,T_2^4+175\,T_1^2\,T_2^4-190\,T_1^3\,T_2^4-12\,T_1^4\,T_2^4+218\,T_1^5\,T_2^4-113\,T_1^6\,T_2^4+20\,T_1^7\,T_2^4+15\,T_1^8\,T_2^4+26\,T_1\,T_2^5-70\,T_1^2\,T_2^5-43\,T_1^3\,T_2^5+218\,T_1^4\,T_2^5-304\,T_1^5\,T_2^5+65\,T_1^6\,T_2^5+38\,T_1^7\,T_2^5-46\,T_1^8\,T_2^5-3\,T_1^2\,T_2^6+74\,T_1^3\,T_2^6-113\,T_1^4\,T_2^6+65\,T_1^5\,T_2^6+139\,T_1^6\,T_2^6-142\,T_1^7\,T_2^6+69\,T_1^8\,T_2^6-22\,T_1^3\,T_2^7+20\,T_1^4\,T_2^7+38\,T_1^5\,T_2^7-142\,T_1^6\,T_2^7+116\,T_1^7\,T_2^7-46\,T_1^8\,T_2^7+15\,T_1^4\,T_2^8-46\,T_1^5\,T_2^8+69\,T_1^6\,T_2^8-46\,T_1^7\,T_2^8+15\,T_1^8\,T_2^8\right)\Big\},$$

$$\text{Knot}[10,\,137]\to\left\{\frac{\left(1-3\,T+T^2\right)^2}{T^2},\ -\frac{1}{T_1^5\,T_2^4}\,2\left(1-3\,T_1+T_1^2\right)\left(1-3\,T_2+T_2^2\right)\right.$$

$$\left(1-3\,T_1\,T_2+T_1^2\,T_2^2\right)\left(-1+3\,T_1-T_1^2+3\,T_2-8\,T_1\,T_2+4\,T_1^2\,T_2-T_2^2+4\,T_1\,T_2^2-6\,T_1^2\,T_2^2-2\,T_1^3\,T_2^2+T_1^4\,T_2^2-2\,T_1^2\,T_2^3+10\,T_1^3\,T_2^3-3\,T_1^4\,T_2^3+T_1^2\,T_2^4-3\,T_1^3\,T_2^4+T_1^4\,T_2^4\right)\Big\},$$

$$\text{Knot}[10,\,140]\to\left\{\frac{\left(1-T+T^2\right)^2}{T^2},\ -\frac{1}{T_1^5\,T_2^4}\,2\left(-1+2\,T_1-3\,T_1^2+2\,T_1^3-T_1^4+2\,T_2-3\,T_1\,T_2+4\,T_1^2\,T_2-T_1^3\,T_2+T_1^5\,T_2-3\,T_2^2+4\,T_1\,T_2^2+T_1^2\,T_2^2-8\,T_1^3\,T_2^2+7\,T_1^4\,T_2^2-2\,T_1^5\,T_2^2+2\,T_2^3-T_1\,T_2^3-8\,T_1^2\,T_2^3+6\,T_1^3\,T_2^3-8\,T_1^5\,T_2^3+2\,T_1^6\,T_2^3-T_1^7\,T_2^3-T_2^4+7\,T_1^2\,T_2^4+4\,T_1^5\,T_2^4+5\,T_1^6\,T_2^4+T_1^8\,T_2^4+T_1\,T_2^5-2\,T_1^2\,T_2^5-8\,T_1^3\,T_2^5+4\,T_1^4\,T_2^5-2\,T_1^5\,T_2^5-8\,T_1^6\,T_2^5+T_1^7\,T_2^5-2\,T_1^8\,T_2^5+2\,T_1^3\,T_2^6+5\,T_1^4\,T_2^6-8\,T_1^5\,T_2^6+11\,T_1^6\,T_2^6-4\,T_1^7\,T_2^6+\right.$$

$$3\,T_1^8\,T_2^6 - T_1^3\,T_2^7 + T_1^5\,T_2^7 - 4\,T_1^6\,T_2^7 + 3\,T_1^7\,T_2^7 - 2\,T_1^8\,T_2^7 + T_1^4\,T_2^8 - 2\,T_1^5\,T_2^8 + 3\,T_1^6\,T_2^8 - 2\,T_1^7\,T_2^8 + T_1^8\,T_2^8\Big)\Big\},$$

$$\text{Knot}[10,153] \to \Big\{\frac{\left(1-T+T^3\right)\,\left(1-T^2+T^3\right)}{T^3},\ -\frac{1}{T_1^7\,T_2^6}\,\Big(-2+2\,T_1+2\,T_1^2-6\,T_1^3+2\,T_1^4+2\,T_1^5-2\,T_1^6+$$
$$2\,T_2-3\,T_1\,T_2+3\,T_1^3\,T_2-T_1^4\,T_2-T_1^6\,T_2+T_1^7\,T_2+2\,T_2^2-2\,T_1^2\,T_2^2+6\,T_1^3\,T_2^2-2\,T_1^5\,T_2^2+2\,T_1^6\,T_2^2+2\,T_1^7\,T_2^2-$$
$$6\,T_2^3+3\,T_1\,T_2^3+6\,T_1^2\,T_2^3-14\,T_1^3\,T_2^3-2\,T_1^4\,T_2^3+2\,T_1^5\,T_2^3+4\,T_1^6\,T_2^3-5\,T_1^7\,T_2^3-5\,T_1^8\,T_2^3+3\,T_1^9\,T_2^3+2\,T_2^4-$$
$$T_1\,T_2^4-2\,T_1^3\,T_2^4+5\,T_1^4\,T_2^4+7\,T_1^5\,T_2^4-11\,T_1^6\,T_2^4-2\,T_1^7\,T_2^4+10\,T_1^8\,T_2^4-3\,T_1^9\,T_2^4-2\,T_1^{10}\,T_2^4+2\,T_2^5-2\,T_1^2\,T_2^5+$$
$$2\,T_1^3\,T_2^5+7\,T_1^4\,T_2^5-5\,T_1^5\,T_2^5-7\,T_1^6\,T_2^5+18\,T_1^7\,T_2^5-10\,T_1^8\,T_2^5-3\,T_1^9\,T_2^5+8\,T_1^{10}\,T_2^5-3\,T_1^{11}\,T_2^5-2\,T_2^6-T_1\,T_2^6+$$
$$2\,T_1^2\,T_2^6+4\,T_1^3\,T_2^6-11\,T_1^4\,T_2^6-7\,T_1^5\,T_2^6+24\,T_1^6\,T_2^6-13\,T_1^7\,T_2^6-9\,T_1^8\,T_2^6+16\,T_1^9\,T_2^6-6\,T_1^{10}\,T_2^6-3\,T_1^{11}\,T_2^6+$$
$$4\,T_1^{12}\,T_2^6+T_1\,T_2^7+2\,T_1^2\,T_2^7-5\,T_1^3\,T_2^7-2\,T_1^4\,T_2^7+18\,T_1^5\,T_2^7-13\,T_1^6\,T_2^7-15\,T_1^7\,T_2^7+29\,T_1^8\,T_2^7-14\,T_1^9\,T_2^7-$$
$$6\,T_1^{10}\,T_2^7+10\,T_1^{11}\,T_2^7-4\,T_1^{12}\,T_2^7-5\,T_1^3\,T_2^8+10\,T_1^4\,T_2^8-10\,T_1^5\,T_2^8-9\,T_1^6\,T_2^8+29\,T_1^7\,T_2^8-25\,T_1^8\,T_2^8-10\,T_1^9\,T_2^8+$$
$$20\,T_1^{10}\,T_2^8-7\,T_1^{11}\,T_2^8-4\,T_1^{12}\,T_2^8+3\,T_1^3\,T_2^9-3\,T_1^4\,T_2^9-3\,T_1^5\,T_2^9+16\,T_1^6\,T_2^9-14\,T_1^7\,T_2^9-10\,T_1^8\,T_2^9+34\,T_1^9\,T_2^9-$$
$$14\,T_1^{10}\,T_2^9-11\,T_1^{11}\,T_2^9+12\,T_1^{12}\,T_2^9-2\,T_1^4\,T_2^{10}+8\,T_1^5\,T_2^{10}-6\,T_1^6\,T_2^{10}-6\,T_1^7\,T_2^{10}+20\,T_1^8\,T_2^{10}-14\,T_1^9\,T_2^{10}-$$
$$2\,T_1^{10}\,T_2^{10}+10\,T_1^{11}\,T_2^{10}-4\,T_1^{12}\,T_2^{10}-3\,T_1^5\,T_2^{11}-3\,T_1^6\,T_2^{11}+10\,T_1^7\,T_2^{11}-7\,T_1^8\,T_2^{11}-11\,T_1^9\,T_2^{11}+10\,T_1^{10}\,T_2^{11}-$$
$$T_1^{11}\,T_2^{11}-4\,T_1^{12}\,T_2^{11}+4\,T_1^6\,T_2^{12}-4\,T_1^7\,T_2^{12}-4\,T_1^8\,T_2^{12}+12\,T_1^9\,T_2^{12}-4\,T_1^{10}\,T_2^{12}-4\,T_1^{11}\,T_2^{12}+4\,T_1^{12}\,T_2^{12}\Big)\Big\},$$

$$\text{Knot}[10,155] \to \Big\{-\frac{\left(-1+T-2\,T^2+T^3\right)\,\left(-1+2\,T-T^2+T^3\right)}{T^3},$$
$$\frac{1}{T_1^7\,T_2^6}\,\Big(-3+9\,T_1-15\,T_1^2+21\,T_1^3-15\,T_1^4+9\,T_1^5-3\,T_1^6+9\,T_2-21\,T_1\,T_2+27\,T_1^2\,T_2-33\,T_1^3\,T_2+3\,T_1^4\,T_2+$$
$$3\,T_1^5\,T_2-9\,T_1^6\,T_2+6\,T_1^7\,T_2-15\,T_2^2+27\,T_1\,T_2^2-28\,T_1^2\,T_2^2+38\,T_1^3\,T_2^2+16\,T_1^4\,T_2^2-2\,T_1^5\,T_2^2+12\,T_1^6\,T_2^2-$$
$$3\,T_1^7\,T_2^2-5\,T_1^8\,T_2^2+21\,T_2^3-33\,T_1\,T_2^3+38\,T_1^2\,T_2^3-64\,T_1^3\,T_2^3-24\,T_1^4\,T_2^3-12\,T_1^5\,T_2^3-16\,T_1^6\,T_2^3-7\,T_1^7\,T_2^3+$$
$$15\,T_1^8\,T_2^3-15\,T_2^4+3\,T_1\,T_2^4+16\,T_1^2\,T_2^4-24\,T_1^3\,T_2^4+151\,T_1^4\,T_2^4-53\,T_1^5\,T_2^4+63\,T_1^6\,T_2^4+4\,T_1^7\,T_2^4-10\,T_1^8\,T_2^4-$$
$$15\,T_1^9\,T_2^4+5\,T_1^{10}\,T_2^4+9\,T_2^5+3\,T_1\,T_2^5-2\,T_1^2\,T_2^5-12\,T_1^3\,T_2^5-53\,T_1^4\,T_2^5-91\,T_1^5\,T_2^5+29\,T_1^6\,T_2^5-48\,T_1^7\,T_2^5+$$
$$8\,T_1^8\,T_2^5+23\,T_1^9\,T_2^5+3\,T_1^{10}\,T_2^5-6\,T_1^{11}\,T_2^5-3\,T_2^6-9\,T_1\,T_2^6+12\,T_1^2\,T_2^6-16\,T_1^3\,T_2^6+63\,T_1^4\,T_2^6+29\,T_1^5\,T_2^6+$$
$$60\,T_1^6\,T_2^6-45\,T_1^7\,T_2^6+41\,T_1^8\,T_2^6-28\,T_1^9\,T_2^6-16\,T_1^{10}\,T_2^6+9\,T_1^{11}\,T_2^6+3\,T_1^{12}\,T_2^6+6\,T_1\,T_2^7-3\,T_1^2\,T_2^7-7\,T_1^3\,T_2^7+$$
$$4\,T_1^4\,T_2^7-48\,T_1^5\,T_2^7-45\,T_1^6\,T_2^7+75\,T_1^7\,T_2^7-43\,T_1^8\,T_2^7+24\,T_1^9\,T_2^7+18\,T_1^{10}\,T_2^7-3\,T_1^{11}\,T_2^7-9\,T_1^{12}\,T_2^7-$$
$$5\,T_1^2\,T_2^8+15\,T_1^3\,T_2^8-10\,T_1^4\,T_2^8+8\,T_1^5\,T_2^8+41\,T_1^6\,T_2^8-43\,T_1^7\,T_2^8-47\,T_1^8\,T_2^8+36\,T_1^9\,T_2^8-36\,T_1^{10}\,T_2^8-$$
$$3\,T_1^{11}\,T_2^8+15\,T_1^{12}\,T_2^8-15\,T_1^4\,T_2^9+23\,T_1^5\,T_2^9-28\,T_1^6\,T_2^9+24\,T_1^7\,T_2^9+36\,T_1^8\,T_2^9+20\,T_1^9\,T_2^9-22\,T_1^{10}\,T_2^9+$$
$$33\,T_1^{11}\,T_2^9-21\,T_1^{12}\,T_2^9+5\,T_1^4\,T_2^{10}+3\,T_1^5\,T_2^{10}-16\,T_1^6\,T_2^{10}+18\,T_1^7\,T_2^{10}-36\,T_1^8\,T_2^{10}-22\,T_1^9\,T_2^{10}+24\,T_1^{10}\,T_2^{10}-$$
$$27\,T_1^{11}\,T_2^{10}+15\,T_1^{12}\,T_2^{10}-6\,T_1^5\,T_2^{11}+9\,T_1^6\,T_2^{11}-3\,T_1^7\,T_2^{11}-3\,T_1^8\,T_2^{11}+33\,T_1^9\,T_2^{11}-27\,T_1^{10}\,T_2^{11}+21\,T_1^{11}\,T_2^{11}-$$
$$9\,T_1^{12}\,T_2^{11}+3\,T_1^6\,T_2^{12}-9\,T_1^7\,T_2^{12}+15\,T_1^8\,T_2^{12}-21\,T_1^9\,T_2^{12}+15\,T_1^{10}\,T_2^{12}-9\,T_1^{11}\,T_2^{12}+3\,T_1^{12}\,T_2^{12}\Big)\Big\}\Big\}$$

In[•]:=
```
DunfieldKnots = ReadList["../../People/Dunfield/nmd_random_knots"] /. k_Integer :> k + 1;
DK[n_] := DunfieldKnots[[n - 2]]
```

In[•]:=
```
Crossings[DK[576]]
```

Out[•]=
```
576
```

In[•]:=
```
AbsoluteTiming[∅[DK[3]]]
```

Out[•]=
$$\Big\{0.0110907,$$
$$\Big\{\frac{1-T+T^2}{T},\ -\frac{-2+2\,T_1-2\,T_1^2+2\,T_2-T_1\,T_2+T_1^2\,T_2+T_1^3\,T_2-2\,T_2^2+T_1\,T_2^2-T_1^3\,T_2^2+T_1\,T_2^3-T_1^2\,T_2^3+T_1^3\,T_2^3}{T_1^3\,T_2^2}\Big\}\Big\}$$

```
In[•]:= AbsoluteTiming[θ[DK[30]];]
Out[•]=
{2.91933, Null}

In[•]:= AbsoluteTiming[θ[DK[60]];]
Out[•]=
{27.4555, Null}

In[•]:= AbsoluteTiming[θ[DK[90]];]
Out[•]=
{227.389, Null}

In[•]:= AbsoluteTiming[θ120 = θ[DK[120]];]
Out[•]=
{761.428, Null}

In[•]:= Put[θ120, "Theta4DK120.m"]

In[•]:= AbsoluteTiming[θ[DK[150]];]
Out[•]=
{2357.39, Null}
```

(during the previous computation I biked home, so the AbsoluteTiming is too much)

```
In[•]:= AbsoluteTiming[θ[DK[180]];]
Out[•]=
{5391.24, Null}

In[•]:= AbsoluteTiming[θ[DK[210]];]
Out[•]=
{9613.68, Null}

In[•]:= AbsoluteTiming[θ[DK[240]];]
Out[•]=
{22462.4, Null}

In[•]:= AbsoluteTiming[θ[DK[270]];]
```

Mathematica crashed while trying the above computation.

```
In[•]:= AbsoluteTiming[θ[DK[300]];]

In[•]:= Do[Echo@AbsoluteTiming[n → θ22/7,34/21[DK[n]]], {n, 100, 1000, 100}]
```

» $\left\{1.61069,\ 100 \to \left\{\frac{35\,388\,936\,522\,490\,931\,938\,908\,923\,343\,364\,558\,590\,414\,632\,463\,375\,508\,742\,089}{264\,554\,736\,545\,069\,605\,885\,631\,471\,128\,764\,401\,339\,301\,535\,744},\right.\right.$

$\frac{525\,106\,180\,586\,933\,014\,293\,865\,927\,609\,379\,271\,742\,972\,076\,277\,257\,025\,413\,914\,338\,499}{37\,324\,734\,431\,368\,634\,257\,516\,595\,221\,111\,791\,096\,751\,570\,183\,668\,795\,296\,664\,772\,608},$

$50\,463\,574\,955\,913\,231\,815\,385\,186\,261\,134\,862\,814\,456\,979\,779\,055\,953\,806\,229\,018\,368\,595\,827\,102\,502\,222\,063 -$
$117\,299\,430\,053\,887\,387\,799\,738\,329\,099\,644\,807\,147\,011\,110\,057\,363 /$
$78\,995\,482\,272\,843\,339\,527\,758\,555\,299\,340\,636\,345\,228\,530\,305\,737\,655\,210\,586\,135\,944\,082\,585\,735\,874\,960\,483 -$
$459\,404\,024\,632\,393\,880\,311\,552\,802\,816,$

$1\,454\,157\,786\,439\,833\,158\,073\,510\,296\,166\,664\,511\,893\,763\,273\,612\,593\,032\,508\,481\,123\,686\,040\,806\,094\,906\,156 -$
$614\,995\,898\,652\,264\,094\,554\,239\,129\,456\,588\,617\,926\,009\,448\,561\,362\,411\,131\,681\,477\,750\,921\,092\,323\,392\,446 -$
$495\,713\,256\,739\,833\,241\,088\,441\,458\,310\,488\,748\,504\,021\,676\,377\,213\,848\,231\,251\,594\,253\,709\,501\,604\,962\,486 -$
$232\,978\,740\,267\,199\,418\,705 /$
$6\,354\,394\,310\,390\,312\,822\,748\,925\,543\,501\,753\,140\,065\,543\,014\,212\,174\,242\,859\,988\,362\,148\,491\,816\,333\,172\,074 -$
$897\,312\,311\,833\,360\,632\,039\,282\,660\,548\,484\,996\,262\,406\,692\,880\,258\,968\,176\,579\,330\,558\,485\,056\,729\,052\,432 -$
$993\,117\,420\,466\,424\,511\,597\,633\,186\,035\,359\,746\,499\,955\,026\,301\,328\,379\,761\,576\,116\,224\}\}\}$

» $\{7.3406,\ 200 \to$

$\{-(72\,941\,025\,249\,230\,622\,091\,769\,886\,034\,332\,903\,937\,878\,867\,275\,035\,495\,850\,289\,152\,467\,601\,139\,729\,946\,680 -$
$691\,983\,449\,444\,238\,470\,173\,260\,899\,434\,879\,455\,547\,646\,677 /$
$79\,780\,391\,006\,864\,379\,747\,986\,053\,920\,193\,038\,680\,545\,693\,079\,622\,955\,011\,027\,668\,359\,182\,291\,645\,896\,903 -$
$218\,461\,275\,510\,571\,008),$

$13\,469\,039\,288\,358\,770\,844\,889\,186\,746\,410\,419\,403\,949\,987\,382\,833\,567\,787\,469\,752\,570\,946\,087\,488\,964\,056\,464 -$
$083\,956\,449\,441\,872\,952\,430\,656\,158\,262\,269\,810\,083\,547\,830\,189\,003\,289\,443\,154\,125 /$
$4\,240\,161\,130\,043\,882\,037\,823\,084\,995\,205\,726\,632\,691\,185\,572\,237\,933\,032\,456\,552\,833\,243\,815\,216\,744\,170\,971 -$
$881\,548\,991\,957\,331\,738\,797\,061\,590\,095\,303\,559\,046\,326\,968\,215\,750\,967\,296,$

$3\,058\,236\,953\,956\,402\,226\,943\,593\,388\,603\,713\,021\,071\,954\,699\,338\,326\,371\,450\,792\,000\,285\,430\,803\,814\,324\,110 -$
$911\,806\,690\,348\,020\,780\,088\,584\,382\,124\,603\,092\,971\,693\,299\,841\,778\,094\,187\,288\,377\,810\,035\,496\,408\,283\,188 -$
$130\,224\,093\,352\,681\,965\,580\,164\,395\,682\,496\,054\,504\,489\,551\,954\,332\,992\,465\,733\,972\,977\,594\,735\,369\,459\,115 -$
$633\,590\,163\,189\,798\,671\,672\,600\,349\,071\,866\,872\,120\,468\,309\,375 /$
$7\,389\,876\,778\,587\,670\,278\,409\,931\,856\,936\,212\,530\,694\,800\,372\,408\,625\,530\,583\,166\,986\,417\,139\,021\,654\,981\,203 -$
$589\,910\,511\,227\,601\,136\,991\,125\,732\,955\,086\,827\,137\,765\,975\,954\,473\,403\,792\,833\,419\,463\,344\,119\,138\,486\,741 -$
$874\,061\,457\,114\,480\,552\,952\,530\,491\,222\,541\,669\,872\,799\,328\,574\,041\,719\,777\,250\,405\,019\,238\,495\,420\,416,$

$6\,055\,772\,594\,728\,596\,455\,407\,813\,858\,842\,300\,305\,796\,013\,725\,424\,394\,998\,189\,342\,946\,654\,268\,378\,113\,712\,561 -$
$666\,514\,250\,963\,072\,635\,563\,646\,381\,893\,610\,078\,547\,194\,951\,520\,922\,176\,280\,055\,828\,977\,992\,790\,718\,440\,260 -$
$265\,768\,945\,724\,619\,893\,197\,506\,834\,661\,525\,408\,297\,131\,541\,146\,205\,253\,666\,135\,607\,099\,918\,846\,528\,650\,241 -$
$665\,431\,626\,142\,217\,658\,708\,337\,497\,196\,938\,649\,312\,026\,701\,158\,028\,747\,303\,072\,493\,247\,553\,589\,721\,953\,615 -$
$917\,185\,931\,220\,885\,007\,830\,743\,638\,653\,999\,049\,048\,125\,193\,483\,716\,503\,525\,421\,345\,329\,584\,261\,341\,390\,875 -$
$905\,831\,474\,490\,397\,967\,045\,567\,474\,649\,173\,307\,967\,130\,069\,663\,503\,481\,368\,684\,552\,829\,185\,853\,244\,925\,938 -$
$721\,709\,248\,151\,559\,595\,774\,718\,912\,716\,699\,323\,404\,120\,108\,037\,147\,527\,421\,337\,576\,724\,551\,464\,750\,796\,875 /$
$1\,421\,817\,615\,970\,538\,122\,570\,446\,834\,523\,766\,760\,908\,222\,281\,779\,172\,226\,538\,184\,963\,226\,220\,605\,962\,435\,467 -$
$357\,530\,974\,629\,707\,529\,507\,866\,808\,092\,086\,184\,206\,711\,834\,962\,868\,351\,140\,030\,416\,830\,209\,310\,218\,134\,988 -$
$084\,400\,322\,916\,305\,290\,967\,755\,527\,855\,236\,924\,726\,203\,319\,295\,186\,578\,690\,288\,248\,904\,894\,482\,820\,217\,219 -$
$936\,617\,834\,694\,117\,100\,777\,085\,133\,489\,362\,587\,426\,540\,579\,190\,556\,123\,633\,779\,730\,340\,813\,256\,893\,167\,951 -$
$316\,991\,548\,797\,991\,645\,638\,090\,254\,721\,915\,664\,804\,963\,927\,625\,281\,349\,644\,642\,357\,197\,558\,649\,709\,884\,596 -$
$639\,231\,084\,685\,491\,565\,277\,101\,735\,550\,434\,012\,716\,913\,649\,081\,518\,440\,855\,540\,999\,887\,069\,493\,854\,208\}\}$

» {75.9612, 300 →
{54 300 428 014 802 247 763 147 703 343 836 297 447 025 108 824 684 772 425 762 525 822 095 039 545 899 375 981 -
953 473 178 602 586 048 430 534 584 880 163 873 723 541 762 115 735 883 067 341 959 560 581 371 283 178 656 -
972 648 408 925 263 946 669 /
6 741 838 682 197 306 940 008 962 116 848 220 280 436 936 971 437 572 995 472 014 771 688 913 708 639 211 514 -
814 195 885 491 758 038 709 972 366 558 512 006 372 340 250 849 089 814 593 530 683 936 627 298 651 512 766 -
464,
1 084 128 382 249 743 436 824 663 986 171 685 150 273 646 351 713 912 937 150 171 700 202 730 323 922 010 700 -
294 161 035 743 289 238 368 194 879 507 950 682 627 574 784 328 439 797 605 967 434 628 113 238 619 877 448 -
933 104 349 915 804 145 167 106 117 098 828 582 214 168 974 179 /
458 816 114 715 914 322 691 410 371 538 510 819 835 906 604 695 828 488 701 592 446 861 566 683 983 329 916 -
364 046 021 667 534 630 113 436 786 891 827 119 466 479 256 930 424 597 743 983 452 685 367 746 981 696 618 -
500 346 273 956 034 473 567 578 882 100 679 941 606 973 898 752,
-(158 777 874 852 495 582 515 909 215 389 994 852 546 352 653 931 705 508 650 307 891 657 053 561 609 520 779 -
186 320 897 348 004 451 340 565 961 074 347 535 242 136 402 407 084 832 097 701 971 876 894 887 835 991 169 -
195 699 017 190 487 685 513 574 819 025 748 109 103 168 978 452 501 811 090 422 603 306 747 210 926 095 970 -
770 670 185 035 477 605 544 327 410 988 587 473 792 754 126 636 018 339 393 952 001 669 899 686 164 600 864 -
484 927 816 109 847 962 066 717 003 302 534 438 301 515 100 500 581 439 281 502 338 168 771 925 334 310 271 -
437 341 818 561 /
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