Pensieve Header: The Polyak Algebra computations run on sphere.math.toronto.edu on September 25 2009.

```
<< PolyakAlgebra.m
```

Corrections for running on sphere.math.toronto.edu

(sphere is faster so we can do degree 5, and it runs linux so we can use LinBox)

```
Analyze[reltypes_List, opts___Rule] := Analyze[5, reltypes, opts];
SetOptions[Dim, UseLinBox → True];
SetOptions[PSpace, ReduceEquivalences → False]
```

```
{Debug → False, Gr → False, ReduceEquivalences → False}
```

Runs

Runs as in the paper

(See http://www.math.toronto.edu/~drorbn/papers/v-Dims/)

```
Analyze[{"R3b", "R2c", "R1", "Round"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[4, 12] \to 0 & \text{QuotientSpace}[4, 12] \to 0 \\ \text{QuotientSpace}[40, 124] \to 1 & \text{QuotientSpace}[44, 136] \to 1 \\ \text{QuotientSpace}[576, 2064] \to 4 & \text{QuotientSpace}[620, 2200] \to 5 \\ \text{QuotientSpace}[10\,528, 42\,768] \to 17 & \text{QuotientSpace}[11\,148, 44\,968] \to 22 \end{pmatrix}$$

```
Analyze[{"R3b", "R2c", "R1"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[4, 8] \to 2 & \text{QuotientSpace}[4, 8] \to 2 \\ \text{QuotientSpace}[40, 84] \to 7 & \text{QuotientSpace}[44, 92] \to 9 \\ \text{QuotientSpace}[576, 1488] \to 42 & \text{QuotientSpace}[620, 1580] \to 51 \\ \text{QuotientSpace}[10\,528, 32\,240] \to 246 & \text{QuotientSpace}[11\,148, 33\,820] \to 297 \end{pmatrix}$$

```
Analyze[{"R3b", "R2c", "R1", "Descending"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[1, 2] \to 0 & \text{QuotientSpace}[1, 2] \to 0 \\ \text{QuotientSpace}[5, 9] \to 1 & \text{QuotientSpace}[6, 11] \to 1 \\ \text{QuotientSpace}[36, 76] \to 6 & \text{QuotientSpace}[42, 87] \to 7 \\ \text{QuotientSpace}[329, 795] \to 34 & \text{QuotientSpace}[371, 882] \to 41 \end{pmatrix}$$

```
Analyze[{"R3b", "R2c", "Round"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[12, 20] \to 1 & \text{QuotientSpace}[14, 22] \to 2 \\ \text{QuotientSpace}[120, 264] \to 2 & \text{QuotientSpace}[134, 286] \to 4 \\ \text{QuotientSpace}[1680, 4560] \to 7 & \text{QuotientSpace}[1814, 4846] \to 11 \\ \text{QuotientSpace}[30\,240, 97\,440] \to 29 & \text{QuotientSpace}[32\,054, 102\,286] \to 40 \end{pmatrix}$$

```
Analyze[{"R3b", "R2c"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[12, 8] \to 5 & \text{QuotientSpace}[14, 8] \to 7 \\ \text{QuotientSpace}[120, 144] \to 15 & \text{QuotientSpace}[134, 152] \to 22 \\ \text{QuotientSpace}[1680, 2880] \to 67 & \text{QuotientSpace}[1814, 3032] \to 89 \\ \text{QuotientSpace}[30\,240, 67\,200] \to 365 & \text{QuotientSpace}[32\,054, 70\,232] \to 454 \end{pmatrix}$$

```
Analyze[{"R3b", "R2c", "Descending"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[3, 2] \to 1 & \text{QuotientSpace}[4, 2] \to 2 \\ \text{QuotientSpace}[15, 16] \to 2 & \text{QuotientSpace}[19, 18] \to 4 \\ \text{QuotientSpace}[105, 150] \to 8 & \text{QuotientSpace}[124, 168] \to 12 \\ \text{QuotientSpace}[945, 1680] \to 42 & \text{QuotientSpace}[1069, 1848] \to 54 \end{pmatrix}$$

```
Analyze[{"R3b", "R1", "R1d", "R1a", "Round"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[4, 12] \to 0 & \text{QuotientSpace}[4, 12] \to 0 \\ \text{QuotientSpace}[40, 116] \to 1 & \text{QuotientSpace}[44, 128] \to 1 \\ \text{QuotientSpace}[576, 1944] \to 4 & \text{QuotientSpace}[620, 2072] \to 5 \\ \text{QuotientSpace}[10\,528, 40\,464] \to 17 & \text{QuotientSpace}[11\,148, 42\,536] \to 22 \end{pmatrix}$$

```
Analyze[{"R3b", "R1d", "R1a", "R1"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[4, 8] \to 2 & \text{QuotientSpace}[4, 8] \to 2 \\ \text{QuotientSpace}[40, 76] \to 7 & \text{QuotientSpace}[44, 84] \to 9 \\ \text{QuotientSpace}[576, 1368] \to 42 & \text{QuotientSpace}[620, 1452] \to 51 \\ \text{QuotientSpace}[10\,528, 29\,936] \to 246 & \text{QuotientSpace}[11\,148, 31\,388] \to 297 \end{pmatrix}$$

```
Analyze[{"R3b", "R1d", "R1", "Descending"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[1, 2] \to 0 & \text{QuotientSpace}[1, 2] \to 0 \\ \text{QuotientSpace}[5, 7] \to 1 & \text{QuotientSpace}[6, 9] \to 1 \\ \text{QuotientSpace}[36, 61] \to 6 & \text{QuotientSpace}[42, 70] \to 7 \\ \text{QuotientSpace}[329, 651] \to 34 & \text{QuotientSpace}[371, 721] \to 41 \end{pmatrix}$$

```
Analyze[{"R3b", "Round"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[12, 18] \to 2 & \text{QuotientSpace}[14, 20] \to 3 \\ \text{QuotientSpace}[120, 240] \to 5 & \text{QuotientSpace}[134, 260] \to 8 \\ \text{QuotientSpace}[1680, 4200] \to 19 & \text{QuotientSpace}[1814, 4460] \to 27 \\ \text{QuotientSpace}[30\,240, 90\,720] \to 77 & \text{QuotientSpace}[32\,054, 95\,180] \to 104 \end{pmatrix}$$

```
Analyze[{"R3b"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[12, 6] \to 7 & \text{QuotientSpace}[14, 6] \to 9 \\ \text{QuotientSpace}[120, 120] \to 27 & \text{QuotientSpace}[134, 126] \to 36 \\ \text{QuotientSpace}[1680, 2520] \to 139 & \text{QuotientSpace}[1814, 2646] \to 175 \\ \text{QuotientSpace}[30\,240, 60\,480] \to 813 & \text{QuotientSpace}[32\,054, 63\,126] \to 988 \end{pmatrix}$$

```
Analyze[{"R3b", "Descending"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[3, 1] \to 2 & \text{QuotientSpace}[4, 1] \to 3 \\ \text{QuotientSpace}[15, 10] \to 6 & \text{QuotientSpace}[19, 11] \to 9 \\ \text{QuotientSpace}[105, 105] \to 24 & \text{QuotientSpace}[124, 116] \to 33 \\ \text{QuotientSpace}[945, 1260] \to 120 & \text{QuotientSpace}[1069, 1376] \to 153 \end{pmatrix}$$

```
Analyze[{"R2c", "R1", "Round"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[4, 6] \to 0 & \text{QuotientSpace}[4, 6] \to 0 \\ \text{QuotientSpace}[40, 52] \to 4 & \text{QuotientSpace}[44, 58] \to 4 \\ \text{QuotientSpace}[576, 744] \to 44 & \text{QuotientSpace}[620, 802] \to 48 \\ \text{QuotientSpace}[10\,528, 13\,488] \to 648 & \text{QuotientSpace}[11\,148, 14\,290] \to 696 \end{pmatrix}$$

```
Analyze[{"R2c", "R1"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[4, 2] \to 2 & \text{QuotientSpace}[4, 2] \to 2 \\ \text{QuotientSpace}[40, 12] \to 28 & \text{QuotientSpace}[44, 14] \to 30 \\ \text{QuotientSpace}[576, 168] \to 420 & \text{QuotientSpace}[620, 182] \to 450 \\ \text{QuotientSpace}[10\,528, 2960] \to 7808 & \text{QuotientSpace}[11\,148, 3142] \to 8258 \end{pmatrix}$$

```
Analyze[{"R2c", "R1", "Descending"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[1, 1] \to 0 & \text{QuotientSpace}[1, 1] \to 0 \\ \text{QuotientSpace}[5, 3] \to 2 & \text{QuotientSpace}[6, 4] \to 2 \\ \text{QuotientSpace}[36, 21] \to 18 & \text{QuotientSpace}[42, 25] \to 20 \\ \text{QuotientSpace}[329, 185] \to 174 & \text{QuotientSpace}[371, 210] \to 194 \end{pmatrix}$$

```
Analyze[{"R2c", "Round"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[12, 14] \to 3 & \text{QuotientSpace}[14, 16] \to 4 \\ \text{QuotientSpace}[120, 144] \to 16 & \text{QuotientSpace}[134, 160] \to 20 \\ \text{QuotientSpace}[1680, 2040] \to 160 & \text{QuotientSpace}[1814, 2200] \to 180 \\ \text{QuotientSpace}[30\,240, 36\,960] \to 2248 & \text{QuotientSpace}[32\,054, 39\,160] \to 2428 \end{pmatrix}$$

```
Analyze[{"R2c"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[12, 2] \to 10 & \text{QuotientSpace}[14, 2] \to 12 \\ \text{QuotientSpace}[120, 24] \to 96 & \text{QuotientSpace}[134, 26] \to 108 \\ \text{QuotientSpace}[1680, 360] \to 1332 & \text{QuotientSpace}[1814, 386] \to 1440 \\ \text{QuotientSpace}[30\,240, 6720] \to 23\,880 & \text{QuotientSpace}[32\,054, 7106] \to 25\,320 \end{pmatrix}$$

```
Analyze[{"R2c", "Descending"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[3, 1] \to 2 & \text{QuotientSpace}[4, 1] \to 3 \\ \text{QuotientSpace}[15, 6] \to 9 & \text{QuotientSpace}[19, 7] \to 12 \\ \text{QuotientSpace}[105, 45] \to 63 & \text{QuotientSpace}[124, 52] \to 75 \\ \text{QuotientSpace}[945, 420] \to 570 & \text{QuotientSpace}[1069, 472] \to 645 \end{pmatrix}$$

Other runs

```
Analyze[{"R3b", "TC", "R1"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[4, 9] \to 1 & \text{QuotientSpace}[4, 9] \to 1 \\ \text{QuotientSpace}[40, 108] \to 1 & \text{QuotientSpace}[44, 117] \to 2 \\ \text{QuotientSpace}[576, 1980] \to 2 & \text{QuotientSpace}[620, 2097] \to 4 \\ \text{QuotientSpace}[10\,528, 43\,920] \to 2 & \text{QuotientSpace}[11\,148, 46\,017] \to 6 \end{pmatrix}$$

```
Analyze[{"R3b", "TC"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[12, 9] \to 4 & \text{QuotientSpace}[14, 9] \to 6 \\ \text{QuotientSpace}[120, 180] \to 7 & \text{QuotientSpace}[134, 189] \to 13 \\ \text{QuotientSpace}[1680, 3780] \to 12 & \text{QuotientSpace}[1814, 3969] \to 25 \\ \text{QuotientSpace}[30\,240, 90\,720] \to 19 & \text{QuotientSpace}[32\,054, 94\,689] \to 44 \end{pmatrix}$$

```
Analyze[{"R3b", "TC", "R1", "Round"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[4, 13] \to 0 & \text{QuotientSpace}[4, 13] \to 0 \\ \text{QuotientSpace}[40, 148] \to 0 & \text{QuotientSpace}[44, 161] \to 0 \\ \text{QuotientSpace}[576, 2556] \to 0 & \text{QuotientSpace}[620, 2717] \to 0 \\ \text{QuotientSpace}[10\,528, 54\,448] \to 0 & \text{QuotientSpace}[11\,148, 57\,165] \to 0 \end{pmatrix}$$

```
Analyze[{"R3b", "TC", "Round"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[12, 21] \to 1 & \text{QuotientSpace}[14, 23] \to 2 \\ \text{QuotientSpace}[120, 300] \to 1 & \text{QuotientSpace}[134, 323] \to 3 \\ \text{QuotientSpace}[1680, 5460] \to 1 & \text{QuotientSpace}[1814, 5783] \to 4 \\ \text{QuotientSpace}[30\,240, 120\,960] \to 1 & \text{QuotientSpace}[32\,054, 126\,743] \to 5 \end{pmatrix}$$

```
Analyze[{"R3b", "R1", "Round"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[4, 10] \to 0 & \text{QuotientSpace}[4, 10] \to 0 \\ \text{QuotientSpace}[40, 112] \to 1 & \text{QuotientSpace}[44, 122] \to 1 \\ \text{QuotientSpace}[576, 1896] \to 4 & \text{QuotientSpace}[620, 2018] \to 5 \\ \text{QuotientSpace}[10\,528, 39\,808] \to 17 & \text{QuotientSpace}[11\,148, 41\,826] \to 22 \end{pmatrix}$$

```
Analyze[{"R3b", "R1"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[4, 6] \to 2 & \text{QuotientSpace}[4, 6] \to 2 \\ \text{QuotientSpace}[40, 72] \to 7 & \text{QuotientSpace}[44, 78] \to 9 \\ \text{QuotientSpace}[576, 1320] \to 42 & \text{QuotientSpace}[620, 1398] \to 51 \\ \text{QuotientSpace}[10\,528, 29\,280] \to 246 & \text{QuotientSpace}[11\,148, 30\,678] \to 297 \end{pmatrix}$$

```
Analyze[{"R3b", "R1", "Descending"}] // MatrixForm
```

$$\begin{pmatrix} \text{QuotientSpace}[1, 1] \to 0 & \text{QuotientSpace}[1, 1] \to 0 \\ \text{QuotientSpace}[5, 6] \to 1 & \text{QuotientSpace}[6, 7] \to 1 \\ \text{QuotientSpace}[36, 55] \to 6 & \text{QuotientSpace}[42, 62] \to 7 \\ \text{QuotientSpace}[329, 610] \to 34 & \text{QuotientSpace}[371, 672] \to 41 \end{pmatrix}$$