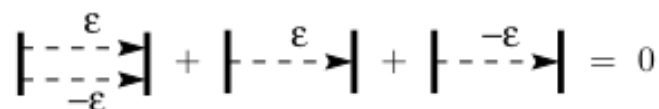
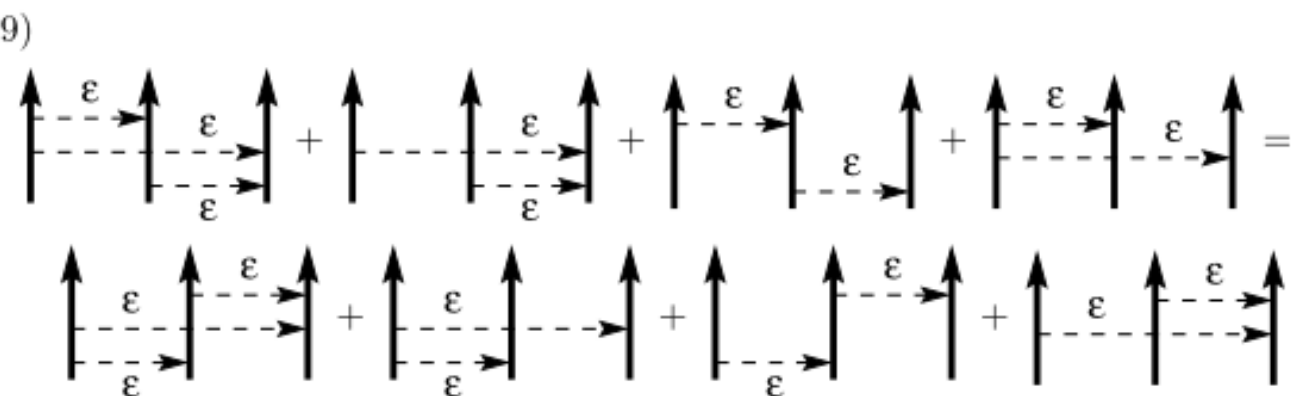


2.5. **The Polyak Algebra.** The *Polyak algebra* is the quotient of  $\mathcal{A}$  by the following relations:

(7)  = 0

(8)  = 0

(9)  =

R3b

## Program

```

SetAttributes[Diag, Orderless];
Place[{Ar}, {i_, j_}] := {Diag[Ar[i, j]], Diag[Ar[j, i]]};
Place[{Ar, objs_}, {i_, rest_}] := Flatten[Table[
  Outer[Join,
    Place[{Ar}, {i, {rest}[[k]]}],
    Place[{objs}, Delete[{rest}, k]]
  ],
  {k, Length[{rest]}]
]];
Diagrams[k_.*Ar] := Place[Table[Ar, {k}], Range[2 k]]

```

```

Draw[expr_] := expr /. diag_Diag => Draw[diag];
Draw[diag_Diag] := Module[
  {n = Length[diag]},
  Graphics[{
    Line[{{0, 0}, {2 n + 1, 0}}],
    (List @@ diag) /. {
      Ar[i_, j_] => {
        BezierCurve[{
          {i, 0}, {(i + j) / 2, Abs[i - j]}, {j, 0}
        ]},
        Line[{{j - 0.2 + 0.2 Sign[i - j], 0.4}, {j, 0}, {j + 0.2 + 0.2 Sign[i - j], 0.4}]]
      }
    ]
  ]
]

Place[{r: (TC | R3b), objs__}, {i_, rest__}] := Flatten[Table[
  Outer[Join,
    Place[{r}, {i, {rest}[[j]], {rest}[[k]]}],
    Place[{objs}, Delete[{rest}, {j}, {k}]]
  ],
  {k, 2, Length[{rest}]}, {j, 1, k - 1}
]];

Place[{R3b}, {i_, j_, k_}] :=
  Permutations[{i, j, k}] /. {i1_, j1_, k1_} => Diag[R3b[i1, j1, k1]];
Diagrams[R3b] := Place[{R3b}, {1, 2, 3}];
Diagrams[R3b + k_. * Ar] /; k > 0 := Flatten[
  Place[#, Range[2 k + 3]] & /@ Permutations[Table[Ar, {k}] ~ Append ~ R3b]
];
Place[{TC}, {i_, j_, k_}] := Diag /@ {TC[i, j, k], TC[j, k, i], TC[k, i, j]};
Diagrams[TC] := Place[{TC}, {1, 2, 3}];
Diagrams[TC + k_. * Ar] /; k > 0 := Flatten[
  Place[#, Range[2 k + 3]] & /@ Permutations[Table[Ar, {k}] ~ Append ~ TC]
];
Diagrams[TC + k_. * Ar] /; k < 0 := {};

Place[{R2c}, {i_, j_}] := {Diag[R2c[i, j]], Diag[R2c[j, i]]};
Place[{r: (R2c), objs__}, {i_, rest__}] := Flatten[Table[
  Outer[Join,
    Place[{r}, {i, {rest}[[j]]}],
    Place[{objs}, Delete[{rest}, j]]
  ],
  {j, 1, Length[{rest}]]
]];
Diagrams[R2c] := Place[{R2c}, {1, 2}];
Diagrams[R2c + k_. * Ar] /; k > 0 := Flatten[
  Place[#, Range[2 k + 2]] & /@ Permutations[Table[Ar, {k}] ~ Append ~ R2c]
];
Diagrams[R2c + k_. * Ar] /; k < 0 := {};

```

```

NormalizeDiag[diag_Diag] := Module[
  {indices = Union@@ (List @@ diag /. Ar → List)},
  diag /. Thread[indices → Range[Length[indices]]]
];
R[Diag[lft___, R3b[i_, j_, k_], rgt___]] := (
  (* RHS of (9) *)
  NormalizeDiag[Diag[lft, Ar[i, j], Ar[i + 0.5, k], Ar[j + 0.5, k + 0.5], rgt]]
  + NormalizeDiag[Diag[lft, Ar[i, j], Ar[i + 0.5, k], rgt]]
  + NormalizeDiag[Diag[lft, Ar[i, j], Ar[j + 0.5, k], rgt]]
  + NormalizeDiag[Diag[lft, Ar[i, k], Ar[j, k + 0.5], rgt]]
  (* LHS of (9) *)
  - NormalizeDiag[Diag[lft, Ar[j, k], Ar[i, k + 0.5], Ar[i + 0.5, j + 0.5], rgt]]
  - NormalizeDiag[Diag[lft, Ar[i, k], Ar[i + 0.5, j], rgt]]
  - NormalizeDiag[Diag[lft, Ar[i, j + 0.5], Ar[j, k], rgt]]
  - NormalizeDiag[Diag[lft, Ar[i, k + 0.5], Ar[j, k], rgt]]
);
RWrong1[Diag[lft___, R3b[i_, j_, k_], rgt___]] := (
  (* RHS of (9) *)
  NormalizeDiag[Diag[lft, Ar[i, j], Ar[i + 0.5, k], Ar[j + 0.5, k + 0.5], rgt]]
  + NormalizeDiag[Diag[lft, Ar[i, j], Ar[i + 0.5, k], rgt]]
  + NormalizeDiag[Diag[lft, Ar[i, j], Ar[j + 0.5, k], rgt]]
  + NormalizeDiag[Diag[lft, Ar[i, k], Ar[j, k + 0.5], rgt]]
  (* LHS of (9) *)
  + NormalizeDiag[Diag[lft, Ar[j, k], Ar[i, k + 0.5], Ar[i + 0.5, j + 0.5], rgt]]
  - NormalizeDiag[Diag[lft, Ar[i, k], Ar[i + 0.5, j], rgt]]
  - NormalizeDiag[Diag[lft, Ar[i, j + 0.5], Ar[j, k], rgt]]
  - NormalizeDiag[Diag[lft, Ar[i, k + 0.5], Ar[j, k], rgt]]
);
RWrong2[Diag[lft___, R3b[i_, j_, k_], rgt___]] := (
  (* RHS of (9) *)
  NormalizeDiag[Diag[lft, Ar[i, j], Ar[i + 0.5, k], Ar[j + 0.5, k + 0.5], rgt]]
  + NormalizeDiag[Diag[lft, Ar[i, j], Ar[i + 0.5, k], rgt]]
  + NormalizeDiag[Diag[lft, Ar[i, j], Ar[j + 0.5, k], rgt]]
  - NormalizeDiag[Diag[lft, Ar[i, k], Ar[j, k + 0.5], rgt]]
  (* LHS of (9) *)
  - NormalizeDiag[Diag[lft, Ar[j, k], Ar[i, k + 0.5], Ar[i + 0.5, j + 0.5], rgt]]
  - NormalizeDiag[Diag[lft, Ar[i, k], Ar[i + 0.5, j], rgt]]
  - NormalizeDiag[Diag[lft, Ar[i, j + 0.5], Ar[j, k], rgt]]
  - NormalizeDiag[Diag[lft, Ar[i, k + 0.5], Ar[j, k], rgt]]
);
RMinus[n_, Diag[lft___, R3b[i_, j_, k_], rgt___]] := Module[
  {Expand2, Expand3, n1},

```

```

n1 = n - Length[{lft, rgt}];
Expand2[Diag[Ar[i1_, j1_], Ar[i2_, j2_]]] := Sum[
  (-1)^(k1 + k2) * NormalizeDiag[Flatten[Diag[
    lft,
    Diag@@Table[Ar[i1 + 0.01 q, j1 + 0.01 q], {q, k1}],
    Diag@@Table[Ar[i2 + 0.01 q, j2 + 0.01 q], {q, k2}],
    rgt
  ]]],
  {k1, n1 - 1}, {k2, n1 - k1}
]; 28 686
Expand3[Diag[Ar[i1_, j1_], Ar[i2_, j2_], Ar[i3_, j3_]]] := Sum[
  (-1)^(k1 + k2 + k3) * NormalizeDiag[Flatten[Diag[
    lft,
    Diag@@Table[Ar[i1 + 0.01 q, j1 + 0.01 q], {q, k1}],
    Diag@@Table[Ar[i2 + 0.01 q, j2 + 0.01 q], {q, k2}],
    Diag@@Table[Ar[i3 + 0.01 q, j3 + 0.01 q], {q, k3}],
    rgt
  ]]],
  {k1, n1 - 2}, {k2, n1 - k1 - 1}, {k3, n1 - k1 - k2}
];
Expand[Plus[
  (
    Diag[Ar[i, j], Ar[i + 0.5, k]] +
    Diag[Ar[i, j], Ar[j + 0.5, k]] + Diag[Ar[i, k], Ar[j, k + 0.5]]
    - Diag[Ar[i, k], Ar[i + 0.5, j]] - Diag[Ar[i, j + 0.5], Ar[j, k]] -
    Diag[Ar[i, k + 0.5], Ar[j, k]]
  ) /. d_Diag => Expand2[d],
  (Diag[Ar[i, j], Ar[i + 0.5, k], Ar[j + 0.5, k + 0.5]] -
    Diag[Ar[j, k], Ar[i, k + 0.5], Ar[i + 0.5, j + 0.5]]) /. d_Diag => Expand3[d]
]]
];
R[n_, Diag[lft___, R2c[i_, j_], rgt___]] := Module[
  {n1},
  n1 = n - Length[{lft, rgt}];
  Sum[
    Expand[(-1)^k * s * NormalizeDiag[Flatten[Diag[
      lft,
      Diag@@Table[Ar[i + 0.01 q, j + s * 0.01 q], {q, k}],
      rgt
    ]]]],
    {k, 2, n1}, {s, -1, 1, 2}
  ]
]

```

```
];  
R[Diag[lft___, TC[i_, j_, k_], rgt___]] := (  
  +NormalizedDiag[Diag[lft, Ar[k, i], Ar[k+0.5, j], rgt]]  
  -NormalizedDiag[Diag[lft, Ar[k+0.5, i], Ar[k, j], rgt]]  
);  
ContainsShortArrow[diag_Diag] := (1 == Min[List@@diag /. Ar[i_, j_] -> Abs[i - j]]);
```

```

PSpace[m_] /; m < 2 := QuotientSpace[Diagrams[m Ar], {}];
PSpace[m_, reotypes_List, opts___] /; m ≥ 2 := Module[
{
  debug = Debug /. {opts} /. Debug → False,
  gr = Gr /. {opts} /. Gr → False,
  diags, rels, i
},
If[debug, Print[Date[], ": Starting work..."]];
diags = Join@@Table[Diagrams[k Ar], {k, If[gr, m, 1], m}];
reotypes /. "R1" => (
  diags = Select[diags, !ContainsShortArrow[#] &]
);
If[debug, Print[Date[], ": Computed ", Length[diags], " diagrams..."]];
rels = Join@@(reotypes /. {
  "R1" → {},
  "Round" → (diags /.
    d_Diag → (NormalizeDiag[d /. {1 → 2 Length[d], i_Integer → i - 1}] - d)
  ),
  "R3b" → (
    R /@ Join@@Table[Diagrams[R3b + k Ar], {k, If[gr, m - 2, 0], m - 2}] /.
    diag_Diag /; Length[diag] > m => 0
  ),
  "RWrong1" → (
    (RWrong1 /@ Join@@Table[Diagrams[R3b + k Ar], {k, If[gr, m - 2, 0], m - 2}] /.
    diag_Diag /; Length[diag] > m => 0
  ),
  "RWrong2" → (
    (RWrong2 /@ Join@@Table[Diagrams[R3b + k Ar], {k, If[gr, m - 2, 0], m - 2}] /.
    diag_Diag /; Length[diag] > m => 0
  ),
  "R3bMinus" → (
    RMinus[m, #] & /@ Join@@Table[Diagrams[R3b + k Ar], {k, If[gr, m - 2, 0], m - 2}]
  ),
  "R2c" → (
    R[m, #] & /@ Join@@Table[Diagrams[R2c + k Ar], {k, If[gr, m - 2, 0], m - 2}]
  ),
  "TC" → (
    R /@ Join@@Table[Diagrams[TC + k Ar], {k, If[gr, m - 2, 0], m - 2}]
  )
});
reotypes /. "R1" => (
  rels = DeleteCases[rels /. {d_Diag /; ContainsShortArrow[d] => 0}, 0]
);
If[debug, Print[Date[], ": Computed ", Length[rels], " relations..."]];
QuotientSpace[diags, rels]
]

```

```

Options[Dim] = {Debug → False, UseLinBox → False};
Dim[QuotientSpace[diags_List, rels_List], opts___] := Module[
{
  debug = Debug /. {opts} /. Options[Dim],
  uselinbox = UseLinBox /. {opts} /. Options[Dim],
  diagtoindex, mat, rel, matfile, rank
},
diagtoindex = Dispatch[Thread[Rule[diags, Range[Length[diags]]]];
If[uselinbox,
  matfile = OpenWrite["matfile.matrix"];
  WriteString[matfile, Length[rels], " ", Length[diags], " S\n"];
  Do[
    rel = rels[[i]];
    WriteString[matfile, StringJoin @@ Flatten[{
      ToString[Count[{rel}, _Diag, Infinity]], " ",
      {
        ToString[(# /. diagtoindex) - 1], " ",
        ToString[Coefficient[rel, #]], " "
      } & /@ Cases[{rel}, diag_Diag, Infinity],
      "\n"
    }]],
    {i, Length[rels]}
  ];
  matfile = Close[matfile];
  rank = RunThrough["./ComputeRank " <> matfile, Random[]][[2]];
  Length[diags] - rank,
  (* else *) mat = SparseArray[
    Join @@ Table[
      rel = rels[[i]];
      {i, # /. diagtoindex} → Coefficient[rel, #] & /@
      Cases[{rel}, diag_Diag, Infinity],
      {i, Length[rels]}
    ],
    {Length[rels], Length[diags]}
  ];
  If[debug, Print[Date[], ": Computed mat..."]];
  Length[diags] - MatrixRank[mat]
]
];
DimP[m_, reltypes_List, opts___] := Dim[PSpace[m, reltypes, opts], opts]

```

## Runs

```
Analyze[relytypes_] := Module[
  {qs},
  Table[
    {
      qs = PSpace[n, relytypes, Gr → True];
      (Length /@ qs) → Dim[qs, UseLinBox → True],
      qs = PSpace[n, relytypes, Gr → False];
      (Length /@ qs) → Dim[qs, UseLinBox → True]
    },
    {n, 2, 5}
  ]
]
```

```
Analyze[{"R3b", "R2c", "R1"}] // MatrixForm
```

$$\left( \begin{array}{cc} \text{QuotientSpace}[4, 8] \rightarrow 2 & \text{QuotientSpace}[4, 8] \rightarrow 2 \\ \text{QuotientSpace}[40, 84] \rightarrow 7 & \text{QuotientSpace}[44, 92] \rightarrow 9 \\ \text{QuotientSpace}[576, 1488] \rightarrow 42 & \text{QuotientSpace}[620, 1580] \rightarrow 51 \\ \text{QuotientSpace}[10\,528, 32\,240] \rightarrow 246 & \text{QuotientSpace}[11\,148, 33\,820] \rightarrow 297 \end{array} \right)$$

```
Analyze[{"R3b", "R2c"}] // MatrixForm
```

$$\left( \begin{array}{cc} \text{QuotientSpace}[12, 8] \rightarrow 5 & \text{QuotientSpace}[14, 8] \rightarrow 7 \\ \text{QuotientSpace}[120, 144] \rightarrow 15 & \text{QuotientSpace}[134, 152] \rightarrow 22 \\ \text{QuotientSpace}[1680, 2880] \rightarrow 67 & \text{QuotientSpace}[1814, 3032] \rightarrow 89 \\ \text{QuotientSpace}[30\,240, 67\,200] \rightarrow 365 & \text{QuotientSpace}[32\,054, 70\,232] \rightarrow 454 \end{array} \right)$$

```
Analyze[{"R3b", "R2c", "R1", "Round"}] // MatrixForm
```

$$\left( \begin{array}{cc} \text{QuotientSpace}[4, 12] \rightarrow 0 & \text{QuotientSpace}[4, 12] \rightarrow 0 \\ \text{QuotientSpace}[40, 124] \rightarrow 1 & \text{QuotientSpace}[44, 136] \rightarrow 1 \\ \text{QuotientSpace}[576, 2064] \rightarrow 4 & \text{QuotientSpace}[620, 2200] \rightarrow 5 \\ \text{QuotientSpace}[10\,528, 42\,768] \rightarrow 17 & \text{QuotientSpace}[11\,148, 44\,968] \rightarrow 22 \end{array} \right)$$

```
Analyze[{"R3b", "R2c", "Round"}] // MatrixForm
```

$$\left( \begin{array}{cc} \text{QuotientSpace}[12, 20] \rightarrow 1 & \text{QuotientSpace}[14, 22] \rightarrow 2 \\ \text{QuotientSpace}[120, 264] \rightarrow 2 & \text{QuotientSpace}[134, 286] \rightarrow 4 \\ \text{QuotientSpace}[1680, 4560] \rightarrow 7 & \text{QuotientSpace}[1814, 4846] \rightarrow 11 \\ \text{QuotientSpace}[30\,240, 97\,440] \rightarrow 29 & \text{QuotientSpace}[32\,054, 102\,286] \rightarrow 40 \end{array} \right)$$

```
Analyze[{"R3b", "R1"}] // MatrixForm
```

$$\left( \begin{array}{cc} \text{QuotientSpace}[4, 6] \rightarrow 2 & \text{QuotientSpace}[4, 6] \rightarrow 2 \\ \text{QuotientSpace}[40, 72] \rightarrow 7 & \text{QuotientSpace}[44, 78] \rightarrow 9 \\ \text{QuotientSpace}[576, 1320] \rightarrow 42 & \text{QuotientSpace}[620, 1398] \rightarrow 51 \\ \text{QuotientSpace}[10\,528, 29\,280] \rightarrow 246 & \text{QuotientSpace}[11\,148, 30\,678] \rightarrow 297 \end{array} \right)$$

```
Analyze[{"R3b"}] // MatrixForm
```

$$\left( \begin{array}{cc} \text{QuotientSpace}[12, 6] \rightarrow 7 & \text{QuotientSpace}[14, 6] \rightarrow 9 \\ \text{QuotientSpace}[120, 120] \rightarrow 27 & \text{QuotientSpace}[134, 126] \rightarrow 36 \\ \text{QuotientSpace}[1680, 2520] \rightarrow 139 & \text{QuotientSpace}[1814, 2646] \rightarrow 175 \\ \text{QuotientSpace}[30\,240, 60\,480] \rightarrow 813 & \text{QuotientSpace}[32\,054, 63\,126] \rightarrow 988 \end{array} \right)$$



**Analyze[{"R3b", "R1", "Round"}] // MatrixForm**

$$\left( \begin{array}{ll} \text{QuotientSpace}[4, 10] \rightarrow 0 & \text{QuotientSpace}[4, 10] \rightarrow 0 \\ \text{QuotientSpace}[40, 112] \rightarrow 1 & \text{QuotientSpace}[44, 122] \rightarrow 1 \\ \text{QuotientSpace}[576, 1896] \rightarrow 4 & \text{QuotientSpace}[620, 2018] \rightarrow 5 \\ \text{QuotientSpace}[10\,528, 39\,808] \rightarrow 17 & \text{QuotientSpace}[11\,148, 41\,826] \rightarrow 22 \end{array} \right)$$

**Analyze[{"R3b", "Round"}] // MatrixForm**

$$\left( \begin{array}{ll} \text{QuotientSpace}[12, 18] \rightarrow 2 & \text{QuotientSpace}[14, 20] \rightarrow 3 \\ \text{QuotientSpace}[120, 240] \rightarrow 5 & \text{QuotientSpace}[134, 260] \rightarrow 8 \\ \text{QuotientSpace}[1680, 4200] \rightarrow 19 & \text{QuotientSpace}[1814, 4460] \rightarrow 27 \\ \text{QuotientSpace}[30\,240, 90\,720] \rightarrow 77 & \text{QuotientSpace}[32\,054, 95\,180] \rightarrow 104 \end{array} \right)$$

**Analyze[{"R3b", "TC", "R1"}] // MatrixForm**

$$\left( \begin{array}{ll} \text{QuotientSpace}[4, 9] \rightarrow 1 & \text{QuotientSpace}[4, 9] \rightarrow 1 \\ \text{QuotientSpace}[40, 108] \rightarrow 1 & \text{QuotientSpace}[44, 117] \rightarrow 2 \\ \text{QuotientSpace}[576, 1980] \rightarrow 2 & \text{QuotientSpace}[620, 2097] \rightarrow 4 \\ \text{QuotientSpace}[10\,528, 43\,920] \rightarrow 2 & \text{QuotientSpace}[11\,148, 46\,017] \rightarrow 6 \end{array} \right)$$

**Analyze[{"R3b", "TC"}] // MatrixForm**

$$\left( \begin{array}{ll} \text{QuotientSpace}[12, 9] \rightarrow 4 & \text{QuotientSpace}[14, 9] \rightarrow 6 \\ \text{QuotientSpace}[120, 180] \rightarrow 7 & \text{QuotientSpace}[134, 189] \rightarrow 13 \\ \text{QuotientSpace}[1680, 3780] \rightarrow 12 & \text{QuotientSpace}[1814, 3969] \rightarrow 25 \\ \text{QuotientSpace}[30\,240, 90\,720] \rightarrow 19 & \text{QuotientSpace}[32\,054, 94\,689] \rightarrow 44 \end{array} \right)$$

**Analyze[{"R3b", "TC", "R1", "Round"}] // MatrixForm**

$$\left( \begin{array}{ll} \text{QuotientSpace}[4, 13] \rightarrow 0 & \text{QuotientSpace}[4, 13] \rightarrow 0 \\ \text{QuotientSpace}[40, 148] \rightarrow 0 & \text{QuotientSpace}[44, 161] \rightarrow 0 \\ \text{QuotientSpace}[576, 2556] \rightarrow 0 & \text{QuotientSpace}[620, 2717] \rightarrow 0 \\ \text{QuotientSpace}[10\,528, 54\,448] \rightarrow 0 & \text{QuotientSpace}[11\,148, 57\,165] \rightarrow 0 \end{array} \right)$$

**Analyze[{"R3b", "TC", "Round"}] // MatrixForm**

$$\left( \begin{array}{ll} \text{QuotientSpace}[12, 21] \rightarrow 1 & \text{QuotientSpace}[14, 23] \rightarrow 2 \\ \text{QuotientSpace}[120, 300] \rightarrow 1 & \text{QuotientSpace}[134, 323] \rightarrow 3 \\ \text{QuotientSpace}[1680, 5460] \rightarrow 1 & \text{QuotientSpace}[1814, 5783] \rightarrow 4 \\ \text{QuotientSpace}[30\,240, 120\,960] \rightarrow 1 & \text{QuotientSpace}[32\,054, 126\,743] \rightarrow 5 \end{array} \right)$$

**Analyze[{"R2c", "R1"}] // MatrixForm**

$$\left( \begin{array}{ll} \text{QuotientSpace}[4, 2] \rightarrow 2 & \text{QuotientSpace}[4, 2] \rightarrow 2 \\ \text{QuotientSpace}[40, 12] \rightarrow 28 & \text{QuotientSpace}[44, 14] \rightarrow 30 \\ \text{QuotientSpace}[576, 168] \rightarrow 420 & \text{QuotientSpace}[620, 182] \rightarrow 450 \\ \text{QuotientSpace}[10\,528, 2960] \rightarrow 7808 & \text{QuotientSpace}[11\,148, 3142] \rightarrow 8258 \end{array} \right)$$

**Analyze[{"R2c"}] // MatrixForm**

$$\left( \begin{array}{ll} \text{QuotientSpace}[12, 2] \rightarrow 10 & \text{QuotientSpace}[14, 2] \rightarrow 12 \\ \text{QuotientSpace}[120, 24] \rightarrow 96 & \text{QuotientSpace}[134, 26] \rightarrow 108 \\ \text{QuotientSpace}[1680, 360] \rightarrow 1332 & \text{QuotientSpace}[1814, 386] \rightarrow 1440 \\ \text{QuotientSpace}[30\,240, 6720] \rightarrow 23\,880 & \text{QuotientSpace}[32\,054, 7106] \rightarrow 25\,320 \end{array} \right)$$

```
Analyze[{"R2c", "R1", "Round"}] // MatrixForm
```

$$\left( \begin{array}{ll} \text{QuotientSpace}[4, 6] \rightarrow 0 & \text{QuotientSpace}[4, 6] \rightarrow 0 \\ \text{QuotientSpace}[40, 52] \rightarrow 4 & \text{QuotientSpace}[44, 58] \rightarrow 4 \\ \text{QuotientSpace}[576, 744] \rightarrow 44 & \text{QuotientSpace}[620, 802] \rightarrow 48 \\ \text{QuotientSpace}[10\,528, 13\,488] \rightarrow 648 & \text{QuotientSpace}[11\,148, 14\,290] \rightarrow 696 \end{array} \right)$$

```
Analyze[{"R2c", "Round"}] // MatrixForm
```

Part::partd: Part specification Null[2] is longer than depth of object. >>

Part::partd: Part specification Null[2] is longer than depth of object. >>

Part::partd: Part specification Null[2] is longer than depth of object. >>

General::stop: Further output of Part::partd will be suppressed during this calculation. >>

$$\left( \begin{array}{ll} \text{QuotientSpace}[12, 14] \rightarrow 3 & \text{QuotientSpace}[14, 16] \rightarrow 4 \\ \text{QuotientSpace}[120, 144] \rightarrow 16 & \text{QuotientSpace}[134, 160] \rightarrow 20 \\ \text{QuotientSpace}[1680, 2040] \rightarrow 160 & \text{QuotientSpace}[1814, 2200] \rightarrow 180 \\ \text{QuotientSpace}[30\,240, 36\,960] \rightarrow 30\,240 - \text{Null}[2] & \text{QuotientSpace}[32\,054, 39\,160] \rightarrow 32\,054 - \text{Null}[2] \end{array} \right)$$

## Save

```
NotebookSave []
```