Pensieve header: Testing suite for the FullDoPeGDO project.

In[∘]:=
```
Date[]
SetDirectory["C:\\Users\\T15Roland\\Wiskunde\\Bn\\ProgramFullDoPegDO"];
Once[<< KnotTheory`];
(*Once[Get@"../Profile/Profile.m"];*)
PP_ = Identity;
$k = 1;
<< Engine.m
<< ObjectsR.m
```

Out[∘]= {2021, 9, 10, 1, 29, 57.0775723}

ParentDirectory: Argument File should be a positive machine-size integer, a nonempty string, or a File specification.

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ToFileName: String or list of strings expected at position 1 in ToFileName[{File, WikiLink, mathematica}].

ToFileName: String or list of strings expected at position 1 in ToFileName[{File, QuantumGroups}].

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.
Read more at http://katlas.org/wiki/KnotTheory.

In[∘]:=
```
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, ■, ■]];
```

Testing

co-associativity

In[∘]:=
```
(dΔ_{1→1,2} // dΔ_{2→2,3}) ≡ (dΔ_{1→2,3} // dΔ_{2→1,2})
```

Out[∘]= True

algebra morphism

In[∘]:=
```
(dΔ_{i→1,2} dΔ_{j→3,4} // dm_{1,3→i} // dm_{2,4→j}) ≡ (dm_{i,j→k} // dΔ_{k→i,j})
```

Out[∘]= True

associativity

In[∘]:=
```
(dm_{1,2→k} // dm_{k,3→k}) ≡ (dm_{2,3→k} // dm_{1,k→k})
```

Out[∘]= True

antipode

In[∘]:=
```
dΔ_{i→1,2} // dS_1 // dm_{1,2→1}
dΔ_{i→1,2} // dS_2 // dm_{1,2→1}
```

Out[∘]= $\mathbb{E}_{\{i\}\to\{1\}}$[0, 0]

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quasi-triangular axioms

In[∘]:= $(R_{1,3}\ //\ d\Delta_{1\to1,2}) \equiv (R_{1,3}\, R_{2,4}\ //\ dm_{3,4\to3})$
$(R_{1,3}\ //\ d\Delta_{3\to2,3}) \equiv (R_{1,3}\, R_{0,2}\ //\ dm_{1,0\to1})$
$(d\Delta_{i\to k,j}\, R_{1,2}\ //\ dm_{j,1\to1}\ //\ dm_{k,2\to2}) \equiv (R_{1,2}\, d\Delta_{i\to j,k}\ //\ dm_{1,j\to1}\ //\ dm_{2,k\to2})$

Out[∘]= True

Out[∘]= True

Out[∘]= True

In[∘]:= $(R_{1,2}\ //\ aS_2) \equiv (\overline{R}_{1,2})$

Out[∘]= True

In[∘]:= {(*Rotational Reidemeister moves*)
$(\overline{C}_1\, \overline{C}_2\, R_{3,4}\, C_5\, C_6\ //\ dm_{1,3\to1}\ //\ dm_{1,5\to1}\ //\ dm_{2,4\to2}\ //\ dm_{2,6\to2}) \equiv R_{1,2}$,
$(\overline{C}_1\, \overline{C}_2\, \overline{R}_{4,3}\, C_5\, C_6\ //\ dm_{1,3\to1}\ //\ dm_{1,5\to i}\ //\ dm_{2,4\to2}\ //\ dm_{2,6\to j}) \equiv \overline{R}_{j,i}$,
$(R_{3,1}\, C_2\ //\ dm_{1,2\to1}\ //\ dm_{1,3\to i}) \equiv (R_{1,3}\, \overline{C}_2\ //\ dm_{1,2\to1}\ //\ dm_{1,3\to i})$,
$(\overline{R}_{1,2}\, R_{3,4}\ //\ dm_{1,3\to j}\ //\ dm_{2,4\to i}) \equiv d\eta_i\, d\eta_j$,
$(\overline{R}_{1,6}\, R_{3,2}\, C_4\ //\ dm_{1,3\to i}\ //\ dm_{2,4\to2}\ //\ dm_{2,6\to j}) \equiv d\eta_i\, C_j$,
$(R_{1,2}\, R_{4,3}\, R_{5,6}\ //\ dm_{1,4\to i}\ //\ dm_{2,5\to j}\ //\ dm_{3,6\to k}) \equiv (R_{1,6}\, R_{2,3}\, R_{4,5}\ //\ dm_{1,4\to i}\ //\ dm_{2,5\to j}\ //\ dm_{3,6\to k})$}

Out[∘]= $\Big\{\text{True},\ \{i, j\} == \{j, i\},$
$\hbar\, a_i\, b_i + \frac{1}{2}\left(\text{Log}\left[\frac{1}{B_i^2}\right] - \hbar\, b_i\right) + \hbar\, x_i\, y_i == \frac{\hbar\, b_i}{2} + \hbar\, a_i\, b_i + \hbar\, x_i\, y_i,\ \text{True},\ \text{True},\ \text{True}\Big\}$

In[∘]:= $(R_{3,1}\, C_2\ //\ dm_{1,2\to1}\ //\ dm_{1,3\to i})$
$(R_{1,3}\, \overline{C}_2\ //\ dm_{1,2\to1}\ //\ dm_{1,3\to i})$

Out[∘]= $\mathbb{E}_{\{\}\to\{i\}}\left[\hbar\, a_i\, b_i + \frac{1}{2}\left(\text{Log}\left[\frac{1}{B_i^2}\right] - \hbar\, b_i\right) + \hbar\, x_i\, y_i,\ \frac{\hbar\, a_i}{2} - \frac{1}{4}\hbar^3\, x_i^2\, y_i^2\right]$

Out[∘]= $\mathbb{E}_{\{\}\to\{i\}}\left[\frac{\hbar\, b_i}{2} + \hbar\, a_i\, b_i + \hbar\, x_i\, y_i,\ \frac{\hbar\, a_i}{2} - \frac{1}{4}\hbar^3\, x_i^2\, y_i^2\right]$

In[∘]:= $R_{1,2}$

Out[∘]= $\mathbb{E}_{\{\}\to\{1,2\}}\left[\hbar\, a_2\, b_1 + \hbar\, x_2\, y_1,\ -\frac{1}{4}\hbar^3\, x_2^2\, y_1^2\right]$

In[∘]:= $k = 2; $cm_{1,2\to1}\ //\ cm_{1,3\to1}$

In[∘]:= Timing@Block[{$k = 2}, HL[$(cm_{1,2\to1}\ //\ cm_{1,3\to1}) \equiv (cm_{2,3\to2}\ //\ cm_{1,2\to1})$]]

Out[∘]= {0.125, True}

In[∘]:= Timing@Block[{$k = 3}, HL[$(cm_{1,2\to1}\ //\ cm_{1,3\to1}) \equiv (cm_{2,3\to2}\ //\ cm_{1,2\to1})$]]

Out[∘]= {0.265625, True}

In[∘]:= Timing@Block[{$k = 4}, HL[$(cm_{1,2\to1}\ //\ cm_{1,3\to1}) \equiv (cm_{2,3\to2}\ //\ cm_{1,2\to1})$]]

Out[∘]= {0.640625, True}

In[∘]:= `Timing@Block[{$k = 5}, HL[(cm_{1,2→1} // cm_{1,3→1}) ≡ (cm_{2,3→2} // cm_{1,2→1})]]`

Out[∘]= {6.21875, True}

In[∘]:= `Timing@Block[{$k = 6}, HL[(cm_{1,2→1} // cm_{1,3→1}) ≡ (cm_{2,3→2} // cm_{1,2→1})]]`

Out[∘]= {11.8125, True}

In[∘]:= `Timing@Block[{$k = 7}, HL[(cm_{1,2→1} // cm_{1,3→1}) ≡ (cm_{2,3→2} // cm_{1,2→1})]]`

Out[∘]= {3.20313, True}

In[∘]:= `Timing@Block[{$k = 8}, HL[(cm_{1,2→1} // cm_{1,3→1}) ≡ (cm_{2,3→2} // cm_{1,2→1})]]`

Out[∘]= {5.53125, True}

In[∘]:= $a\sigma_{1\to 2}$

Out[∘]= $\mathbb{E}_{\{1\}\to\{2\}}[a_2\,\alpha_1 + x_2\,\xi_1, 0, 0]$

In[∘]:= $am_{1,2\to 3}$

Out[∘]= $\mathbb{E}_{\{1,2\}\to\{3\}}\left[a_3\,(\alpha_1+\alpha_2) + x_3\left(\frac{\xi_1}{\mathcal{A}_2}+\xi_2\right), 0, 0\right]$

In[∘]:= $bm_{1,2\to 3}$

Out[∘]= $\mathbb{E}_{\{1,2\}\to\{3\}}\left[b_3\,(\beta_1+\beta_2) + y_3\,(\eta_1+\eta_2), -y_3\,\beta_1\,\eta_2, \frac{1}{2}\,y_3\,\beta_1^2\,\eta_2\right]$

In[∘]:= `Block[{$k = 8}, R_{1,2}]`

Out[∘]= $$\mathbb{E}_{\{\}\to\{1,2\}}\Big[\hbar\,a_2\,b_1 + \hbar\,x_2\,y_1, -\frac{1}{4}\,\hbar^3\,x_2^2\,y_1^2, \frac{1}{9}\,\hbar^5\,x_2^3\,y_1^3, \frac{1}{48}\left(\hbar^5\,x_2^2\,y_1^2 - 3\,\hbar^7\,x_2^4\,y_1^4\right),$$
$$-\frac{1}{36}\,\hbar^7\,x_2^3\,y_1^3 + \frac{1}{25}\,\hbar^9\,x_2^5\,y_1^5, -\frac{1}{480}\,\hbar^7\,x_2^2\,y_1^2 + \frac{1}{32}\,\hbar^9\,x_2^4\,y_1^4 - \frac{1}{36}\,\hbar^{11}\,x_2^6\,y_1^6,$$
$$\frac{7\,\hbar^9\,x_2^3\,y_1^3}{1080} - \frac{1}{30}\,\hbar^{11}\,x_2^5\,y_1^5 + \frac{1}{49}\,\hbar^{13}\,x_2^7\,y_1^7, \frac{17\,\hbar^9\,x_2^2\,y_1^2}{80\,640} - \frac{17\,\hbar^{11}\,x_2^4\,y_1^4}{1280} + \frac{5}{144}\,\hbar^{13}\,x_2^6\,y_1^6 - \frac{1}{64}\,\hbar^{15}\,x_2^8\,y_1^8,$$
$$-\frac{809\,\hbar^{11}\,x_2^3\,y_1^3}{544\,320} + \frac{9}{400}\,\hbar^{13}\,x_2^5\,y_1^5 - \frac{1}{28}\,\hbar^{15}\,x_2^7\,y_1^7 + \frac{1}{81}\,\hbar^{17}\,x_2^9\,y_1^9\Big]$$

In[∘]:=
```
Timing@Block[{$k = 8},
  HL /@ {(am_{1,2→1} // am_{1,3→1}) ≡ Echo@(am_{2,3→2} // am_{1,2→1}),
    (bm_{1,2→1} // bm_{1,3→1}) ≡ Echo@(bm_{2,3→2} // bm_{1,2→1})}
 ]
```

» $\mathbb{E}_{\{1,2,3\}\to\{1\}}\left[a_1\ (\alpha_1+\alpha_2+\alpha_3)+\frac{x_1\ \xi_1}{\mathcal{A}_2\ \mathcal{A}_3}+\frac{x_1\ \xi_2}{\mathcal{A}_3}+x_1\ \xi_3,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right]$

» $\mathbb{E}_{\{1,2,3\}\to\{1\}}\Big[b_1\ \beta_1+b_1\ \beta_2+b_1\ \beta_3+y_1\ \eta_1+y_1\ \eta_2+y_1\ \eta_3,$

$-y_1\ \beta_1\ \eta_2-y_1\ \beta_1\ \eta_3-y_1\ \beta_2\ \eta_3,\ \frac{1}{2}\ y_1\ \beta_1^2\ \eta_2+\frac{1}{2}\ y_1\ \beta_1^2\ \eta_3+y_1\ \beta_1\ \beta_2\ \eta_3+\frac{1}{2}\ y_1\ \beta_2^2\ \eta_3,$

$-\frac{1}{6}\ y_1\ \beta_1^3\ \eta_2-\frac{1}{6}\ y_1\ \beta_1^3\ \eta_3-\frac{1}{2}\ y_1\ \beta_1^2\ \beta_2\ \eta_3-\frac{1}{2}\ y_1\ \beta_1\ \beta_2^2\ \eta_3-\frac{1}{6}\ y_1\ \beta_2^3\ \eta_3,$

$\frac{1}{24}\ y_1\ \beta_1^4\ \eta_2+\frac{1}{24}\ y_1\ \beta_1^4\ \eta_3+\frac{1}{6}\ y_1\ \beta_1^3\ \beta_2\ \eta_3+\frac{1}{4}\ y_1\ \beta_1^2\ \beta_2^2\ \eta_3+\frac{1}{6}\ y_1\ \beta_1\ \beta_2^3\ \eta_3+\frac{1}{24}\ y_1\ \beta_2^4\ \eta_3,$

$-\frac{1}{120}\ y_1\ \beta_1^5\ \eta_2-\frac{1}{120}\ y_1\ \beta_1^5\ \eta_3-\frac{1}{24}\ y_1\ \beta_1^4\ \beta_2\ \eta_3-\frac{1}{12}\ y_1\ \beta_1^3\ \beta_2^2\ \eta_3-\frac{1}{12}\ y_1\ \beta_1^2\ \beta_2^3\ \eta_3-\frac{1}{24}\ y_1\ \beta_1\ \beta_2^4\ \eta_3-\frac{1}{120}\ y_1\ \beta_2^5\ \eta_3,$

$\frac{1}{720}\ y_1\ \beta_1^6\ \eta_2+\frac{1}{720}\ y_1\ \beta_1^6\ \eta_3+\frac{1}{120}\ y_1\ \beta_1^5\ \beta_2\ \eta_3+\frac{1}{48}\ y_1\ \beta_1^4\ \beta_2^2\ \eta_3+\frac{1}{36}\ y_1\ \beta_1^3\ \beta_2^3\ \eta_3+\frac{1}{48}\ y_1\ \beta_1^2\ \beta_2^4\ \eta_3+$

$\frac{1}{120}\ y_1\ \beta_1\ \beta_2^5\ \eta_3+\frac{1}{720}\ y_1\ \beta_2^6\ \eta_3,\ -\frac{y_1\ \beta_1^7\ \eta_2}{5040}-\frac{y_1\ \beta_1^7\ \eta_3}{5040}-\frac{1}{720}\ y_1\ \beta_1^6\ \beta_2\ \eta_3-\frac{1}{240}\ y_1\ \beta_1^5\ \beta_2^2\ \eta_3-\frac{1}{144}\ y_1\ \beta_1^4\ \beta_2^3\ \eta_3-$

$\frac{1}{144}\ y_1\ \beta_1^3\ \beta_2^4\ \eta_3-\frac{1}{240}\ y_1\ \beta_1^2\ \beta_2^5\ \eta_3-\frac{1}{720}\ y_1\ \beta_1\ \beta_2^6\ \eta_3-\frac{y_1\ \beta_2^7\ \eta_3}{5040},\ \frac{y_1\ \beta_1^8\ \eta_2}{40320}+\frac{y_1\ \beta_1^8\ \eta_3}{40320}+\frac{y_1\ \beta_1^7\ \beta_2\ \eta_3}{5040}+$

$\frac{y_1\ \beta_1^6\ \beta_2^2\ \eta_3}{1440}+\frac{1}{720}\ y_1\ \beta_1^5\ \beta_2^3\ \eta_3+\frac{1}{576}\ y_1\ \beta_1^4\ \beta_2^4\ \eta_3+\frac{1}{720}\ y_1\ \beta_1^3\ \beta_2^5\ \eta_3+\frac{y_1\ \beta_1^2\ \beta_2^6\ \eta_3}{1440}+\frac{y_1\ \beta_1\ \beta_2^7\ \eta_3}{5040}+\frac{y_1\ \beta_2^8\ \eta_3}{40320}\Big]$

Out[•]= {0.25, {True, True}}

In[•]:= **$k = 1; $R_{i,j}$**

Out[•]= $\mathbb{E}_{\{\}\to\{i,j\}}\left[\hbar\ a_j\ b_i+\hbar\ x_j\ y_i,\ -\frac{1}{4}\ \hbar^3\ x_j^2\ y_i^2\right]$

In[•]:= **$k = 3; $\overline{R}_{i,j}$**

Out[•]= $\mathbb{E}_{\{\}\to\{i,j\}}\Big[-\hbar\ a_j\ b_i-\frac{\hbar\ x_j\ y_i}{B_i},\ -\frac{\hbar^2\ a_j\ x_j\ y_i}{B_i}-\frac{3\ \hbar^3\ x_j^2\ y_i^2}{4\ B_i^2},$

$-\frac{\hbar^3\ a_j^2\ x_j\ y_i}{2\ B_i}+\frac{\hbar^4\ x_j^2\ y_i^2}{2\ B_i^2}-\frac{3\ \hbar^4\ a_j\ x_j^2\ y_i^2}{2\ B_i^2}-\frac{10\ \hbar^5\ x_j^3\ y_i^3}{9\ B_i^3},$

$-\frac{\hbar^4\ a_j^3\ x_j\ y_i}{6\ B_i}-\frac{3\ \hbar^5\ x_j^2\ y_i^2}{16\ B_i^2}+\frac{\hbar^5\ a_j\ x_j^2\ y_i^2}{B_i^2}-\frac{3\ \hbar^5\ a_j^2\ x_j^2\ y_i^2}{2\ B_i^2}+\frac{2\ \hbar^6\ x_j^3\ y_i^3}{B_i^3}-\frac{10\ \hbar^6\ a_j\ x_j^3\ y_i^3}{3\ B_i^3}-\frac{35\ \hbar^7\ x_j^4\ y_i^4}{16\ B_i^4}\Big]$

In[•]:= **$k = 3; $P_{i,j}$**

Out[•]= $\mathbb{E}_{\{i,j\}\to\{\}}\left[\frac{\alpha_j\ \beta_i}{\hbar}+\frac{\eta_i\ \xi_j}{\hbar},\ \frac{\eta_i^2\ \xi_j^2}{4\ \hbar},\ \frac{1}{8}\ \eta_i^2\ \xi_j^2+\frac{5\ \eta_i^3\ \xi_j^3}{36\ \hbar},\ \frac{1}{24}\ \hbar\ \eta_i^2\ \xi_j^2+\frac{1}{6}\ \eta_i^3\ \xi_j^3+\frac{5\ \eta_i^4\ \xi_j^4}{48\ \hbar}\right]$

In[∘]:= **$k = 3; aS$_i$**

Out[∘]= $\mathbb{E}_{\{i\}\to\{i\}}\Big[-a_i\,\alpha_i - x_i\,\mathcal{A}_i\,\xi_i,\ -\hbar\,a_i\,x_i\,\mathcal{A}_i\,\xi_i - \frac{1}{2}\,\hbar\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2,$
$-\frac{1}{2}\,\hbar^2\,a_i^2\,x_i\,\mathcal{A}_i\,\xi_i + \frac{1}{4}\,\hbar^2\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2 - \hbar^2\,a_i\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2 - \frac{1}{2}\,\hbar^2\,x_i^3\,\mathcal{A}_i^3\,\xi_i^3,\ -\frac{1}{6}\,\hbar^3\,a_i^3\,x_i\,\mathcal{A}_i\,\xi_i -$
$\frac{1}{12}\,\hbar^3\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2 + \frac{1}{2}\,\hbar^3\,a_i\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2 - \hbar^3\,a_i^2\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2 + \frac{2}{3}\,\hbar^3\,x_i^3\,\mathcal{A}_i^3\,\xi_i^3 - \frac{3}{2}\,\hbar^3\,a_i\,x_i^3\,\mathcal{A}_i^3\,\xi_i^3 - \frac{2}{3}\,\hbar^3\,x_i^4\,\mathcal{A}_i^4\,\xi_i^4\Big]$

In[∘]:= **$k = 3; $\overline{\text{aS}}_i$**

Out[∘]= $\mathbb{E}_{\{i\}\to\{i\}}\Big[-a_i\,\alpha_i - x_i\,\mathcal{A}_i\,\xi_i,\ \hbar\,x_i\,\mathcal{A}_i\,\xi_i - \hbar\,a_i\,x_i\,\mathcal{A}_i\,\xi_i - \frac{1}{2}\,\hbar\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2,$
$-\frac{1}{2}\,\hbar^2\,x_i\,\mathcal{A}_i\,\xi_i + \hbar^2\,a_i\,x_i\,\mathcal{A}_i\,\xi_i - \frac{1}{2}\,\hbar^2\,a_i^2\,x_i\,\mathcal{A}_i\,\xi_i + \frac{5}{4}\,\hbar^2\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2 - \hbar^2\,a_i\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2 - \frac{1}{2}\,\hbar^2\,x_i^3\,\mathcal{A}_i^3\,\xi_i^3,$
$\frac{1}{6}\,\hbar^3\,x_i\,\mathcal{A}_i\,\xi_i - \frac{1}{2}\,\hbar^3\,a_i\,x_i\,\mathcal{A}_i\,\xi_i + \frac{1}{2}\,\hbar^3\,a_i^2\,x_i\,\mathcal{A}_i\,\xi_i - \frac{1}{6}\,\hbar^3\,a_i^3\,x_i\,\mathcal{A}_i\,\xi_i - \frac{19}{12}\,\hbar^3\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2 +$
$\frac{5}{2}\,\hbar^3\,a_i\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2 - \hbar^3\,a_i^2\,x_i^2\,\mathcal{A}_i^2\,\xi_i^2 + \frac{13}{6}\,\hbar^3\,x_i^3\,\mathcal{A}_i^3\,\xi_i^3 - \frac{3}{2}\,\hbar^3\,a_i\,x_i^3\,\mathcal{A}_i^3\,\xi_i^3 - \frac{2}{3}\,\hbar^3\,x_i^4\,\mathcal{A}_i^4\,\xi_i^4\Big]$

In[∘]:= $(\overline{\text{aS}}_1 \mathbin{//} \text{aS}_1)$

Out[∘]= $\mathbb{E}_{\{1\}\to\{1\}}[a_1\,\alpha_1 + x_1\,\xi_1,\ 0,\ 0,\ 0]$

In[∘]:= $(\overline{\text{aS}}_1 \mathbin{//} \text{aS}_1)$

Out[∘]= $\mathbb{E}_{\{1\}\to\{1\}}[a_1\,\alpha_1 + x_1\,\xi_1,\ 0,\ 0,\ 0]$

In[∘]:= $(\overline{\text{bS}}_1 \mathbin{//} \text{bS}_1)$

Out[∘]= $\mathbb{E}_{\{1\}\to\{1\}}[b_1\,\beta_1 + y_1\,\eta_1,\ 0,\ 0,\ 0]$

In[∘]:= **$k = 1**

Out[∘]= 1

In[∘]:= **dS$_1$**

Out[∘]= $\mathbb{E}_{\{1\}\to\{1\}}\Big[-a_1\,\alpha_1 - b_1\,\beta_1 - \frac{y_1\,\mathcal{A}_1\,\eta_1}{B_1} - x_1\,\mathcal{A}_1\,\xi_1 + \frac{(\mathcal{A}_1 - B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar\,B_1},$
$\frac{\hbar\,y_1\,\mathcal{A}_1\,\eta_1}{B_1} - \frac{y_1\,\mathcal{A}_1\,\beta_1\,\eta_1}{B_1} - \frac{\hbar\,y_1^2\,\mathcal{A}_1^2\,\eta_1^2}{2\,B_1^2} - \hbar\,a_1\,x_1\,\mathcal{A}_1\,\xi_1 - x_1\,\mathcal{A}_1\,\beta_1\,\xi_1 + \frac{a_1\,\mathcal{A}_1\,\eta_1\,\xi_1}{B_1} -$
$\frac{\hbar\,x_1\,y_1\,\mathcal{A}_1^2\,\eta_1\,\xi_1}{B_1} + \frac{(-\mathcal{A}_1 + B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{B_1} + \frac{(\mathcal{A}_1 - B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar\,B_1} + \frac{y_1\,\left(3\,\mathcal{A}_1^2 - B_1\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1}{2\,B_1^2} -$
$\frac{1}{2}\,\hbar\,x_1^2\,\mathcal{A}_1^2\,\xi_1^2 + \frac{x_1\,\left(3\,\mathcal{A}_1^2 - B_1\,\mathcal{A}_1^2\right)\,\eta_1\,\xi_1^2}{2\,B_1} + \frac{\left(-3\,\mathcal{A}_1^2 + 4\,B_1\,\mathcal{A}_1^2 - B_1^2\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1^2}{4\,\hbar\,B_1^2}\Big]$

In[∘]:= $\mathbb{F} = \mathbb{E}$; $dS_1 \equiv \mathbb{F}_{\{1\}\to\{1\}}\Big[-a_1\,\alpha_1 - b_1\,\beta_1 - \frac{y_1\,\mathcal{A}_1\,\eta_1}{B_1} - x_1\,\mathcal{A}_1\,\xi_1 + \frac{(\mathcal{A}_1 - B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar\,B_1},$

$$\frac{\hbar\,y_1\,\mathcal{A}_1\,\eta_1}{B_1} - \frac{y_1\,\mathcal{A}_1\,\beta_1\,\eta_1}{B_1} - \frac{\hbar\,y_1^2\,\mathcal{A}_1^2\,\eta_1^2}{2\,B_1^2} - \hbar\,a_1\,x_1\,\mathcal{A}_1\,\xi_1 - x_1\,\mathcal{A}_1\,\beta_1\,\xi_1 + \frac{a_1\,\mathcal{A}_1\,\eta_1\,\xi_1}{B_1} -$$

$$\frac{\hbar\,x_1\,y_1\,\mathcal{A}_1^2\,\eta_1\,\xi_1}{B_1} + \frac{(-\mathcal{A}_1 + B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{B_1} + \frac{(\mathcal{A}_1 - B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar\,B_1} + \frac{y_1\,(3\,\mathcal{A}_1^2 - B_1\,\mathcal{A}_1^2)\,\eta_1^2\,\xi_1}{2\,B_1^2} -$$

$$\frac{1}{2}\,\hbar\,x_1^2\,\mathcal{A}_1^2\,\xi_1^2 + \frac{x_1\,(3\,\mathcal{A}_1^2 - B_1\,\mathcal{A}_1^2)\,\eta_1\,\xi_1^2}{2\,B_1} + \frac{(-3\,\mathcal{A}_1^2 + 4\,B_1\,\mathcal{A}_1^2 - B_1^2\,\mathcal{A}_1^2)\,\eta_1^2\,\xi_1^2}{4\,\hbar\,B_1^2}\Big]$$

Out[∘]= True

In[∘]:= $\overline{dS}_1 \equiv \mathbb{F}_{\{1\}\to\{1\}}\Big[-a_1\,\alpha_1 - b_1\,\beta_1 - \frac{y_1\,\mathcal{A}_1\,\eta_1}{B_1} - x_1\,\mathcal{A}_1\,\xi_1 + \frac{(\mathcal{A}_1 - B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar\,B_1},$

$$-\frac{y_1\,\mathcal{A}_1\,\beta_1\,\eta_1}{B_1} - \frac{\hbar\,y_1^2\,\mathcal{A}_1^2\,\eta_1^2}{2\,B_1^2} + \hbar\,x_1\,\mathcal{A}_1\,\xi_1 - \hbar\,a_1\,x_1\,\mathcal{A}_1\,\xi_1 - x_1\,\mathcal{A}_1\,\beta_1\,\xi_1 + \frac{a_1\,\mathcal{A}_1\,\eta_1\,\xi_1}{B_1} -$$

$$\frac{\hbar\,x_1\,y_1\,\mathcal{A}_1^2\,\eta_1\,\xi_1}{B_1} + \frac{(-\mathcal{A}_1 + B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{B_1} + \frac{(\mathcal{A}_1 - B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar\,B_1} + \frac{y_1\,(3\,\mathcal{A}_1^2 - B_1\,\mathcal{A}_1^2)\,\eta_1^2\,\xi_1}{2\,B_1^2} -$$

$$\frac{1}{2}\,\hbar\,x_1^2\,\mathcal{A}_1^2\,\xi_1^2 + \frac{x_1\,(3\,\mathcal{A}_1^2 - B_1\,\mathcal{A}_1^2)\,\eta_1\,\xi_1^2}{2\,B_1} + \frac{(-3\,\mathcal{A}_1^2 + 4\,B_1\,\mathcal{A}_1^2 - B_1^2\,\mathcal{A}_1^2)\,\eta_1^2\,\xi_1^2}{4\,\hbar\,B_1^2}\Big]$$

Out[∘]= True

In[∘]:= dS_1 // $\overline{dS}_1$

Out[∘]= $\mathbb{E}_{\{1\}\to\{1\}}[a_1\,\alpha_1 + b_1\,\beta_1 + y_1\,\eta_1 + x_1\,\xi_1,\ 0]$

In[∘]:=

$$\mathbb{E}_{\{1\}\to\{1\}}\Big[-a_1\,\alpha_1-b_1\,\beta_1-\frac{{\color{blue}\blacksquare}\,y_1\,\mathcal{A}_1\,\eta_1}{B_1}-{\color{green}\blacksquare}\,x_1\,\mathcal{A}_1\,\xi_1+\frac{(\mathcal{A}_1-B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar\,B_1},$$

$$\Big(\frac{\hbar\,y_1\,\mathcal{A}_1\,\eta_1}{B_1}-\frac{y_1\,\mathcal{A}_1\,\beta_1\,\eta_1}{B_1}-\frac{\hbar\,y_1^2\,\mathcal{A}_1^2\,\eta_1^2}{2\,B_1^2}-\hbar\,a_1\,x_1\,\mathcal{A}_1\,\xi_1-x_1\,\mathcal{A}_1\,\beta_1\,\xi_1+\frac{a_1\,\mathcal{A}_1\,\eta_1\,\xi_1}{B_1}-$$

$$\frac{\hbar\,x_1\,y_1\,\mathcal{A}_1^2\,\eta_1\,\xi_1}{B_1}+\frac{(-\mathcal{A}_1+B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{B_1}+\frac{(\mathcal{A}_1-B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar\,B_1}+\frac{y_1\,\left(3\,\mathcal{A}_1^2-B_1\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1}{2\,B_1^2}-$$

$$\frac{1}{2}\,\hbar\,x_1^2\,\mathcal{A}_1^2\,\xi_1^2+\frac{x_1\,\left(3\,\mathcal{A}_1^2-B_1\,\mathcal{A}_1^2\right)\,\eta_1\,\xi_1^2}{2\,B_1}+\frac{\left(-3\,\mathcal{A}_1^2+4\,B_1\,\mathcal{A}_1^2-B_1^2\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1^2}{4\,\hbar\,B_1^2}\Big)\Big]\ //$$

$$\mathbb{E}_{\{1\}\to\{1\}}\Big[-a_1\,\alpha_1-b_1\,\beta_1-\frac{y_1\,\mathcal{A}_1\,\eta_1}{B_1}-x_1\,\mathcal{A}_1\,\xi_1+\frac{(\mathcal{A}_1-B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar\,B_1},$$

$$-\frac{y_1\,\mathcal{A}_1\,\beta_1\,\eta_1}{B_1}-\frac{\hbar\,y_1^2\,\mathcal{A}_1^2\,\eta_1^2}{2\,B_1^2}+\hbar\,x_1\,\mathcal{A}_1\,\xi_1-\hbar\,a_1\,x_1\,\mathcal{A}_1\,\xi_1-x_1\,\mathcal{A}_1\,\beta_1\,\xi_1+\frac{{\color{red}\blacksquare}\,a_1\,\mathcal{A}_1\,\eta_1\,\xi_1}{B_1}-$$

$$\frac{\hbar\,x_1\,y_1\,\mathcal{A}_1^2\,\eta_1\,\xi_1}{B_1}+\frac{(-\mathcal{A}_1+B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{B_1}+\frac{{\color{purple}\blacksquare}\,(\mathcal{A}_1-B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar\,B_1}+\frac{y_1\,\left(3\,\mathcal{A}_1^2-B_1\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1}{2\,B_1^2}-$$

$$\frac{1}{2}\,\hbar\,x_1^2\,\mathcal{A}_1^2\,\xi_1^2+\frac{x_1\,\left(3\,\mathcal{A}_1^2-B_1\,\mathcal{A}_1^2\right)\,\eta_1\,\xi_1^2}{2\,B_1}+\frac{\left(-3\,\mathcal{A}_1^2+4\,B_1\,\mathcal{A}_1^2-B_1^2\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1^2}{4\,\hbar\,B_1^2}\Big]$$

Out[∘]=

$$\mathbb{E}_{\{1\}\to\{1\}}\Big[a_1\,\alpha_1+b_1\,\beta_1+{\color{blue}\blacksquare}\,y_1\,\eta_1+{\color{green}\blacksquare}\,x_1\,\xi_1+\frac{\left(-\mathcal{A}_1+{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,\mathcal{A}_1+B_1\,\mathcal{A}_1-{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1\right)\,\eta_1\,\xi_1}{\hbar},$$

$$\left(-\hbar+\hbar\,{\color{blue}\blacksquare}\right)\,y_1\,\eta_1+\left(1-{\color{blue}\blacksquare}\right)\,y_1\,\beta_1\,\eta_1+\frac{1}{2}\,\left(-\hbar+\hbar\,{\color{blue}\blacksquare}^2\right)\,y_1^2\,\eta_1^2+\left(\hbar-\hbar\,{\color{green}\blacksquare}\right)\,x_1\,\xi_1+\left(-\hbar+\hbar\,{\color{green}\blacksquare}\right)\,a_1\,x_1\,\xi_1+$$

$$\left(1-{\color{green}\blacksquare}\right)\,x_1\,\beta_1\,\xi_1+\left(-\hbar+\hbar\,{\color{blue}\blacksquare}\right)\,x_1\,y_1\,\eta_1\,\xi_1+a_1\,\left(-{\color{blue}\blacksquare}\,\mathcal{A}_1+{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,{\color{red}\blacksquare}\,\mathcal{A}_1-B_1\,\mathcal{A}_1+{\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1\right)\,\eta_1\,\xi_1+$$

$$\Big(\big({\color{blue}\blacksquare}\,\mathcal{A}_1-{\color{green}\blacksquare}\,\mathcal{A}_1-{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,\mathcal{A}_1+{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,\mathcal{A}_1-{\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1+{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1+{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1-{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1\big)\,\eta_1\,\xi_1+$$

$$\big(-\mathcal{A}_1+{\color{blue}\blacksquare}\,\mathcal{A}_1+{\color{green}\blacksquare}\,\mathcal{A}_1-{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,\mathcal{A}_1+B_1\,\mathcal{A}_1-{\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1-{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1+{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1\big)\,\beta_1\,\eta_1\,\xi_1\Big)\Big/\hbar+$$

$$\frac{1}{2}\,y_1\,\big(\mathcal{A}_1-2\,{\color{blue}\blacksquare}\,\mathcal{A}_1+2\,{\color{blue}\blacksquare}^2\,\mathcal{A}_1-2\,{\color{green}\blacksquare}\,\mathcal{A}_1-{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}\,\mathcal{A}_1+2\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,\mathcal{A}_1-3\,B_1\,\mathcal{A}_1+$$

$$6\,{\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1-2\,{\color{blue}\blacksquare}^2\,B_1\,\mathcal{A}_1+2\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1-{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1-2\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1\big)\,\eta_1^2\,\xi_1+$$

$$\frac{1}{2}\,\left(-\hbar+2\,\hbar\,{\color{green}\blacksquare}-\hbar\,{\color{green}\blacksquare}^2\right)\,x_1^2\,\xi_1^2+\frac{1}{2}\,x_1\,\big(\mathcal{A}_1-2\,{\color{blue}\blacksquare}\,\mathcal{A}_1-2\,{\color{green}\blacksquare}\,\mathcal{A}_1+4\,{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,\mathcal{A}_1-{\color{blue}\blacksquare}\,{\color{green}\blacksquare}^2\,\mathcal{A}_1-$$

$$3\,B_1\,\mathcal{A}_1+2\,{\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1+6\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1-4\,{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1-{\color{blue}\blacksquare}\,{\color{green}\blacksquare}^2\,B_1\,\mathcal{A}_1\big)\,\eta_1\,\xi_1^2+$$

$$\frac{1}{4\,\hbar}\,\big(-\mathcal{A}_1^2+2\,{\color{blue}\blacksquare}\,\mathcal{A}_1^2-2\,{\color{blue}\blacksquare}^2\,\mathcal{A}_1^2+2\,{\color{green}\blacksquare}\,\mathcal{A}_1^2-4\,{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,\mathcal{A}_1^2+4\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}\,\mathcal{A}_1^2-2\,{\color{green}\blacksquare}^2\,\mathcal{A}_1^2-3\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}^2\,\mathcal{A}_1^2+4\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}^2\,{\color{purple}\blacksquare}\,\mathcal{A}_1^2+$$

$$4\,B_1\,\mathcal{A}_1^2-8\,{\color{blue}\blacksquare}\,B_1\,\mathcal{A}_1^2+4\,{\color{blue}\blacksquare}^2\,B_1\,\mathcal{A}_1^2-8\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1^2+12\,{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1^2-8\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}\,B_1\,\mathcal{A}_1^2+4\,{\color{green}\blacksquare}^2\,B_1\,\mathcal{A}_1^2+$$

$$4\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}^2\,B_1\,\mathcal{A}_1^2+4\,{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1^2-8\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}^2\,{\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1^2-3\,B_1^2\,\mathcal{A}_1^2+6\,{\color{blue}\blacksquare}\,B_1^2\,\mathcal{A}_1^2-2\,{\color{blue}\blacksquare}^2\,B_1^2\,\mathcal{A}_1^2+6\,{\color{green}\blacksquare}\,B_1^2\,\mathcal{A}_1^2-$$

$$8\,{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,B_1^2\,\mathcal{A}_1^2+4\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}\,B_1^2\,\mathcal{A}_1^2-2\,{\color{green}\blacksquare}^2\,B_1^2\,\mathcal{A}_1^2-{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}^2\,B_1^2\,\mathcal{A}_1^2-4\,{\color{blue}\blacksquare}\,{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,B_1^2\,\mathcal{A}_1^2+4\,{\color{blue}\blacksquare}^2\,{\color{green}\blacksquare}^2\,{\color{purple}\blacksquare}\,B_1^2\,\mathcal{A}_1^2\big)\,\eta_1^2\,\xi_1^2\Big]$$

In[∘]:=
$$\mathbb{E}_{\{1\}\to\{1\}}\Big[-a_1\alpha_1-b_1\beta_1-\frac{y_1\mathcal{A}_1\eta_1}{B_1}-x_1\mathcal{A}_1\xi_1+\frac{(\mathcal{A}_1-B_1\mathcal{A}_1)\eta_1\xi_1}{\hbar B_1},$$
$$\Big(\frac{\hbar y_1\mathcal{A}_1\eta_1}{B_1}-\frac{y_1\mathcal{A}_1\beta_1\eta_1}{B_1}-\frac{\hbar y_1^2\mathcal{A}_1^2\eta_1^2}{2B_1^2}-\hbar a_1x_1\mathcal{A}_1\xi_1-x_1\mathcal{A}_1\beta_1\xi_1+\frac{a_1\mathcal{A}_1\eta_1\xi_1}{B_1}-$$
$$\frac{{\color{blue}\blacksquare}\,\hbar x_1y_1\mathcal{A}_1^2\eta_1\xi_1}{B_1}+\frac{(-\mathcal{A}_1+B_1\mathcal{A}_1){\color{red}\blacksquare}\,\eta_1\xi_1}{B_1}+\frac{(\mathcal{A}_1-B_1\mathcal{A}_1)\beta_1\eta_1\xi_1}{\hbar B_1}+\frac{y_1(3\mathcal{A}_1^2-B_1\mathcal{A}_1^2)\eta_1^2\xi_1}{2B_1^2}-$$
$$\frac{1}{2}\hbar x_1^2\mathcal{A}_1^2\xi_1^2+\frac{x_1(3\mathcal{A}_1^2-B_1\mathcal{A}_1^2)\eta_1\xi_1^2}{2B_1}+\frac{(-3\mathcal{A}_1^2+4B_1\mathcal{A}_1^2-B_1^2\mathcal{A}_1^2)\eta_1^2\xi_1^2}{4\hbar B_1^2}\Big)\Big]\ //$$

$$\mathbb{E}_{\{1\}\to\{1\}}\Big[-a_1\alpha_1-b_1\beta_1-\frac{y_1\mathcal{A}_1\eta_1}{B_1}-x_1\mathcal{A}_1\xi_1+\frac{(\mathcal{A}_1-B_1\mathcal{A}_1)\eta_1\xi_1}{\hbar B_1},$$
$$-\frac{y_1\mathcal{A}_1\beta_1\eta_1}{B_1}-\frac{\hbar y_1^2\mathcal{A}_1^2\eta_1^2}{2B_1^2}+\hbar x_1\mathcal{A}_1\xi_1-\hbar a_1x_1\mathcal{A}_1\xi_1-x_1\mathcal{A}_1\beta_1\xi_1+\frac{a_1\mathcal{A}_1\eta_1\xi_1}{B_1}-$$
$$\frac{\hbar x_1y_1\mathcal{A}_1^2\eta_1\xi_1}{B_1}+\frac{{\color{green}\blacksquare}\,(-\mathcal{A}_1+B_1\mathcal{A}_1)\eta_1\xi_1}{B_1}+\frac{{\color{purple}\blacksquare}\,(\mathcal{A}_1-B_1\mathcal{A}_1)\beta_1\eta_1\xi_1}{\hbar B_1}+\frac{y_1(3\mathcal{A}_1^2-B_1\mathcal{A}_1^2)\eta_1^2\xi_1}{2B_1^2}-$$
$$\frac{1}{2}\hbar x_1^2\mathcal{A}_1^2\xi_1^2+\frac{x_1(3\mathcal{A}_1^2-B_1\mathcal{A}_1^2)\eta_1\xi_1^2}{2B_1}+\frac{(-3\mathcal{A}_1^2+4B_1\mathcal{A}_1^2-B_1^2\mathcal{A}_1^2)\eta_1^2\xi_1^2}{4\hbar B_1^2}\Big]$$

Out[∘]=
$$\mathbb{E}_{\{1\}\to\{1\}}\Big[a_1\alpha_1+b_1\beta_1+y_1\eta_1+x_1\xi_1,$$
$$(\hbar-\hbar{\color{blue}\blacksquare})x_1y_1\eta_1\xi_1+(-{\color{blue}\blacksquare}\mathcal{A}_1-{\color{green}\blacksquare}\mathcal{A}_1+{\color{purple}\blacksquare}\mathcal{A}_1+{\color{red}\blacksquare}\mathcal{A}_1+{\color{blue}\blacksquare}B_1\mathcal{A}_1+{\color{green}\blacksquare}B_1\mathcal{A}_1-{\color{purple}\blacksquare}B_1\mathcal{A}_1-{\color{red}\blacksquare}B_1\mathcal{A}_1)\eta_1\xi_1+$$
$$\frac{(\mathcal{A}_1-{\color{purple}\blacksquare}\mathcal{A}_1-B_1\mathcal{A}_1+{\color{purple}\blacksquare}B_1\mathcal{A}_1)\beta_1\eta_1\xi_1}{\hbar}+$$
$$y_1(-{\color{blue}\blacksquare}\mathcal{A}_1+{\color{purple}\blacksquare}\mathcal{A}_1+{\color{blue}\blacksquare}B_1\mathcal{A}_1-{\color{purple}\blacksquare}B_1\mathcal{A}_1)\eta_1^2\xi_1+x_1(\mathcal{A}_1-{\color{blue}\blacksquare}\mathcal{A}_1-B_1\mathcal{A}_1+{\color{blue}\blacksquare}B_1\mathcal{A}_1)\eta_1\xi_1^2+$$
$$\frac{(-{\color{blue}\blacksquare}\mathcal{A}_1^2+{\color{purple}\blacksquare}\mathcal{A}_1^2-B_1\mathcal{A}_1^2+2{\color{blue}\blacksquare}B_1\mathcal{A}_1^2-{\color{purple}\blacksquare}B_1\mathcal{A}_1^2+B_1^2\mathcal{A}_1^2-{\color{blue}\blacksquare}B_1^2\mathcal{A}_1^2)\eta_1^2\xi_1^2}{\hbar}\Big]$$

In[∘]:=
$$\mathbb{E}_{\{1\}\to\{1\}}\Big[a_1\alpha_1+b_1\beta_1+y_1\eta_1+x_1\xi_1,$$
$$(\hbar-\hbar{\color{blue}\blacksquare})x_1y_1\eta_1\xi_1+(-{\color{green}\blacksquare}\mathcal{A}_1+{\color{purple}\blacksquare}\mathcal{A}_1+{\color{red}\blacksquare}\mathcal{A}_1+{\color{green}\blacksquare}B_1\mathcal{A}_1-{\color{purple}\blacksquare}B_1\mathcal{A}_1-{\color{red}\blacksquare}B_1\mathcal{A}_1-{\color{blue}\blacksquare}\mathcal{A}_1^2+{\color{blue}\blacksquare}B_1\mathcal{A}_1^2)\eta_1\xi_1+$$
$$\frac{(\mathcal{A}_1-{\color{purple}\blacksquare}\mathcal{A}_1-B_1\mathcal{A}_1+{\color{purple}\blacksquare}B_1\mathcal{A}_1)\beta_1\eta_1\xi_1}{\hbar}+$$
$$y_1(-{\color{blue}\blacksquare}\mathcal{A}_1+{\color{purple}\blacksquare}\mathcal{A}_1+{\color{blue}\blacksquare}B_1\mathcal{A}_1-{\color{purple}\blacksquare}B_1\mathcal{A}_1)\eta_1^2\xi_1+x_1(\mathcal{A}_1-{\color{blue}\blacksquare}\mathcal{A}_1-B_1\mathcal{A}_1+{\color{blue}\blacksquare}B_1\mathcal{A}_1)\eta_1\xi_1^2+$$
$$\frac{(-{\color{blue}\blacksquare}\mathcal{A}_1^2+{\color{purple}\blacksquare}\mathcal{A}_1^2-B_1\mathcal{A}_1^2+2{\color{blue}\blacksquare}B_1\mathcal{A}_1^2-{\color{purple}\blacksquare}B_1\mathcal{A}_1^2+B_1^2\mathcal{A}_1^2-{\color{blue}\blacksquare}B_1^2\mathcal{A}_1^2)\eta_1^2\xi_1^2}{\hbar}\Big]\ /.\ \{\text{Red}\mid\text{Green}\mid\text{Blue}\to 1\}$$

Out[∘]=
$$\mathbb{E}_{\{1\}\to\{1\}}\Big[a_1\alpha_1+b_1\beta_1+y_1\eta_1+x_1\xi_1,$$
$$({\color{purple}\blacksquare}\mathcal{A}_1-{\color{purple}\blacksquare}B_1\mathcal{A}_1-\mathcal{A}_1^2+B_1\mathcal{A}_1^2)\eta_1\xi_1+\frac{(\mathcal{A}_1-{\color{purple}\blacksquare}\mathcal{A}_1-B_1\mathcal{A}_1+{\color{purple}\blacksquare}B_1\mathcal{A}_1)\beta_1\eta_1\xi_1}{\hbar}+$$
$$y_1(-\mathcal{A}_1+{\color{purple}\blacksquare}\mathcal{A}_1+B_1\mathcal{A}_1-{\color{purple}\blacksquare}B_1\mathcal{A}_1)\eta_1^2\xi_1+\frac{(-\mathcal{A}_1^2+{\color{purple}\blacksquare}\mathcal{A}_1^2+B_1\mathcal{A}_1^2-{\color{purple}\blacksquare}B_1\mathcal{A}_1^2)\eta_1^2\xi_1^2}{\hbar}\Big]$$

In[∘]:= $\mathbb{E}_{\{1\}\to\{1\}}\left[-a_1\,\alpha_1 - b_1\,\beta_1 - \frac{{\color{green}\blacksquare}\; y_1\,\mathcal{A}_1\,\eta_1}{B_1} - x_1\,\mathcal{A}_1\,\xi_1 + \frac{(\mathcal{A}_1 - B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar\,B_1},\ 0\right]$ //

$\mathbb{E}_{\{1\}\to\{1\}}\left[-a_1\,\alpha_1 - b_1\,\beta_1 - \frac{y_1\,\mathcal{A}_1\,\eta_1}{B_1} - x_1\,\mathcal{A}_1\,\xi_1 + \frac{(\mathcal{A}_1 - B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar\,B_1},\ \frac{{\color{purple}\blacksquare}\;(\mathcal{A}_1 - B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar\,B_1}\right]$

Out[∘]= $\mathbb{E}_{\{1\}\to\{1\}}\Big[a_1\,\alpha_1 + b_1\,\beta_1 + {\color{green}\blacksquare}\,y_1\,\eta_1 + x_1\,\xi_1 + \frac{(-\mathcal{A}_1 + {\color{green}\blacksquare}\,\mathcal{A}_1 + B_1\,\mathcal{A}_1 - {\color{green}\blacksquare}\,B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1}{\hbar},$

$({\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,\mathcal{A}_1 - {\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1)\,\eta_1\,\xi_1 + \frac{(-{\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,\mathcal{A}_1 + {\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1)\,\beta_1\,\eta_1\,\xi_1}{\hbar} + y_1\,\left({\color{green}\blacksquare}^2\,{\color{purple}\blacksquare}\,\mathcal{A}_1 - {\color{green}\blacksquare}^2\,{\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1\right)\,\eta_1^2\,\xi_1 +$

$\frac{\left({\color{green}\blacksquare}^2\,{\color{purple}\blacksquare}\,\mathcal{A}_1^2 + {\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1^2 - 2\,{\color{green}\blacksquare}^2\,{\color{purple}\blacksquare}\,B_1\,\mathcal{A}_1^2 - {\color{green}\blacksquare}\,{\color{purple}\blacksquare}\,B_1^2\,\mathcal{A}_1^2 + {\color{green}\blacksquare}^2\,{\color{purple}\blacksquare}\,B_1^2\,\mathcal{A}_1^2\right)\,\eta_1^2\,\xi_1^2}{\hbar}\Big]$

In[∘]:= $\left(kR_{1,4}\,\overline{kR}_{5,2}\,\overline{kC}_3\right)$ // $km_{2,4\to2}$ // $km_{1,3\to1}$ // $km_{1,5\to1}$

Out[∘]= $\mathbb{E}_{\{\}\to\{1,2\}}[0,\ \hbar\,a_1,\ 0]$

In[∘]:= $\overline{kC}_1\,d\eta_2$

Out[∘]= $\mathbb{E}_{\{\}\to\{1,2\}}\left[-\frac{\hbar\,t_1}{2},\ \hbar\,a_1,\ 0\right]$

In[∘]:= $(\mathbb{E}_{\{\}\to\{1,2\}}[0,\ a_2\,x_1]$ // $am_{1,2\to1})$

Out[∘]= $\mathbb{E}_{\{\}\to\{1\}}[0,\ -x_1 + a_1\,x_1]$

In[∘]:= $k = 2; $\mathbb{E}2\Lambda[\mathbb{E}_{\{\}\to\{1,2\}}[0,\ y_2\,b_1]$ // $bm_{1,2\to1}]$

Out[∘]= $b_1\,y_1$

In[∘]:= $\mathbb{E}_{\{\}\to\{1,2\}}[0,\ y_2\,b_1]$

Out[∘]= $\mathbb{E}_{\{\}\to\{1,2\}}[0,\ b_1\,y_2]$