Quillen's Theorem

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$G_1 = G$; $G_{n+1} = (G, G_n)$ $\quad L_n := \mathbb{Q} \otimes (G_n / G_{n+1})$

$$L_n \longrightarrow \mathcal{P}_n(\mathcal{A}) =: \mathcal{L}_n$$

s.t.

$A = S(L_*) \twoheadrightarrow \mathcal{A}$ surjective because it is surjective in degree 1.

maybe $\mathcal{U}(L_*) \twoheadrightarrow \mathcal{A}$

Can I construct an A-expansion?

$G_1/G_2 \underset{\pi_1}{\overset{\sigma_1}{\rightleftarrows}} G_1$

$G_2/G_3 \underset{\pi_2}{\overset{\sigma_2}{\rightleftarrows}} G_2$

$$\zeta(g) = \left(0, \pi_0(g), \pi_1\left(g\sigma_1(\pi_0(g))^{-1}\right), \dots\right) \in L_*$$

So there is $\zeta: G \longrightarrow L_*$. And also $\sigma: L_* \longrightarrow G$

<u>Claim</u> $Z = e^\zeta$ is an A-expansion. (?)

Why is it even a filtered map?

Is the exponential of an L expansion

necessarily an $\mathcal{A}$-expansion?

Likely I will need a proposition of the form:

<u>Prop</u> There is a product $\mu: L_* \times L_* \to L_*$ defined by

is this the BCH product?

$$\mu(\lambda_1, \lambda_2) = \sigma(\lambda_1)\sigma(\lambda_2) /\!/ \mathcal{J}.$$

The product μ is "polynomial", in the sense that the degree d part of $\mu(\lambda_1, \lambda_2)$ is an "appropriate" polynomial in lower-degree parts of λ_1 & λ_2.

must be made explicit