$\mathcal{U}(\mathsf{dk}_{m+n})$ uses an "expansion" $Z\colon PB_{m+n} \to \mathcal{U}(\mathsf{dk}_{m+n})$ defined using the Kontsevich integral and/or using holonomies of the Knizhnik-Zamolodchikov connection, and whose associated graded is an isomorphism $\operatorname{gr} Z\colon \mathcal{A}_{PB_{m+n}} \to \mathcal{U}(\mathsf{dk}_{m+n})$. That proof easily restricts and descends to a proof of $\mathcal{A}_{PB^{em}_{m,n}} \cong \mathcal{U}(\mathsf{edk}_{m,n})$.

## 1.4 Organization of the paper

1st version.

In Sections 2 to 4, we discuss emergent braids. Actually, it is more natural to consider more general objects, and so we begin by introducing the concepts of mixed braids and chord diagrams in Section 2. ~~There is a filtration on the category of mixed braids which comes from the theory of finite type invariants. Taking the quotient by the second stage of this filtration, we obtain the emergent braids. We also obtain the corresponding algebra of chord diagrams that we call the emergent Drinfeld-Kohno Lie algebra.~~ In Section 3 we study the structure of the emergent Drinfeld-Kohno Lie algebra. In Section 4, we introduce the parenthesized version of mixed braids and emergent braids, formulate the notion of 1-formality isomorphisms (homomorphic expansions) of the categories of parenthesized mixed/emergent braids, and extract the emergent version of the Drinfeld associator equations.

Contained in 1,3

The last two sections are devoted to the proof of Theorem 1.1. In Section 5, we recall necessary materials from [2, 3] on the (linearized) Goldman-Turaev Lie bialgebra of a punctured disk and its relation to the KV theory. In Section 6, we prove Theorem 1.1.

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### Notation

- Throughout this paper we work over the rationals $\mathbb{Q}$, though all of our argument holds true over any field of characteristic zero.
- For a nonnegative integer $n$, let $\mathsf{ass}_n$ be the free associative algebra on $n$ free generators. When we need to specify generators, we write $\mathsf{ass}_n = \mathsf{ass}(x_1, \dots, x_n)$ for example.
- The algebra $\mathsf{ass}_n$ has a structure of Hopf algebra whose coproduct, antipode and augmentation are given on generators by $\Delta(x_i) = x_i \otimes 1 + 1 \otimes x_i$, $\iota(x_i) = -x_i$ and $\varepsilon(x_i) = 1$. We also use the notation $\overline{a} = \iota(a)$ for the antipode.