

Pensieve header: Finding  $Z^w$ , especially testing to what extent the cap is unique. Continues VCapSolution.nb of pensieve://Projects/WKO4/.

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Duflo"];
<< ../WKO4/FreeLie.m;
<< ../WKO4/AwCalculus.m;
Rs[a_, b_] := Es[⟨a → LS[0], b → LS[LW@a]⟩, CWS[0]];
α = LS[{x, y}, αs]; β = LS[{x, y}, βs]; γ = CWS[{x, y}, γs];
V = Es[⟨x → α, y → β⟩, γ];
κ = CWS[{x}, κs]; Cap = Es[⟨x → LS[0]⟩, κ];
R4Eqn = V ** (Rs[x, z] // dΔ[x, x, y]) ≡ Rs[y, z] ** Rs[x, z] ** V;
UnitarityEqn = (V ** (V // dA[x] // dA[y]) ≡ Es[⟨x → LS[0], y → LS[0]⟩, CWS[0]]);
CapEqn = ((V ** (Cap // dΔ[x, x, y]) // dc[x] // dc[y]) ≡
(Cap * (Cap // dσ[x, y]) // dc[x] // dc[y]));
βs[x] = 1/2; βs[y] = 0;
```

FreeLie` implements / extends

{\*, +, \*\*, \$SeriesShowDegree, ⟨, ⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords, AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw, CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE, Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization, Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support, t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ℓ, Δ, σ, ħ, ↦, ↠}.

FreeLie` is in the public domain. Dror Bar-Natan is committed to support it within reason until July 15, 2022. This is version 150814.

AwCalculus` implements / extends {\*, \*\*, ≡, dA, dc, deg, dm, dS, dΔ, dη, dσ, El, Es, hA, hm, hS, hΔ, hη, hσ, RandomElSeries, RandomEsSeries, tA, tha, tm, tS, tΔ, tη, tσ, Γ, Δ}.

AwCalculus` is in the public domain. Dror Bar-Natan is committed to support it within reason until July 15, 2022. This is version 150909.

```
In[ ]:= κs[x] = κ1;
κs[x, x, x] = κ3;
SeriesSolve[{α, β, γ, κ}, (ħ-1 R4Eqn) && UnitarityEqn && CapEqn];
{V, κ}
```

$$Out[ ]:= \left\{ Es \left[ \left\langle x \rightarrow LS \left[ 0, -\frac{\overline{xy}}{24}, 6 \kappa^3 \overline{xyy}, \dots \right], y \rightarrow LS \left[ \frac{\overline{x}}{2}, -\frac{\overline{xy}}{12}, 6 \kappa^3 \overline{xxy}, \dots \right] \right\rangle, \right. \\ \left. CWS \left[ 0, -\frac{\overline{xy}}{48}, 3 \kappa^3 \overline{xyy} + 3 \kappa^3 \overline{xyy}, \dots \right], CWS \left[ \kappa^1 \overline{x}, -\frac{\overline{xx}}{96}, \kappa^3 \overline{xxx}, \dots \right] \right\}$$

```
In[ ]:= κs[x, x, x, x, x] = κ5; κs[x, x, x, x, x, x, x] = κ7;
```

```
In[ ]:= V@{8} // Timing
```

SeriesSolve: In degree 8 arbitrarily setting {αs[x, x, x, x, y, x, y, y] → 0}.

$$Out[ ]:= \{13.1406, \\ Es \left[ \left\langle x \rightarrow LS \left[ 0, -\frac{\overline{xy}}{24}, 6 \kappa^3 \overline{xyy}, \frac{7 \overline{xxy}}{5760} - \frac{7 \overline{xyy}}{5760} + \frac{\overline{xyyy}}{1440}, \frac{1}{4} (-\kappa^3 + 40 \kappa^5) \overline{xyyy} + \dots \right] \right\rangle, \right.$$

$$\begin{aligned}
 & \frac{1}{4} (\kappa^3 - 40 \kappa^5) \overline{\overline{\overline{\overline{x \overline{xy} y}}} + \frac{1}{8} (\kappa^3 - 120 \kappa^5) \overline{\overline{\overline{\overline{xy \overline{xy} y}}} - 20 \kappa^5 \overline{\overline{\overline{\overline{x \overline{xy} \overline{xy}}} + 10 \kappa^5 \overline{\overline{\overline{\overline{xy y y}}},} \\
 & - \frac{31 \overline{\overline{\overline{\overline{x x x \overline{xy}}} + \frac{31 \overline{\overline{\overline{\overline{x x x \overline{xy} y}}} + \frac{(-83 + 17418240 \kappa^3)}{967680} \overline{\overline{\overline{\overline{x x \overline{xy} y}}} + \\
 & \frac{(-31 + 13063680 \kappa^3)}{725760} \overline{\overline{\overline{\overline{x \overline{xy} \overline{xy} y}}} - \frac{31 \overline{\overline{\overline{\overline{x x \overline{xy} \overline{xy}}} + \frac{(13 - 4354560 \kappa^3)}{241920} \overline{\overline{\overline{\overline{x \overline{xy} y y}}} + \\
 & \frac{(101 - 52254720 \kappa^3)}{1451520} \overline{\overline{\overline{\overline{xy \overline{xy} y}}} + \frac{(527 - 209018880 \kappa^3)}{5806080} \overline{\overline{\overline{\overline{x \overline{xy} \overline{xy}}} - \frac{\overline{\overline{\overline{\overline{xy y y}}}}{60480},} \\
 & \frac{1}{960} (7 \kappa^3 - 400 \kappa^5 + 13440 \kappa^7) \overline{\overline{\overline{\overline{x x x x \overline{xy} y}}} + \frac{1}{480} (-7 \kappa^3 + 400 \kappa^5 - 13440 \kappa^7) \overline{\overline{\overline{\overline{x x x \overline{xy} y}}} + \\
 & \frac{(-21 \kappa^3 + 400 \kappa^5 - 26880 \kappa^7)}{1920} \overline{\overline{\overline{\overline{x x \overline{xy} \overline{xy} y}}} + \frac{1}{6} (5 \kappa^5 - 336 \kappa^7) \overline{\overline{\overline{\overline{x x \overline{xy} \overline{xy}}} + \\
 & \frac{1}{960} (11 \kappa^3 - 800 \kappa^5 + 40320 \kappa^7) \overline{\overline{\overline{\overline{x x \overline{xy} y y}}} + \frac{1}{480} (11 \kappa^3 - 500 \kappa^5 + 21420 \kappa^7) \overline{\overline{\overline{\overline{x \overline{xy} \overline{xy} y}}} + \\
 & \frac{(21 \kappa^3 - 4400 \kappa^5 + 215040 \kappa^7)}{1920} \overline{\overline{\overline{\overline{x x \overline{xy} y \overline{xy}}} + \frac{1}{240} (-\kappa^3 + 100 \kappa^5 - 6720 \kappa^7) \overline{\overline{\overline{\overline{x \overline{xy} y y y}}} + \\
 & \frac{(77 \kappa^3 - 1800 \kappa^5 + 408240 \kappa^7)}{5760} \overline{\overline{\overline{\overline{xy \overline{xy} \overline{xy} y}}} + \frac{1}{240} (-3 \kappa^3 + 50 \kappa^5 - 16800 \kappa^7) \overline{\overline{\overline{\overline{xy \overline{xy} y y}}} + \\
 & \frac{(7 \kappa^3 - 400 \kappa^5 + 134400 \kappa^7)}{1920} \overline{\overline{\overline{\overline{x \overline{xy} x \overline{xy} y}}} + 42 \kappa^7 \overline{\overline{\overline{\overline{x x \overline{xy} x \overline{xy}}} + \\
 & \frac{(-17 \kappa^3 - 75600 \kappa^7)}{1920} \overline{\overline{\overline{\overline{x \overline{xy} y \overline{xy} y}}} + \frac{1}{240} (-3 \kappa^3 + 50 \kappa^5 - 29610 \kappa^7) \overline{\overline{\overline{\overline{x \overline{xy} y y \overline{xy}}} + \\
 & \frac{1}{96} (-\kappa^3 - 2688 \kappa^7) \overline{\overline{\overline{\overline{xy y \overline{xy} y}}} + \frac{1}{12} (5 \kappa^5 - 336 \kappa^7) \overline{\overline{\overline{\overline{x \overline{xy} \overline{xy} \overline{xy}}} + \\
 & 14 \kappa^7 \overline{\overline{\overline{\overline{xy y y y y}}}, \frac{127 \overline{\overline{\overline{\overline{x x x x x \overline{xy}}} + \frac{127 \overline{\overline{\overline{\overline{x x x x x \overline{xy} y}}} + \\
 & \frac{(2399 - 348364800 \kappa^3 + 27869184000 \kappa^3 \kappa^5)}{464486400} \overline{\overline{\overline{\overline{x x x x \overline{xy} y}}} + \frac{127 \overline{\overline{\overline{\overline{x x x \overline{xy} \overline{xy}}} + \\
 & \frac{(-2893 + 696729600 \kappa^3 - 55738368000 \kappa^3 \kappa^5)}{464486400} \overline{\overline{\overline{\overline{x x x \overline{xy} y y}}} +
 \end{aligned}$$

$$\frac{(-727 + 212\,284\,800 \kappa^3 - 16\,982\,784\,000 \kappa^5) x x \overline{x y} \overline{x y} y y}{87\,091\,200} +$$

$$\frac{(-1397 + 232\,243\,200 \kappa^3 - 27\,869\,184\,000 \kappa^5) x x x \overline{x y} y \overline{x y}}{77\,414\,400} +$$

$$\frac{(271 - 43\,545\,600 \kappa^3 + 6\,967\,296\,000 \kappa^5) x x \overline{x y} y y y y}{58\,060\,800} +$$

$$\frac{(-15\,389 + 5\,486\,745\,600 \kappa^3 - 355\,332\,096\,000 \kappa^5) x \overline{x y} \overline{x y} \overline{x y} y}{1\,393\,459\,200} +$$

$$\frac{(12\,809 - 2\,351\,462\,400 \kappa^3 + 438\,939\,648\,000 \kappa^5) x \overline{x y} \overline{x y} y y y}{1\,393\,459\,200} +$$

$$\frac{(-113 + 9\,331\,200 \kappa^3 - 3\,732\,480\,000 \kappa^5) x x \overline{x y} \overline{x y} y}{24\,883\,200} - \frac{127 x x \overline{x y} \overline{x y} y}{77\,414\,400} +$$

$$\frac{(13\,103 - 4\,441\,651\,200 \kappa^3 + 271\,724\,544\,000 \kappa^5) x x \overline{x y} y \overline{x y} y}{1\,393\,459\,200} +$$

$$\frac{(12\,409 - 3\,004\,646\,400 \kappa^3 + 365\,783\,040\,000 \kappa^5) x x \overline{x y} y y \overline{x y}}{696\,729\,600} +$$

$$\frac{(1207 - 783\,820\,800 \kappa^3 + 29\,262\,643\,200 \kappa^5) x \overline{x y} y \overline{x y} y y}{278\,691\,840} +$$

$$\frac{(11 - 3\,110\,400 \kappa^3 + 248\,832\,000 \kappa^5) x x \overline{x y} \overline{x y} \overline{x y} y}{1\,382\,400} +$$

$$\frac{(-19 - 580\,608\,000 \kappa^3) x \overline{x y} y y y y}{9\,676\,800} +$$

$$\frac{(67 + 10\,886\,400 \kappa^3 + 3\,483\,648\,000 \kappa^5) \overline{x y} \overline{x y} \overline{x y} y y}{7\,257\,600} +$$

$$\frac{(-1327 - 261\,273\,600 \kappa^3 - 62\,705\,664\,000 \kappa^5) \overline{x y} \overline{x y} y y y y}{348\,364\,800} +$$

$$\frac{(1 + 38\,400 \kappa^3 + 58\,368\,000 \kappa^5) \overline{x y} \overline{x y} y y}{204\,800} +$$

$$\begin{aligned}
 & \frac{(181 - 58\,060\,800 \kappa^3 + 4\,644\,864\,000 \kappa^5) \overline{\overline{\overline{\overline{x\overline{xy}}\overline{x\overline{xy}}\overline{xy}}}}}{38\,707\,200} + \\
 & \frac{(2021 + 522\,547\,200 \kappa^3 + 125\,411\,328\,000 \kappa^5) \overline{\overline{\overline{\overline{xx\overline{xy}y}\overline{x\overline{xy}}}}}}{696\,729\,600} + \\
 & \frac{(-239 + 261\,273\,600 \kappa^3 + 14\,929\,920\,000 \kappa^5) \overline{\overline{\overline{\overline{x\overline{xy}}\overline{xy}y}\overline{xy}}}}{199\,065\,600} + \\
 & \frac{(-2171 - 1\,654\,732\,800 \kappa^3 - 118\,444\,032\,000 \kappa^5) \overline{\overline{\overline{\overline{x\overline{xy}y}y}\overline{xy}}}}{464\,486\,400} + \\
 & \frac{(-631 - 17\,418\,240 \kappa^3 - 26\,475\,724\,800 \kappa^5) \overline{\overline{\overline{\overline{x\overline{xy}y}yy}\overline{xy}}}}{92\,897\,280} + \\
 & \frac{(-1751 - 1\,306\,368\,000 \kappa^3 - 125\,411\,328\,000 \kappa^5) \overline{\overline{\overline{\overline{xy}y}\overline{xy}y}y}}{696\,729\,600} + \\
 & \frac{(-451 - 280\,627\,200 \kappa^3 - 27\,869\,184\,000 \kappa^5) \overline{\overline{\overline{\overline{xy}\overline{xy}y}\overline{xy}y}}}{154\,828\,800} + \\
 & \frac{(-997 + 319\,334\,400 \kappa^3 - 30\,191\,616\,000 \kappa^5) \overline{\overline{\overline{\overline{x\overline{xy}y}\overline{xy}y}\overline{xy}y}}}{77\,414\,400} + \frac{\overline{\overline{\overline{\overline{xy}y}yyyy}}}{2419200}, \dots], \\
 y \rightarrow LS & \left[ \frac{\overline{x}}{2}, -\frac{\overline{xy}}{12}, 6\kappa^3 \overline{\overline{\overline{\overline{xy}}}}, \frac{(1 + 8640\kappa^3) \overline{\overline{\overline{\overline{xx\overline{xy}}}}}}{5760} + \frac{1}{720} (-1 - 2160\kappa^3) \overline{\overline{\overline{\overline{x\overline{xy}y}}}} + \frac{1}{720} \overline{\overline{\overline{\overline{xy}y}y}}, \right. \\
 & \frac{(-1 + 960\kappa^3 + 76\,800\kappa^5) \overline{\overline{\overline{\overline{xx\overline{xy}}}}}}{7680} + \frac{(1 - 1920\kappa^3 - 38\,400\kappa^5) \overline{\overline{\overline{\overline{xx\overline{xy}y}}}}}{3840} + \\
 & \frac{1}{2} (\kappa^3 + 20\kappa^5) \overline{\overline{\overline{\overline{x\overline{xy}y}y}}} + \frac{1}{2} (\kappa^3 - 20\kappa^5) \overline{\overline{\overline{\overline{xy}\overline{xy}y}}} + \frac{(-1 + 1728\kappa^3 - 34\,560\kappa^5) \overline{\overline{\overline{\overline{x\overline{xy}}\overline{xy}}}}}{6912}, \\
 & \frac{(-1 + 1\,612\,800 \kappa^5) \overline{\overline{\overline{\overline{xxx\overline{xy}}}}}}{645\,120} + \frac{(23 + 8\,709\,120 \kappa^3 - 3\,628\,800 \kappa^5) \overline{\overline{\overline{\overline{xxx\overline{xy}y}}}}}{483\,840} + \\
 & \frac{(-13 - 2\,903\,040 \kappa^3 + 1\,209\,600 \kappa^5) \overline{\overline{\overline{\overline{xx\overline{xy}y}y}}}}{161\,280} + \frac{(-1 - 408\,240 \kappa^3 + 113\,400 \kappa^5) \overline{\overline{\overline{\overline{x\overline{xy}}\overline{xy}y}}}}{22\,680} + \\
 & \frac{(-41 + 24\,192 \kappa^3 + 5\,080\,320 \kappa^5) \overline{\overline{\overline{\overline{xx\overline{xy}}\overline{xy}}}}}{580\,608} + \frac{(1 - 75\,600 \kappa^5) \overline{\overline{\overline{\overline{x\overline{xy}y}y}}}}{15\,120} + \\
 & \frac{\overline{\overline{\overline{\overline{xy}\overline{xy}y}y}}}{12\,096} + \frac{(71 - 8\,709\,120 \kappa^3) \overline{\overline{\overline{\overline{x\overline{xy}y}\overline{xy}}}}}{483\,840} - \frac{\overline{\overline{\overline{\overline{xy}y}yyy}}}{30\,240},
 \end{aligned}$$

$$\begin{aligned}
 & \frac{(5 - 672 x^3 + 268800 x^5 + 18063360 x^7) x x x x x^x y}{1290240} + \\
 & \frac{(-25 + 2016 x^3 + 8709120 x^3^2 - 2016000 x^5 - 54190080 x^7) x x x x x^x y y}{1935360} + \\
 & \frac{(5 - 3024 x^3 - 4354560 x^3^2 + 604800 x^5 + 13547520 x^7) x x x x^x y y y}{322560} + \\
 & \frac{(5 - 4536 x^3 - 6531840 x^3^2 + 100800 x^5 + 1270080 x^7) x x x x^x y y y}{483840} + \\
 & \frac{(5 + 574560 x^5 + 4064256 x^7) x x x x^x y y}{290304} + \\
 & \frac{(-5 + 8064 x^3 + 4354560 x^3^2 - 806400 x^5 - 13547520 x^7) x x x x^x y y y y}{483840} + \\
 & \frac{(-17 + 42336 x^3 + 26127360 x^3^2 - 3024000 x^5 - 60963840 x^7) x x x x^x y y y y}{1451520} + \\
 & \frac{(-1 + 945 x^3 + 816480 x^3^2 - 327600 x^5 - 2381400 x^7) x x x x^x y y y}{60480} + \\
 & \frac{1}{120} (-x^3 + 100 x^5 + 1680 x^7) x x x x^x y y y y y + \frac{1}{120} (3 x^3 - 100 x^5 + 1680 x^7) x x x x^x y y y y y + \\
 & \frac{1}{120} (-3 x^3 + 100 x^5 - 1680 x^7) x x x x^x y y y y y + \\
 & \frac{(-1 + 1080 x^3 + 1866240 x^3^2 - 259200 x^5 + 8709120 x^7) x x x x^x y y}{207360} + \\
 & \frac{(-7 + 864 x^3 + 2488320 x^3^2 - 691200 x^5 + 23224320 x^7) x x x x^x y y}{1658880} + \\
 & \frac{1}{60} (-x^3 + 540 x^3^2 + 840 x^7) x x x x^x y y y y + \\
 & \frac{(1 - 6048 x^3 + 259200 x^5 - 5806080 x^7) x x x x^x y y y y}{207360} + \frac{1}{60} (-x^3 + 840 x^7) x x x x^x y y y y + \\
 & \frac{1}{288} (x^3 + 420 x^5 - 3780 x^7) x x x x^x y y y y y, \frac{(1 + 270950400 x^7) x x x x x x x^x y}{77414400} +
 \end{aligned}$$

$$\frac{(-587 - 362\,880\kappa^3 + 174\,182\,400\kappa^3^2 + 27\,869\,184\,000\kappa^3\kappa^5 - 6\,502\,809\,600\kappa^7) \times \times \times \times \times \overline{xy}y}{464\,486\,400} +$$

$$\frac{(253 + 155\,520\kappa^3 - 124\,416\,000\kappa^3^2 - 7\,962\,624\,000\kappa^3\kappa^5 + 1\,625\,702\,400\kappa^7) \times \times \times \times \overline{xy}y}{66\,355\,200} +$$

$$\frac{(59 + 3\,265\,920\kappa^3 - 3\,396\,556\,800\kappa^3^2 - 104\,509\,440\,000\kappa^3\kappa^5 + 10\,668\,672\,000\kappa^7) \times \times \times \overline{xy} \overline{xy}y}{1\,393\,459\,200} +$$

$$\frac{1}{154\,828\,800} (541 - 53\,760\kappa^3 + 116\,121\,600\kappa^3^2 + 10\,752\,000\kappa^5 -$$

$$9\,289\,728\,000\kappa^3\kappa^5 + 4\,877\,107\,200\kappa^7) \times \times \times \times \overline{xy} \overline{xy} +$$

$$\frac{(-43 - 11\,340\kappa^3 + 21\,772\,800\kappa^3^2 + 870\,912\,000\kappa^3\kappa^5 - 203\,212\,800\kappa^7) \times \times \times \overline{xy}y \overline{yy}}{7\,257\,600} +$$

$$\frac{1}{1\,393\,459\,200} (-12\,143 - 5\,322\,240\kappa^3 + 9\,144\,576\,000\kappa^3^2 + 271\,724\,544\,000\kappa^3\kappa^5 -$$

$$45\,722\,880\,000\kappa^7) \times \times \overline{xy} \overline{xy}y \overline{yy} + \frac{1}{1\,393\,459\,200} (-24\,457 - 4\,475\,520\kappa^3 +$$

$$2\,351\,462\,400\kappa^3^2 - 96\,768\,000\kappa^5 + 229\,920\,768\,000\kappa^3\kappa^5 - 91\,750\,579\,200\kappa^7) \times \times \times \overline{xy}y \overline{xy} +$$

$$\frac{(73 - 21\,772\,800\kappa^3^2 - 870\,912\,000\kappa^3\kappa^5 + 254\,016\,000\kappa^7) \times \times \overline{xy}y \overline{yyy}}{14\,515\,200} +$$

$$\frac{1}{99\,532\,800} (-1217 + 86\,400\kappa^3 + 410\,572\,800\kappa^3^2 + 3\,456\,000\kappa^5 +$$

$$14\,929\,920\,000\kappa^3\kappa^5 - 3\,178\,828\,800\kappa^7) \times \times \overline{xy} \overline{xy} \overline{xy}y +$$

$$\frac{(1423 - 609\,638\,400\kappa^3^2 - 13\,934\,592\,000\kappa^3\kappa^5 + 3\,657\,830\,400\kappa^7) \times \times \overline{xy} \overline{xy}y \overline{yy}}{116\,121\,600} +$$

$$\frac{1}{29\,030\,400} (-199 - 50\,400\kappa^3 + 81\,648\,000\kappa^3^2 - 2\,016\,000\kappa^5 +$$

$$1\,306\,368\,000\kappa^3\kappa^5 - 711\,244\,800\kappa^7) \times \times \overline{xy} \overline{xy} \overline{xy}y +$$

$$\frac{(-157 - 4320\kappa^3 + 37\,324\,800\kappa^3^2 - 5\,184\,000\kappa^5 - 870\,912\,000\kappa^7) \times \times \times \overline{xy} \overline{xy} \overline{xy}y}{49\,766\,400} +$$

$$\frac{(7367 - 1\,088\,640\kappa^3 - 1\,828\,915\,200\kappa^3^2 - 62\,705\,664\,000\kappa^3\kappa^5 + 16\,155\,417\,600\kappa^7) \times \times \overline{xy} \overline{xy} \overline{xy}y}{696\,729\,600}$$

$$\begin{aligned} & + \frac{1}{278\,691\,840} \left( 5939 + 48\,384 \kappa_3 - 2\,037\,934\,080 \kappa_3^2 + 19\,353\,600 \kappa_5 - \right. \\ & \left. 12\,541\,132\,800 \kappa_3 \kappa_5 + 15\,240\,960\,000 \kappa_7 \right) \times \overline{\overline{\overline{\overline{\overline{\overline{xy\,y\,xy}}}}}}} + \\ & \frac{(3263 - 3\,919\,104\,000 \kappa_3^2 - 62\,705\,664\,000 \kappa_3 \kappa_5 + 12\,192\,768\,000 \kappa_7) \times \overline{\overline{\overline{\overline{\overline{\overline{xy\,y\,xy\,y}}}}}}}{696\,729\,600} + \\ & \frac{(271 - 1\,075\,200 \kappa_5 + 331\,914\,240 \kappa_7) \times \overline{\overline{\overline{\overline{\overline{\overline{xy\,xy}}}}}}} + \frac{(-1 - 2\,822\,400 \kappa_7) \times \overline{\overline{\overline{\overline{\overline{\overline{xy\,y\,yyy}}}}}}}{30\,965\,760} + \frac{403\,200}{172\,800} + \\ & \frac{(583 - 87\,091\,200 \kappa_3^2 + 2\,322\,432\,000 \kappa_3 \kappa_5) \times \overline{\overline{\overline{\overline{\overline{\overline{xy\,xy\,xy\,y\,y}}}}}} - \frac{\overline{\overline{\overline{\overline{\overline{\overline{xy\,xy\,y\,yyy}}}}}}}{172\,800} + \\ & \frac{(47 + 13\,440 \kappa_3 - 24\,192\,000 \kappa_3^2 + 224\,000 \kappa_5 + 580\,608\,000 \kappa_3 \kappa_5) \times \overline{\overline{\overline{\overline{\overline{\overline{xy\,x\,xy\,y\,y}}}}}}}{6\,451\,200} + \\ & \frac{(4057 + 302\,400 \kappa_3 - 566\,092\,800 \kappa_3^2 - 12\,096\,000 \kappa_5 + 41\,803\,776\,000 \kappa_3 \kappa_5) \times \overline{\overline{\overline{\overline{\overline{\overline{xy\,xy\,xy}}}}}}}{696\,729\,600} + \\ & \frac{(2281 + 967\,680 \kappa_3 - 696\,729\,600 \kappa_3^2 + 16\,128\,000 \kappa_5 + 41\,803\,776\,000 \kappa_3 \kappa_5) \times \overline{\overline{\overline{\overline{\overline{\overline{xy\,xy\,xy}}}}}}}{464\,486\,400} + \\ & \frac{(27 + 89\,600 \kappa_3 - 164\,505\,600 \kappa_3^2 - 1\,792\,000 \kappa_5 + 5\,419\,008\,000 \kappa_3 \kappa_5) \times \overline{\overline{\overline{\overline{\overline{\overline{xy\,xy\,xy\,xy}}}}}}}{51\,609\,600} + \\ & \frac{(-1 - 276\,480 \kappa_3^2) \times \overline{\overline{\overline{\overline{\overline{\overline{xy\,y\,xy\,y}}}}}}} + \frac{(-151 + 21\,772\,800 \kappa_3^2 - 870\,912\,000 \kappa_3 \kappa_5) \times \overline{\overline{\overline{\overline{\overline{\overline{xy\,y\,yyy\,xy}}}}}}}{92\,160} + \frac{14\,515\,200}{172\,800} + \\ & \frac{\overline{\overline{\overline{\overline{\overline{\overline{xy\,y\,xy\,y\,y}}}}}}} + \frac{(-4589 - 783\,820\,800 \kappa_3^2 + 20\,901\,888\,000 \kappa_3 \kappa_5) \times \overline{\overline{\overline{\overline{\overline{\overline{xy\,xy\,xy\,xy}}}}}}}{172\,800} + \frac{696\,729\,600}{1\,393\,459\,200} + \\ & \left. \frac{\overline{\overline{\overline{\overline{\overline{\overline{xy\,y\,yyy\,y}}}}}}}{1\,209\,600}, \dots \right] , \end{aligned}$$

$$\begin{aligned} & \text{CWS} \left[ 0, -\frac{\overline{xy}}{48}, 3 \kappa_3 \overline{\overline{xyy}} + 3 \kappa_3 \overline{\overline{xyy}}, \frac{\overline{\overline{\overline{xxxxy}}}}{2880} + \frac{\overline{\overline{\overline{xyyy}}}}{2880} + \frac{\overline{\overline{\overline{xyxy}}}}{5760} + \frac{\overline{\overline{\overline{xyyy}}}}{2880}, \right. \\ & \left. 5 \kappa_5 \overline{\overline{\overline{\overline{xxxxy}}}} + 5 \kappa_5 \overline{\overline{\overline{\overline{xxxxy}}}} + 5 \kappa_5 \overline{\overline{\overline{\overline{xyxyy}}}} + \right. \\ & \left. 5 \kappa_5 \overline{\overline{\overline{\overline{xyyyy}}}} + 5 \kappa_5 \overline{\overline{\overline{\overline{xyxyy}}}} + 5 \kappa_5 \overline{\overline{\overline{\overline{xyyyy}}}}, \right. \\ & -\frac{\overline{\overline{\overline{\overline{\overline{\overline{xxxxxy}}}}}}}{120\,960} - \frac{\overline{\overline{\overline{\overline{\overline{\overline{xxxxxy}}}}}}}{120\,960} - \frac{\overline{\overline{\overline{\overline{\overline{\overline{xxxxyy}}}}}}}{120\,960} - \frac{\overline{\overline{\overline{\overline{\overline{\overline{xxxxyy}}}}}}}{120\,960} - \frac{\overline{\overline{\overline{\overline{\overline{\overline{xyxyxy}}}}}}}{241\,920} - \frac{\overline{\overline{\overline{\overline{\overline{\overline{xyxyxy}}}}}}}{120\,960} - \frac{\overline{\overline{\overline{\overline{\overline{\overline{xyyyxy}}}}}}}{120\,960} \end{aligned}$$

$$\begin{aligned}
 & \frac{\overline{xyyy}}{120960} - \frac{\overline{xyxy}}{362880} - \frac{\overline{xyxy}}{120960} - \frac{\overline{xyxy}}{241920} - \frac{\overline{xyyy}}{120960}, \\
 & 7 \kappa^7 \overline{xxxxxy} + 7 \kappa^7 \overline{xxxxxy} + 7 \kappa^7 \overline{xxxxxy} + 7 \kappa^7 \overline{xxxxxy} + 7 \kappa^7 \overline{xxxxxy} + \\
 & 7 \kappa^7 \overline{xxxxyx} + 7 \kappa^7 \overline{xxxxyx} + 7 \kappa^7 \overline{xxxxyx} + 7 \kappa^7 \overline{xxxxyx} + \\
 & 7 \kappa^7 \overline{xyxyxy} + 7 \kappa^7 \overline{xyxyxy} + 7 \kappa^7 \overline{xyxyxy} + 7 \kappa^7 \overline{xyxyxy} + \\
 & 7 \kappa^7 \overline{xyxyxy} + 7 \kappa^7 \overline{xyxyxy} + 7 \kappa^7 \overline{xyxyxy} + 7 \kappa^7 \overline{xyxyxy} + 7 \kappa^7 \overline{xyxyxy}, \\
 & \frac{\overline{xxxxxy}}{4838400} + \frac{\overline{xxxxxy}}{4838400} + \frac{\overline{xxxxxy}}{4838400} + \frac{\overline{xxxxxy}}{4838400} + \frac{\overline{xxxxxy}}{4838400} + \frac{\overline{xxxxxy}}{4838400} + \\
 & \frac{\overline{xxxxyx}}{4838400} + \frac{\overline{xxxxyx}}{4838400} + \frac{\overline{xxxxyx}}{9676800} + \frac{\overline{xxxxyx}}{4838400} + \frac{\overline{xxxxyx}}{4838400} + \frac{\overline{xxxxyx}}{4838400} + \\
 & \frac{\overline{xyxyxy}}{4838400} + \frac{\overline{xyxyxy}}{4838400} + \frac{\overline{xyxyxy}}{4838400} + \frac{\overline{xyxyxy}}{4838400} + \frac{\overline{xyxyxy}}{4838400} + \frac{\overline{xyxyxy}}{4838400} + \\
 & \frac{\overline{xyxyxy}}{4838400} + \frac{\overline{xyxyxy}}{4838400} + \frac{\overline{xyxyxy}}{4838400} + \frac{\overline{xyxyxy}}{9676800} + \frac{\overline{xyxyxy}}{4838400} + \frac{\overline{xyxyxy}}{4838400} + \\
 & \frac{\overline{xyxyxy}}{4838400} + \frac{\overline{xyxyxy}}{4838400} + \frac{\overline{xyxyxy}}{4838400} + \frac{\overline{xyxyxy}}{19353600} + \frac{\overline{xyxyxy}}{4838400} + \\
 & \frac{\overline{xyxyxy}}{4838400} + \frac{\overline{xyxyxy}}{4838400} + \frac{\overline{xyxyxy}}{4838400} + \frac{\overline{xyxyxy}}{9676800} + \frac{\overline{xyxyxy}}{4838400}, \dots \}}
 \end{aligned}$$

In[ ]:=  $\kappa@{8}$

$$\text{Out[ ]:= CWS} \left[ \kappa^1 \overline{x}, -\frac{\overline{xx}}{96}, \kappa^3 \overline{xxx}, \frac{\overline{xxxx}}{11520}, \kappa^5 \overline{xxxxx}, -\frac{\overline{xxxxxx}}{725760}, \kappa^7 \overline{xxxxxxx}, \frac{\overline{xxxxxxxx}}{38707200}, \dots \right]$$