250702 **Q.** tDer, sDer, etc., are they the gr of anything? Do they have a tensorial interpretation?

250701 **UGP:** Implement Fiedler's arXiv:2506.17738.

250624c **Q.** Do cyclic words inject into their profiles?

250624b **Q.** Is there a relationship between SolKV and 1-loop associators? Between 1-loop associators and emergent associators?

250624a **Q.** Does $\mathcal{A}^w_p$ act on $\mathcal{A}^{em}_{p,s}$? (Probably not).

250603 **Q.** Is there an ideal triangulation formula for $\Delta$? And $\theta$?

250529 Talk Idea: "Piggyback Matchbox Burau Representations". Needs a few further root system examples.

250516 **Q.** Is there a tangle proof that all Seifert surfaces of a knot are tube-equivalent?

250515 **Proj.** Dirty-implement normal surfaces.

231013b **Def.** A Measured Partial Quadratic (MPQ) on a v.s. $A$ is a triple $Q = (D, v, q)$ where $q$ is a non-degenerate quadratic on a subspace $D \subset A$ and $v$ is a volume element on $D$. Given such $Q$ and a $\varphi \in A^*$, $\int_D dv \exp(-q/2 + \varphi)$ makes sense. **Conj.** Given a linear $L\colon A \to B$ and an MPQ $Q$ on $A$ there is a unique MPQ $L_*Q = (D_*, v_*, q_*)$ on $B$ such that for every $\varphi \in B^*$ we have $\int_{D_*} dv_* \exp(-q_*/2 + \varphi) = \int_D dv \exp(-q/2 + L^*\varphi)$.

250414b Boninger: The torus is flat so rotation numbers make sense so YB+R invariants of knots make sense for knots on a torus.

250414a **Proj.** Find formulas for the weight systems of $\rho_1$ and $\theta$.

250309b **Q.** Is there a direct {expansions for emergent KO} $\to$ {expansions for wKO}?

250309a **Chal.** Understand non-orientable surfaces and the Goeritz Alexander formula.

250227 **Q.** Is there a (semi)-Seifert formula for the MVA?

250131 **Proj.** Find a Seifert-slides invariant face formula for $\theta$.

241218 **Proj.** Use BF to solve KV.

241214 **Talk.** "Goldman-Turaev by ambience and constraint".

241112 **Q.** What means Alekseev-Torossian's $\nu\colon \mathfrak{grt}_1 \to \mathfrak{krv}_2$ by $\varphi \mapsto (\varphi(y,x), \varphi(x,y))$? A Lie algebra map by their Prop. 4.9.

241127 **Proj.** Write $\Theta$ in Sage.

241113 **Proj.** Find an *ss* vertex that solves the *pss* R4 (and unitarity and cap) in $\mathcal{A}^{wgh}$ and in $\mathcal{A}^{wem}$.

221028 Needed. A homological interpretation of $g_{\alpha\beta}(K)$. Is it the ?-invariant of a 3-component tangle $K \cup L_{\alpha\beta}$?

241105 **Q.** Do all homotopy invariants satisfy the $H_1$ relation? Is this an equivalence?

160330 **UGP:** An "Insolubility of the Quintic" web site.

241105b **UGP:** Implement many families of knots, compute their $\theta$.

241104a **UGP:** Properly draw knots, even large.

241029 **Prof.** Figure out Chern-Simons with $\mathfrak{g}^\epsilon_+$.

241022 **Q.** Is there a –○– invariant for a pair of homology classes on a knotted surface?

240918 **Proj.** Study "emergent KV equations". What's the *ss* filtration? By tails on strands? By arrows on strands? By chords on strands?

240904 Is there a determinant formula for the Conway polynomial?

240721 Needed: A groupoid view of $\pi_1$, $\Delta$, $\rho_1$, and $\theta$.

240715 $\mathcal{K}^{wem}$: w-knotted objects with a marked "emergent" subgraph, meaning we mod out by having two semi-virtuals / arrows whose over / tail side is emergent. **Proj.** Use pole dancing ideas to find Goldman-Turaev within $\mathcal{K}^{wem}$ and construct a homomorphic expansion for $\mathcal{K}^{wem}$ (hence for Goldman-Turaev) using a homomorphic expansion for $\mathcal{K}^w$ (hence using Kashiwara-Vergne).

240628 **Q.** Is $\mathfrak{grt} \simeq \mathfrak{krv}$ a version of "braids act nearly transitively on simple curves"?

240624 **Proj.** Study "timelined tangles". Are there homomorphic expansions?

240523 What if Conway preceded Alexander?

240516 AP: Projects: HigherRank: TheCast has the Feynman ring for $sl_3$, Heisenberg version.

240510 Started AP: People: Kuno: CheatSheet.

240502 The Drinfel'd Double: Given bialgebras $A$, $B$, an invertible non-degenerate $R \in A \otimes B$ defines a double-meta-bialgebra morphism $\{PVOUB_{X,Y}\} \to \{A^{\otimes X} \otimes B^{\otimes Y}\}$. **Thm?** Up to ??, there is a unique bialgebra structure on $D := A \otimes B$ such that the diagram on the right commutes.

$$\begin{array}{ccc} \{PVOUB_{X,Y}\} & \xrightarrow{Z:=X\sqcup Y} & \{PVB_Z\} \\ \downarrow & & \downarrow \\ \{A^{\otimes X} \otimes B^{\otimes Y}\} & \xrightarrow{Z:=X\sqcup Y} & \{D^{\otimes Z}\} \end{array}$$

240322 AP: Projects: HigherRank: Rank-2 Gassner:

```
R2G_{i_,j_}[ℰ_] := Expand[ℰ /. {
    e_j :→ T_i e_j + (1 - T_i) e_i, f_j :→ S_i f_j + (1 - S_i) f_i,
    g_j :→ T_i S_i g_j + (1 - T_i S_i) g_i
      + (S_i + T_i - 2 S_i T_i) e_i f_i + S_i (T_i - 1) e_i f_j + T_i (S_i - 1) e_j f_i}]
```

The last line is a "cross-rep glow". **Prob.** Find all cross-rep glows between products of Gassners at different $T$ parameters. There too: a more general rank 2 Burau.

240415 What are burves, plane curves mod braid moves?

240404 MAT 198 Cryptology dreams: Enigma, compression, DES, RSA, Bible codes, Bitcoin, homomorphic computation.

240401 Projects: APAI: APAILinks2.nb: Why does $\rho_1$ makes sense for links? Does it always vanish when the MVA does? An MV$\rho_1$?

240328 **Proj.** Say "Rasmussen's $s$" w/o saying "spectral sequence".

240305a Boninger's conjecture: The coefficients of $\rho_1(K)$ in $z$ are uniform-sign for positive $K$. Tested in `Rho_d-Positivity.nb` in AP: T-: Oaxaca-2210. True for $\Delta$ by Cromwell's "Homogeneous Links".

240319 Hartley: *The Conway Potential Function for Links*. Benheddi, Cimasoni: *Link Floer Homology Categorifies the Conway Function*.

240311 2024-03: Implementation of Kauffman States for tangles. Is there a pull-push formalism for PA operations?

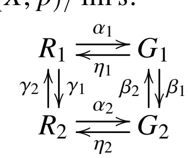
231114 Ozsvàth, Szabò: arXiv:1603.06559, *Kauffman states, bordered algebras, and a bigraded knot invariant*, arXiv:2212.11885, *The pong algebra*, arXiv:2311.07503, *Planar graphs deformations of bordered knot algebras*. Zibrowius: arXiv:1601.04915, *Kauffman states and Heegaard diagrams for tangles*. Roberts: arXiv:math/0607244, AGT 2009, *Heegaard-Floer homology and string links*.

240305b : Kauffman states ↔ tree-dual-tree pairs in the checkerboard graphs ↔ one-long-path smoothings.

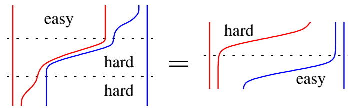
240304 Is the a local / size respecting description of knots ↪ bnots? In some completion? Being liberal about $R1$s? Allowing virtuals?

240302 **JFF.** Compute the sign of a length $10^6$ permutation, 10 of whose entries are hidden.

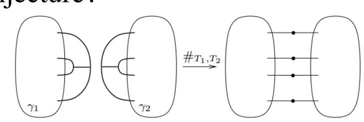
240301 **Q.** A local description of the Wirtinger-Alexander det-signs?
240221b In Kauffman's *Formal Knot Theory*, A state model for the *normalized* Alexander polynomial. Implementation: `VxF_Alexander.nb` in AP: Pe-: Martchenkov. →p3:**230609**.
240221a "The Alexander module dogma is wrong".
170317 Wikipedia: $q$-derivative: $D_{q,x}f(x) = \frac{f(qx)-f(x)}{qx-x}$; has $D_{q,x}\mathbb{e}_q^x = \mathbb{e}_q^x$ (and $\mathbb{e}_q^0 = 1$); seek it and $\mathbb{e}_q^x$ and $xy = qyx$ in nature. Finds: $[a,x] = x \Rightarrow \mathbb{e}^{ta}x = \mathbb{e}^t x \mathbb{e}^{ta}$. Also, in tensor powers with $X_k := \mathbb{e}^{t(a_1+\ldots+a_{k-1})}x_k$, have $X_kX_l = \mathbb{e}^t X_lX_k$ for $k<l$.
240213 **Q.** Do braids act on multi-Fox profiles of free words?
190320 **Proj.** Analyze 2016-06/Turbo-Gassner (also Talks/Toronto-1912/GvIExamples). Is it homological? Find it in Artin's representation. Following Ito@M19, relations with Garside lengths?
240209b Jim Davis: A Witt-valued quadratic pushforward story?
240209a Calaque, Roca i Lucio arXiv:2402.05539 *Associators from an operadic point of view*, a survey of associators.
240205 **Q.** Is there a *(poly-)computable* functor from measured v.s. to Sets, extending Gaussian integration?
240130 Garoufalidis, Kashaev arXiv:2311.11528 *Multivariable knot polynomials from braided Hopf algebras with automorphisms.* Also has a Drinfel'd double alternative.
240122 : With $Z$ a domain and $Q$ its field of fractions, given $\partial\colon R \to G$ invertible over $Q$ and $(\cdot,\cdot)\colon R\otimes G \to Z$ with $(r_1,\partial r_2) = (r_2,\partial r_1)$ get a symmetric $\langle\cdot,\cdot\rangle\colon (\operatorname{coker}_Z\partial)^{\otimes 2} \to Q/Z$ by $\langle \bar g_1, \bar g_2\rangle := (\partial_Q^{-1}g_1, g_2)/Z$. **Q.** When do two presentations yield equivalent $\langle\cdot,\cdot\rangle$'s?
240118 **Proj.** Understand "tangle nullities" (cf. knot nullities).
240115 Borodzik, A. Conway, Politarczyk arXiv:2111.10632, Section 1.1: TL signatures from matrices presenting Blanchfield. Kearton (1978): A edge-centric matrix presenting Blanchfield.
240103 Formal perturbed Gaussian integration, sans the determinant prefactor, is invariant under *all* linear coordinate changes.
240102b Projects: APAI: PerturbedGaussianIntegration.nb: With $K$ a knot diagram, $w$ its writhe, $\pi K$ its rotation by $180°$, $\mathcal{R}_1$ the integrand for $\rho_1$, $\mathcal{P} = \left\{\begin{smallmatrix} x_{2n+1}\to p_1, \\ p_{2n+1}\to x_{2n+1}\end{smallmatrix}\right\} \cup \bigcup_{c:(s,i,j)\in K}\left\{\begin{smallmatrix} x_i\to T^{\nu_i}(p_1-p_{i+1}),\ p_i\to T^{-\nu_i}x_i,\ p_j\to T^{-\nu_i-s}x_j, \\ x_j\to T^{\nu_i}((1-T^s)p_{i+1}+T^sp_1-p_{j+1})\end{smallmatrix}\right\}$ with $\nu$ the Alexander numbering, get $\overline{\int\mathcal{R}_1(\pi K)} = T^{-w}\int\mathcal{R}_1(K)/\mathcal{P}$.
240102a 2023-12: BridgesAndTunnels.nb: Palindromicity using bridges and tunnels.
231214b **Q.** Are there "dual presentations" signature formulas?
231214a **Q.** Do virtual knots have a meaningful Dehn fundamental group? Is it related to the two Wirtinger ones? Are there others?
231212 In the knot complement cyclic cover context, $0 \to H_1(\tilde X) \to H_1(\tilde X,\tilde p) \underset{s}{\overset{\partial}{\rightleftarrows}} \ker(i_*) \to 0$ is split exact, where $\mathbb{Z}[T^{\pm1}] = H_0(\tilde p) \xrightarrow{i_*} H_0(\tilde X) = \mathbb{Z}$ so $H_1(\tilde X) \simeq \ker\partial \simeq H_1(\tilde X,\tilde p)/\operatorname{im} s$.
230811 **Thm** (cf. Lickorish pp. 50). Module presentations $R_i \xrightarrow{\alpha_i} G_i$ $(\to M,\ i=1,2)$ are equivalent iff $\exists\beta_i,\gamma_i,\eta_i$ as here s.t. $\beta_1\alpha_1 = \alpha_2\gamma_1$, $\beta_2\alpha_2 = \alpha_1\gamma_2$, $\beta_2\beta_1 + \alpha_1\eta_1 = I$, and $\beta_1\beta_2 + \alpha_2\eta_2 = I$. Then $\begin{pmatrix} I & 0 \\ \beta_1 & \alpha_2\end{pmatrix} = \begin{pmatrix}\alpha_1 & \beta_2 \\ 0 & I\end{pmatrix}\begin{pmatrix}\eta_1 & -\gamma_2 \\ \beta_1 & \alpha_2\end{pmatrix}$ and $\begin{pmatrix}\alpha_1 & \beta_2 \\ 0 & I\end{pmatrix} = \begin{pmatrix} I & 0 \\ \beta_1 & \alpha_2\end{pmatrix}\begin{pmatrix}\alpha_1 & \beta_2 \\ -\gamma_1 & \eta_2\end{pmatrix}$ so elementary ideals



make sense and (if $\exists\alpha_i^{-1}$) $\alpha_1^{-1} = \gamma_2\alpha_2^{-1}\beta_1 + \eta_1$ and $\alpha_2^{-1} = \gamma_1\alpha_1^{-1}\beta_2 + \eta_2$. Implement for all Alexander presentations!
230406 The AKKN operational envelope: Cuts, then doubles (and reversals), then letter substitutions, then infusion of constants, then merges.
231205 López Neumann, van der Veen: arXiv:2312.02070, "Genus bounds from unrolled quantum groups at roots of unity".
230404 **Q.** Is the pentagon in emergent 2-poles 2-strands equivalent to the standard *FL*-pentagon? (The spaces grow slower!) Can $\Phi_{pps}$ be found degree by degree? Is there a GT group? Isomorphic to a known variant?

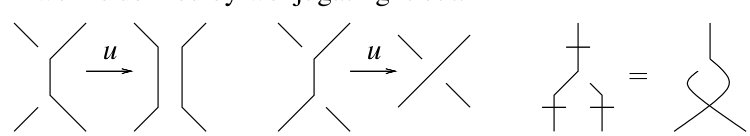



220720 In framed $\Sigma^2$, mod homotopy, curves are the same as "sailing curves", immersions whose tangent avoids direct front winds.
231105a Implicit in Chrisman, Todd arXiv:2307.09387: $\rotatebox{180}{\text{K}}(D) = D\cup\omega$, where $\omega\in H^1(\underline{D})$ is the self-intersection 1-form of $\underline{D}$, the underlying graph of $D$. Is there a rotational version? →p5:**141113b**
231103 Tautology: Alexander numbering ⇔ homologically trivial.
210818 Abbasi's $u\mathcal{K} \hookrightarrow v\mathcal{K}$, "opposite inner-most pairs of non-local R2s can be removed": An "even set of xings" has even incidence with every face. Non-empty ones transport through local R-moves, get created when a non-local R2 is performed, and unions of even sets can be projected preserving R-moves. Now use Abbasi's zigzag trick to reduce the xing-number profile between non-local moves. **Qs.** Links? Rotational virtuals? Braid-like R-moves? Does $u\mathcal{K} \hookrightarrow v\mathcal{K}$ preserve 1-cycles? Can these ideas be used to prove Satoh's conjecture?
231014 Papers/ktgs: Why the dots? $\bullet \to \nu^{1/2}$, with $\nu = Z(\bigcirc)$.

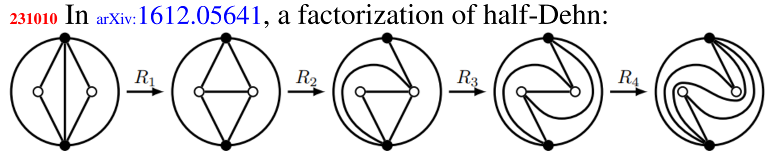



231013a Turaev's arXiv:math/0310218 "Virtual Strings" has "based matrices" and sliceness criteria.
231009 Best $w$-practices:

1. Trivalent tangles are end-labeled, to make a circuit algebra.
2. Tubes are bare (no colours and/or orientations).
3. Crossings have no signs; filtration is by comparison with virtual crossings.
4. Vertices are oriented and have marked legs: stem ($s$), upper ($u$), and lower ($l$). They are classical: they satisfy both R4s.
5. Wenjugating interchanges the two vertex types, and adds a virtual $l \leftrightarrow u$ crossing.
6. Only stems can be unzipped. Unzipping unwened edges connects $u$ to $u$ above a connection of $l$ to $l$. Unzipping through a wen is defined by wenjugating it out.




231010 In arXiv:1612.05641, a factorization of half-Dehn:




231008 Is there a finitely presented algebraic structure made of unoriented pure tangles?
231002b Projects: FullDoPeGDO: cR.nb: in $sl^0_{2+}$, $(x_1,x_2)\cdot R_{12} = R_{12}\cdot(x_1 + (1-B_1)x_2, B_1x_2)$, and $(y_1,y_2)\cdot R_{12} = R_{12}\cdot$

$(y_1, \frac{b_2(1-B_1^{-1})}{b_1}y_1 + B_1^{-1}y_2)$.

231002a **Proj.** "Seven formulas for $\rho_1$".

221228 **Missing.** A fully defined theory of pushing forward Gaussians (better with determinants and signatures). **Q.** Does the signature pushforward work also for det′?

230915 Frohlich's trace: Let $A = \mathcal{U}(sl^0_{2+})$ and $\phi\colon \mathbb{Q}[b,z,a] \to A_A$ by $b^kz^na^m \mapsto b^ky^na^mx^n/n!$ (surjective as $0 = [a, b^ky^na^mx^l] = (l-n)b^ky^na^mx^l$ in $A_A$). In $A$ with $f = f(a)$ and $\nabla f := f(a) - f(a-1)$, $[x, f] = -\nabla f \cdot x$ so in $A_A$, $0 = [x, y^{n+1}fx^n] = (n+1)by^nfx^n - y^{n+1}\nabla f x^{n+1}$ so $\phi(bz^nf) = \phi(z^{n+1}\nabla f)$. Ergo $\mathcal{G}(\mathrm{tr})$: $\mathrm{tr}(\mathbb{e}^{\beta b}\mathbb{e}^{\eta y}\mathbb{e}^{\alpha a}\mathbb{e}^{\xi x}) = \phi(\mathbb{e}^{\beta b}\mathbb{e}^{\eta\xi z}\mathbb{e}^{\alpha a}) = \phi(\mathbb{e}^{\beta z\nabla}\mathbb{e}^{\eta\xi z}\mathbb{e}^{\alpha a}) = \phi(\mathbb{e}^{\alpha a + (\eta\xi + \beta(1-\mathbb{e}^{-\alpha}))z})$. →p6:**200906**, →p7:**180909a**.

230911 $\mathcal{A}(*_*)$ is a contraction algebra.

230904 If cobrackets come from the asymmetry of coproducts, wherefore the Turaev cobracket?

230823 With $\Lambda := \mathbb{Z}[T^{\pm 1}]$ and $\Lambda_0 := \mathbb{Z}[T + T^{-1}]$, is it that for every $\Lambda$-module $A$ there is a $\Lambda_0$-module $B$ with $A \oplus \bar{A} \equiv \Lambda \otimes_{\Lambda_0} B$?

230822 Figure out the Kashaev-formula weight system (for $\Delta^2$?).

230817 Is there a Goeritz formula for $\Delta$?

230228b In VanDerVeen_Journal: Given a diagram $D$ for a long $K$, the phase $\phi$ along a curve $\gamma \subset D^c$ multiplies by $T^s$ whenever $\gamma$ passes over $D$ with sign $s$. **Conj.** $\mathrm{lk}_K(\alpha,\beta) = (T-1)\langle$flow: generated by $\alpha$, measured by $\beta\rangle + \langle$phased $\alpha$ over $\beta$ count$\rangle$. $\alpha$ generates $\pm\phi$-flow when it runs over $D$. $\beta$ measures $\pm\phi^{-1}$-flow when it runs under $D$.

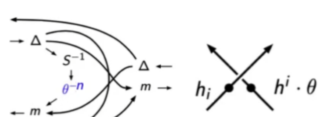
230804b Fox derivatives: in $H_1(\tilde{D}_n, \tilde{p})$, $[\gamma] = \sum(\partial_i\gamma)[x_i]$.

230804a Given a long $K$, is $K = (-K) \bmod \Delta\Delta$?

140422 Yajima's "On Simply Knotted Spheres in $\mathbb{R}^4$": all are ribbon. *Pf.* Enough: every simply-knotted balloon forest is equivalent to a ribbon-certificate. Take an inner-most double line on a balloon, slide out string ends and string transverses, and compress to a new string. When no double lines are left, float balloons to unnest them, and iron wens to make string ends external. See BBS: KAL-140520. I don't fully understand the case of tubes.

230725 In w, what about $\mathfrak{g} \ltimes (\mathfrak{g}^* \oplus \mathfrak{g}^*)$? And flying graphs?

230702 BBS with Suzuki on Heisenberg doubles. Also her arXiv:1612.08262, Kashaev's arXiv:q-alg/9503005, Baseilhac's arXiv:math/0202272 and arXiv:1101.3440, and Baseilhac-Benedetti's arXiv:1101.1851.




230425b López-Neumann@[K-OS], twisted Drinfel'd doubles: Let $H$ be a f.d. $\mathbb{N}$-graded Hopf algebra and let $\theta(h) = t^{|h|}h$, $h \in H$. Virelizier (2000): for each $n \in \mathbb{Z}$ let $D_n = H^* \otimes H$ with multiplication as on the right. This is NOT a Hopf algebra (if $n \neq 0$) but there is a "coproduct" $\Delta_{n,m}\colon D_{n+m} \to D_n \otimes D_m$ and antipode $S_n\colon D_n \to D_{-n}$ satisfying graded versions of Hopf axioms.

230425a López-Neumann@[K-OS]: $X \in \mathrm{Rep}(DH)$ iff $X \in \mathrm{Rep}(H)$ together with a collection of *half-braidings* $\{\sigma_{Y,X}\colon Y \otimes X \to X \otimes Y\}_{Y \in \mathrm{Rep}(H)}$. Thus, $\mathrm{Rep}(DH)$ is the *Drinfel'd Centre* of $(\mathrm{Rep}(H), \otimes)$ (centre as for monoids, but it's a monoidal category!). It is a braided monoidal category.

230711 What's the canopoly envelope of Khovanov's Frobenius algebra as generated by $\{\rangle\!\langle, \odot_\pm, \otimes_\pm\}$?

230612b For matrices $A$, $B$, is $\{\lambda\colon \exists v\ Av = \lambda Bv \neq 0\}$ always finite? (No; take $A, B\colon \mathbb{R}^2_{x,y} \to \mathbb{R}$ with $A = x + y$, $B = x$. Then with $v = (1, \lambda - 1)$, $Av = \lambda Bv = \lambda \neq 0$.)

230609 `AlexanderUsingDehn.nb` in AP: P–: Martchenkov. →p2:**240221b**.

230607 Abstract version of $\det\begin{pmatrix} A & B \\ C & U \end{pmatrix} = \det(A)\det(U - CA^{-1}B)$?

230606 Alexander: $tA - A^T$. Tristram-Levine: $(1-\omega)A + (1-\bar{\omega})A^T = (\bar{\omega} - 1)(\omega A - A^T)$.

230525 Are there other nearly-linear invariants of tangles, beyond linking numbers and signatures? →p6:**210114a**

230419 **Sydney, Mar-Apr 2023.** In AP: A–: Alekseev_Kawa–: Annotated AKKN1. In BBS: Hogan: Multiplication, exp, and ⬡ in $\mathcal{A}^{1p2s}$ (1 pole 2 strands); implementation in AP: —: Hogan: Comp1ss.nb. In BBS: Dancso: "weakly parenthesized" tangles in a *PDS*, searching for div in strand doubling $\mathcal{A}^{2p1s} \to \mathcal{A}^{2p2s}$ (more likely KV2 is in pole doubling $\mathcal{A}^{1p1s} \to \mathcal{A}^{2p1s}$, in degree 1), the pentagon in $\mathcal{A}^{2p2s}$ (→p2:**230404**).

230417 **Proj.** Extend knot colouring to tangles in Zombian language.

200611a **Q.** What conditions on $(\mathbb{A}, \mathbb{B}, \langle\cdot\rangle)$ are enough to make the Drinfel'd double associative? Examples beyond Hopf algebras? Restrict attention to braids?

230321b **Proj.** Develop a language to describe $\mathcal{A}^{/(k+1)\text{-co}}$ in terms of *FA*/*FL*. What are the atomic spaces and operations?

230321a López-Neumann and van der Veen, arXiv:2211.15010, [K-OS], email: A "twisted double" that may replace the need to deform the co-product.

230317b **Q.** In PDS, is there an expansion for $v$-tangles?

230317a **Proj.** In PDS, study emergent $v$-tangles and/or $v_1$-tangles.

230309 In PDS, is $\mathcal{A}^{*/*+1}$ isomorphic to "*FL*-dirty $\mathcal{A}$"? Is there a homomorphic $ss$-degree expansion $\mathcal{A}^* \to \mathcal{A}^{*/*+1}$? Does the sequence $0 \to \langle\text{wheels}\rangle \otimes FA \to \mathcal{A}^{/2}_H \to FA \to 0$ split?

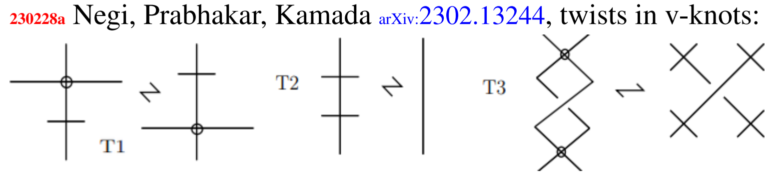
230315 In 2023-03: AlexanderLogConcavity.nb: Testing log-concavity for alternating knots, inspired by arXiv:2303.04733 by Hafner, Mészéros, Vidinas.

230313 Hiroe, Negami, arXiv:2303.05770 "Long-Moody Construction of Braid Representations and Katz Middle Convolution".

220712b Construct homomorphic expansions for tangles in $PDS_n$ $ss$-degree by $ss$-degree starting from $FG_n \to FA_n$.

230227 Equivariant linking numbers: • Kearton (∼1978): Blanchfield formulas. • Kojima and Yamasaki (1979): First definition. • Kricker's and Garoufalidis' arXiv:math/0105028: Definition from the Kontsevich Integral. • Garoufalidis' and Teichner's arXiv:math/0206023: Abstract definition (also, indication that $\rho_1$ sees genus). • Ohtsuki 2007 "Invariants ... Surgery Presentations": Some computations, probably no PD formula. • Lescop's arXiv:1001.4474, arXiv:1008.5026: Abstract definitions (also appearing: $(\log\Delta)'$). • Friedl's and Powell's arXiv:1512.04603: Blanchfield formulas from Seifert surfaces.

230228a Negi, Prabhakar, Kamada arXiv:2302.13244, twists in v-knots:




230217 **Groningen, Jan-Feb 2023.** In Projects: MetaCalculi: UnitaryGamma.nb: A pairing with a Lagrangian property and an $\omega/\bar{\omega}$ = det property for Γ-calculus (single variable). In Talks: MoscowByWeb-2104: Hodge.nb: Hodge infrastructure for $\mathcal{A}$-calculus; exponentials are preserved, but unfinished unitarity property. Lashing fails at Projects: APAI: Lashings.nb.

221118 **Do.** Unify Goldman-Turaev (GT) with equivariant intersection numbers. Understand the action of the braid group on GT (and its expansions). Related to the unitarity of Gassner?

230213b **Do.** Digest $S(V)/\Lambda(V)$-exponentiation as "unfurling".

230213a Re. ωεβ/mo21, is there a fast zero-test for linear combinations of Fermionic Gaussians?

230103 **Q.** Are there "$R$-matrix" long knot invariants in which a distinction is made between $R_{i<j}$ and $R_{i>j}$? "Scheduled tangles"?

230209 **Chal.** Find topological interpretations for QA Alexander formulas. I.e., what do specific matrix entries mean?

230203b **Q.** Might it be that determinant formulas for Alexander should depend on a presentation matrix along with a certificate that it presents a torsion module?

230203a **Do.** Analyze Alexander determinants for tangles similarly to the signature analysis.

230201b In Γ, is there an infinitesimal version? Are there $\text{im}(\alpha)$ "classicality" conditions?

230201a In Γ, for classical tangles $\bar{A} = \mathcal{L}_0 A$ with analytic $\mathcal{L}_0$. Is $(\bar{A}, \bar{\omega}) = \mathcal{L}(A, \omega)$ with analytic $\mathcal{L}$?

230118 **Proj.** Write `ValidatePD` and `PlanarQ` for `KnotTheory‘`.

230131 In Γ, truly understand the "flatness property" of $\omega$.

221213 The signed number of ascending crossings is invariant under R2/R3.

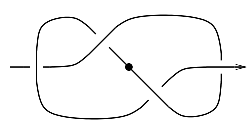
221212 **Q.** Is Alexander a counting of representations?

221207 Which Gauss diagram formulas vanish on classical knots / tangles? Tanglify "the linking matrix of a classical link is symmetric".

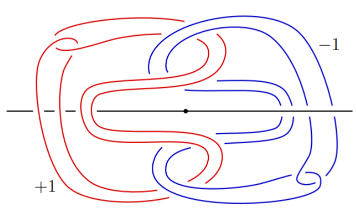
221128 **Q.** Is there a 3D understanding of the balancing of $\Delta(K)$?

221114 Burton tabulated knots to 19 crossings.

221025b Boyle@Oaxaca: Develop a finite type theory for SNACKs, Strongly Negative AmphiChiral Knots (long knots invariant under 180° rotation with vertical flip, as $4_1$). Probable examples in arXiv:2206.03598. **Q.** If SNACKs are equivalent as knots, are they equivalent within SNACKs? Are they the same as knots in $\mathbb{RP}^3$? (No and no).



2210a5a Learn how to compute invariants of knots presented by surgeries. E.g., Boyle's@Oaxaca knot, arXiv: 2207.12593.




221007 With $V$ the Burau representation of $w\mathcal{B}_n$, there is a non-trivial splitting of $\mathbb{Q}w\mathcal{B}_n \ltimes \mathbb{H}(V \otimes V^*)[\![\epsilon]\!]_{\text{doc}} \to \mathbb{Q}w\mathcal{B}_n$. So what?

220928 **Q.** Given a classical $K$, ∃? automorphism $\phi\colon \pi(K) = \pi \circlearrowleft$ s.t. $\bar{\phi}\colon \pi/[\pi,\pi] \cong \mathbb{Z} \circlearrowleft$ is $(-1)$?

220920 In Projects: RibbonKnots: Rho4Ribbons.nb: $(p = 1 - 3T + T^2)^2 \mid \Delta(11_{n66})$ yet $p \nmid \rho_1(11_{n66})$. Is there a Fox-Milnor $K$ and a $p$ s.t. $p^2 \mid \Delta(K)$ yet $p \nmid \rho_1(K)$? →p4:**220816**

220712a A quadratic form $Q$ on $V$ induces $Q'$ on $V/\langle u\rangle$ via $Q'(x,y) = Q(u,u)Q(x,y) - Q(x,u)Q(u,y)$. Best if $Q(u,u) \neq 0$. Leads to a characterization of signatures?

220821 **Proj.** Find Duflo in Goldman-Turaev.

220816 **Q.** When the Alexander polynomial factors, is there a reason? (It factors more than Jones). Is every ribbon knot the sum of a w-knot with its mirror, modulo Alexander skein moves?

220807 **Proj.** Develop a "universal" Goldman-Turaev theory: With $G$ a group and $F$ a free group, with $|G| := G/\operatorname{Ad} G$ and $|GG| := G \times G/\operatorname{Ad} G$ (note the maps $|GG| \to |G| \times |G|$ and $|GG| \to |G|$) there are $b\colon \mathbb{Q}|F| \wedge \mathbb{Q}|F| \to \mathbb{Q}|FF|$ and $\delta\colon \mathbb{Q}|F| \to \mathbb{Q}|FF|/\text{alt}$. (1) Really? (2) What Lie-bialgebra-like properties hold? (3) Expansions? (4) KV?

150224b Fadell-Neuwirth: For $0 < r < n$, $m \geqslant 0$, and $M = \Sigma^2_{g\geqslant 1} \mid D^2$, $1 \to P\mathcal{B}_{n-r}(M \backslash \underline{m+r}) \to P\mathcal{B}_n(M \backslash \underline{m}) \to P\mathcal{B}_r(M \backslash \underline{m}) \to 1$ is exact. **Q.** An infinitesimal version?

220615b **Q.** Which maps $\varphi\colon \mathbb{Q}F_n \to \mathbb{Q}F_n$ preserve the filtration(s)? Just homomorphisms combined with translations? Which decrease the filtration? Riffled splicings with homomorphisms with intersection countings?

220615a Does the recovery formula of HUJI-1912,

$$P^{(1)} = T\omega\dot{\omega} - p_1(T-1)^2/T + 2T\omega\dot{\omega}a + 2T\omega\dot{\omega}xy/(1-T),$$

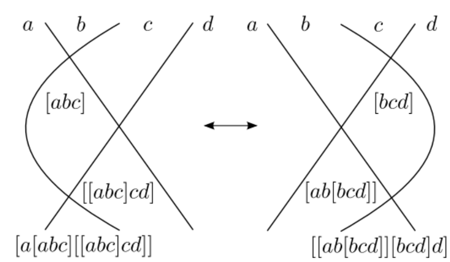
hold for virtuals?

220530 Is the ⋄-algebra generating function poly-computable?

220426 Lamm lists ribbons with up to 12 crossings.

220421 **Proj.** Use AI to rediscover Fox-Milnor.

220413 Niebrzydowski, Pilitowska, Zamojska-Dzienio arXiv:1805.07817 "Knot-theoretic Ternary Groups". Also, Chavez, Nelson arXiv:2204.05851 "Δ-Tribrackets and Link Homotopy".




140312 Minsky: Fary-Milnor: $\gamma\colon S^1 \to \mathbb{R}^3$, $\kappa$ its total curvature. Then $\kappa < 4\pi \Rightarrow \gamma$ unknotted. *Pf.* Find a projection direction $p \in S^2$ in which $\gamma$ has $\leqslant 3$ criticals, hence 2 criticals, hence $\gamma$ is 1-bridge. As $\dot{\gamma}$ travels length $\kappa$ on $S^2$, $(\dot{\gamma})^\perp$ spans area $< 4\kappa$, hence some $p$ is covered $< 4$ times. □ *Further proofs.* Petrunin, Stadler arXiv:2203.15137. *Generalization.* $2\pi(\text{bridge number}) = (\text{infimal } \kappa)$.

220321 **Q.** What are the ADO (Akutsu-Deguchi-Ohtsuki) invariants?

220302 Giroux, Goodman, arXiv:math/0509555: fibered links all come from plumbing.

201225b **Proj.** Develop poly-dimensional braid representations via "Burau/Gassner homology" (cf. 2020-07: HeisenbergPerturbations.nb).

220128 Miller: All the Ways I Know to Define the Alexander Polynomial.

220102 **Dream.** (1) An extension theorem for Heisenberg modules. (2) All quantum invariants arise in this way.

211225 **Q.** A quantum mechanical interpretation of coloured Jones?

211222b Owens' arXiv:2112.10706 has a list of sliceness criteria.

211222a Owens and Swenton's "An Algorithm to Find Ribbon Disks for Alternating Knots", arXiv:2102.11778 and here, has programs, data, and stats. →p5:**210517**

211221 **Q.** Mirror and glue a slice disk $(\mathbb{R}^2_+, S = \mathbb{R}) \subset (\mathbb{R}^4_+, \mathbb{R}^3)$ to get a w-knot $K = \mathbb{R}^2 \subset \mathbb{R}^4 = \mathbb{R}^4_+ \cup \mathbb{R}^4_-$. Is $A(S) = |A(K)|^2$?

211206 Signatures: I know neither the domain (tangles as what algebraic structure?) nor the range (as algebro-geometric object, without evaluation in $\mathbb{R}/\mathbb{C}$). Are signatures compatible with strand doubling? What's the multi-variable version?

211114 Is there a universal way to express the quotient MVA→Alexander?

211014 Aizawa, Harada, Kawaguchi, Otsuki, arXiv:math/0612781: All Link Invariants for Two Dimensional Solutions of YBE.

210930 In Ruppik's talk: Any epimorphism $\pi_1(\Sigma_g) \to FG_g$ is realized by a handlebody.

210923 Roland: $e^{\hbar x}$ makes sense in $(\mathbb{Z}/p)[\hbar]/(\hbar^p)$.

210918 Compact ⇔ Every open cover that's closed under pair unions contains the full set.

210916 **Do.** Implement and double the Taft Hopf algebra (e.g. sec. 4 of Montgomery, Schneider, "Skew derivations of finite-dimensional algebras and actions of the double of the Taft Hopf algebra"). Compute invariants. Compare with arXiv:2103.01081 by Feng, Hu, and Li, arXiv:1805.10340 by Cline, and Cline's thesis.

210908 Farley's arXiv:2109.02815: The Planar Pure Braid Group, $\Gamma_n := \pi_1(\mathbb{R}^n \backslash \{\text{triple intersections}\})$.

210905b The thickening $\{\text{signed bipartite planar graphs}\} \to \mathcal{K}$ is surjective. Is there a local theory? Finite-type? Virtual?

210905a Checkerboard / Alexander knots: Reidemeister theory? Algebraic structure? Finite-type theory? Relations with almost classical? Signatures? Other invariants?

141113b Boden: A v-knot is "Almost Classical (AC)" if it is homologically trivial on a surface. Equivalent to "image in $\mathcal{K}(\bigcirc_v \uparrow_w)$ splits"? Is there a FT theory for AC knots? Is there "$H^1$ of the carrying surface", an invariant of v-knots? →p2:**231105a**

210806 Y-C. Tang: u-width$(K) = 6 > 4 =$ v-width$(K)$ for $K \in \{9_{18}, 9_{27}, 10_{24}, 10_{25}, 10_{42}, 10_{44}\}$.

210802 Write Duflo in **DoPeGDO** language.

170316 Wikipedia: Wigner-Weyl transform, arXiv:hep-th/0011137 (& within): Gaussians compose hyperbolically, $\exp\left(-a(q^2+p^2)\right) \star \exp\left(-b(q^2+p^2)\right) = \frac{1}{1+\hbar^2 ab} \exp\left(-\frac{a+b}{1+\hbar^2 ab}(q^2+p^2)\right)$.

210414 Signatures: The "standard" à la Tristram–Levine, `KnotTheory``, Goeritz (e.g. Ozsváth-Stipsicz-Szabó), Kashaev's arXiv:1801.04632, A. Conway's arXiv:1903.04477. Also Cimasoni-Conway arXiv:1507.07818 and Merz' arXiv:2104.02993 on additivity defects of signatures of braid closures.

210106 The "Iterates Completion" $\mathcal{M}$ of a **Vect**-enriched category $C$ with an ideal $\mathcal{I}$: adjoin "iterates" $a^\star := (1-a)^{-1}$ for endomorphic $a \in \mathcal{I}$. Commutes with additive completion, with positive coefficients: $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^\star = \begin{pmatrix} (a+bd^\star c)^\star & a^\star b(d+ca^\star b)^\star \\ d^\star c(a+bd^\star c)^\star & (d+ca^\star b)^\star \end{pmatrix}$! If $C$ is monoidal, contains $\det(1-A)^{-1}$ for $A \in \mathcal{I}$? Relations beyond $a^\star b(d+ca^\star b)^\star = (a+bd^\star c)^\star bd^\star$ with $a, (b|c), d \in \mathcal{I}$ at $\circlearrowleft_a \bullet \overset{b}{\underset{c}{\rightleftarrows}} \bullet \circlearrowright_d$? Also/just $(a+b)^\star = b^\star(ab^\star)^\star = a^\star(ba^\star)^\star$? Canonical forms for morphisms? In poly-time? Carries Alexander? 2021-01: Det3x3.nb, →p6:**181222b**, "weighted automata", "rational series".

151216 **Proj.** Braidors & weak associators: $B = \Phi^{012} R^{12}_u \Phi^{-021}$,
$$B^{012}B^{02,1,3}B^{023} = B^{01,2,3}B^{013}B^{03,1,2}.$$
Extensibility / uniqueness in $\mathcal{A}^u$, $\mathfrak{sder}$, Γ? A KV/WKO3 variant? Are they enough to quantize Lie bialgebras? To solve YB?

210525 **TIL** (Kuperberg on FB, 2021-05: FubiniCounterexample.nb) $\int_{[0,\infty]^2} \frac{(x-y)dxdy}{(1+(x-y)^2)^2}$, a nice Fubini counterexample.

210517 Manolescu@BIRS: Sherry Gong has a program to find ribbons for (ribbon) knots. →p4:**211222a**

210512 **Q.** What is docility for maps between polynomial rings?

210506 $\pi_1(SL_2(\mathbb{R})) = \mathbb{Z}$.

210429 On the Blanchfield pairing: mo:7411.

210408 Lobb's talk, "Four-Sided Pegs Fitting Round Holes Fit All Smooth Holes", inspires plotting Möb $=$ Conf$_2(S^1 \subset \mathbb{C}) \to \mathbb{C}^2$ via $(z,w) \mapsto ((z+w)/2, (z-w)^2)$ to get a circle-boundary Möbius band in $\mathbb{R}^3$.

210329 **Q.** What values take singular Gaussian Fermionic integrals?

210318c **Chal.** Interpret Leclerc's "On Identities Satisfied by Minors of a Matrix" in terms of Fermionic Gaussians.

210318b Grinberg's book, "Notes on the Combinatorial Fundamentals of Algebra".

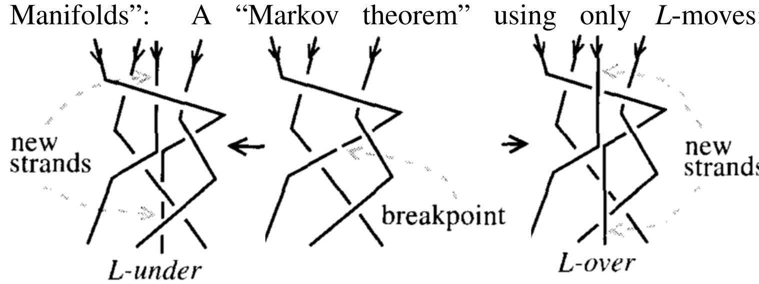
210316 **Q.** Why are Fermionic Gaussian compositions so similar to Bosonic ones?

180821b I don't understand the MVA (and yet it's there). E.g., does the Halacheva meta-trace have a tensorial representation
$$\begin{array}{c|cc} \omega & c & S \\ \hline c & \alpha & \theta \\ S & \psi & \Xi \end{array} \xrightarrow[\mu:=1-\alpha]{\Gamma::\mathrm{tr}_c} \begin{array}{c|c} \mu\omega & S \\ \hline S & \Xi + \psi\theta/\mu \end{array}$$
→p7:**180909a**? Why is it independent of the component left open?

210211a $\Lambda^*(X^* \cup Y)$ with $\deg X^* = -1$, $\deg Y = 1$ is a contraction algebra with $c_{x,y} := e^{\partial_\eta \partial_x}\colon \Lambda^*(X^* \cup Y) \to \Lambda^*((X-x)^* \cup (Y-y))$ hence $\mathcal{A}(X) := \Lambda^*(X^* \cup X)_0$ is a traced meta-monoid with $m^{xy}_z := c_{xy} /\!/ (\zeta \to \xi, y \to z)$ and $\mathrm{tr}_x(A) := c_{xx}$. Halacheva (∼): Contains Γ (w/ fixed colours) via $\Upsilon_X\colon (\omega, M) \mapsto \omega e^{\sum \xi M_{\xi y} y}$. Predict $\mathcal{A}$ from Γ? Interpret $\mathcal{A}$ in *ybax*? Related to super-algebras? Raise $\mathcal{A}$ to meta-Hopf? Understand im(Υ)?

210217 **Do.** In **DoPeGDO** for *ybax*, do *a* first?

201217a **Proj.** A concise proof of Alexander / Markov (including an $n^{3/2}$ complexity bound and an implementation). **Q.** Is there a framed Markov theorem? Lambropoulou, Rourke "Markov's Theorem in 3-Manifolds": A "Markov theorem" using only *L*-moves:




Also: Sundheim, Traczyk, Morton, Birman, Birman-Brendle.

210211c **Do.** Super-**DoPeGDO**.

210211b **Do.** $\mathcal{A}^w$ and super-Lie-algebras.

190314b Wherefore $\left(\sum_{n\geq 0} \mathrm{tr}\, \Lambda^n A\right) \exp\left(\sum_{n>0} \frac{(-1)^n}{n} \mathrm{tr}\, A^n\right) = 1$? 2021-02/rdet.nb: use for quick det computations in rings in which non-zero integers are invertible. →p6:**210130**

200703a **Q.** An Archibald calculus that includes strand doubling? A common generalization of Archibald- and Γ-calculus? →p7:**200804**, →p5:**210211a**

210204 Conway's talk, "Knotted Surfaces with Infinite Cyclic Knot Group": a topological classification of surfaces Σ with $\pi_1(\Sigma^c) = \mathbb{Z}$ in a simply-connected 4-manifold.

210130 Itai: Computing det over a ring $R$ in poly-time: compute $\det(1-\epsilon(1-A))$ in $R[\![\epsilon]\!]$ by Gaussian elimination and formal inversions. But it's a polynomial in $\epsilon$! Now set $\epsilon = 1$. →p5:**190314b**

210114a Feller's talk: the fractional Dehn twist coefficient, the unique homogeneous quasimorphism $\omega\colon B_n \to \frac{1}{n}\mathbb{Z}$ with defect $\sup_{g,h\in G}|f(gh)-f(g)-f(h)| = 1$ s.t. $\omega(\Delta^2)=1$, $\omega(B_{n-1})=0$. Invariant under conjugation, probably under strand doubling. Also Malyutin, "Twist Number of (Closed) Braids".

210128b **Do.** Use ☺ to compute $\mathcal{A}^w(\bigcirc_n)$. →p6:**200906**

210128a Hom's talk, "Infinite Order Rationally Slice Knots": Rationally slice := bounds a smooth disk in a $\mathbb{Q}HS^4$. **Thm.** Using Heegaard-Floer, there is a $\mathbb{Z}^\infty \oplus (\mathbb{Z}/2)^\infty$ in rationally slice knots modulo concordance. Levine 69': algebraic concordance group := {Seifert forms}/(metabolic, vanishes on half-dimensional subspace) $\equiv \mathbb{Z}^\infty \oplus (\mathbb{Z}/2)^\infty \oplus (\mathbb{Z}/4)^\infty$.

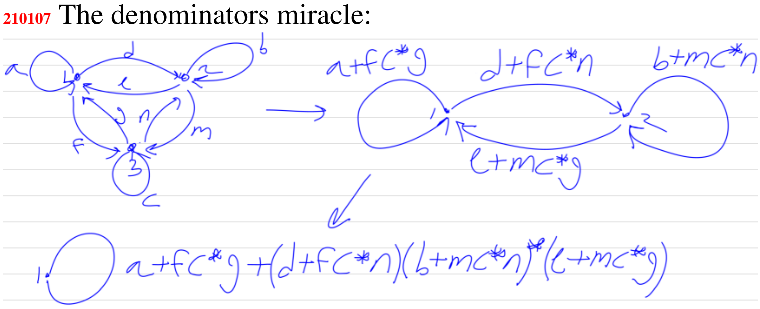
210124b Manturov's philosophy: "If something is wrong, it's because it's not drawable on the plane. This must be because of a homological obstruction, which leads to a parity, leading to projections, brackets, coverings."

210124a Manturov (zoom): his "Parity and Projection from Virtual Knots to Classical Knots" has a combinatorial proof of Kuperberg's theorem.

210121 Khovanov's talk, "Bilinear Pairings and Topological Theories": "near TQFTs" regressed from arbitrary invariants; especially in 2D.

210114b **TIL.** "Digit ratio".

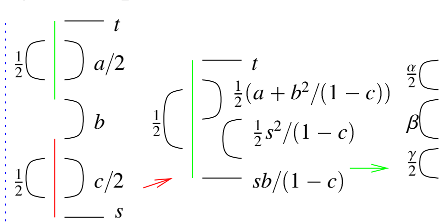
210107 The denominators miracle:




$= a+((1-c)de+(1-b)fg+dmg+fne)/((1-b)(1-c)-mn)$
$= a+(db^\star e+fc^\star g+db^\star mc^\star g+fc^\star nb^\star e)(mb^\star nc^\star)$

200917 **Do.** Zipping in the 1PI context. Convert diagram from "external assembly" to "merging of completed".

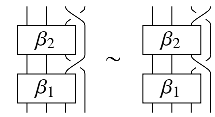
201225a Zipping twice:



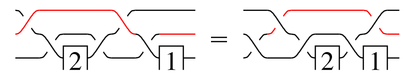

$\alpha = \frac{t^2}{1-\left(a+\frac{b^2}{1-c}\right)} = \frac{t^2(1-c)}{(1-a)(1-b)-b^2}$, $\beta = t\frac{sb}{1-c}\frac{1}{1-\left(a+\frac{b^2}{1-c}\right)} = \frac{tsb}{(1-a)(1-b)-b^2}$, $\gamma = \frac{s^2b^2}{(1-c)^2}\frac{1}{1-\left(a+\frac{b^2}{1-c}\right)} + \frac{s^2}{1-c} = \frac{s^2b^2+s^2((1-a)(1-c)-b^2)}{(1-c)((1-a)(1-c)-b^2)} = \frac{s^2(1-a)(1-c)}{(1-c)((1-a)(1-c)-b^2)} = \frac{s^2(1-a)}{(1-a)(1-c)-b^2}$.

201230 People: VanDerVeen: SingularGamma.nb: $S_{ab}$ (on right) is a general singular point in Γ-calculus, with $S_{ab}|_{e=1} = R_{ab}$ and $S_{ab}|_{e=t^{-1}} = R^{-1}_{ba}$. $\begin{pmatrix}1 & a & b\\ a & e & 1-et\\ b & 1-e & et\end{pmatrix}$

201217b **Q.** If $\beta_{1,2}\in B_n$ then $\beta_1\sigma_n\beta_2\sigma_n^{-1}$ and $\beta_1\sigma_n^{-1}\beta_2\sigma_n$ have the same closure. How are they related by Markov moves?




Sol'n from Birman-Brendle arXiv:math/0409205, Fig. 13: "Markov" the red strands on the right.




201214b Polyak: {planar diagrams}/($R1r, R2b, R3b$) describes knots via braids, but by counting counterclockwise cycles in the oriented smoothing, are more than knots.

201214a **Q.** What are {planar curves}/($R1l, R1r, R2b, R3b$)?

201209 Bolan: $\sqrt{2}^{\log_2 9}\in\mathbb{Q}$.

201117 **Q.** Are there GPV formulas for tangles and links?

201109 There is a "crossing change" construction of Seifert surfaces, and a construction starting from an immersed bounding disk.

201112 Piccirollo's talk, "A Users Guide to Straightforward Exotica": has a calculus for handle decompositions of 4-manifolds.

201105 Dynnikov's talk, "An Algorithm for Comparing Legendrian Knots": Legendrian knot: in $\ker(xdy+dz)$. Have Reidemeister theory. Related to grid diagrams. Topological meaning?

201023 **TIL.** Merge "only" in "she told him that she loved him".

181222b For $A := \begin{pmatrix}a & b\\ c & d\end{pmatrix} = \begin{pmatrix}1 & b\bar d\\ 0 & 1\end{pmatrix}\begin{pmatrix}a-b\bar dc & 0\\ 0 & d\end{pmatrix}\begin{pmatrix}1 & 0\\ \bar dc & 1\end{pmatrix}\in M_{(p+q)\times(p+q)}$, $|A| = |d||a-b\bar dc|$, and $A^{-1} = \begin{pmatrix}(a-b\bar dc)^{-1} & -(a-b\bar dc)^{-1}b\bar d\\ -\bar dc(a-b\bar dc)^{-1} & \bar d+\bar dc(a-b\bar dc)^{-1}b\bar d\end{pmatrix}$, with $\bar d := d^{-1}$. 2019-03, →p8:**190312**, →p5:**210106**, Bareiss' "Sylvester's Identity…".

181222a If $A\in M_{n\times n}$ and $\omega = |A|$, then each entry of $A^{-1}$ has denominator $\omega$, so expect $|A^{-1}|\propto\omega^{-n}$. Yet $|A^{-1}| = \omega^{-1}$.

201011 The Steinberg relations between elementary matrices: $e_{ij}(\lambda)e_{ij}(\mu) = e_{ij}(\lambda+\mu)$, $[e_{ij}(\lambda), e_{jk}(\mu)] = e_{ik}(\lambda\mu)$ for $i\neq k$, and $[e_{ij}(\lambda), e_{kl}(\mu)] = 1$ for $i\neq l$ and $j\neq k$.

201006 **Q.** Let $I_{v\downarrow w} := \ker(P{\it v}B \to P{\it w}B)$ and let $\mathcal{A}_{v\downarrow w} := \prod I^m_{v\downarrow w}/I^{m+1}_{v\downarrow w}$. What is $\mathcal{A}_{v\downarrow w}$? Are there "expansions"? Are there "Burau/Gassner expansions"?

200925 **Riddle** (Matthew Bolan) ∃? cont. $f\colon\mathbb{R}\to\mathbb{R}$ s.t. $f\circ f = \cos$?

200216 Direct proof that $uB_n \hookrightarrow vB_n$ (cf. Gaudreau, arXiv:2008.09631): For $w = \prod_{\alpha=1}^{l}\sigma_{i_\alpha j_\alpha}^{s_\alpha}$ define "depths" $(d_{k,\alpha})_{1\leqslant k\leqslant n, 0\leqslant\alpha\leqslant l}$ inductively by $d_{k0} = k$ and for $\alpha>0$, $d_{k\alpha} = d_{k,\alpha-1}+s_\alpha(\delta_{ki_\alpha}-\delta_{kj_\alpha})$. Drop from $w$ every $\sigma_{i_\alpha j_\alpha}^{s_\alpha}$ for which $s_\alpha\neq d_{j_\alpha,\alpha-1}-d_{i_\alpha,\alpha-1}$ (well defined on $vB_n$!). Iterate. The result is a retraction $vB_n\to uB_n$.

200906 With $\mathfrak{g} = sl^0_{2+} = \langle y,b,a,x\rangle/([a,x] = x, [a,y] = -y, [x,y] = b, [b,-] = 0)$, $\mathcal{U}(\mathfrak{g})_\mathfrak{g}$ is freely generated by $\{b^ka^n\}_{k\leqslant n}$, also $\{y^ka^nx^k\}_{k,n\geqslant 0}$, for $[a, y^la^mx^n] = (n-l)y^la^mx^n$ forces $l = n$ and $xa = (a-1)x$ implies $xf(a) = f(a-1)x$ so $[x, f(a)] = (f(a-1)-f(a))x$ so with $a^{(k)} := \binom{a+k}{k} = (a+1)(a+2)\cdots(a+k)/k!$ and $a^{(-1)} := 0$, $[x, a^{(k)}] = -a^{(k-1)}x$ so $[x, y^na^{(k)}x^{n-1}] = nby^{n-1}a^{(k)}x^{n-1} - y^na^{(k-1)}x^n$. Thus in $\mathcal{U}(\mathfrak{g})_\mathfrak{g}$, $by^nx^n = 0$ and $y^na^{(k)}x^n = n!b^na^{(k+n)}$. So $\{b^ka^n\}$ generates and $b^{n+1}a^n = 0$. The same relations also follow from

$[y^{n-1}a^{(k)}x^n, y] = -y^n a^{(k-1)}x^n + nby^{n-1}a^{(k)}x^{n-1}$, and these are all the relations in $\mathcal{U}(\mathfrak{g})_{\mathfrak{g}}$. →p3:**230915**

**180909a** The meta-trace (simpler before quantization?): At $k=0$, $\mathrm{tr}_m = \mathbb{E}\left(\beta_m b_m, \frac{\mathcal{A}_m(1-B_m)}{\mathcal{A}_m-1}\xi_m\eta_m\right)$. Then ruled by $C_{abc} \xrightarrow{d^2} C_{12} \xrightarrow{d^0} C_0$ via $d^1 = \mu_0^{12} - \mu_0^{21}$ and $d^2 = (\sigma_1^a\mu_2^{bc} + \text{cyc. perm.})$, where $\mu_k^{ij} = (\alpha_k \to \alpha_i + \alpha_j, \xi_k \to \mathcal{A}_j\xi_i + \xi_j, \eta_k \to \eta_i + \mathcal{A}_i\eta_j, \beta_k \to \beta_i + \beta_j - \xi_i\eta_j)$. More generally, $\mathrm{tr}_m = \mathbb{E}\left(r\alpha_m\beta_m/\hbar + s\beta_m b_m + t\alpha_m a_m, \frac{\mathcal{A}_m^{1-r}(\mathcal{A}_m^r - B_m^s)}{\mathcal{A}_m-1}\xi_m\eta_m\right)$. Pensieve: 2018-08: Trace.nb, →p5:**180821b**, →p3:**230915**.

**200827** Baby **DoPeGDO** in 2020-07: HeisenbergPerturbations.nb and People: VanDerVeen: TimidHeisenbergRGeneralForm@.nb.

**190304** Exponential zipping: With $[f]_\lambda := \mathbb{e}^{\lambda\partial_z\partial_\zeta}f = \langle f|_{z\to z+z',\zeta\to\zeta+\zeta'}\rangle_{\lambda;z'\leftrightarrow\zeta'}$ (so $\langle f\rangle_\lambda = [f]_\lambda|_{z=\zeta=0}$) and $P_\lambda := \log[\mathbb{e}^f]_\lambda$, have $P_0 = f$ and $\partial_\lambda P_\lambda = (\partial_z\partial_\zeta P_\lambda) + (\partial_z P_\lambda)(\partial_\zeta P_\lambda)$. **Proj.** Extend **DoPeGDO** to **EDDO** (Exponentiated Docile Differential Operators). 2019-03: requires the "wake equation" $\partial_\lambda W^j = W^i\partial_{z_i}W^j$.

**200811** **Q.** In $CU$, if $Q$ is quadratic and $P$ is docile, are $\log\mathbb{O}(\mathbb{e}^Q)$ quadratic and $\log\mathbb{O}(\mathbb{e}^P)$ docile?

**160312** **Proj.** Truly understand $Kh(K) = 1 \Rightarrow K = 1$. If $Kh(D) = 1$, constructively reduce $D$ to 1. Kronheimer, Mrowka arXiv:1005.4346, "Khovanov homology is an unknot-detector".

**200807** **Q.** Is there a unique factorization $K = \kappa\# K'$, with $K, K'$ virtual knots, and $\kappa$ maximal classical?

**200806** **Q.** Is there a link invariant $UMVA$ such that $UMVA(L)$ dominates all the $MVA$s of satellites of $L$? →p5:**200703a**

**200804** Costantino-Le arXiv:1907.11400: $O_{q^2}(SL(2))$ in skein theory.

**20721** **Q.** Is there a unique factorization $T = \beta' T'$, with $T, T' \in \mathcal{T}_n$, and $\beta' \in \mathcal{B}_{2n}$ "maximal"? What is $\mathcal{T}_n/\mathcal{B}_{2n}$?

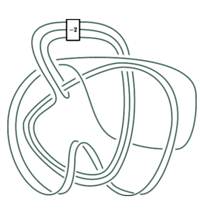
**200703b** Kassel-Turaev: $\Sigma$ a punctured disk with basepoint $d$, $\varphi\colon \pi_1(\Sigma) \to G$ a surjective homomorphism, $(\tilde\Sigma, \tilde d)$ the cover of $(\Sigma, d)$ corresponding to $\ker\varphi$. Then $\tilde H := H_1(\tilde\Sigma, \tilde d)$ is a left module over $\mathbb{Z}G$ of free rank $\mathrm{rk}\, H_1(\Sigma)$.

**200625** Beliakova@KOS: "Partial trace property".

**200618** Meusburger@KOS: Mapping class groups presentations by Gervais, Penner, Bene. "Pivotal Hopf monoids". How much of her work is OU / sutured 3-manifolds? **Q.** Are there virtual versions of other mapping class groups? Will they have simpler presentations, like $P\!v\!B$ is simpler than $PB$?

**200611b** D. Long: Given $\rho\colon AB_n = F_n \rtimes B_n \to \mathrm{End}(V)$ constructs $\rho^+$ acting on $(\mathrm{aug}\,\mathbb{Z}F_n)\otimes_{\mathbb{Z}F_n} V = \mathbb{Z}^n \otimes_{\mathbb{Z}} V$.

**200523** **TIL.** $E_1(x) := \int_x^\infty \frac{\mathbb{e}^{-t}dt}{t} \sim \sum_{n\geqslant 0}\frac{(-)^n n!}{x^n}$. Also $\int_0^\infty \frac{\mathbb{e}^{-x}dx}{1+\epsilon x} \sim \sum_{n\geqslant 0} n!(-\epsilon)^n$.



**200521b** Piccirillo's knot arXiv:1808.02923 requires $Kh$(55xings).

**200521a** **TIL.** Naisse: "graded monoidal category".

**200520** Is the gr of acyclic tangles acyclic arrow diagrams? What's the right algebraic structure?

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**180406** **Proj.** A volume $V$ yarn-ball knot has $\sim V^{4/3}$ xings. Can compute linking numbers in $\sim V$ time. $\pi_1$ has $\sim V$ generators/relations. Is the degree of Alexander is bounded by $\sim V$? Same for $\rho_1$? How big is a KTG presentation? How high the genus? The hyperbolic volume? The degree of Jones?

**170401** **Proj.** over-then-under "Ⓤ-Tangles". Closed under compositions; (v-)braids are Ⓤ; non-braid Ⓤ-tangles? Relations in Ⓤ? In $\mathcal{A}^{Ⓤ}$? Not all tangles are Ⓤ? Alexander properties; v-version. Associators in $\mathcal{A}^u \cap \mathcal{A}^{Ⓤ}$: Constructible? Sufficient for EK? Relations with Chterental's "virtual curve diagrams"? **Q.** Is there a quotient-completion-extension $\mathcal{T} \to \tilde{\mathcal{T}}$ of $\mathcal{T} = \{\text{tangles}\}$ s.t. in $\tilde{\mathcal{T}}$ every tangle is Ⓤ and s.t. all Reshetikhin-Turaev invariants factor through $\tilde{\mathcal{T}}$? The w reduction (with pacifiers?)? Does Ⓤ$/R1, R2 \hookrightarrow v\mathcal{T}$?

**200326** Meadow: commutative ring with unit with $x \mapsto x^{-1}$ s.t. $(x^{-1})^{-1} = x$ and $x(xx^{-1}) = x$.

**200320** Audoux, Bellingeri, Meilhan, Wagner in arXiv:1507.00202: Is $P\!v\!B \to P\!v\!T$ injective?

**200205** The Hopf axiom $S * I = I * S = \eta\epsilon$ is too strong, so for involutive-Hopf-algebra invariants R2b ⇒ R2c. →p10:**180909b**.

**200204a** $\hat{\mathfrak{g}} := \mathfrak{g}[t^{\pm 1}] \oplus \langle c\rangle$ with $[c, -] = 0$, $[t^n a, t^m b] = t^{n+m}[a,b] + \delta_{n+m} n\langle a, b\rangle c$.

**170126d** **Wanted.** A finite-dimensional representation of $gl^\epsilon_{n,+}$.

**191127** **Q.** A "representation" theory $\mathfrak{g}^\epsilon_+ \to gl^\epsilon_{n,+}$? Weyl actions?

**191119** Kopparty on Berlekamp-Welch: The values of a degree $n$ polynomial $p$ are given on a set with $10n$ points with $n$ lies. Can recover $p$ in poly time! Used in "Reed-Solomon Codes".

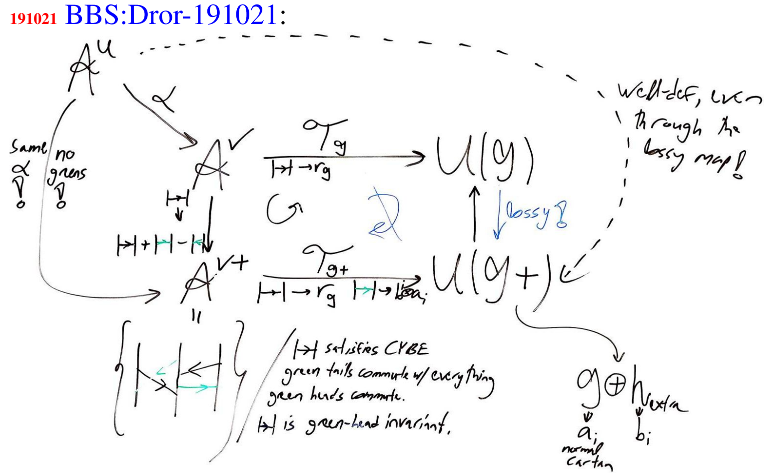
**191112b** **Do.** GDO for all Lie algebras.

**191112a** **TIL.** PIT (Polynomial Identity Testing) is BPP (use random evaluations on scalars and Zariski density) but unknown if in P.

**191107b** **Def.** A $Q$-module: An assignment $Q \mapsto \mathcal{P}_Q$ taking $\mathcal{S}(V^{\otimes 2}) \to$ Set, along with maps $\mathcal{P}_{Q_1} \times \mathcal{P}_{Q_2} \to \mathcal{P}_{Q_1+Q_2}$ and $\mathcal{P}_Q \to \mathcal{P}_{[F:\,Q]}$ for $F \in \mathcal{S}((V^*)^{\otimes 2})$. Interesting examples?

**191107a** **Do.** Put co-Poisson-Hopf-algebras in **DoPeGDO**.

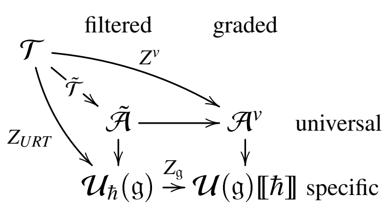
**191021** BBS:Dror-191021:



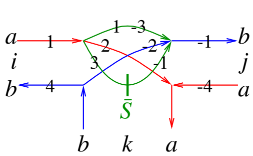

• Requires $[r, a_i\otimes 1 + 1\otimes a_i] = 0$. • Geometric meaning for $\mathcal{A}^{v+}$? • A diagrammatic quantization in $\mathcal{A}^{v+}$? • An Ⓤ version?

**191104** Whirling (2019-11): $W\colon \begin{pmatrix}\Xi & \phi \\ \theta & \alpha\end{pmatrix} \mapsto \frac{1}{\alpha}\begin{pmatrix}1 & -\theta \\ \phi & \alpha\Xi - \phi\theta\end{pmatrix}$ satisfies $W^n(A) = A^{-1}$ for $A \in M_{n\times n}$. Why won't denominators explode?

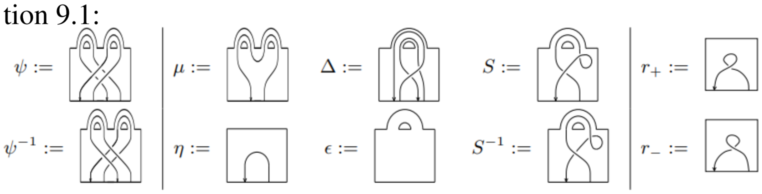
191030 A direct sum Lie algebra can degenerate to a non direct sum.
191029 **Do.** 6T and TC solutions in 3D are too weak for Alexander.
141114a **Proj.** Short paper on "crossing the crossings", $ⵣ\!K\colon v\mathcal{K}_n \to w\mathcal{K}_{n+1}$: definition, invariance, u-neutrality, associated graded. Domination of Manturov, Bardakov, Boden, Brendenbursky. Relation with Medina-Revoy. →p13:**141107**.
191020 **Do. DoPeGDO** directly with $sl_2^\epsilon$ (or why it can't be done).
190808 **Do.** For $D \in \mathcal{A}^u$, recover $\mathcal{T}_{\mathfrak{g}+}(D)$ from $\mathcal{T}_{\mathfrak{g}}(D)$. Recover the $sl_{2+}$ invariant from the $sl_2$ one.
191001 "The *n*-Category Cafe is the left adjoint of the inclusion functor of obscure mathematics into all mathematics" (credits on file).
190914 Are there "homological virtual knots", where only the homology of a carrying surface matters?
190906 Pulmann-Ševera @arXiv:1906.10616: In $C$ a $BMC$, $[HA(C)] \equiv$ [lax monoidal nerves $N\colon BrCom \to C$]. In $C$ an $R$-$iBMC$, $[PHA(C)] \equiv$ [lax monoidal nerves $N\colon iCom(R) \to C$]. Drinfel'd: ($C$ an $R$-$iBMC$) $\rightsquigarrow$ ($C_\hbar^\Phi$ a $BMC$).
190722 **Do.** Develop the ⓪/$\mathbb{AB}R$ narrative to a solution of $R2c$. Find a linear-complexity embedding of tangle theory into diagrams mod braid-like moves.
190723 Friedl, Powell @arXiv:1907.09031: if a knot $J$ is homotopy ribbon concordant to $K$ then $A(J) \mid A(K)$. An AKT view?
190711 **Q.** Wherefore the Heisenberg Double $H(A)$? Kashaev (95'): The Drinfel'd double $D(A) \subset H(A) \otimes H(A)^{op}$.
190709 Naef: If $gl(U) \supset G \circlearrowright V$, a faithful representation of $(g,v) \in G \ltimes V$ in $U \oplus V \oplus \mathbb{1}$: $\begin{pmatrix} g|_U & 0 & 0 \\ 0 & g|_V & v \\ 0 & 0 & 1 \end{pmatrix}$
190612 Boot up to Ševera: • Implement $RI = \mathbb{e}^{t_{12}/2}$. . . .
190606 **Needed.** A precedent for "two-stage Gaussian integration".
171001 What the world should look like. $Z_{\mathfrak{g}}$: A representation theoretic construction? A homological construction? A soft construction from $Z^v$? A torsor? $GT/GRT$? $\tilde{\mathcal{A}}$: a universal $\mathcal{U}_\hbar(\mathfrak{g})$. An a priori meaning? $\tilde{\mathcal{T}}$: A universal extension/quotient of $\mathcal{T}$. Rotational virtual tangles? A quotient thereof? $v$-Claspers? A closure of ⓪? →p9:**190105a**.




190603 Kofman in Da Nang: The Vol-Det Conjecture: For a hyperbolic alternating link $K$, $\mathrm{Vol}(K) \leqslant 2\pi \log\det(K)$.
190530 Porti in Da Nang: For a hyperbolic $K \subset S^3$ and $|\zeta| = 1$, $\lim_{N\to\infty} N^{-2}\log|\Delta_k^{\rho_N}(\zeta)| = \mathrm{vol}(S^3\backslash K)/4\pi$, with $\rho_N$ the $N$-dim representation of $SL_2(\mathbb{C})$.
190523 BBS:Itai-190523+: $L^p$ inequalities: $\|u\|_q \leqslant C_{p,d}\|\nabla u\|_p$ in $\mathbb{R}^d$ with $\frac{1}{q} = \frac{1}{p} - \frac{1}{d}$; $\|u\|_p \leqslant \|u\|_{p_0}^{1-\theta}\|u\|_{p_1}^{\theta}$.
190113 The non-linear Schrödinger eqn: $\beth\partial_t\psi = -\frac{1}{2}\Delta\psi - |\psi|^{p-1}\psi$. BBS:Itai-190523: conserves $\int|\psi|^2$ and $\int\left(\frac{1}{2}|\nabla\psi|^2 - \frac{2}{p+1}|\psi|^{p+1}\right)$.
180619 Does the Drinfel'd double generate all "suppressed-cycle diagrams"?




190509 Snyder, Tingley @arXiv:0810.0084: $T$ with $R = T_1^{-1}T_2^{-1}\Delta_{12}T$. In **DoPeGDO**?
190501b Merkulov @arXiv:1904.13097: "Grothendieck-Teichmüller Group, Operads and Graph Complexes: a Survey".
190501a Livingston @arXiv:1504.03368: "Doubly Slice Knots with Low Crossing Number". An AKT description? →p9:**181218b**
190425b **DoPeGDO**$_2$: Quadratics are of weight precisely 2, interactions of weight at most 2, with $\mathrm{wt}(x,y,\xi,\eta,a,b,\alpha,\beta,\epsilon) = (1,1,1,1,2,0,0,2,-2)$.
190425a Darné @arXiv:1904.10677: w-braids up to homotopy.
190417 **Riddle.** If a box of sides $(b_i)$ is contained in a box of sides $(a_i)$, then $\sum b_i \leqslant \sum a_i$. Khesin's and Itai's Sol'ns in %.
190414 With Ens. • Establish a cluster⇔DK dictionary. • Do syzygy operators always eliminate the image of some "error operator", like $\tilde{d}^1/\!\!/\tilde{d}^2 = 0$ in papers/GT1?
190412b **Chal.** Mix braidors and solvable approximation.
190412a **Q.** Is there an interesting "braidor-liberator" algebra?
190407 DCA := Directed Circuit Algebra = Symmetric Strict Spherical Category with singly-generated set of objects.
190409 Khovanskii @arXiv:1904.03341: "One Dimensional Topological Galois Theory".
190404 **Do.** Implement Habiro-Massuyeau arXiv:1702.00830, section 9.1:




190324 In rec-tangles: rectangle = handle = hand = sleeve = protected zone = input of op. Is there a 3D description?
190325 **TIL.** Mathematica's Notation package.
190322 Fiedler @arXiv:1902.06091: "A refinement of the first Vassiliev invariant can distinguish the orientation of knots".
171205 (Approx.) On $H^{*cop} \otimes H$ with $R = Id = \rho \otimes r$ (summed), $\int \phi \otimes x := \langle \phi\bar\rho \mid xr\rangle$ is an integral. $\frac{1}{2}$**Pf.** $x_1\int\phi\otimes x_2 = x_1\langle\phi\bar\rho \mid x_2 r\rangle = x_1 r^a r^b\langle\phi\bar\rho\bar\rho^a\rho^b \mid x_2 r\rangle \sim x_1 r_1 r^b\langle\phi\bar\rho\rho^b \mid x_2 r_2\rangle \sim (xr)_1 r^b\langle\phi\bar\rho \mid (xr)_3\rangle\langle\rho^b \mid (xr)_2\rangle \sim (xr)_1(\overline{xr})_2\langle\phi\bar\rho \mid (xr)_3\rangle = \langle\phi\bar\rho \mid xr\rangle = \int\phi\otimes x$. Verify! Attempt in Projects/SL2Portfolio2/DoubleIntegration.nb.
190314a **Do.** A graphical calculus for **DoPeGDO** and **DoPeGDO**$_2$.
190321 After Rushworth: Is Jones on $\mathbb{Q}HS$ a polynomial? A skein relation? A Kauffman bracket? Categorifies? →p16:**140116**
190312 **Def.** $\omega\|A$ means $\forall k\, \omega^{k-1}|\Lambda^k(A)$. **Ex.** With $\omega = |A|$, $\omega\|(\omega B + \mathrm{adj}(A))$ (tested 2018-12). **Q.** An effective $\omega\|A$ certificate strong enough to certify the example? →p6:**181222b**
190310b Tree generation: $\partial_\lambda T_\lambda = (\partial_z T_\lambda)(\partial_\zeta T_\lambda)$. Legendre transform: $(Lf)(\eta) := \mathrm{crit}_y(\eta y - F(y)) = \eta y_0 - F(y_0)$, with $dF_{y_0} = \eta$.
190310a KV in Γ in Projects/MetaCalculi:

| $\dfrac{\left(\frac{T_1-1}{t_1}\right)^{\frac14}\left(\frac{T_2-1}{t_2}\right)^{\frac14}}{\left(\frac{T_1 T_2-1}{t_1+t_2}\right)^{\frac14}}$ | $s_1$ | $s_2$ |
|---|---|---|
| $s_1$ | $\dfrac{t_1+\sqrt{\frac{t_1 t_2 (t_1+t_2) T_1 (T_2-1)}{(T_1-1)(T_1 T_2-1)}}}{t_1+t_2}$ | $\dfrac{t_1-T_2\sqrt{\frac{t_1 t_2 (t_1+t_2)(T_1-1) T_1}{(T_2-1)(T_1 T_2-1)}}}{t_1+t_2}$ |
| $s_2$ | $\dfrac{t_2-\sqrt{\frac{t_1 t_2 (t_1+t_2) T_1 (T_2-1)}{(T_1-1)(T_1 T_2-1)}}}{t_1+t_2}$ | $\dfrac{t_2+T_2\sqrt{\frac{t_1 t_2 (t_1+t_2)(T_1-1) T_1}{(T_2-1)(T_1 T_2-1)}}}{t_1+t_2}$ |
| $\Gamma$ | 1 | $\sqrt{T_1}$ |

190225 "Set-theoretically-induced delusions of greatness".
131203 Ševera quantization, arXiv:1401.6164: Given a BMC $\mathcal{D}$ (with Manin $(\partial, \mathfrak{g}, \mathfrak{g}^\star)$, set $\mathcal{D} := \mathcal{U}(\partial)$-Mod$^\Phi$), a co-braided co-

algebra $(M,\Delta\colon M\to M^2,\epsilon\colon M\to 1_{\mathcal D})$ in it ($M:=\mathcal{U}(\mathfrak g)=\mathcal{U}(\partial)/\mathcal{U}(\partial)\mathfrak g^*$), a second BMC $\mathcal C$ (Vect), a functor $F\colon\mathcal D\to\mathcal C$ ($F(X):=X/\mathfrak g X$) and a comonoidal structure $c$ (natural $c_{X,Y}\colon F(XY)\to F(X)F(Y)$ and $c_1\colon F(1_{\mathcal D})\to 1_{\mathcal C}$ respecting braiding and associativity) so that

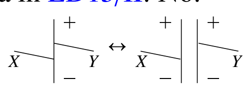
$$F(XMY)\xrightarrow{F(1\Delta 1)}F(XMMY)\xrightarrow{c_{XM,MY}}F(XM)F(MY)$$
$$\text{and}\quad F(M)\xrightarrow{F(\epsilon)}F(1_{\mathcal D})\xrightarrow{c_1}1_{\mathcal C}$$

are isomorphisms (the clear $c_{X,Y}\colon XY/\mathfrak g(XY)\to(X/\mathfrak gX)(Y/\mathfrak gY)$), construct a Hopf algebra structure on $H:=F(M^2)$:

$$\Delta_H\colon\ F(M^2)\xrightarrow{F(\Delta\Delta)}F(M^4)\xrightarrow{F(1R1)}F(M^4)\xrightarrow{c_{M,M}}F(M^2)^2,$$
$$m_H\colon\ F(M^2)^2\xleftarrow[\sim]{c_{M^2,M^2}\circ F(1\Delta 1)}F(M^3)\xrightarrow{F(1\epsilon 1)}F(M^2),$$
$$S_H\colon\ F(M^2)\xrightarrow{F(R)}F(M^2).$$

Set also $G\colon X\mapsto F(MX)$ ($G\colon X\mapsto\frac{\mathcal U(\mathfrak g)X}{\mathfrak g(\mathcal U(\mathfrak g)X)}$), the "twist".
– Is $H$ the symmetry algebra of something? – In the non-quasi case, can we reconstruct $\mathcal U(\mathfrak g)$ from the category of $\partial$-modules?
– In the abstract context, what is the relation between $H$ and $M$?
– How does this restrict to AT/AET in the commutative case?
– $H$ pairs with the quantization of $\mathfrak g^\star$? Ševera in LD15/II: No.

190221 **Do.** Understand "two tube surgery":



190205 **Q.** When/why does the universal Verma module see all invariants?

190124 **TIL.** "Groupoid algebras" are "weak Hopf algebras".

181106a Guo@Hefei: Algebras: • Rota-Baxter: Associative with unary $P$ with $P(f)*P(g)=P(f*P(g))+P(P(f)*g))$. Think $P(f)=\int f$, $f*g=\int f'g'$. • Dendriform; pre-Lie; averaging; diassociative.

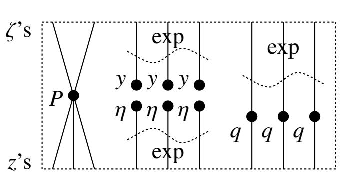
190105b **Q.** Why is there a Cartan involution for classical $sl_{2,\epsilon}$ (multiplicative co-multiplicative $\theta$ preserving $r+r^{21}$)? Is there a quantum analog?

190105a **Q.** If $A_h$ is the quantization of a Lie bialgebra $\mathfrak a$, is there always a multiplicative expansion $A_h\to\mathcal U(\mathfrak a)[\![h]\!]$? →p8:**171001**.

190104 **Wanted.** Examples around "$\partial_x(x^{-1})\big|_{x=0}$ is undefined, yet $\mathbb{e}^{\xi\partial_x}(x^{-1})\big|_{x=0}=\xi^{-1}$".

190102 Etingof-Schiffmann 2.2.2: $\langle a,x\rangle/([a,x]=x)$ is also a Lie bialgebra with $\delta(a,x)=(a\wedge x,0)$.

180629 The Zipping Thm (verification 2018-12). If $P$ has a finite $\zeta$-degree and $\tilde q$ is the inverse matrix of $1-q$: $(\delta^i_j-q^i_j)\tilde q^j_k=\delta^i_k$, then



$$\left\langle P(z_i,\zeta^j)\mathbb{e}^{c+\eta^iz_i+y_j\zeta^j+q^i_jz_i\zeta^j}\right\rangle=|\tilde q|\left\langle P(z_i,\zeta^j)\mathbb{e}^{c+\eta^iz_i}\Big|_{z_i\to\tilde q^k_i(z_k+y_k)}\right\rangle$$
$$=|\tilde q|\mathbb{e}^{c+\eta^i\tilde q^k_iy_k}\left\langle P\left(\tilde q^k_i(z_k+y_k),\zeta^j+\eta^i\tilde q^j_i\right)\right\rangle.$$

**Do.** • Sort $\langle\cdot\rangle_{\text{int}}\leftrightarrow\langle\cdot\rangle\leftrightarrow\oint$. • Sort denominators.

181218b Freedman: an AKT description of slice knots? →p8:**190501a**

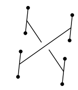
181218a **Q** (following Boden via Gaudreau). Is the crossing number of a virtual link equal to that of its irreducible representative?

181202 **TIL.** Vibration modes of mugs (Tadashi Tokieda).

181201 **Do.** A better narrative for $\wp$.

181123 **Proj.** Clasp number $k$ knots: AKT-definable? Alexander properties? →p13:**161009**.

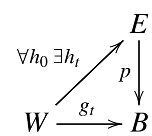
181119 **Do.** EK in terms of pegged tangles:



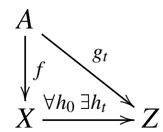
181116 Chang, Cui @arXiv:1710.09524 relate Kuperberg/Turaev-Viro-Barrett-Westbury with Hennings-Kauffman-Radford/Witten-Reshetikhin-Turaev.

181104 AC⇒Zorn: Assume by contradiction that in $(X,<)$ every chain $C$ has a (chosen) *strict* bound $M(C)$, and let $\mathcal W:=\{W\subset X\colon W$ well ordered, $\forall x\in W\ M(\{w\in W\colon w<x\})=x\}$. Then $\bigcup\mathcal W$ is a maximal element of $\mathcal W$ (effort here), contrary to the existence of $M(\bigcup\mathcal W)$. (The key: transfinite constructions have a "maximal extent" $\mathcal M$; here leading to a contradiction. AC is not needed for $\mathcal M$, yet it has a busy beaver feel.)

150609d Fibrations $p\colon E\to B$ on right. Any $X\xrightarrow{\phi}Y$ is $X\xrightarrow{i_0}E_\phi\xrightarrow{p_1}Y$ with $i_0$ a homotopy equivalence and $p_1$ a fibration. Here $E_\phi\sim\{(x\in X,\gamma\colon[0,1]\to Y)\colon\gamma(0)=\phi(x)\}$; $i_0(x)=(x,\bar x)$, $p_1(x,\gamma)=\gamma(1)$, and the "homotopy fiber" is $p_1^{-1}(y)=\{(x,\gamma)\colon\gamma(0)=\phi(x),\ \gamma(1)=y\}$. E.g., for $\text{Emb}(\mathbb R\hookrightarrow\mathbb R^3)\hookrightarrow\text{Imm}(\mathbb R\to\mathbb R^3)$, the homotopy fiber is framed knots.

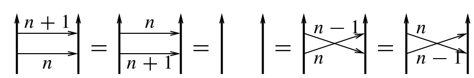


Cofibrations $f\colon A\to X$ on right (e.g. cones, $A\to CA$). Any $X\xrightarrow{\phi}Y$ is $X\xrightarrow{i_0}M_\phi\xrightarrow{p_1}Y$ with $i_0$ a cofibration and $p_1$ a homotopy equivalence.



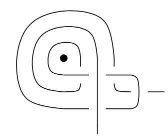
181106b Drummond-Cole knows dimensions of the BiAlg PROP.

181028 Wherefore these relations, with $n\in 2\mathbb Z$?



181017a "Topological Expansionism": "Quantum Topology" is a mix of topology, algebra, representation theory, and quantum field theory. I will explain how to expand the territory of topology within that mix at the expense of representation theory and algebra.

181022 **Do.** Understand the swirl:



181019 Bruguieres, Virelizier @arXiv:math/0505119: "Hopf diagrams and quantum invariants".

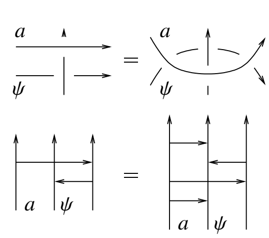
181017b Hass, Thompson, Tsvietkova @arXiv:1809.10996 "alternating links have at most polynomially many Seifert surfaces of fixed genus": In AKT language?

181014 **Do.** Show that KV is a full triangularity equation, as Duflo is triangularity in co-invariants.

181013 Khanin: $\sum 1/n^2=\pi^2/6$ by comparing coefficients of $x^2$ in $\frac{\sin x}{x}=\prod(1-x^2/\pi^2n^2)$, itself true by comparing roots and constant terms.

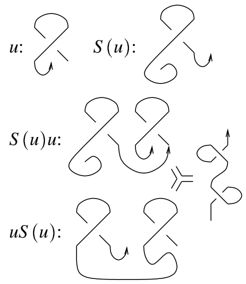
181009 de Mesmay, Rieck, Sedgwick, Tancer @arXiv:1810.03502: realizations of some logic circuits in KO ⇒ many KT problems are NP-hard.

160611 Majid's Primer §8.1: the quantum double $\mathcal D A:=A^{*,op}\otimes A$ with $(\phi a)(\psi b):=\langle Sa_1,\psi_1\rangle\langle a_3,\psi_3\rangle(\psi_2\phi)(a_2b)$ ("$op$" for multiplication). What problem does it solve? Two layers of wrong: 1. $R$ isn't in the result so it shouldn't be in the motivation. 2. The result is degree-non-decreasing, the formula should be the same. →p10:**180725**. **Q.** It's an image construction. A kernel one?



181001 **Riddle** (Dylan). Warden to 100 prisoners: I've chosen a permutation $\pi$ of your names and tomorrow I will place each of you in an isolated room with 100 boxes storing $\pi$. You will each get to open 50 boxes and each must open their "own" box. Maximize the probability of success. Sol'n 2018-10.

180424 BBS:VanDerVeen-170622, verification Doubling.pdf, in $R_{12}^{-1} = S_2(R_{12})$ conventions. The Drinfel'd's cuap: $u := R_{12}/\!\!/S_1/\!\!/m^{21}$, $v^2 := S(u)u$, $C := uv^{-1}$. Properties: $S^2(z) = u^{-1}zu$ (pf?), $uS(u) = S(u)u$ is central. Issues: Invariance property of $C$? $R$ is in the total-rotation-0 subspace, and all operations preserve it. How can they generate $C$? Perhaps it's the distinction between the pre- and post-doubling $S$? →p10:**180724**.




180910 "Locally Euclidean Knotted Objects", leKO. **Thm.** leKO ⇔ $\mathbb{R}$-rotation vKO; and $\mathbb{Z}$-rotation vKO ⇔ leKO with all measurable rotation numbers in $\mathbb{Z}$.

180905 **Riddle** (Itai). Fairly select 1 in 1,001 in 2 tosses of some fair $p$-dice, $p < 1,000$ prime. Sol'n in %.

180909b Green, Nichols, Taft: "Left Hopf Algebras", $S * I = \eta\epsilon$, $I * S \neq \eta\epsilon$. Also Lauve, Taft @arXiv:0908.3718: "[Left $SL_q(n)$]". →p7:**200205**.

180904b **Riddle** (Tsimerman). How many not-necessarily-fair coins to fairly select 1 of $n$ in finitely many tosses? Sol'n in %.

180830 **Riddle** (Khovanova) Among 14 coins, 7 weigh $a$ each, and 7 weigh $b < a$ each. You're told which is which. Confirm this with three uses of a balance scale. ("leverage")

180904c What's "tangle planarity" in evaluation diagrams?

180904a Dylan's sutured 3-manifolds: $\partial M = R_+ \cup_\sigma R_-$. Balanced: $\chi(R_+) = \chi(R_-)$ (per component?). Taut: if $\Sigma \subset M$ with $\partial\Sigma = \sigma$ then $\chi(\Sigma) \leqslant \chi(R_\pm)$ (wherefore?). Equivalence: add or remove a $D_1 \times D_2$ with suture $\{0\} \times D_2$ along $S^0 \times D_2$ or along $D_1 \times S^1$. Marking: by multiple disjoint arrows in $R_+$ and in $R_-$, with ends on $\sigma$. Gluing with 90° rotation! Contains $m, \Delta, S$ if $S^2 = 1$. **Q.** Faithfulness and completeness? Separation algorithm? $S^2 \neq 1$? **Proj.** Write "3-manifolds and the Drinfel'd double construction".

180821a The open Hopf ⌽ maps dual co-invariants to invariants.

180827 **Riddle** (Ido). On an $n$-vertex directed graph with letter-marked vertices, every length $2^n$ word can be formed with walks. Prove that the same is true for all words. Sol'n 2018-08.

180818 Kirk's unitarity: $X$ is the complement of a pure tangle in $D^2 \times [0,1]$, $X_i := X \cap (D^2 \times \{i\})$ for $i = 0, 1$. Choose a generic $U(1)$ representation $\alpha\colon H_1(X;\mathbb{Z}) = \mathbb{Z}^n \to U(1)$ with $\alpha(m_k) = t_k$. Then $\alpha|_{X_0} = \alpha|_{X_1}$. Since $t_k \neq 1$, the cohomologies (with coefficients in $\alpha$) of meridians and tubes in $\partial X$ vanish. By Mayer-Vietoris the restriction map

$$H^1(\partial X;\alpha) \to H^1(X_0;\alpha) \oplus H^1(X_1;\alpha) \qquad (*)$$

is an isomorphism. By the long exact sequence $H^1(X_i, \partial X_i;\alpha) \simeq H^1(X_i;\alpha)$. Thus the non-degenerate skew-Hermitian (ndsH) cup product $H^1(\partial X;\alpha)^2 \to H^2(\partial X;\mathbb{C})$ decomposes using (*) as $\mathrm{diag}(A, -A)$ with $A$ the ndsH inner product $H^1(X_0, \partial X_0;\alpha)^2 \to \mathbb{C}$, as the $X_i$'s are disjoint with opposite orientations in $\partial X$.

**Claim.** $x^T Ay = g(x)^T Ag(y)$ for $g$ the Gassner representation.

**Pf.** 1. The image $H^1(X) \to H^1(\partial X)$ is a Lagrangian $L$ (Poincaré duality). 2. The two composites $H^1(X) \to H^1(\partial X) \to H^1(X_i)$ on the summands in (*) are isomorphisms (Le Dimets, also [KLW]). Using 1 and 2, $x \in H^1(X_0)$ has a unique lift to $(x, g(x)) \in L$. If $x, y \in H^1(X_0)$, as $L$ is Lagrangian,

$$0 = (x, g(x)) \cup (y, g(y)) = x \cup y - g(x) \cup g(y).$$

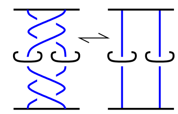
(the cross terms vanish because of the 0s off diagonal.) but $x \cup y = x^T Ay$ and $g(x) \cup g(y) = g(x)^T(-A)g(y)$, hence $x^T Ay = g(x)^T Ag(y)$ □

180815 Blair, Sack @arXiv:1801.00230: the tangle category is Karoubi complete: $f^2 = f \Rightarrow f = gh$ with $hg = 1$. **Q.** v,w?

180809 Meta-monoids aren't equivalent to monoid objects in a monoidal category: $m_k^{ij}[S]\colon M_{S\sqcup\{i,j\}} \to M_{S\sqcup\{k\}}$ isn't induced from $m_k^{ij}[\emptyset]$. Yet if $M$ is a monoid object in a symmetric strict monoidal $(C, \otimes, \mathbb{1})$, then $M_S := \mathrm{mor}(\mathbb{1}, M^{\otimes S})$ is a meta-monoid.

180812 **Riddle** (Ido). In an $\underline{n}$-cards game of war, can the sides jointly ensure finiteness from any initial position? **Q.** Effective? Polytime?

170928 **Riddle** (Chterental, May 2014). Get left to right moving only blue.



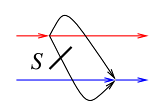
180811 Tangloids, Medusas. **Q.** Expansions? Intermediate to u and v, so implied by neither; yet implies $\mathcal{U}(\mathfrak{g})$.

150610 Andrews-Curtis Conjecture: balanced group presentations differ by Nielsen trans.: $(g_i) \to (g_{\sigma i})$, $g_i \to g_i^{-1}$, $g_i \to g_i g_j$. Myasnikov×2, Shpilrain @arXiv:math/0302080: potential counterexamples.

180806 Schwinger-Dyson is translation invariance of the path integral measure, written using post-integration handles:

$$0 = \int \mathcal{D}\phi\, \partial_\phi \mathbb{e}^{\phi\cdot Q\phi/2 + V(\phi) + J\cdot\phi} = \big(Q(\delta_J) + (\partial_\phi V)(\delta_J) + J\big)\, Z(J).$$

180803 **Q.** What axioms befall meta-Hopf-actions?




180722 $\mu\colon$ {DK-Associators} → SolKV is injective as $\mu/\!\!/(- \to \{\mathfrak{sder}-\text{Associators}\})$ is the identity modulo wheels (also Furusho, Schneps, Enriquez@LD16). Surjectivity? →p10:**180721a**.

180725 **Wanted.** A limited-foresight narrative for (want a stitching- and cabling-compatible invariant of u-tangles) ⟹ (look at $am$, $bm$, $R$, $P$, rotational virtual knots, and the meta-Drinfel'd double procedure). →p9:**160611**.

180724 (w/ Dylan). $(l+)$-kink= $(r+)$-kink in $RVT$ implies $s$ = ⫯ in $\mathcal{A}^{rvt}$. →p11:**170520**, →p10:**180424**.

180721a **Dream** (w/ Zsuzsi). $d^3\gamma\pi d^2 f = 0$ when $d^n = \sum_{k=0}^{n+1}(-)^k d_k^n$ the co-Hochschield differential, $\pi\colon \mathfrak{sder}_n \to FL_{n-1}$ the projection on $x_n$-degree 1, $\gamma\colon FL_{n-1} \to \mathfrak{sder}_n$ the Lie morphism with $x_i \mapsto t_{in}$, and with $f \in \mathfrak{sder}_2$. Leads to an algebraic construction of DK associators? →p10:**180722**.

180721b $FL(V \oplus W) \cong FL(FA(V) \otimes W) \oplus FL(V)$ like $FA(V \oplus W) \cong FA(FA(V) \otimes W) \oplus FA(V)$.

180721c **Conj** (w/ Zsuzsi). $\mathfrak{sder}_n \cong \oplus_{k=1}^n FL_k^{\text{palindromic}}$.

180716 **Proj.** Direct proofs of $u\mathcal{T} \hookrightarrow v\mathcal{T}$ and $u\mathcal{T} \hookrightarrow w\mathcal{T}$.

180708 With $\mathcal{Z}_\lambda = \{(Z\colon B \to A = \mathrm{gr}\, B)\colon \mathrm{gr}\, Z = \lambda^{\deg}\}$, have $Z_0 \in \mathcal{Z}_0$ by $b \mapsto [b]_I \oplus 0 \ldots$. Hence GRT → $\mathcal{Z}_0$. When injective? Surjective? Bijective for $B = PaB$! For $\epsilon^2 = 0$, $\mathcal{Z}_\epsilon \neq \emptyset$ iff there is $\beta_\kappa \in B_\kappa$ for every kind $\kappa$, such that for every op $\rho$, $\rho(\beta \ldots \beta) - \beta \in I^2$ (set $Z_1(b) = [b]_I \oplus \epsilon[b - \beta]_{I^2} \oplus 0 \ldots$). In that case, GRT → $\mathcal{Z}_\epsilon$. Injective? Surjective? Always $\mathcal{Z}_\epsilon \to \mathcal{Z}_0$; a bijection for $PaB$.

180615 **Q.** For the BMC crowd, are tangles a Hopf algebra variant?

180611 **Q.** Does $t = \epsilon a - \gamma b$ have a topological meaning?

180528b Not every v-knot has a Seifert surface. Are there "Seifert v-tangles"? Is this related to unitarity and to Fox-Milnor?

180515 **Q.** Wherefore the zip algebra, $\langle z_n \rangle_{n\geqslant 0}$ with $z_m z_n = \sum_{k=0}^{\min(m,n)} k!\binom{m}{k}\binom{n}{k} z_{m+n-k}$? Representation: $z_n \mapsto \hat{x}^n \partial_x^n$. Isomorphic to $\mathbb{Z}[y]$ via $z_n \mapsto y(y-1)\cdots(y-n+1)$.

180508 Etingof: Mod $\epsilon^2$, co-Jacobi is not needed for $\delta$.

180507 **Proj.** Unravel the topology behind Enqriquez-Furusho arXiv:1605.02838 *A Stabilizer Interpretation of Double Shuffle Lie Algebras*; also at SCGP.

180423 **Riddle** (Masbaum). Can you partition a rectangle exactly one of whose sides is irrational into finitely many squares?

180413 Itai: {partitions of $n$ into odd numbers} $\leftrightarrow$ {partitions of $n$ into distinct numbers}.

180411 **Riddle** (Tsimerman). The area of a projection of a unit cube on a plane $P$ is the length of its projection on $P^\perp$.

180402 BBS:VanDerVeen-180402: $y \rightsquigarrow x^{-1}$ via $y = (\omega + (T - T^{-1})a)x^{-1}$.

180320 $M_{2\times 2}(\mathbb{C})/(\text{conj})$ isn't Hausdorff.

180318 In CU/QU, $t_i$'s are "semi-scalars" — scalars for one tensor factor, Lie elements for $m$, $\Delta$, $S$. Reduction by $\langle t_i \rangle$ kills interest. $\langle t_i = t_j \rangle$ is interesting for $m$ but fully destructive for $\Delta/S$.

180317 Burton arXiv:1712.05776: HOMFLY-PT is $\mathbb{e}^{O(\sqrt{n}\log n)}$.

140405 In Projects: Mathematica: Localization.pdf: `a=1; c:=b; Command[{a=a, b=2}, x:=a; y=c]; ?x; ?y`. `Block`: local values, `x:=a, y=2`. `Module`: local symbols, `x:=a$1, y=b` with `a$1=1` and unset `b`. `With`: internal replacements, `x:=1, y=b`.

170427 Faddeev arXiv:math/9912078 (10), then Quesne arXiv:math-ph/0305003: $\log \mathbb{e}_q^x = \sum_{k\geqslant 1} \frac{(1-q)^k x^k}{k(1-q^k)}$. Readable proof in Zagier's "The Dilogarithm Function", pp. 28-30.

180206 "Diagrammatic" $\Rightarrow$ depends on $\mathfrak{g}$ continuously; meaningful "group-like"; an intrinsic upper bound on what can be done.

180208 The Benkart-Witherspoon representation of 2017-06/BW.nb is commented out here.

180205 **Task.** Re-examine the relation between 2-parameter and 1-parameter quantum groups. Wherefore moding out by $t$? Can it be recovered? Where is $\epsilon$ hiding in the 1-parameter picture?

180126 LuaTEX marries Lua and TEX.

180118 Outreach talk idea "Humans' Art and God's Art": pattern recognition in Schwartz's factorization, then in an `ArrayPlot` of primes, then a word on crypto, then K250 poster.

170520 ◹ is graded! Related to spinners (sol's of $[a_{12}, s_1 + s_2] = 0$ and $\delta/\!\!/[\cdot,\cdot] = [-, s]$). $s$ is inseparable from the tadpole ⫫ $= (a_{12} - a_{21})/\!\!/m_{-}^{12}$ by products, co-products, primitivity, and degree. →p10:**180724**.

180113 Dylan: "elevator pitch".

180106b Refined BCH: What's $\log \mathbb{e}^{x\to z}\mathbb{e}^{y\to z}$ in $\mathcal{A}^{v,ac}(x^*y^*z)$? Is it of $z$-degree 1? What if replacing $\mathbb{e}^{?\to z}$ by another exp?

180106a In $\mathcal{A}^{v,ac}$, can bring all $c$ vertices to before all $b$ vertices.

180104b **Q.** Let $B_L^U$ be the RAAG generated by $(R_x^a)_{a\in U, x\in L}$ modulo $(R_x^a, R_y^b) = 1$ whenever locality, $a \neq b$ and $x \neq y$. Let $A_L^U = \operatorname{gr} B_L^U$, the RAAA generated by $(r_x^a)_{a\in U,x\in L}$ modulo $[r_x^a, r_y^b] = 0$ whenever locality. Let $Z\colon B_L^U \to A_L^U$ by $R_x^a \mapsto \mathbb{e}^{r_x^a}$. Are there $\{\Delta_a^{bc}, \Delta_{yz}^x\}$ on $A_L^U$ compatible with the natural ones on $B_L^U$?

180104a Costello, Witten, Yamazaki arXiv:1709.09993: "Gauge Theory and Integrability, I".

171228b Nosaka arXiv:1712.02060: "the Orr invariant of degree $k$ is equivalent to the tree [] Kontsevich invariant of degree $< 2k$".

171228a Gonzàlez-Meneses, Silvero arXiv:1712.01552: Polynomial braid combing.

170829b Przytycki arXiv:1707.07733: With HOMFLYPT $P(\bigcirc) = 1$, $aP(\nearrow\!\!\!\nwarrow) + a^{-1}P(\nwarrow\!\!\!\nearrow) = zP()\,()$, expand $P = \sum_i P_{2i}(a)z^{2i}$. Then $P_{2i}$ is of complexity $O(n^{2+3i})$, likely $O(n^{2+2i})$. **Q.** Does $P_{2i}$ factor through type $2+2i$ invariants? Ito arXiv:1710.09969: (1) Related to "low genus invariants" in the $\mathcal{A} \to \mathcal{M}$ sense. (2) Even the cables of $P_0$ are mutation invariant.

171214 **Wanted.** In $\mathcal{A}^{\sim v}$, a tail-strand/head-strand pairing $P$, a co-product $P$-dual to $m$ and a $P$-compatible antipode ($R$ would then be the $P$-inverse of $P$?). Breaks: (1) A spinner / a homotopy $w$-strand. (2) The Cartan-criterion relation.

171213 **Q.** What's transmutation? (In Majid and in Habiro's "Bottom Tangles").

171212 **Q.** An $\mathcal{A}^v$ analog of the $\mathcal{A}^u$ notion "$gl(N)$ genus 0"?

170923 **Q.** Is $\hat{\hbar}$ injective on $\mathcal{U}_\hbar(\mathfrak{g})$? What's $\operatorname{gr}\mathcal{U}_\hbar(\mathfrak{g})$? Expansion? Is it inductive? Can I trust a non-universal inductivity proof?

171202 If $(C(S), \Delta_{bc}^a)$ is a meta coalgebra (less is enough; "symmetric set comodule?") and $C^n := C(\{0\} \cup \underline{n})$ then $d\colon C^n \to C^{n+1}$ by $dE := \sum_{k=1}^{n+1} (-)^k E/\!\!/\sigma_{k+1,\ldots,n+1}^{k,\ldots,n}/\!\!/\Delta_{0k}^0$ has $d^2 = 0$. **Q.** Find $H^n$ when $\Delta_{tx}^t f(t,S)$ is $D_1 = f(t,S)$, $D_2 = f(t+x, S)$ or $D_3 = D_2 - D_1$ on $FA(*, S)$. For $D_3$, a spectral seq. with $D_{1,2}$?

170625 $\mathcal{U}_{\hbar;\gamma\epsilon}$ conventions in Projects: PPSA: CS-PPSA.pdf.

171117 Kotorii arXiv:1705.10490: $n$-equivalence on $v\mathcal{K}(\uparrow) \Leftrightarrow$ equivalence modulo LCS($PvB$).

171116 The proofreader's clasper transpose.

171109 Cheng, Jackson, Stanley arXiv:1601.01377: With $q = \mathbb{e}^{\hbar/2}$, $(n)_q = (q^n - q^{-n})/(q - q^{-1})$ under $[h,x] = 2x$, $[h,y] = -2y$, $[x,y] = (h)_q$, have $\frac{x^a}{(a)_q!}\frac{y^b}{(b)_q!} = \sum_{i\geqslant 0}\binom{h+b-a}{i}_q \frac{y^{b-i}}{(b-i)_q!}\frac{x^{a-i}}{(a-i)_q!}$ (and more). Also in People: VanDerVeen: Generalxy.nb.

171108b **Def.** $\mathcal{K} = \mathcal{K}^{gcs\text{-}rvt}$: ground (some components are on the ground) ceiling (some are ceiling) surgery (some $g/c$ components are surgery-slippery) rotational virtual tangles. $\mathcal{K}$ is Ⓤ. **Conj.** (1) $\mathcal{A}^{gcs\text{-}rvt}$ has a combinatorial description. (2) $\mathcal{K}$ has a surgery-compatible expansion; v-Hopf surgeries split. (3) $\mathcal{K}$ has a stitching-compatible "universal dequantizator" expansion.

171108a **Q.** Do volumes / homologies extend to rotational v-knots?

171018 Taylor: If $f(a+x) = \sum_{k=0}^n \frac{f^{(k)}(a)x^k}{k!} + R_{n,a}(x)$ then $\exists \xi_{1,2} \in (0,x)$ s.t. $R_{n,a}(x) = \int_0^x \frac{f^{(n+1)}(a+\xi)}{n!}(x-\xi)^n d\xi = \frac{f^{(n+1)}(a+\xi_1)}{n!}x(x-\xi_1)^n = \frac{f^{(n+1)}(a+\xi_2)}{(n+1)!}x^{n+1}$.

171107 Manin '89 "multiparametric quantum deformation". Garcia, Gavarini arXiv:1708.05760 "multiparameter quantum groups" (MpQG).

140213 \$: $F$, `mathtools`: $\coloneqq$, `mathabx`: ⟳ $\Psi$, `babel`: ϡϟ, `txfonts`: ⫯. Hupfer knows jumplines. pdfcomment.

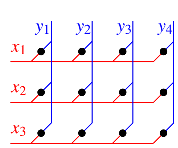
171102 Khovanskii: an algebraic formula is expressible in radicals iff its monodromy group is solvable. $\Rightarrow$: Arnol'd argument. $\Leftarrow$ middle lemma: a finite Abelian group $A$ acts on a ring $R$ that contains all roots of 1. Then every element of $R$ is a linear combination of roots of elements in $R^A$.

171015 Ito's arXiv:1411.5418 "Topological formula of the loop expansion of the colored Jones polynomial" has (multi-)forks.
171010 An expansion $\mathcal{U}_{\hbar;\gamma\epsilon} \to \operatorname{gr}_\epsilon \mathcal{U}_{\hbar;\gamma\epsilon}$?
171009b **Q.** A name for $\mathbb{e}^{\nu(\xi x+\eta y+\delta xy-t\xi\eta)}$?
171009a **Q.** If $\phi\colon (V=\mathbb{R}^n_{\xi^i}) \to (W=\mathbb{R}^m_{\eta^j})$ and $W=\langle y_j\rangle$ with $\eta^j(y_k)=\delta^j_k$, what do you call $\Phi(\xi^i,y_j) := \sum y_j\phi^*(\eta^j)$?
141226b **Proj.** FT invariants of fixed-linking-numbers (uvw)-KO. A Goussarov view? FT relative to CO/CU (Commute Overcrossings/Undercrossings)? Is there a good presentation of tangles with fixed linking numbers?
170919 Talk idea: "Ⓤ-Tangles & the Quantum Groups Conspiracy".
170128 Cartan's criterion: $\mathfrak{g}\subset \operatorname{End}(V)$ is solvable iff $\forall x\in\mathfrak{g}$, $y\in[\mathfrak{g},\mathfrak{g}]$, $\operatorname{tr}_V(xy)=0$. Induces a quotient of $\mathcal{A}^v$; what is it?
170917 Gautam: an explicit $\mathcal{U}(sl_n)[\![\hbar]\!]\cong\mathcal{U}_\hbar(sl_n)$.
170914 **Q.** Is there a canonical isomorphism between quantizations of $sl_2$ with varying $r$?
170913b **Do.** Center poly-poly at Lie algebra contractions.
170908 Livingston arXiv:1709.00732: $\sigma\colon S^1\to\mathbb{Z}$ is a knot signature function iff all discontinuities are at roots of an Alexander polynomial and [...]. **Q.** How fits with w and with Γ-calculus?
170519 Bonahon's "miraculous cancellations" arXiv:1708.07617 link Ito with PPSA?
170813 Given a Hopf $H$, is there a "pair one" op $H^*\otimes H\to H^*\otimes H$?
170309 In 2017-03/geps.nb: $w,u,b,c = \begin{pmatrix}0&1\\0&0\end{pmatrix}, \begin{pmatrix}0&0\\-\epsilon&0\end{pmatrix}, \begin{pmatrix}-1&0\\0&-1\end{pmatrix}, \frac12\begin{pmatrix}-1-\epsilon^{-1}&0\\0&1-\epsilon^{-1}\end{pmatrix}$ obey $\mathfrak{g}^\epsilon$: $[w,c]=w$, $[c,u]=u$, $[u,w]=b-2\epsilon c$. Then $r=(b_1-\epsilon c_1)c_2+u_1w_2 = \frac{1}{4\epsilon}\begin{pmatrix}1-\epsilon&0\\0&1+\epsilon\end{pmatrix}\otimes\begin{pmatrix}1+\epsilon&0\\0&1-\epsilon\end{pmatrix} - \epsilon\begin{pmatrix}0&0\\1&0\end{pmatrix}\otimes\begin{pmatrix}0&1\\0&0\end{pmatrix}$.
170807 Naef: For $D\in\operatorname{der}(FL)$ (any!), $\operatorname{div}(D) := \sum_x \operatorname{tr}\partial_x D(x)$ satisfies $\operatorname{div}[D_1,D_2]=D_1\operatorname{div}(D_2)-D_2\operatorname{div}(D_1)$.
170804 Given $f(x)$, $g(\xi)$, have $f(\partial_\xi)g(\xi)|_{\xi=0}=g(\partial_x)f(x)|_{x=0}$.
170802 Does every infinitesimal deformation of a solvable Lie algebra globalize? Does some $H^2$ parameterise knot invariants?
170708 In w, inner $q\Delta$'s automatically lead to $\mathfrak{sder}$- hence tree-level u- associators.
170707 Which doubling makes the diagram commute?
170703 What characterizes "PBW" maps $\mathcal{S}(\mathfrak{g})\to\mathcal{U}(\mathfrak{g})$?

$$\begin{array}{ccc}\mathcal{A}^w(\uparrow) & \xrightarrow{?} & \mathcal{A}^w(\uparrow_2)\\ \downarrow\mathcal{T} & & \downarrow\mathcal{T}\\ \mathcal{U}(sl_2^0) & \xrightarrow{q\Delta} & \mathcal{U}(sl_2^0)^{\otimes 2}\end{array}$$

170702 In $\mathcal{U}_{\hbar;\gamma\beta}$, $\prod_i \mathbb{e}^{\eta_i y}\mathbb{e}^{\alpha_i a}\mathbb{e}^{\xi_i x} = \mathbb{e}^{\eta y}\mathbb{e}^{\alpha a}\mathbb{e}^{\xi x}\mathbb{e}^{\sigma}\left(1+\sum_{k\geqslant1}\Lambda_k\beta^k\right)$, with $\alpha=\sum\alpha_i$, $\eta=\sum_i\eta_i\mathbb{e}^{-\gamma\sum_{j<i}\alpha_i}$, $\xi=\sum_i\xi_i\mathbb{e}^{-\gamma\sum_{j>i}\alpha_i}$, $\sigma=\frac{1-T}{\hbar}\sum_{i<j}\xi_i\eta_j\mathbb{e}^{-\gamma\sum_{l:\,i<l<j}\alpha_l}$ and $\Lambda_k$ is ...
170412b Rote (2001), BBS:Dancso-170529: An $n^4$-time division-free algorithm for det.
170602 Word-pairing in Hopf algebras:

$$\left\langle\prod_{i\in\underline{n}}x_i,\prod_{j\in\underline{m}}y_j\right\rangle=\prod_{i\in\underline{n},\,j\in\underline{m}}\left\langle x_i^{(j)},y_j^{(i)}\right\rangle.$$




170529 Is "$(\mathfrak{g},[],\delta)$ non-negatively graded with Abelian degree 0" same as "$D\mathfrak{g}$ is a sum of Kac-Moody and inhomogeneous factors"? A sense by which these are precisely "the quantizeables"?
170323b Is there a "Heisenberg-Drinfel'd Double Construction"?
170224 $\mathfrak{g}$ solvable $\Leftrightarrow$ $[\mathfrak{g},\mathfrak{g}]$ nilpotent $\overset{?}{\Leftrightarrow}$ $\mathfrak{g}\cong\mathfrak{a}\ltimes\mathfrak{n}$ with Abelian $\mathfrak{a}$ and nilpotent $\mathfrak{n}$. Also, $\mathfrak{g}$ solvable $\Leftrightarrow$ there is a finite decreasing filtration $(\mathfrak{g}_k)_{k\geqslant0}$, $\mathfrak{g}_0=\mathfrak{g}$, with $\mathfrak{g}_0/\mathfrak{g}_1$ Abelian and $[\mathfrak{g}_k,\mathfrak{g}_l]\subset\mathfrak{g}_{k+l}$. Then $\mathcal{U}(\mathfrak{g})$ also has a multiplicative decreasing filtration.
131009 Let $\Gamma_{1,2,3}$ be thickened surfaces. Is there an expansion for the structure $\mathcal{K}(\Gamma_1\hookrightarrow\Gamma_2)\times\mathcal{K}(\Gamma_2\hookrightarrow\Gamma_3)\xrightarrow{/\!/}\mathcal{K}(\Gamma_1\hookrightarrow\Gamma_3)$? (Handlebodies in Habiro-Massuyeau arXiv:1702.00830).
170518 Given unital algebras $B$, $C$ and $R\in B\otimes C$, when is there a swap $s\colon C\otimes B\to B\otimes C$ so that $R^{12}R^{13}R^{23}=R^{23}R^{13}R^{12}$ would hold in $B\bowtie_s C$?
170512 Le: "Every $\mathcal{U}_q(\mathfrak{g})$ embeds in some quantum torus". A smidge version? Does "every $\mathfrak{g}$ embeds in a Heisenberg" quantize to Le's?
170508 Majid's Primer §4: $A\mapsto M(A)$, algebras to bialgebras.
131023 Markl: "like a bottle under a waterfall". Psychology buzzword: "cognitive overload".
170413a Shortcut / characterize / replace / generalize $\mathcal{U}(\mathfrak{g})^*\cong\mathcal{S}(\mathfrak{g})^*\cong\mathcal{S}(\mathfrak{g}^*)\cong\mathcal{U}(\mathfrak{g}^*)$. Are "doubling the Cartan" a/o "Δ-conjugations" more fundamental than "pairing"?
131130b **Proj.** "Alexander Recovery". Conway relation; relations as in Archibald; factorization as in Levine arXiv:q-alg/9711007, Tsukamoto-Yasuhara arXiv:math/0405481; cabling; Fox-Milnor (is there for links?); genus property; crossing-number property; split-link property; u-range; w-range; unitarity; concordance; mutations; behaviour under mirror/strand reversal; Torres conditions; Hartley's property; cheirality properties as in arXiv:1608.04453; Alexander/Thurston norms as in McMullen:alex.pdf.
170413b In 2009-01/KAL-090128....pdf and BBS:KAL-090128:

$$\begin{array}{ccc}\mathcal{A}^v(\uparrow) & \xrightarrow{T_{\mathfrak{g}}} & \mathcal{U}(\mathfrak{g})\\ \downarrow\Delta & & \updownarrow\\ \mathcal{A}^v(\uparrow\uparrow^{ab}) & \xrightarrow{T_{\mathfrak{g}^+}} & \mathcal{U}(\mathfrak{g}^+)\end{array}$$

170411 Vogtmann@MSRI: action of trivalent trees on the $\operatorname{gr}(\pi_1(\Sigma_g))$ by derivations via contractions; relation between $H^*(\operatorname{Out}(F_n))$ and some $H^*$ of trees modulo AS & IHX.
170211b Gaussian pairing: $\left\langle\exp\left(\frac{x\subset}{2}\right)\,\middle|\,\exp\left(\frac{\supset y}{2}+\sum_i -\!\bullet i\right)\right\rangle = \exp\left(\frac12\log(\frac{1}{1-xy})\bigcirc+\sum_{i,j}\frac{x\, i\bullet\!\!-\!\!\bullet j}{1-xy}\right).$
131014 Problems with the projectivization paradigm: No room for negative degrees and for degree-decreasing ops. No built-in $\hat\hbar$.
170325b **Q.** Is there a "Vogel group" acting on $\mathcal{A}^w(S)$? In general, is there topology behind the Vogel action?
170325a Generalized Weyl: If $f\in\mathcal{S}(V)$ and $\psi\in\mathcal{S}(V^*)$ then in $\mathcal{U}(HV)$, $f\psi=\psi_1f_1\langle\psi_2,f_2\rangle$, where $\Delta f=\sum f_1\otimes f_2$ and $\Delta\psi=\sum\psi_1\otimes\psi_2$. Is there a version with $\mathcal{U}(\mathfrak{g})$ replacing $\mathcal{S}(V)$?
170325c **Q.** Is there a $\mathcal{K}^u$ interpretation of the Vogel action on $\mathcal{A}^u$?
170318 **Q.** Are there easy $\theta$-invariant braidors for $\mathfrak{g}_0$?
170323a **Proj.** Study $\mathcal{K}^w/\mathcal{A}^w$ with "solvable heads".
170322 With $f_t=\mathbb{e}^t-1$, $f_{x+y}=f_x+\mathbb{e}^xf_y=\mathbb{e}^yf_x+f_y$.
170320a **Q.** Is there a good algebraic structure of "groups with a fixed Abelianization"?
170310 Tentative ID: I'm a selfish rationalist atheist permissive individual-rights free-market socialist global citizen. Note to self: read defs! (Liberal? Democrat?)
170308 Mine "$\mathcal{S}(\mathfrak{g}):\mathcal{S}(\mathfrak{g}^*)$ pairing" $\Leftrightarrow$ "commutation in $\mathcal{U}(H\mathfrak{g})$".
170302 **Conj.** Every rotational classical YB structure can be quantized. Related to spectral parameters? (Chari-Pressley 15.2.B).

170306 For $sl_2^+ = \langle e, f, h, c\rangle/([h,e] = 2e, [h,f] = -2f, [e,f] = h, [c,\cdot] = 0)$, $r_{ij} := e_i f_j + h_i h_j/4 + \alpha(h_i c_j - c_i h_j)$ solves CYBE.

170301 **Prob.** Given ad and a solvable structure on $\mathfrak{g}$, fully implement the group-likes in $\hat{\mathcal{U}}(\mathfrak{g})$.

170223d VdV on gmail/161122: A $\mathfrak{g}_0$ / $gl(1|1)$ relationship.

170223c **Q.** What's the internal kernel in $\mathcal{A}^v$ for 2-loop $Z^u$? What's the nearest-dual Lie bialgebra?

170223b **Do.** Compute $Z_{I\mathfrak{g}}$ for general $\mathfrak{g}$ w/o back reference to $\mathcal{A}^w$.

170223a By nilpotent approximation, all semi-simple weight systems come from nilpotent Lie algebras. Do the latter make more?

170222 Is there a Chern-Simons theory for degenerate Casimirs?

141107 Claim. $\mathfrak{g}$ a Lie algebra, $d \in \mathfrak{g}$ fixed, $c$ a "new" central element, $\mathfrak{g}_1 := \mathfrak{g} \oplus \langle c\rangle$, $\delta\colon \mathfrak{g}_1 \to \mathfrak{g}_1 \otimes \mathfrak{g}_1$ by $c \mapsto 0$ and $\mathfrak{g} \ni x \mapsto [d,x] \otimes c + c \otimes [d,x]$, then $\mathfrak{g}_1$ is a Lie bialgebra. Extends to a non-cocommutative seed? Eckhard: may be related to Medina-Revoy "double extensions", a structure theorem for metrized Lie algebras. →p8:**141114a**.

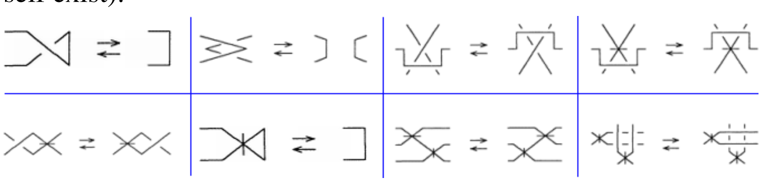
170221 Teichner on mo:7052: $K$ is slice iff $K\#R$ is ribbon for some ribbon $R$.

170126b **Q.** What does Ado give for $\mathfrak{g}_1$? For $I\mathfrak{g}$ (→p8:**190709**)?

161027a Describe $\mathcal{B}^{rv}$ and $\alpha\colon \mathcal{A}^u \to \mathcal{A}^{rv} := \mathcal{A}^v/\langle[a_{ij}, s_i + s_j]\rangle$.

170126a From Roland's `Poly.pdf`: Under $[F,E] = 1-t-(1+t)\epsilon L$, $[F,L] = F$, $[L,E] = E$, $t = e^c$, $s = 1-t$ and $\nu = (1-s\delta)^{-1}$, have $\mathbb{O}(FE|e^{\alpha E+\beta F+\delta EF+\epsilon(s-2)P(E,F)}) =$ $\mathbb{O}\big(ELF|(1+\epsilon(s-2)(P(\partial_\alpha,\partial_\beta)+\partial_s((\partial_\alpha+\partial_\beta)/2+\partial_\delta+L)+s\partial_s^2/4))$ $\nu e^{\nu(\alpha\beta s+\alpha E+\beta F+\delta EF)}\big)$

131103 The Yoshikawa moves (usefulness limited by the before/after unknottedness condition; Chterental: the Swenton proof of completeness may be broken, new ones by Kearton-Kurlin and himself exist):



170108a `Table[As[n,6], As[k,0,n], Binomial[n,k]]` in 2017-01/As.nb.

170107b What do coverings of the annulus say about annular braids?

170107a Is there strand-doubling for handle-strands ($|_h$'s)? In general, what do maps between surfaces of (possibly different) genera say about $\mathcal{A}(|_h^g \uparrow^n)$?

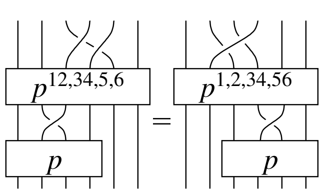
161122 What's $\theta$, in $\mathcal{A}^w$ language?

161101 Stein's paradox: with $\theta \in \mathbb{R}^{n\geq 3}$, given a single measurement $x_i$ of $n$ independent normal Gaussians with mean $\theta_i$ and variance 1, the estimator $\hat\theta = x$ for $\theta$ is dominated by another (across all $\theta$), if aiming to minimize $E\left[\|\theta - \hat\theta\|^2\right]$.

161027b What's the abstract relation between Roland's $\mathfrak{g}_0$ and mine?

161009 Kondo (1979): Any Alexander polynomial is attained with unknotting number 1.

160911 **T/F?** The only solutions to Morrison's equation for $p \in PB_4$ are 1 and $\sigma_2^{-2}$.




160801 Is there a topological meaning to primitive-exponential hybrids like $\Phi^{-1}t_{14}\Phi$?

160721 **Q.** Are braidors related to quandle cohomology?

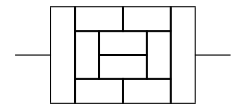
160616 Representing $\mathcal{A}^w$ on functions on a 2D Lie algebra, what is the functional representation of the braid group we get?

160613 Find Gassner and dual-Gassner in the topology of $PwB_n$.

160609 rad $\mathfrak{g}$ :=(maximal solvable ideal). Levi: rad $\mathfrak{g}$ has a complementary Lie algebra.

160519 **Proj.** Extendibles extend to extendibles, the group case.

160508 Mrowka's dodecahedron:



160505 Chterental: Brunnian 2-component links have manifestly Brunnian diagrams.

160503b **Prob.** Find a methodology for promoting invariants of braid-like virtuals to full invariants of classicals.

160503a **Q.** What's $\omega$, as seen from linear control theory?

160414b **Q.** In $\mathbb{Q}G$, is $\varprojlim_{m\geq n} I^n/I^m = \left(\varprojlim_m I/I^m\right)^n$?

160414a **Q.** Suppose $A \otimes B \twoheadrightarrow C$ and all are filtered compatibly. Does $\widehat{A \otimes B} \twoheadrightarrow \hat{C}$? Does $\hat{A} \otimes \hat{B} \twoheadrightarrow \hat{C}$?

160410 **Q.** Can the filtrations of $\mathbb{Q}G^{(n)}$ and of $\widehat{\mathbb{Q}G}$ be defined from their (Hopf-)algebraic structures?

160408 **Q.** Integrate Lie algebra 2-cocycles to Lie group 2-cocycles.

160403b **Prob.** Characterize $\{\exp F(\log X, \log Y)\}$ in $\widehat{\mathbb{Q}FG_2}$ language.

160315b Is $FG_{a,b,c,d}/\{a = c^b, b = d^a, c = e^f, d = f^e\}$ free? Expansion faithful?

160315a $\pi_2(n\text{-ring complement}) = \mathbb{Z}(FG_n \times \underline{n})$?

160311 For $wB$, why does the $\pi_1$ action determine the $\pi_2$ action? For $w\mathcal{T}$, it doesn't.

160308 Naor's $\sqrt{2} \notin \mathbb{Q}$: Else $0 < (\sqrt{2}-1)^n \to 0$ but with $\sqrt{2} = p/q$, $(\sqrt{2}-1)^n = a_n\sqrt{2} + b_n = (pa_n + qb_n)/q \geq 1/q$.

160304 **Riddle** (saw $\Omega$ by Gracia-Saz after M. Bernstein). 100 prisoners strategize, then are sealed in rooms with the same countable sequence of "boxes with reals" in each. Can each open all but one of their boxes and guess the remaining one so that at most one prisoner would be wrong? Hint: 0-1 boxes, finitely many 1s.

160113 A meta-monoid $M$ is *factored* if it has a $\square$ compatible with all operations. What structure form the primitives $P$ of $M$? Does $P$ determine $M$?

151214 BBS:Schneps-151209, the spherical ⬠:

$$\varphi(t_{12},t_{23})\varphi(t_{34},t_{45})\varphi(t_{51},t_{12})\varphi(t_{23},t_{34})\varphi(t_{45},t_{51}) = 1.$$

151209 Are there solutions of R4 (+more?) in $\langle a_{12}, a_{21}\rangle$? No?

150924 Schneps in Les Diablerets: For $f \in FL(x,y)$, $\pi_y(f)$ proj. on words ending with $y$, $f_* := \pi_y(f) - \sum\frac{(-)^n}{n}(f \mid x^{n-1}y)y^n$ rewritten in $y_i := x^{i-1}y$. $\partial s := \{f\colon \Delta_*(f_*) = f_* \otimes 1 + 1 \otimes f_*\}$, with $\Delta_*(y_i) := \sum_{k+l=i} y_k \otimes y_l$ (group version in BBS:Schneps-151209). BBS:Ens-150923: Write $u, v \in FA(x,y)y$ in $y_i := x^{i-1}y$ and set $St(1,u) = St(u,1) = 1$, $St(y_i u, y_j v) = y_i St(u, y_j v) + y_j St(y_i u, v) + y_{i+j} St(u,v)$. Then $\partial s = \{f \in FL_{\geq 3}(x,y)\colon (f \mid St(u,v)) = 0\}$, where not both $u$ and $v$ are powers of $y$. For $f \in \partial s$ set $F(x,y) = f(-x-y,-y) = xF^x + yF^y$, $G(x,y) = \sum_{i\geq 0}\frac{(-)^i}{i!}\partial_x^i(F^x)yx^i$. Then $f \mapsto D_{F,G}$ is $\partial s \hookrightarrow \mathfrak{krv}_2$.

151003a BBS:Ens-151002: In $FA(u_d)$ with $\deg u_d = d$, if $\exp\sum u_d = \sum y_k$ with $\deg y_k = k$, then $\Delta y_k = \sum_{i+j=k} y_i \otimes y_j$.

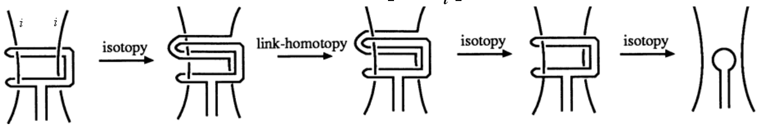
151126 $A$ a Hopf algebra, $b$ a primitive derivation $b/\!\!/\square = \square/\!\!/(b \otimes 1 + 1 \otimes b)$, $B := \{D \in A\colon (bD = 0) \wedge (\square D = D \otimes 1 + 1 \otimes D)\}$. Characterize the subalgebra $\langle B\rangle$ generated by $B$.

151118c Wikipedia: Schur multiplier: "A projective representation of $G$ can be pulled back to a linear representation of a central extension $C$ of $G$".

151118b Hopf: $F$ free, $G = F/R$, $H_2(G,\mathbb{Z}) \cong (R \cap [F,F])/[F,R]$.

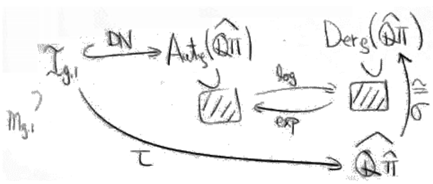
151118a Hillman's Alg. Inv. of Links, pp. 238: "Cochran, Orr conjectured that if all Milnor invariants of length $< r$ vanish then all to length $2r$ are well-defined".

151110 The Milnor homotopy trick $[x_i, x_i^g] = 1$ by Habegger-Lin:

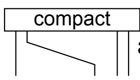


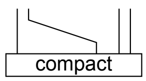

151012 The Goldman Lie algebra, which is its group?

151006 Kuno@LD15: (missing: the archetypical model for "$\sigma$ is an isomorphism")



151003b The Alexander quandle: $t\colon A \to A$ an automorphism of an Abelian group, $a \uparrow b := ta + (1-t)b$.

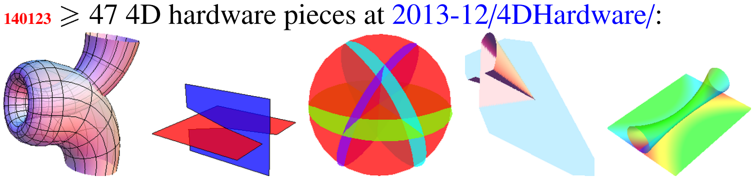
131122c Overhand/underbelly, 

and 

, abstain. Sep. 2015, ~~stronger: "4-end bottom tangles are unparenthesized"!~~

150925 König's lemma: in an infinite connected graph with finite valencies there's an infinite simple path.

140123 $\geqslant$ 47 4D hardware pieces at 2013-12/4DHardware/:



131106a XII $\Leftrightarrow$ FiC $\Leftrightarrow$ "bra-cobra" $\Leftrightarrow$ "involutive" (Chas, arXiv:math/0105178) $\Leftrightarrow$ "infinitesimal of $S^2 = 1$". Ševera in 2015 Les Diablerets talk: this globalizes.

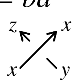
150829 Massuyeau: Passi, Passman: over $\mathbb{Q}$, dimension=LCS.

150806 Two 1/3 rotations $\rho_{3|3'}$ on $\mathcal{A}^w(\uparrow_x\uparrow_y)$: $S^y /\!/ \Delta^y_{yz} /\!/ m^{xz|zx}_x /\!/ \sigma^{xy}_{yx}$. In general, $\operatorname{Aut}(FG_n) \hookrightarrow \mathcal{A}^v(\uparrow^n)$ and $\operatorname{Out}(FG_n) \hookrightarrow \mathcal{A}^u(\uparrow^n)$.

150804 **TIL.** `ctrl-alt-F1` through `ctral-alt-F7`, `pstree`.

150729 Habiro ring: $\widehat{\mathbb{Z}[q]} := \varprojlim \mathbb{Z}[q]/(q)_n$ w/ $(q)_n := \prod_{i=1}^n (1-q^i)$.

150719b Odd quandle-from-group: $a \uparrow b := ba^{-1}b$.

150719a Gordon on Wada: With $\pi(K) :=$ 

$\to x^{-1}zx^{-1}y = 1$, $\pi(K) \cong \pi_1(\Sigma_2(K)) * \mathbb{Z}$.

150205 Abe-Tagami arXiv:1502.01102, Gompf-Scharlemann-Thompson arXiv:1103.1601: slice-ribbon counterexamples?

150624 "Set function $\varphi\colon G \to H$ is affine" means $\varphi(I^n_G) \subset I^n_H$. Makes a category. Group morphisms and translations are affine. $\varphi_i\colon G_i \to H$ affine $\Rightarrow$ $\varphi_1\varphi_2$ affine, so "sorting" on $FG$ is affine, $Id\colon G \times H \to G \ltimes H$ is affine. If $G \ltimes H$ is almost-direct, $Id\colon G \ltimes H \to G \times H$ is affine, so combing braids is bi-affine.

150517 $G =: G_1$ a group, $G_{n+1} := (G, G_n)$, $\pi_n\colon G_n \to L_n := G_n/G_{n+1}$. Given affine sections $\varphi_n\colon L_n \to G_n$ let $\zeta\colon G \to \hat{L} := \prod L_n$, the "LCS-expansion using $\varphi$." ("group-PBW for $\varphi_*$"?), by $\zeta_1 := \pi_1$ and $\zeta_n(g) := \pi_n(\varphi\zeta_{<n}(g)^{-1}g)$ where $\varphi(\lambda_1, \lambda_2, \dots) := \varphi_1(\lambda_1)\varphi_2(\lambda_2)\cdots$. Then $\zeta_{<n}(h) = 0$ iff $h \in G_n$ and $g = \varphi\zeta_{<n}(g)$ in $G/G_n$. Is $\zeta_n$ of type $n$?

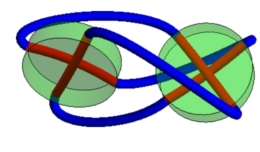
150522 Automatic structure on a group $G$: $A$ a set of semigroup generators, "acceptor" automaton $M$ on $A$ accepts $L$ s.t. $\pi\colon L \twoheadrightarrow G$, for each $x \in A \cup \{e\}$ "multiplier" automaton $M_x$ on $(A, A) = (A \times \$) \times (A \times \$) \setminus (\$, \$)$ accepting $(u_1, u_2)$ iff $\exists v_i \in L$ with $u_i \in v_i\$*$ and $\pi(v_2) = \pi(v_1 x)$.

150609c Budney's arXiv:math/0309427: Long knot space is $\coprod_{n=0}^{\infty} (C_2(n) \times \mathcal{P}^n)/S_n$, with $C_2(n)$ the space of $n$ little 2-cubes and $\mathcal{P}$ the set of prime knots, with $[0,1]^2_{t,s}$ parameterizing arc length ($t$) and scale ($s$).

150609b BBS:Lambrechts-150603: $H_*(\mathrm{Emb}(\mathbb{R}, \mathbb{R}^{\geqslant 4})) \cong H_*$(a graph complex). BBS:Lambrechts-150603 Goodwillie-Sinha: At $n \geqslant 4$, $\operatorname{holim}_{p\to\infty} \overline{Conf}^f(\mathrm{p}, \mathbb{R}^n) \simeq \mathrm{Emb}^f(\mathbb{R}, \mathbb{R}^n)$. Naive? Intuition?

150609a Is $\pi_{>0}$ ever useful to understand $\pi_0$?

150608 A `PogForm` in 2015-06:



150502 BBS:Martins: • A Crossed Module (CM, e.g. arXiv:0801.3921) models $\partial\colon \pi_2(X, A) \to \pi_1(A)$: a group homomorphism $\partial\colon E \to G$ with an action $\triangleright\colon G \circlearrowleft E$ s.t. (1) $\partial(g \triangleright e) = g(\partial e)g^{-1}$, (2) $(\partial e) \triangleright f = efe^{-1}$ (contains $\pi_1 := \operatorname{coker}\partial$, $\pi_2 := \ker\partial$, and the Postnikov $k$-invariant in $H^3(\pi_1, \pi_2)$ when $A = X^1$; equivalent to a "2-group"). There are homotopies of CMs, the free CM over a set-to-group $\partial_0\colon C \to G$, quotients of CMs, the "actor" CM $G \to \operatorname{Aut}(G)$. Whitehead (JHC): $\pi_2(X^2, X^1)$ is the free CM over the attaching maps.

• A Differential Crossed Module (DCM, Baez-Crans arXiv:math/0307263, Cirio-Martins arXiv:1309.4070) is a Lie algebra morphism $\partial\colon \mathfrak{h} \to \mathfrak{g}$ with an action $\triangleright\colon \mathfrak{g} \circlearrowleft \mathfrak{h}$ by derivations, s.t. (1) $\partial(g \triangleright h) = [g, \partial h]$, (2) $(\partial h) \triangleright h' = [h, h']$. Assign CM to DCM by $G := \{e^\gamma\colon \gamma \in \mathfrak{g}\}$, $H := \{e^\eta\colon \eta \in \mathfrak{h}\}$, $\partial\colon e^\eta \mapsto e^{\partial\eta}$, $e^\gamma \triangleright e^\eta := e^{e^{\gamma\triangleright}\eta}$. There's an analytic $\{CM\} \to \{DCM\}$; not yet algebraic. Use Rker, $s, t\colon E \rtimes G \to G$ (BBS:Martins-150501)?

• There's a DCM $\mathcal{GL}(\mathcal{V}) = (\mathfrak{gl}_1(\mathcal{V}) \to \mathfrak{gl}_0(\mathcal{V}))$ for a chain complex $\mathcal{V}$.

• Braided surfaces have ()( → ⤫) (see Khovanov-Thomas arXiv:math/0609335).

• BBS:Martins-150501: A CM $\pi_{12}(K^c)$ for virtual 2-knots.

• BBS:Martins-150501: Rker for Hopf morphisms.

150417 LaTeX displayed equations: `equation(*)`, `align(*)` `BT(&T)*\\...E)`, `multline(*)`, `split` (inner for displayed, `BT&T\\...E`), `aligned` (inner `align`). Related: `\label`, `\tag`, `\nonumber`, `\notag`. [WB→].

150422 Lambert's dreaded $W$ function: $y = xe^x \Leftrightarrow x = W(y)$.

150412 Deriving Gassner: In 2015-04/OneCo.pdf.

150409 2Dv: In 2015-04/OneCo.pdf.

150107 $\square\colon \mathcal{A}(G) \to \mathcal{A}(G) \otimes \mathcal{A}(G)$ wrong sketch: • If $V$ is doubly filtered, the associated graded of the diagonally-associated single filtration of $V$ is isomorphic to the diagonal single-gradation of the associated doubly-graded of $V$. **False.** Take $V = \mathbb{Q}\langle x, y\rangle$, $F_{0,0} = F_{1,0} = F_{0,1} = V$, $F_{2,0} = \langle x\rangle$, $F_{1,1} = F_{0,2} = \langle y\rangle$. Then $0 + \langle [x]\rangle = V_{1,0} \oplus V_{0,1} \neq V_1 = 0$. • $\mathcal{A}(G \times H) \cong \mathcal{A}(G) \otimes \mathcal{A}(H)$ as the associated single filtration of the double filtration of $\mathbb{Q}(G \times H)$ is its single filtration. • $g \mapsto (g, g)$ induces $\square\colon \mathcal{A}(G) \to \mathcal{A}(G \times G) \cong \mathcal{A}(G) \otimes \mathcal{A}(G)$.

140723 w-meaning for $\sigma_{ij} \mapsto \begin{pmatrix} 1-t_j & 1 \\ t_i & 0 \end{pmatrix}$? u-meaning for $\sigma_{ij} \mapsto$

$\begin{pmatrix} 1-t_i & 1 \\ t_i & 0 \end{pmatrix}$? Using the "other" Artin rep. BBS:Dalvit-150318?

150307 Georgetown vocabulary: control theory, zinbieL algebra, Fliess operators, shuffle algebra, dendriform algebra.

150227 **Infinitesimal** $G = \langle X_i \mid R_j \rangle$ **definitions** [Br→], [DPS→]. • Pro-unipotent? • Malcev completion: $\mathrm{Mal}(G) := \varprojlim \mathbb{Q} \otimes_{\mathbb{Z}} (G/G^{(n)})$. • $\operatorname{gr} G := \mathbb{Q} \otimes_{\mathbb{Z}} \bigoplus G^{(n)}/G^{(n+1)}$. • Malcev Lie algebra: roughly, $\mathrm{mal}(G) := \hat{FL}(x_i)/(\log R_j)$, with $x_i := \log X_i$. Is filtered. • 1-formal: $\mathrm{mal}(G)$ isomorphic as filtered to a quadratic Lie algebra. • Holonomy Lie algebra of $X$: ∼ quadratic generated by $H_1$ modulo $\operatorname{im} H_2$.

150224a Surface braids: Bardakov, Bellingeri, Birman, Funar, Gervais, Gonzalez-Meneses, Guaschi, Juan-Pineda.

141226a With Dalvit: for $(+,+) \neq (s_1, s_2) \in \{\pm\}^2$, is there $\phi \in \mathrm{Aut}(FG(x,y))$ s.t. $\phi(y^{-1}xy) = y^{-s_1}xy^{s_1}$, $\phi(x^{-1}yx) = x^{-s_2}yx^{s_1}$? Sela: $\mathrm{Out}(FG_2) := \mathrm{Aut}(FG_2)/\mathrm{Inn}(FG_2) = \mathrm{Aut}(FG_2/[FG_2, FG_2]) = GL_2(\mathbb{Z})$, hence easily not. Chterental: That's easily within "Whitehead's algorithm". Why bother? Otherwise the 4 distinct handshake w-links of BBS:Dalvit-140617 could be equal as 2-knots contradicting Satoh's conjecture & showing that $Z^w$ doesn't extend via BF.

150219 Jones ribbon conditions from the Oberwolfach-1405 AKT?

150206 Study annular braids / tangles. Canonical forms?

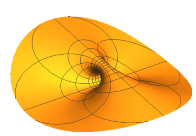
131104 Humbert's thesis pp 22: The relations of $\mathfrak{t}_n^1$: $[v_i, w_j] = \langle v, w \rangle t_{ij}$, $[v_i, t_{jk}] = 0$, $[x_i, y_i] = -\sum_{j \neq i} t_{ij}$. Imply centrality of $\sum_j v_j$ and $t_{ij} = t_{ji}$, $[v_i + v_j, t_{ij}] = 0$, $[t_{ij}, t_{kl}] = 0$, and $[t_{ij}, t_{ik} + t_{jk}] = 0$. Canonical forms?

150217 Enriquez: $B_n^1$ is $\langle \sigma_i, X_1^{\pm} \rangle$ mod $(\sigma_1^{\pm 1} X_1^{\pm})^2 = (X_1^{\pm} \sigma_1^{\pm 1})^2$, $[X_1^{\pm}, \sigma_i] = 1$ for $i \geqslant 2$, $[X_1^-, (X_2^+)^{-1}] = \sigma_1^2$, $X_1^{\pm} \cdots X_n^{\pm} = 1$, and braid relations, where $X_{i+1}^{\pm} = \sigma_i^{\pm 1} X_i^{\pm} \sigma_i^{\pm 1}$.

150210 Reidemeister-Schreier: 1. $H < G \rightsquigarrow$ groupoid $H \backslash G$ with objects cosets $H\gamma$, morphisms $(H\gamma, g)\colon H\gamma \to H\gamma g$, and compositions $(H\gamma, g_1) /\!\!/ (H\gamma g_1, g_2) := (H\gamma, g_1 g_2)$. With this, $H = \mathrm{Aut}(He)$. 2. If $G = \langle X \colon R \rangle$, $H \backslash G$ is presented with $X \times (H \backslash G)$ generators and $R \times (H \backslash G)$ relations. 3. There's a same-size presentation of $\mathrm{Aut}(He)$.

150208 Kohno knew elliptic KZ in 1996.

150201a In 2015-01:



141204a **Prob.** Find a simple description of simple 2-knots. Done in Kawauchi's *A Chord Diagram of a Ribbon Surface-Link?*

150131 Katz 5.1: $R\mathcal{A}^s(|_h \uparrow^n) \hookrightarrow \mathcal{A}^s(|_h \uparrow^n)/C$. $R$: nothing on last strand. $|_h$: a handle line.

150130b Katz 5.2: $L\mathcal{A}_1^{s\curlywedge}(\uparrow^n) \cong \mathcal{A}_1^{s\curlywedge}(\uparrow^n)/C$. $\square_1$: elliptic. $\curlywedge$: strutless. $s$: skeleton-connected. $L$: only lonely vertices on last strand. $C$: closed surface.

150130a **Q.** Why is $PB^g$ related to non-tangential differential operators on $\mathrm{Fun}(\mathfrak{g}^g)$?

141224 Katz points (BBS:Katz-141224, arXiv:1412.7848): • Cheptea-Habiro-Massuyeau's arXiv:math/0701277 has a Clifford-like relation in sec. 8 (earlier, in Habiro's arXiv:math/0001185, fig. 48). • LMO for Lagrangian cobordisms partially interprets leg-gluing in $\mathcal{B}$. • $\mathcal{B}^g$-grading: # of trivalent vertices (excluding univalents).

150123 $\operatorname{gr}\left(PB_n^0 \to PB_n^1\right)$ is $0$: $\mathcal{A}^{pb,0} \to \mathcal{A}^{pb,1}$ for a degree mismatch. Likely $[PB_n^1, PB_n^1] \supsetneq PB_n^0$.

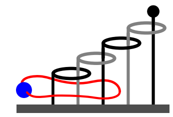
150121 Quillen: $\mathcal{U}(\mathbb{Q} \otimes \operatorname{gr} G) \cong \operatorname{gr} \mathbb{Q}G$, where $\operatorname{gr} G$ uses lower central series, and $\operatorname{gr} \mathbb{Q}G$ uses the augmentation ideal.

150112 Whitney's trick, loosely: In high dimensions at $\pi_1 = 0$, algebraic intersection numbers have precise geometric realizations.

141221 A-S super-CS: (uncertainties highlighted) $(d = \theta^\mu \partial_\mu)$

$$\mathcal{A}(\theta) = c + \theta^\mu A_\mu + \theta^\mu \theta^\nu \epsilon_{\mu\nu\rho} \partial_\rho^{(0)} \bar{c} + \theta^\mu \theta^\nu \theta^\rho \epsilon_{\mu\nu\rho} \phi,$$
$$SCS(\mathcal{A}) = \int dx d\theta \operatorname{tr} \left( \frac{1}{2} \mathcal{A} \cdot d^{(0)} \mathcal{A} + \frac{1}{6} \mathcal{A}^3 \right).$$

150106 Przytycki, Sikora's arXiv:math/0007134 "Chinese Rings", Wikipedia: Baguenaudier:



141226c Presentations of $[FG, FG]$, $[FL, FL]$?

141226d **Q.** If $G \rightsquigarrow \operatorname{gr} G = \bigoplus I^n/I^{n+1}$ is understood, is $\operatorname{gr}_2 G := \bigoplus I^{2n}/I^{2(n+1)}$ interesting? (**A.** Likely not.).

141209 **Plan.** Understand simple circle-pair diagrams, then attempt to generalize to ones with intersections. *Diagrams:* Planar multiple paired oriented circles, AS in said orientation. *Relations:* subdivision, $4T_{1,2}$ as in BBS:Dalvit-141212. Relation with $\mathcal{A}^w$ at BBS:Dalvit-141217.

141208 **Proj.** Milnor/trees / Alexander/MVA / $\pi_1$ for 2-knots with boundary.

140210 **Proj.** A quick paper on a quick combinatorial construction of the wheels invariant following Talks: Hamilton-1412.

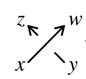
141204b **Proj.** Write up "combinatorial KV"⇒"convolutions".

141127a **Q.** Are intersection graphs mod 4T the gr of something?

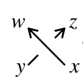
141127b Repeat talks: Watch previous video, repartition handout.

141114b **Proj.** Exposition of Enriquez' solution of YB.

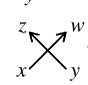
141113a Boden: Brandenbursky has 2 Alexander polys on $v\mathcal{K}$.

140726 Boden's $VG_K$: $[s, q] = 1$ and 

$\to$ $z = xysx^{-1}s^{-1}$, $w = sxs^{-1}$



$\to$ $z = s^{-1}x^{-1}syx$, $w = s^{-1}xs$



$\to$ $z = q^{-1}yq$, $w = qxq^{-1}$

At $q = 1 \neq s$, not basis-conjugating. At $q = s$, OC holds. At $s = 1$ this is Manturov's $vB_n \to \mathrm{Aut}(F(x_1, \ldots, x_n, q))$:

$$\sigma_i \mapsto \begin{cases} x_i \mapsto x_i x_{i+1} x_i^{-1} \\ x_{i+1} \mapsto x_i \end{cases} \text{ or } \begin{cases} x_i \mapsto x_i q^{-1} x_{i+1} q x_i^{-1} \\ x_{i+1} \mapsto q x_i q^{-1} \end{cases}$$
$$\tau_i \mapsto \begin{cases} x_i \mapsto q x_{i+1} q^{-1} \\ x_{i+1} \mapsto q^{-1} x_i q \end{cases} \text{ or } \begin{cases} x_i \mapsto x_{i+1} \\ x_{i+1} \mapsto x_i \end{cases}$$

141102a Zung's visit: • We don't fully understand Configuration Space Integrals (CSI) for curves in the punctured plane. • It's likely that every CSI has a Gauss Diagram Formula (GDF), as winding numbers are computable as intersection numbers. • Degree $n$ GDFs are FT of type $\frac{3}{2}n$, with simple Weight Systems (WS). Likely they are not determined by their WS. • The Merkov quotient of the Feynman-diagram space $\mathcal{A}^t$ is: ∗ internal trivalent vertices vanish. ∗ "Split" arrow-exchange relation. • Is there a $\delta$ like in KBHs?

140923 Manturov's rep. for $PvB_n$: $\sigma_{ij} \mapsto \begin{cases} x_i \mapsto q x_i q^{-1} \\ x_j \mapsto x_i^{-1} q^{-1} x_j q x_i \end{cases}$ (and hence there's a map $PvB_n \to PwB_{n+1}$).

141102b Assaf's riddle: $k$ kids share a loot of $n$ indivisible candies. The first proposes a split; if not accepted by a strict majority, she leaves and the second proposes, etc. How is the loot split?

131213a **Proj.** G-FT invariants of plane curves, 2014-01/PlaneCurves.pdf.

140821 Fiedler: There may exist a "new"* non-oracle map $\mathcal{K} \to \mathbb{Z}\mathcal{K}$. (*) Poly-time, multi-local, low profile, high rank.

140909 **Question.** Is 2-component 2D linking in 4D non-trivial, modulo subdivision and melding? **A.** Likely trivial.

131112b A map $\mathcal{A}^w(\uparrow\updownarrow) \to \mathcal{A}^u(\uparrow\circledast)$ arises in deducing wheeling from the full Duflo (not $\alpha^{-1}$ for $\alpha$ is not well-defined!?); There's a pairing $\mathcal{A}^w(\uparrow\updownarrow) \otimes \mathcal{A}^u(\uparrow) \to \mathcal{A}^u(\uparrow)$. Topological meaning?

140831 **Proj.** Paper: "Why I care about virtual knot theory?".

140731 **Proj.** Make the polynomiality of $B_n$ ridiculously easy.

140725 Two permutations to the virtual braid: $S_n \xleftarrow{s} PvB_n \xrightarrow{\varsigma} S_n$ via $(e, \tau_i, (ij))) \xleftarrow{s} (\sigma_i, \tau_i, \sigma_{ij}) \xrightarrow{\varsigma} (\tau_i, \tau_i, e)$.

140708 Fox $\partial_i\colon F_n \to \mathbb{Z}F_n$: $\partial_i x_j = \delta_{ij}$, $\partial_i(uv) = \partial_i u + u\partial_i v$ (a 1-cocycle). Gassner: $b \mapsto \pi\partial_j b(x_i)$, with $\pi\colon \mathbb{Z}F_n \to \mathbb{Z}\mathbb{Z}^n$ the Abelianization.

140622 Burau: $\Sigma := D^2\backslash\{n \text{ pts}\}$, $p\colon \tilde\Sigma \to \Sigma$ its $\mathbb{Z}$-cover w/ basic deck transformation $t$, $1 \in \partial\Sigma$ a basepoint, $\tilde 1 \in \tilde\Sigma$ a lift, $1^* = p^{-1}(1)$ all lifts, $\tilde H := H_1(\tilde\Sigma, 1^*; \mathbb{Z})$ is a $(\Lambda := \mathbb{Z}[t^{\pm 1}])$-module. $Bu\colon B_n \to \mathrm{Aut}_\Lambda(\tilde H)$ is Burau.

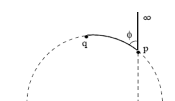
140721 **Proj.** Hilden braids: expansions, the $a/\alpha$-map, tangles?

140716 The $\bar\mu$ invariants are (homology-) concordance invariant.

140713 **Proj.** Low v-algebra: Lie bi-algebras & arrow diagrams.

140604 In $\mathbb{R}^4$, framing a hoop is whatever makes tubing well defined, framing a balloon is whatever makes doubling well defined, and framing a vertex is the interaction between the two.

140424 Mathematica-WikiLink re-implementation: `CreateWikiConnection`, `WikiUserName`, `WikiGetPageText`, `WikiSetPageText`, `WikiSetPageTexts`, `WikiUploadFile`.




140422a The Kontsevich propagator $d\phi$.

140422b Find $\int_{\mathbb{C}_z} \bigwedge_{i=1}^n d\,\mathrm{Arg}(z - z_i) \in \Omega^{n-2}(\mathbb{C}^n_{z_i})$.

140422c Khovanskii's "On a Lemma of Kontsevich" proves Kontsevich's vanishing lemma in 3 pages.

140309 **Proj.** Low degree BF.

140419 **Proj.** Too many definitions of the Alexander polynomial.

140417a Kervaire: $G$ is an $(n \geqslant 3)$-knot group iff it is f.p., normally generated by one element, and $H_{1,2}(G) = (\mathbb{Z}, 0)$.

140417b Does every simple decker set come from a (ribbon) 2-link? Is a 2-link group a LOF group? A decker group a 2-link group?

140413 Are there "spherical w-braids"?

140316 **Proj.** What are all internal quotients of $FL$ (compare "PI-Rings")? Which are of polynomial growth?

140325 Monty Hall: A prize is in 1 of 3 envelopes. You choose one, an oracle shows another to be empty. Will you switch? Deliberate oracle: Yes. Chance oracle: No. What makes it so confusing?

140318 **Proj.** Study Vogel's weight system in the context of $\mathcal{A}^v$.

140304 Itai's iterated mean value theorem: with $(\delta f)(x) := f(x+1) - f(x)$, $\forall x_0, n\,\exists x \in [x_0, x_0+n]$ s.t. $(\delta^n f)(x_0) = (\partial^n f)(x)$. *Pf.* With $\chi := 1_{[0,1]}$, $\delta = \chi\star\partial$ hence $\delta^n = \chi^{\star n}\star\partial^n$ and as $\|\chi^{\star n}\|_{L^1} = 1$, $(\delta^n f)(x_0)$ is bound between the extremals of $(\partial^n f)(x)$.

140302 Assaf: The fundamental group and the fundamental groupoid of a path-connected space are naturally equivalent.

140228b What's the relation between quandle cocycles and 2-knots?

140227 Itai: For at least some quadruples of lines in $\mathbb{R}^3$, there are at least two lines that intersect all of them.

140106 Is the $\vee$-invariant of gnots trivial on 2-knots? Is there a multiplicative Alexander duality? Alexander: $X \subset S^n$ compact, locally contractible $\Rightarrow H^q(X) \simeq H_{n-1-q}(X^c)$.

140218a Vienna vocabulary: cobordism hypothesis, WKB approximation, Fukaya category, gerbes, fusion categories, differential cohomology.

140218b **Proj.** Clean and write up the shielding story.

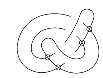
140218c **Proj.** A note on how $DG$ arises in the context of KBHs.

140217 nLab: $C$ monoidal category. Its Drinfel'd centre is the BMC with objects pairs $(X, \beta)$ of $X \in C$ and natural isomorphism $\beta_-\colon X\otimes(-) \to (-)\otimes X$ such that $\forall Y, Z \in C$, $\beta_{Y\otimes Z} = (I_Y \otimes \beta_Z)/\!\!/(\beta_Y \otimes I_Z)$, with $\mathrm{Hom}((X,\beta),(X',\beta')) := \{f \in \mathrm{Hom}(X, X')\colon \forall Z, \beta_Z/\!\!/(I_Z\otimes f) = (f\otimes I_Z)/\!\!/\beta'_Z\}$, $(X,\beta)\otimes(X',\beta') := (X\otimes X', (I_X\otimes\beta')/\!\!/(\beta\otimes I_{X'}))$, and $R_{(X,\beta),(X',\beta')} := \beta_{X'}$ (?).

140211 Ogasa's *Local Move Identities...* — some skein relations for high-dimensional Alexander.

140120 How exactly do normal Euler numbers relate to branch points? Can the latter be avoided?

140130 Satoh's w-knot has the same $\pi_1$ and the same $Z$-polynomial (Sawollek) as the trefoil.



140115 Chterental: $vB_n$ acts faithfully on "virtual curve diagrams", and with run-length compression, this is describable in poly time.

140126 Is tube-bypass an unknotting operation for 2-knots?

140117 Carter: A spun Hopf link with an additional orthogonal plane running once above and once below it makes a knotted $2T^2 + S^2$ with 4 triple points.

140116 Using LMO, FT invariants of links in $S^3$ extend to links in arbitrary $\mathbb{Q}HS$. A simple description? →p8:**190321**

140114 Gavish: "Singular value decompositions".

140113 Many papers by Seiichi Kamada.

131229 Smale ('57): Long immersions $\mathbb{R}^k \hookrightarrow \mathbb{R}^m$ are classified by $\pi_k(V_{m,k})$ where $V_{m,k}$ is the Stiefel manifold (linear embeddings $\mathbb{R}^k \hookrightarrow \mathbb{R}^m$). Paechter (I, '56): $\pi_2(V_{4,2}) = \mathbb{Z}$.

131219 $Gr(\mathbb{R}^2 \hookrightarrow \mathbb{R}^4) = S^2 \times S^2$. Yael: • There's a $\mathbb{C}P^2 = S^2$ of complex lines in $\mathbb{C}\times\mathbb{C}$ and in $\mathbb{C}\times\bar{\mathbb{C}}$. • It is the product of the moduli $C(\mathbb{R}^4) \times C(\bar{\mathbb{R}}^4)$ of metric complex structures on $\mathbb{R}^4$ / $\bar{\mathbb{R}}^4$. For $(I, \bar I) \in C(\mathbb{R}^4)\times C(\bar{\mathbb{R}}^4)$ there is a unique $\mathbb{R}^2 \hookrightarrow \mathbb{R}^4$ which is complex relative to both, and a given $P = \mathbb{C} = \mathbb{R}^2 \hookrightarrow \mathbb{R}^4$ determines two metric complex structures on $\mathbb{R}^4/\bar{\mathbb{R}}^4$ by multiplication by $i$ on $P$ and by $\pm i$ on $P^\perp$. Finally $C(\mathbb{R}^4) = SO(4)/U(2) = \{$left multiplications $L_u$ by unit imaginary quaternions $u\} = S^2$ and $C(\bar{\mathbb{R}}^4) = \{R_v\}_{v\in S^2\subset\mathbb{R}^3\subset\mathbb{H}}$. • $P(u,v) = \mathrm{span}(u+v, uv-1)$ or $\mathrm{span}(u-v, uv+1)^\perp$ and for orthonormal $(\alpha,\beta)$, $\mathrm{span}(\alpha,\beta) \mapsto (\beta\bar\alpha, \bar\alpha\beta) = ((\alpha\wedge\beta)^+, (\alpha\wedge\beta)^-)$, the last using the self-dual and anti-self-dual projections $\Lambda^2 \to \Lambda^{2\pm}$.

131218 Bjorndahl: $N$ prisoners each wears $\infty$-many b/w hats. Simultaneously each needs to point at a black hat on her head. How can they maximize the chance that they will *all* get it right?

131217 Goryunov's *finite order ... $J^+$ ...*: Generic smooth plane curves, allowing triple points and opposite self-intersections, map to knots in the solid torus $ST^\star\mathbb{R}^2$ inducing an isomorphism of projectivizations.

131213b Stallings' theorem: $h\colon A \to B$ a group homomorphism w/ $h\colon H_1(A) \simeq H_1(B)$ and $h\colon H_2(A) \twoheadrightarrow H_2(B)$. Then $h\colon A/A_n \simeq B/B_n$, where $A_n$, $B_n$ denote lower central series (+ more...).

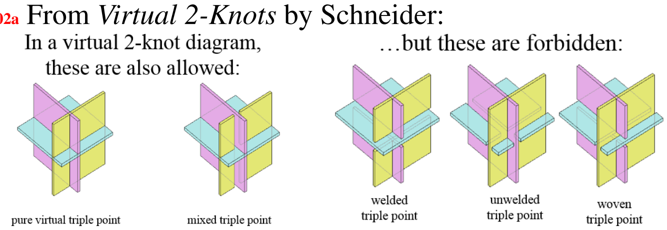
131213c Getzler (bbs): Homotopy 2-types are determined by the action $\pi_1 \circlearrowleft \pi_2$ and a class in $H^3(\pi_1, \pi_2)$.

131126b Anton: is there a triality for solutions of the KV equation? — Yes, `2013-11/DoubleTree/TrialityComputations.nb`. Minor: does $\alpha$ intertwine the triality of $\mathfrak{sder}_2$ with that of $\mathfrak{tder}_2 \ltimes \mathfrak{tr}_2$? — `2013-12/`: Most likely not.

131211 Is the nilpotent completion of the fundamental group of a gnot complement always free-nilpotent? — No; Abelian it is for $\mathbb{R}^2 \times \{0\} \cup \{0\} \times \mathbb{R}^2$.

131122a **Proj.** Figure out the bubble-wrap-finite-type invariants of *all* knotted objects in $\mathbb{R}^4$.

131202a From *Virtual 2-Knots* by Schneider:




In light of "virtual doodles", perhaps this should be modified?

131205a Are there 3 embedded surfaces in $\mathbb{R}^4$ so that any 3 immersed handlebodies bounding them have a common point?

131205b Cimasoni: Levine's *Poly. Inv. of Knots of Codimension 2.*

131204 Coboundary: $\delta(x) = [r, \Delta(x)]$, with invariant $r + r^{21}$.

131202b Farber's *Noncommutative Rational Functions and Boundary Links,* continued Retakh, Reutenauer, Vaintrob, arXiv:math/0004112.

131130c Yanagawa ('69): Ribbon 2-knot $K$ is trivial iff $\pi_1(K) \simeq \mathbb{Z}$.

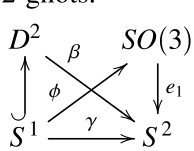
131130d Meilhan: 2-knots papers by Yajima ('62, '64), Yanagawa ('69$^3$), Omae ('71).

131126a Crainic arXiv:math/0403266 on Homological perturbations: A *Homological Homotopy Equivalence (HHE)* is a pair of complexes with quasi-isomorphisms $(L,b) \underset{p}{\overset{i}{\rightleftarrows}} (M,b)$, with a homotopy $h$ between $1 = 1_M$ and $ip$, so $ip = 1 + bh + hb$. A *perturbation* is $\delta\colon M \to M$ with $\deg b = \deg \delta$ and $(b+\delta)^2 = 0$; it is *small* if $(1 - \delta h)^{-1}$ exists. **Claim.** $(L,b_1) \underset{p_1}{\overset{i_1}{\rightleftarrows}} (M, b+\delta)$ is again an HHE, with $A := (1-\delta h)^{-1}\delta$, $b_1 := b + pAi$, $i_1 := i + hAi$, $p_1 := p + pAh$, and with $h_1 := h + hAh$.

131212 Cimasoni: There is a 1-double-point gnot.

131209 Cimasoni: There is a natural smoothing of 2-gnots.

131122b Moskovich, arXiv:math/0211223: On the right, $\phi$ and $\beta$ pair to an integer. Indeed $D^2 \ni x \mapsto \beta(x)^\perp$ is a circle bundle on $D^2$ which must be trivial, inducing a trivialization of the circle bundle $S^1 \ni x \mapsto \beta(x)^\perp$. But $\phi /\!/ e_2$ is a section of that bundle, hence an integer.




131121 Burke, Koytcheff: arXiv:1311.4217, *A colored operad for string link infection*.

131114 Bar-Hillel's Simpson's paradox: In Israel in every age bracket death rates for Arabs are higher than for Jews, yet overall death rates for Jews are higher.

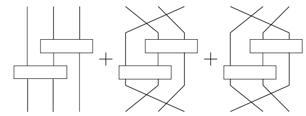
131111 Massuyeau (bbs). Given an algebra $A$ and $N \geqslant 1$, $\exists$ commutative algebra $A_N$ s.t. $\forall$ commutative algebra $B$,

$$\mathrm{Hom}_{\mathrm{Alg}}(A, \mathrm{Mat}_N(B)) \simeq \mathrm{Hom}_{\mathrm{C\text{-}Alg}}(A_N, B).$$

Indeed

$$A_N \simeq \frac{\langle a_{ij}\colon a \in A,\ 1 \leqslant i,j \leqslant N\rangle}{(a+\lambda b)_{ij} = a_{ij} + \lambda b_{ij},\ 1_{ij} = \delta_{ij},\ (ab)_{ik} = \sum_j a_{ij} b_{jk}}.$$

131110 Massuyeau (bbs, eprints), after Van den Bergh: A double bracket in an algebra $A$ is $[\![-,-]\!]\colon A\otimes A \to A\otimes A$ s.t. (1) $[\![b,a]\!] = -[\![a,b]\!]^{op}$. (2) $[\![a, b_1b_2]\!] = (b_1 \otimes 1)[\![a,b_2]\!] + [\![a,b_1]\!](1 \otimes b_2)$. It is Poisson if

$[\![-,-,-]\!] :=$  $= 0$

131107 Enriquez/EllipticAssociators: with $\bar{e}(z) := \frac{\operatorname{ad} z}{e^{\operatorname{ad} z} - 1}$,

$$(\mu, \Phi) \mapsto A = \Phi\left(-\bar{e}(x)y, t^{12}\right) e^{-\mu\bar{e}(x)y} \Phi^{-1}\left(-\bar{e}(x)y, t^{12}\right),$$
$$B = e^{\mu t^{12}/2} \Phi\left(\bar{e}(-x)y, t^{21}\right) e^{x} \Phi^{-1}\left(-\bar{e}(x)y, t^{12}\right).$$

131109 C. Frohman, A. Nicas, *The Alexander Polynomial via topological quantum field theory,* Differential Geometry, Global Analysis, and Topology, Canadian Math. Soc. Conf. Proc. **12**, Amer. Math. Soc. Providence, RI, (1992) 27–40.

131106b Massuyeau (bbs, eprints, easy): $\exists$ "symplectic expansion" — a group-like expansion $Z\colon FG(x_i, y_i) \to FA(\bar{x}_i, \bar{y}_i)$ with $Z\left(\prod_i [x_i, y_i]\right) = \exp\left(-\sum_i [\bar{x}_i, \bar{y}_i]\right)$. Thus surface groups are quadratic and have homomorphic expansions.

131027a Cattaneo: "BV is the 'right' de-Rham differential on supermanifolds."

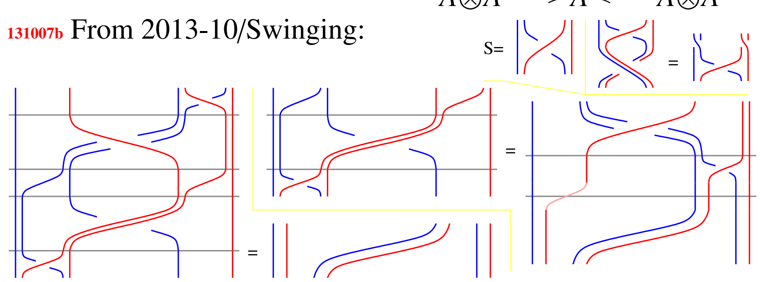
131027b The Hilbert basis theorem: An ideal in the ring of multivariable polynomials over a Noetherian ring is finitely generated. *Pf.* Enough, $R$ Noetherian $\Rightarrow$ any $I \subset R[x]$ is finitely generated. Let $p_n \in I \backslash \langle p_1, \ldots, p_{n-1}\rangle$ be of minimal degree. As $R$ is Noetherian, for large $N$ the leading coefficient of $p_N$ is a combination of previous leading coefficients, so it can be killed off contradicting the minimality of $P_N$. $\square$ Can be made constructive using Gröbner bases.

131020 Artin-Wedderburn: A semi-simple ring is uniquely (up to a permutation) isomorphic to a product of finitely many finite matrix rings over division rings.

131007c I don't understand Ševera's

$$\begin{array}{ccccc}(A\otimes A)\otimes A & & \xrightarrow{\ \Phi\ } & & A\otimes(A\otimes A)\\ \downarrow{\scriptstyle m\otimes 1} & & & & \downarrow{\scriptstyle 1\otimes m}\\ A\otimes A & \xrightarrow{\ m\ } & A & \xleftarrow{\ m\ } & A\otimes A\end{array}$$

131007b From 2013-10/Swinging:




131007a Monoblog starts.

**Archived Items.**

231105b Is "almost classical" an appropriate tag for $w$ objects with div $=0$ (KV solutions, horizontal chord associators)? →p2:**231105a** (After archiving: No.)

210511 **Chal.** Reconstruct a category like $\{\mathrm{Hom}(V^{\otimes A}\to V^{\otimes B})\}$ from a "forward contraction algebra" akin to $\{(V^*)^{\otimes A}\otimes V^{\otimes B}\}$ for (some) $\infty D$ vector spaces $V$. **DoPeGDO** Sol'n: L/G variables are chronologically ordered. In $Q$, LL terms are small, and also LG terms if L preceeds G. Allow only these constrained LG contractions.

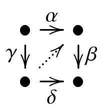
210421 **T/F?** $\sqrt{G}^{-1}\Gamma\sqrt{G}=\Gamma^T$.

170913a A pushforwards challenge in Pushforwards.pdf.

230612a 20m talk: "Rooting the BKT for FTI". Key: $X\subset\underline{n}^d$, $|X|=P$. Using dyadic decompositions, in time $\sim P$ can set a database of size $\sim P$ so $|X\cap R|$ can be computed in time $\sim 1$ for every rectangle $R\subset\underline{n}^d$.

230109 **Def.** Given a v.s. $V$, a Partial Quadratic (PQ) $Q$ on $V$ is a symmetric bilinear form $Q$ on a subspace $\mathcal{D}(Q)\subset V$. For $U\subset\mathcal{D}(Q)$, denote $\mathrm{ann}_Q(U):=\{v\in\mathcal{D}(Q):Q(U,v)=0\}$.
**Def.** $Q_1+Q_2$ is with $\mathcal{D}(Q_1+Q_2)=\mathcal{D}(Q_1)\cap\mathcal{D}(Q_2)$.
**Def.** Given a linear $\psi\colon V\to W$ and a PQ $Q$ on $W$, the pullback is $(\psi^*Q)(v_1,v_2)=Q(\psi v_1,\psi v_2)$ with $\mathcal{D}(\psi^*Q)=\phi^{-1}(\mathcal{D}(Q))$.
**Def.** Given $\phi\colon V\to W$ and a PQ $Q$ on $V$ the pushforward $\phi_*Q$ is with $\mathcal{D}(\phi_*Q)=\phi(\mathrm{ann}_Q(\mathcal{D}(Q)\cap\ker\phi))$ and $(\phi_*Q)(w_1,w_2)=Q(v_1,v_2)$, where $v_i$ are s.t. $\phi(v_i)=w_i$ and $Q(v_i,\mathrm{rad}\,Q|_{\ker\phi})=0$.

**Thm(?).** $\psi^*$ and $\phi_*$ are well-defined and functorial, and if $\alpha/\!\!/\beta=\gamma/\!\!/\delta$, then $\gamma^*/\!\!/\alpha_*=\delta_*/\!\!/\beta^*$. $\psi^*$ is additive but $\phi_*$ isn't. $\phi^*/\!\!/\phi_*$ is restriction to $\mathrm{im}\,\phi$. $\phi_*/\!\!/\phi^*$ is ?.




**Thm(?).** Over $\mathbb{R}$, given $\phi\colon V\to W$ and PQs $Q$ on $V$ and $C$ on $W$,

$$\mathrm{sign}_V(Q+\phi^*C)=\mathrm{sign}_{\ker\phi}(\iota^*Q)+\mathrm{sign}_W(C+\phi_*Q)$$

$$(\Leftrightarrow\ \mathrm{sign}_V(Q)=\mathrm{sign}_{\ker\phi}(\iota^*Q)+\mathrm{sign}_W(\phi_*Q)\quad(\text{no }C,\text{ no }+)).$$

230109 **Def.** Given a v.s. $V$, a Partial Quadratic (PQ) on $V$ is $Q=(D=\mathcal{D}(Q)\subset W,L=\mathcal{L}(Q)\colon D\to D^*)$ with $L=L^\star$. Write $Q(v_1,v_2)=L(v_1)(v_2)$.
**Def.** Given a linear $\psi\colon V\to W$ and a PQ $Q=(D=\mathcal{D}(Q)\subset W,L=\mathcal{L}(Q)\colon D\to D^*,\ L=L^\star)$ on $W$, the pullback $\psi^*Q$ is $(D',L')$ with $D'=\phi^{-1}(D)$ and $L'=\psi/\!\!/L/\!\!/\psi^*$, so $(\psi^*Q)(v_1,v_2)=Q(\psi v_1,\psi v_2)$.
**Def.** If $Q_i$ are PQs on $V$, $Q_1+Q_2$ is with $\mathcal{D}(Q_1+Q_2)=\mathcal{D}(Q_1)\cap\mathcal{D}(Q_2)$.
**Def.** Given a linear $\phi\colon V\to W$ and a PQ $Q=(D,L)$ on $V$ the pushforward $\phi_*Q$ is $Q'=(D',L')$ with $D'=\phi((L(\mathrm{rad}\,Q|_{D\cap\ker\phi}))^\perp)$ and $Q'(w_1,w_2)=Q(v_1,v_2)$, where $v_i$ are s.t. $\phi(v_i)=w_i$ and $Q(v_i,\mathrm{rad}\,Q|_{\ker\phi})=0$.
**Thm.** Pullbacks / pushforwards are well-defined and functorial.
**Thm(?).** Over $\mathbb{R}$, given $\phi\colon V\to W$ and PQs $Q$ on $V$ and $C$ on $W$,

$$\mathrm{sign}_V(Q+\phi^*C)=\mathrm{sign}_{\ker\phi}(\iota^*Q)+\mathrm{sign}_W(C+\phi_*Q).$$

220923 **Q.** For balanced Heisenberg exponentials, does $p$-scattering determine $x$-scattering?

210101 **Proj.** Implement a `PD2ThinMorse` with guaranteed bounds and/or using simulated annealing.

211210 **Proj.** Complexity by ropelength?

180820 $\rho_1=t\left(P|_{e,l,f\to0}-t\omega'\omega^3\right)/(t-1)^2\omega^2$ and

$$P=A^2\frac{(t-1)^3\rho_1+t^2(2vw+(1-t)(1-2c))AA'}{(1-t)t}.$$

210302 **Q.** (w/ Abbasi) Does every contractible but not manifestly contractible curve in the annulus have an odd self-intersection?

170829a **Do.** Find Duflo in Goldman-Turaev.

190108 **Do.** With $P=\sum a_{mn}z^m\zeta^n$, compute $\langle\!\langle\epsilon P\rangle\!\rangle:=\log\langle\exp\epsilon P\rangle$.

150416 Chterental: Is there a Melvin-Morton statement for v-knots?

181031 **Proj.** Verify Kashaev's conjecture @arXiv:1801.04632, re. Tristram–Levine signatures.

170321 For NOE1 with $\Lambda\to0$, are there interesting $R$'s?

170320b **Proj.** $k$-co inductive constructions.

210318a Riba Garcia's talk, "Invariants of Rational Homology 3-Spheres and the Mod $p$ Torelli Group":

210211a-old Halacheva ($\sim$): $\mathcal{A}(X):=\bigoplus_k\mathrm{End}(\Lambda^kX)$ is a traced meta-monoid with $m_z^{xy}(A):=(z\to y)/\!\!/(e_x/\!\!/i_x/\!\!/A/\!\!/e_y/\!\!/i_y-e_x/\!\!/A/\!\!/i_y)/\!\!/(x\to z)$ and $\mathrm{tr}_x(A):=e_x/\!\!/i_x/\!\!/A/\!\!/e_x/\!\!/i_x-e_x/\!\!/A/\!\!/i_x$. Contains $\Gamma$ (w/ fixed colours) via $\Upsilon\colon(\omega,M)\mapsto\omega\Lambda^*(M)$. Predict $\mathcal{A}$ from $\Gamma$? Interpret $\mathcal{A}$ in *ybax*? Related to super-algebras? Raise $\mathcal{A}$ to meta-Hopf? Understand $\mathrm{im}(\Upsilon)$?

180311 **Do.** With strongly docile $L$ and $\Lambda$, compute $\log\langle\mathbb{e}^L\mid\mathbb{e}^\Lambda\rangle$ without exponentiating.

171029 **Do.** Solve $\hbar^{-1}(1-\mathbb{e}^{\hbar(t-2a\epsilon)})=g(a-1,z)+(-\mathbb{e}^{\epsilon\hbar}-(t-2a\epsilon)\partial_z+\epsilon z\partial_z^2)g(a,z)$.

170309 →p18:**170309** Then BBS:AKT17-170317: $\mathbb{e}^{\alpha w}\mathbb{e}^{\beta u}=\mathbb{e}^{au}\mathbb{e}^{d(b-2\epsilon c)}\mathbb{e}^{bw}$ with $\gamma=1-\alpha\beta\epsilon$, $a=\beta/\gamma=\beta+\ldots$, $b=\alpha/\gamma=\alpha+\ldots$, $d=\epsilon^{-1}\log\gamma=-\alpha\beta+\ldots$, so $\mathbb{O}\left(wu\colon\mathbb{e}^{\alpha w+\beta u}\right)=\mathbb{O}\left(ucw\colon\mathbb{e}^{au+bw+d(b-2\epsilon c)}\right)=\mathbb{O}\left(ucw\colon\mathbb{e}^{\lambda_\epsilon(\alpha,\beta)}\mathbb{e}^{\alpha w+\beta u-\alpha\beta b}\right)=\mathbb{O}\left(ucw\colon\mathbb{e}^{\lambda_\epsilon(\partial_w,\partial_u)}\mathbb{e}^{\alpha w+\beta u-\alpha\beta b}\right)$ so $\mathbb{O}\left(wu\colon\mathbb{e}^{\alpha w+\beta u+\delta uw}\right)=\mathbb{O}\left(ucw\colon\mathbb{e}^{\delta\partial_\alpha\partial_\beta}\mathbb{e}^{\lambda_\epsilon(\partial_w,\partial_u)}\mathbb{e}^{\alpha w+\beta u-\alpha\beta b}\right)=\mathbb{O}\left(ucw\colon\mathbb{e}^{\lambda_\epsilon(\partial_w,\partial_u)}\nu\mathbb{e}^q\right)=\mathbb{O}\left(ucw\colon\mathbb{e}^{\Lambda_\epsilon}\nu\mathbb{e}^q\right)$, with $\nu=(1+b\delta)^{-1}$, $q=\nu(\alpha w+\beta u+\delta uw-\alpha\beta b)$ and $\Lambda_\epsilon\in\mathbb{Q}(w,u,b,c,\alpha,\beta,\delta)[\![\epsilon]\!]$.

170522 What is the "sensical" sub-meta-object of

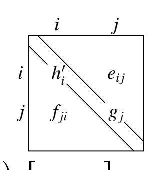
$$(\mathcal{U}(\mathfrak{b}_+),m,\Delta,S,P,R)?$$

180528a Is there an operation-uniformizing "bottom tangles in handlebodies" theory for (rotational) virtuals similar to Habiro-Massuyeau?

131112a The diamond lemma: If $\to$ is a connected Noetherian binary relation (Noetherian: an infinite $a_1\to a_2\to\cdots$ is ultimately constant), and if whenever $a\to b$ and $a\to c$ there is $d$ with $b\Rightarrow d$ and $c\Rightarrow d$ where $\Rightarrow$ is the reflexive transitive closure of $\to$, then $\exists!m\,\forall a\,a\Rightarrow m$.

200204b **Talk.** Over then Under Tangles. **Abstract.** Brilliant wrong ideas should not be buried and forgotten. Instead, they should be mined for the gold that lies underneath the layer of wrong. In my talk I will explain how "over then under tangles" lead to an easy classification of knots, and under the surface, also to some valid mathematics: ...

170126c In $gl_{n+}^\epsilon$: $[◹,◹]=◹$, $[◺,◺]=\epsilon◺$, $[◺,◹]=◺+\epsilon◹$, so with $h_i=h_i'-\epsilon g_i$, $[h_i,\cdot]=0$, $[g_i,g_j]=0$, $[e_{ij},e_{kl}]=\delta_{jk}e_{il}-\delta_{il}e_{kj}$, $[f_{ij},f_{kl}]=\epsilon(\delta_{jk}f_{il}-\delta_{il}f_{kj})$, $[e_{ij},f_{kl}]=\delta_{jk}(\epsilon\delta_{i<l}e_{il}+\delta_{i>l}f_{il})-\delta_{li}(\epsilon\delta_{k<l}e_{kj}+\delta_{k>j}f_{kj})+\delta_{jk}\delta_{li}((h_i-h_j)/2+\epsilon(g_i-g_j))$, $[g_i,e_{jk}]=(\delta_{ij}-\delta_{ki})e_{jk}$, $[g_i,f_{jk}]=(\delta_{ij}-\delta_{ki})f_{jk}$, $\deg(\epsilon,h_i,f_{ij},g_i,e_{ij})=$



$(1,1,1,0,0)$. Verification in 2017-02/glne.nb. Order $n$ symmetry in 2020-01/glne.nb.

171012 Talks/LesDiablerets-1708, esp. PBWDemo.nb, verifications 2017-10/Phi2CR-Classical.nb: In $\hat{\mathcal{U}}(\mathfrak{g}^\epsilon) = \langle t,y,a,x\rangle/([t,\cdot]=0,[a,x]=x,[a,y]=-y,[x,y]=t-2\epsilon a)$, we have $\prod_{i=1}^2 \mathbb{e}^{\tau_i t}\mathbb{e}^{\eta_i y}\mathbb{e}^{\alpha_i a}\mathbb{e}^{\xi_i x} = \mathbb{e}^{\tau t}\mathbb{e}^{\eta y}\mathbb{e}^{\alpha a}\mathbb{e}^{\xi x}$, with

$$\tau = \tau_1+\tau_2 - \frac{\log(1-\epsilon\eta_2\xi_1)}{\epsilon} = \tau_1+\tau_2+\eta_2\xi_1+\frac{\epsilon}{2}\eta_2^2\xi_1^2+\dots,$$
$$\eta = \eta_1 + \frac{\mathbb{e}^{-\alpha_1}\eta_2}{(1-\epsilon\eta_2\xi_1)} = \eta_1+\mathbb{e}^{-\alpha_1}\eta_2+\epsilon\mathbb{e}^{-\alpha_1}\eta_2^2\xi_1+\dots,$$
$$\alpha = \alpha_1+\alpha_2+2\log(1-\epsilon\eta_2\xi_1) = \alpha_1+\alpha_2-2\epsilon\eta_2\xi_1+\dots,$$
$$\xi = \frac{\mathbb{e}^{-\alpha_2}\xi_1}{(1-\epsilon\eta_2\xi_1)}+\xi_2 = \mathbb{e}^{-\alpha_2}\xi_1+\xi_2+\epsilon\mathbb{e}^{-\alpha_2}\eta_2\xi_1^2+\dots.$$

181024 With Ens. The $CD_a$ universe $\mathcal{U} = FA\langle H_k, R_k, \dots, Z_k^i, \dots\rangle * \mathbb{Q}S_*$ has a strand-filtration $\mathcal{F}_n$, a Vassiliev degree deg $\geqslant 0$, a homological degree ht $\geqslant 0$, an ht-odd differential $\delta$ with $\deg\delta = 0$, $\operatorname{ht}\delta = -1$, an endomorphism $c$ with $c\mathcal{F}_n \subset \mathcal{F}_{n+1}$ and $\deg c = \operatorname{ht} c = 0$ and

- $S_* := \bigcup_{n>0} S_n$ with $(\deg,\operatorname{ht}) = (0,0)$, $S_n \subset \mathcal{F}_n$, and $c\colon S_n \to S_{n+1}$ via $(c\sigma)_1 = 1$ and $(c\sigma)_{i>1} = \sigma_{i-1}+1$.
- For $U$ any $H$, $R$, or $Z$, $U_k = c^k U_0 =: c^k U$.
- $H \in \mathcal{F}_1$ with $(\deg,\operatorname{ht}) = (1,0)$. Let $t_{0i} := (1i)H(1i)$, let $t_{12} := H_1 - (12)H(12)$ and at $j>i>0$ let $t_{ij} = (1i)(2j)t_{12}(2j)(1i)$.
- $R \in \mathcal{F}_2$ with $(\deg,\operatorname{ht}) = (1,1)$, $\delta R = \dots$.
- $\dots$

Claim/goal: $H_0(\mathcal{U},\delta) \cong S_* \ltimes DK_{\{0\}\sqcup *}$ and $H_1(\mathcal{U},\delta) = 0$.

190115 The monoidal category with objects $\mathbb{N}$ generated by $\sigma \in \operatorname{Aut}(2)$ with $1_1\otimes\sigma\otimes 1_1 = \sigma\otimes 1_1\otimes\sigma$, possibly with $\sigma^2 = 1_2$.

171104 Roland: Solve $g(a,t)g(-a-1,-t) = P(a,t)$.

170725 (Wrong, see 2017-10/Phi2CR.nb) 2017-07/Multi-beta-yax.nb: In $\mathcal{U}_{\gamma^{-1};\gamma\beta}$ where $q = \mathbb{e}^\beta$, $\prod_{i=1}^2 \mathbb{e}^{\eta_i y}\mathbb{e}^{\alpha_i a}\mathbb{e}^{\xi_i x} = \mathbb{e}^{\eta y}\mathbb{e}^{\alpha a}\mathbb{e}^{\xi x}\mathbb{e}^{\tau t}$, with

$$\begin{aligned}
\eta &= \eta_1+\eta_2 e^{-\gamma\alpha_1} - \beta\gamma\eta_2^2\xi_1 e^{-\gamma\alpha_1}+\dots = \eta_1+\delta\eta_2 e^{\beta-\alpha_1\gamma}\\
\alpha &= \alpha_1+\alpha_2+2\beta\eta_2\xi_1+\dots = \alpha_1+\alpha_2-2\left(\beta+\log\delta\right)/\gamma\\
\xi &= \xi_1 e^{-\gamma\alpha_2}+\xi_2-\beta\gamma\eta_2\xi_1^2 e^{-\gamma\alpha_2}+\dots = \delta\xi_1 e^{\beta-\alpha_2\gamma}+\xi_2\\
\tau &= -\eta_2\xi_1+\beta\eta_2\xi_1\left(\gamma\eta_2\xi_1+1\right)/2+\dots = \left(\beta+\log\delta\right)/(\beta\gamma)
\end{aligned}$$

and $\delta := \left(\left(e^\beta-1\right)\gamma\eta_2\xi_1+e^\beta\right)^{-1} = 1-(1+\gamma\eta_1\xi_1)\beta+\dots$.

170805 With $\Phi = (\phi_j(\alpha_i))$ and $Z = \zeta(\partial_{\alpha_i})$, set $\Phi_* Z := \mathbb{e}^{\sum\partial_{\beta_j}\phi_j(\partial_{a_i})}\zeta(a_i)\Big|_{a_i=0}$. **Do.** With $(a_i,y_i,x_i,t_i) := (\partial_{\alpha_i},\partial_{\eta_i},\partial_{\xi_i},\partial_{\tau_i})$, compute/implement $\Phi_* Z$, with

$$Z = \omega\exp\left(\sum\lambda_{ij}t_i a_j + \sum q_{ij}y_i x_j + \epsilon P_0\right),$$

$\lambda_{ij}\in\mathbb{Z}$, $\omega, q_{ij}\in R := \mathbb{Q}(T_i = \mathbb{e}^{t_i})$, $P_0 \in R[a_i,y_i,x_i]$, and

$$\begin{aligned}
\Phi^*(\bar\alpha_i) &= \sum\psi_{ij}^1\alpha_j+\epsilon P_1,\\
\Phi^*(\bar\eta_i) &= \sum\psi_{ij}^2\eta_j+\epsilon P_2,\\
\Phi^*(\bar\xi_i) &= \sum\psi_{ij}^3\xi_j+\epsilon P_3,\\
\Phi^*(\bar\tau_i) &= \sum\psi_{ij}^4\tau_j+\sum\gamma_{ij}\eta_i\xi_j+\epsilon P_4,
\end{aligned}$$

$\psi_{ij}^{1,4}\in\mathbb{Z}$, $\psi^{2,3}\in R$, $P_{1,4}\in\mathbb{Q}[x_i,y_i]$, $P_{2,3}\in R[x_i,y_i]$, $\gamma_{ij}\in R$.

170713 KZ: $dH = H\sum_{i<j}\frac{dz_i-dz_j}{z_i-z_j}t^{ij}$.

170610 (alt; main: →p11:170625) $\mathcal{U}_{\hbar;\alpha\beta}$ conventions: $q = \mathbb{e}^{\hbar\alpha\beta}$, $H = \langle a,x\rangle/([a,x]=\alpha x)$ with

$$A = \mathbb{e}^{-\hbar\beta a},\quad xA = qAx$$
$$S(a,A,x) = (-a,A^{-1},-A^{-1}x),$$
$$\Delta(a,A,x) = (a_1+a_2, A_1A_2, x_1+A_1x_2)$$

and dual $H^* = \langle b,y\rangle/([b,y]=-\beta y)$ with

$$B = \mathbb{e}^{-\hbar\alpha b},\quad By = qyB$$
$$S(b,B,y) = (-b,B^{-1},-yB^{-1}),$$
$$\Delta(b,B,y) = (b_1+b_2, B_1B_2, y_1B_2+y_2).$$

Pairing by $(a,x)^* = \hbar(b,y)$ making $\langle a^jx^k, y^lb^j\rangle = \delta_{ij}\delta_{kl}i![k]_q!$. Then $\mathcal{U} = H^*\otimes H^{op}$ with $(\phi f)(\psi g) = \langle f_1,\psi_1\rangle\langle f_3, S\psi_3\rangle(\phi\psi_2)(gf_2)$.

170513 (alt; main: →p11:170625) $\mathcal{U}_{\eta,\gamma}$ conventions: $A = \langle g, G = \mathbb{e}^{\eta g}, e\rangle/([g,e]=\gamma e)$ with $S(g,G,e) = (-g,G^{-1},-eG^{-1})$;

$$\Delta(g,G,e) = (g_1+g_2, G_1G_2, e_1G_2+e_2)$$

and dual $A^* = \langle h, H = \mathbb{e}^{\gamma h}, f\rangle/([h,f] = -\eta h)$ with $S(h,H,f) = (-h,H^{-1},-H^{-1}f)$;

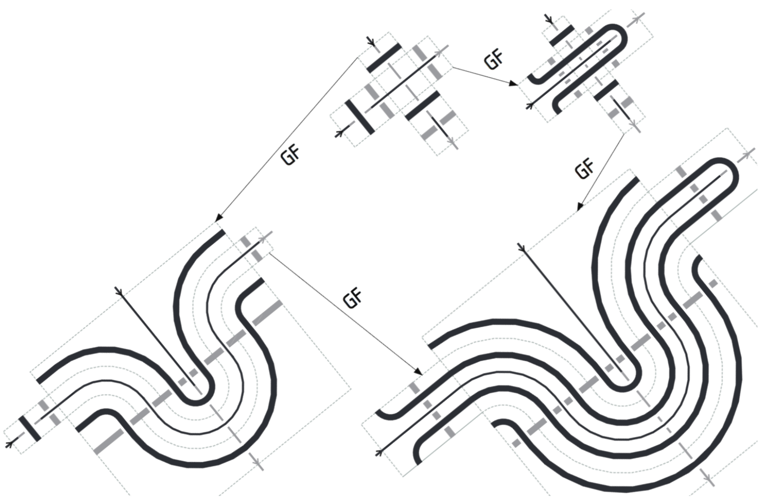
$$\Delta(h,H,f) = (h_1+h_2, H_1H_2, f_1+H_1f_2).$$

Pairing by $(g,e)^* = (h,f)$. Degrees by $\deg(\gamma,g,e,\eta,h,f) = 1$, so ops are degree non-decreasing except the basic pairing lowers 2 degrees.

170528 (alt; main: →p11:170625) $\mathcal{U}_{\hbar,\epsilon}$ conventions: $A = \langle g, G = \mathbb{e}^{\hbar\epsilon g}, e\rangle/([g,e]=\hbar e)$ with $\Delta(g,G,e) = (g_1+g_2,G_1G_2,e_1G_2+e_2)$; $S(g,G,e) = (-g,G^{-1},-eG^{-1})$ and dual $A^* = \langle h, H = \mathbb{e}^{\hbar h}, f\rangle/([h,f] = -\hbar\epsilon h)$ with $\Delta(h,H,f) = (h_1+h_2,H_1H_2,f_1+H_1f_2)$; $S(h,H,f) = (-h,H^{-1},-H^{-1}f)$. Pairing by $(g,e)^* = (h,f)$. Degrees by $\deg(\hbar,g,e,h,f) = 1$, so ops are degree non-decreasing except the basic pairing lowers 2 degrees.

170412a **Title.** The Dogma is Wrong. **Abstract.** It has long been known that there are knot invariants associated to semi-simple Lie algebras, and there has long been a dogma as for how to extract them: "quantize and use representation theory". We present an alternative and better procedure: "centrally extend, approximate by solvable, and learn how to re-order exponentials in a universal enveloping algebra". While equivalent to the old invariants via a complicated process, our invariants are in practice stronger, faster to compute (poly-time vs. exp-time), and clearly carry topological information. ¶ This is joint work with Roland van der Veen and continues work by Rozansky and Overbay.

170401o **Project** over-then-under "Ⓞ-Tangles". Closed under compositions; (v-)braids are Ⓞ; non-braid Ⓞ tangles? Relations in Ⓞ? In $\mathcal{A}^{Ⓞ}$? Not all tangles are Ⓞ. Alexander properties; v-version. Associators in $\mathcal{A}^u \cap \mathcal{A}^{Ⓞ}$: Constructible? Sufficient for EK? Relations with Chterental's "virtual curve diagrams"? Chu's syzygy:




170108b AKT-17 Reality // Plan: **Gentle.** Course introduction (h1). Knots, Reidemeister moves and the Jones polynomial (h2-3). Tangles and a faster Jones program (h4). Tangles and meta-monoids (h5-6). Links, 3-manifolds, Seifert surfaces and genus, ribbon knots and "algebraic knot theory" (h7-8). The Alexander polynomial using Γ-calculus (h9-10). Finite type invariants and expansions (h11-14). / The relationship with metrized Lie algebras and PBW (h15-16). / The variants $v$, $w$, $bv$, and $rv$, and their expansions (h17-18). Lie bialgebras and solvable approximation (h19-20). **Brute.** Knots, algebras, YBE, CYBE, Lie algebras, universal enveloping algebras, formulas (h1) The Lie algebra $\mathfrak{g}_0$, universal enveloping algebras and low degree computations (h2-3). Ordering symbols and commutation relations for $\mathfrak{g}_0$ (h4-5). The $\mathfrak{g}_0$ invariant (h6-7). The Λόγος and $\mathfrak{g}_1$ computations (h8-10). // Morse knots and the $\mathfrak{g}_1$ invariant (h11). $\mathfrak{g}_0$ and $\mathfrak{g}_1$ as approximations of $sl_2$, approximating $sl_3$ (h12). The $sl_3^0$ invariant (h13-14). The $sl_3^1$ invariant, fame, and glory (h15-16).

160513 **Q.** What's Fox-Milnor for links? →p12:**131130b**.

170211a Gaussian pairing:

$$\left\langle \exp\left(\tfrac{x\subset}{2}+\textstyle\sum_{i\in I} i\bullet\!\!-\right) \,\middle|\, \exp\left(\tfrac{\supset y}{2}+\textstyle\sum_{j\in J} -\!\!\bullet j\right)\right\rangle =$$
$$\exp\left(\log\left(\tfrac{1}{1-xy}\right)\bigcirc+\textstyle\sum_{i\in I,j\in J}\frac{i\bullet\!\!-\!\!\bullet j}{1-xy}+\sum_{i_{1,2}\in I}\frac{{}^{i_1}_{i_2}\updownarrow y}{1-xy}+\sum_{j_{1,2}\in J}\frac{x\updownarrow^{j_1}_{j_2}}{1-xy}\right).$$

160314 **Proj.** Visualization of fibred knots. [Done, JB→].

160403a Is □ on unipotent completions (page 7 of my GT1 paper) nonsense? Are Taylor expansions isomorphisms?

160321 **Prob.** Find a quadratic description for the Adjoint rep of $[\bar a_{ik},\bar a_{jk}]=-[\bar a_{ij},\bar a_{jk}]=\bar a_{ik}-\bar a_{jk}$, $[\bar a_{ij},\bar a_{ji}]=\bar a_{ji}-\bar a_{ij}$.

141123 Qinhuangdao: Talk to me about China, America, Taiwan, economy, ecology, religion, democracy, censorship, and all else.

150201b A precise relationship between expansions for $FG$ and "PBW bases" for same?

150401 Halacheva's meta-trace: $\begin{array}{c|cc}\omega & c & S\\ \hline c & \alpha & \theta\\ S & \psi & \Xi\end{array} \xrightarrow[\mu:=1-\alpha]{\Gamma::tr_c} \begin{array}{c|c}\mu\omega & S\\ \hline S & \Xi+\psi\theta/\mu\end{array}$

with $\begin{array}{c|ccc}\omega & a & b & S\\ \hline a & \alpha & \beta & \theta\\ b & \gamma & \delta & \epsilon\\ S & \phi & \psi & \Xi\end{array} \xrightarrow[\substack{\mu:=1-\beta\\ T_a,T_b\to T_c}]{\Gamma::m_c^{ab}} \begin{array}{c|cc}\mu\omega & c & S\\ \hline c & \gamma+\alpha\delta/\mu & \epsilon+\delta\theta/\mu\\ S & \phi+\alpha\psi/\mu & \Xi+\psi\theta/\mu\end{array}$.

When exactly is it defined?

141211 BBS:Alekseev-131108, AT sec. 5.2: $F_1\in$ TAut solving AT $\leftrightarrow F_t := F_1(tx,ty)$ with $F_0=1 \leftrightarrow u_t=\frac{dF_t}{dt}F_t^{-1}$ with $u_t=\frac{1}{t}u(tx,ty)\leftrightarrow \mathfrak{tder}\ni u=(A,B)$ solving KV. What means $F_1(tx,ty)$?

140228a **Proj.** Associator computations using `FreeLie`.

140203 **Project** "expansions and quadraticity for groups": definitions, relations with Hain / Mal'cev / Quillen / Vassiliev, torsion, semi-direct products, $FG$, $PuB$, $PvB$, $PwB$ (and homotopy versions), elliptic / higher genus braids, mapping class groups, right-angled Artin groups, Stallings' theorem, knot groups (u, w, higher D), Hutchings-Lee. ¿Flat braids after Merkov, Hilden braids, $[PuB_n, PuB_n]$ (also $u\to v,w$), $[G,G]$ in general, Aut$(FG_2)$, Aut$(FG_n)$, Torelli following Hain?

150107 **Paperlet.** "An algebraic characterization of the Taylor expansion".

141209

141129 Implement `SeriesSolve`:

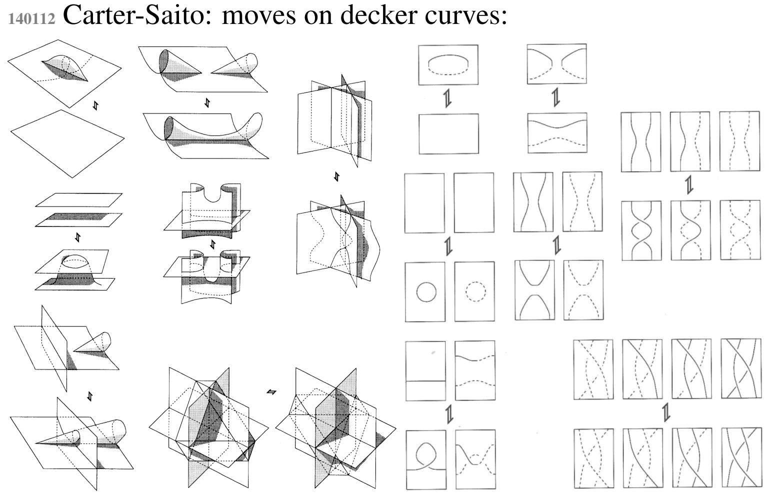
```
SeriesSolve[{
  α = LS[{"1", "2"}, αs], β = LS[{"1", "2"}, βs],
  γ = CWS[{"1", "2"}, γs], κ = CWS[{"1"}, κs]
 },
 V = Eₛ[⟨1 → α, 2 → β⟩, γ];
 ℏ^-1 (ℰₛ[R⁺[2, 3] ** R⁺[1, 3]] ** V ≡ V ** (ℰₛ[R⁺[1, 3]] // dΔ[1, 1, 2]))
    && V ** (V // dA[1] // dA[2]) ≡ dϵ[1] ⋃ dϵ[2]
    && V ** (κ // dΔ[1, 1, 2]) // dc[1] // dc[2] ≡ κ ⋃ (κ // dσ[1, 2])]
```

140627 **Proj.** A 3-page paper on Gassner and its unitarity.

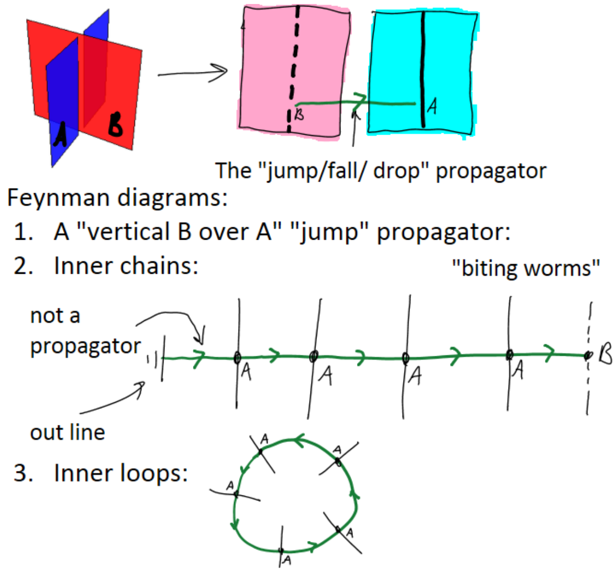
140119 Lie-series Mathematica abstraction challenge (also `2014-01`): `LieSeries`, `MakeLieSeries`, `Crop`, `RandomLieSeries`, $+$, $c\cdot$, $\equiv$, $\int$, $b$, `EulerE`, `adPower`, `adSeries`, `Ad`, `LieDerivation` (also on `CW`, `AW`), $+$, $c\cdot$, `DerivationPower`, `DerivationSeries`, `LieMorphism` (also on `CW`, `AW`, $\langle\rangle$ and into $\langle\rangle$), `StableApply`, `BCH`, `ASeries`, `MakeASeries`, $\iota$, $\sigma$, `CWSeries`, `MakeCWSeries`, `RandomCWSeries`, $+$, $c\cdot$, $\equiv$, $\int$, tr, div `JA`, $\langle\ldots\rangle$, $+$, $c\cdot$, `TangentialDerivation`, $tb$, $\Gamma$, $\Gamma^{-1}$.

140213 Brochier's even associators to degree 9 at `http://abrochier.org/sage.php`.

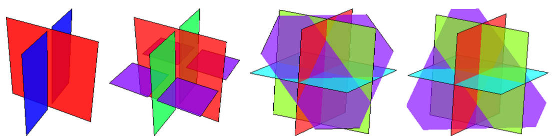
140112 Carter-Saito: moves on decker curves:

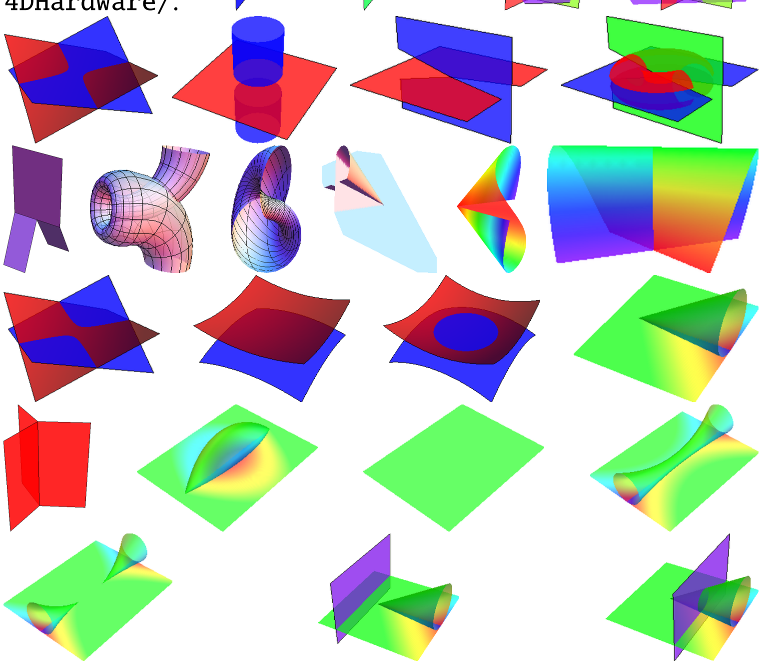


140113 BF perturbation theory in ambient axial gauge ($A$-$B$ propagator is $t$-vertical, $B$ above $A$; inner gauge unspecified):




(131202c) At `2013-12/4DHardware/`:





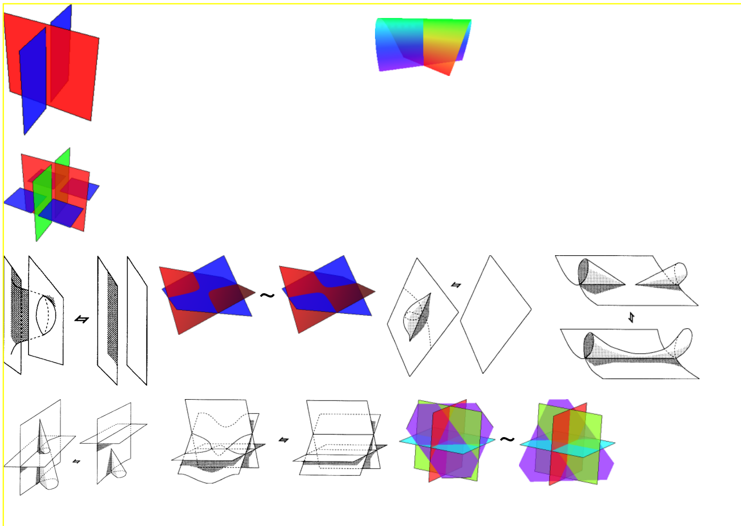
140119 At `2014-01`, an abstraction challenge:

Pensieve header: The Free Lie / Lazy Evaluation Abstraction Challenge.

```
MakeLieSeries[ser_Symbol, expr_] := (
    ser[] = Hold[MakeLieSeries[ser, expr]];
    ser[d_Integer] := ser[d] = Expand[expr /. w_LW /; Deg[w] ≠ d → 0];
    LieSeries[ser]
  );

AddLieSeries[ss___LieSeries] := AddLieSeries[ss] = Module[{ser},
     ser = Unique[AddLieSeries];
     ser[] = Hold[AddLieSeries[ss]];
     ser[d_Integer] := ser[d] = Plus @@ ((#[d]) & /@ {ss});
     LieSeries[ser]
    ];

b[s1_LieSeries, s2_LieSeries] := b[s1, s2] = Module[{ser},
     ser = Unique[b];
     ser[] = Hold[b[s1, s2]];
     ser[d_Integer] := ser[d] = Sum[
        b[s1[k], s2[d - k]],
        {k, 1, d - 1}
       ];
     LieSeries[ser]
    ];

LieDerivation[der_Symbol, rules_List] := (
    der[] = Hold[LieDerivation[der, rules]];
    (der[w_LW] /; Deg[w] == 1) :=
     (der[w] = MakeLieSeries[w /. Append[rules, _LW → 0]]);
    der[w_LW] := der[w] = Module[{x, y},
       {x, y} = LyndonFactorization[w];
       AddLieSeries[b[der[x], y], b[x, der[y]]]
      ];
    der[s_LieSeries] := der[s] = Module[{ser},
       ser = Unique[LieDerivationOnLieSeries];
       ser[] = Hold[der[s]];
       ser[d_] := ser[d] = Sum[
          der[s[k]][d],
          {k, 1, d}
         ];
       LieSeries[ser]
      ];
    der[as_ASeries] := Omitted;
    der[cws_CWSeries] := Omitted;
    der[expr_][d_] :=
     Expand[expr /. {w_LW ⧴ der[w][d], s_LieSeries ⧴ der[s][d]}];
    LieDerivation[der]
  );

BCHBase = Module[{bch},
    bch = Unique["BCHBase"];
    bch[] = Hold[BCHBase];
    bch[1] = ⟨"x"⟩ + ⟨"y"⟩;
    bch[d_Integer] := bch[d] = Expand[Plus[
         adSeries[E^(-ad), MakeLieSeries[⟨"y"⟩]][MakeLieSeries[⟨"x"⟩]][d],
         -adSeries[(1 - E^(-ad)) / ad - 1, LieSeries[bch]][
            EulerE[LieSeries[bch]]][d]
        ] / d];
    LieSeries[bch]
   ];

JA[-1, ___] = MakeCWSeries[0];
JA[n_, y_LW, μ_LieSeries, ss_] := JA[n, y, μ, ss] = Module[
     {s, sμ, μs},
     sμ = ScaleLieSeries[s, μ];
     μs = StableApply[LieMorphism[{y → Ad[ScaleLieSeries[1, sμ]][LW[z]]}], μ];
     μs = μs // LieMorphism[{LW[z] → y}];
     IntegrateCWSeries[
      AddCWSeries[
       JA[n - 1, y, μ, s] // LieDerivation[{y → b[μs, y]}],
       div[y, μs]
      ],
      {s, 0, ss}
     ]
    ];
JA[y_LW, μ_LieSeries] := JA[y, μ] = Module[{cws, s},
     cws = Unique[JA];
     cws[] = Hold[JA[y, μ]];
     cws[d_Integer] := cws[d] = JA[d - 1, y, μ, s][d] /. s → 1;
     CWSeries[cws]
    ];
```

131111 Missing in `FreeLie.m`: The relation with $\mathfrak{tder}_n$: $\exp\left(\sum_i \mathrm{ad}_{u_i}\{\gamma_i\}\right) = C^{\beta_1,\beta_2,\dots}_{u_1,u_2,\dots}$, etc.

140109 Virtual 2-knots:



131229 What are the two winding numbers for immersions $\mathbb{R}^2 \hookrightarrow \mathbb{R}^4$? Is every pair realized? Is there a Whitney-Graustein theorem?

131130a Meilhan: Levine: arXiv:q-alg/9711007 *A Factorization of the Conway Polynomial*. Then Tsukamoto, Yasuhara: arXiv:math/0405481 *A factorization of the Conway polynomial and covering linkage invariants*.

131026 Time to make an "agenda browser".

131017 If $\lambda_{\{ij\}} = 0$, $(\lambda_{ij} dx^i \wedge dx^j)^{n/2} = \sqrt{\det(\lambda_{ij})} \bigwedge_i dx^i$.