

Publication List

Book Chapters

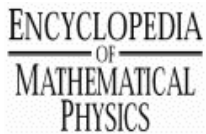
(active links and uncut abstracts at <http://drorbn.net/ARC18/PublicationList.html>)



[OnOnInvariants.pdf](#)

On Raoul Bott's "On Invariants of Manifold" (2 pages, posted August 2015, to appear in Bott's collected works, vol. 5)

I'm not sure how to introduce a review paper. So rather than commenting on the paper as whole, I will concentrate on my subjective view of just one paragraph - a paragraph which I think I



[article's home](#)

Finite Type Invariants (9 pages, posted August 2004, [arXiv:math.GT/0408182](#)).

This is an overview article on finite type invariants, written for the *Encyclopedia of Mathematical Physics*.



Why are we not happy?

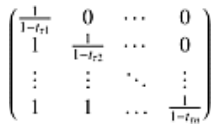
[Fundamental.pdf](#)

The Fundamental Theorem of Vassiliev Invariants (joint with [Alexander Stoimenow](#), *Geometry and Physics*, (J.E. Andersen, J. Dupont, H. Pedersen, and A. Swann, eds.), lecture notes in pure and applied mathematics 184, Marcel Dekker, New-York 1997, pp. 101-134, [arXiv:q-alg/9702009](#)).

An exposition of four approaches to the proof of "The Fundamental Theorem of Vassiliev

Refereed journal articles

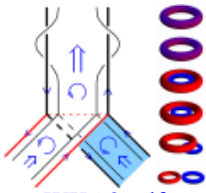
(active links and uncut abstracts at <http://drorbn.net/ARC18/PublicationList.html>)



[UofG.pdf](#)

A Note on the Unitarity Property of the Gassner Invariant (3 pages, posted June 2014, updated August 2014, *Bulletin of Chelyabinsk State University (Mathematics, Mechanics, Informatics)* **3-358-17** (2015) 22-25, [arXiv:1406.7632](#))

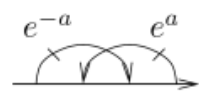
We give a 3-page description of the Gassner invariant (or representation) of braids (or pure braids),



[WKO2.pdf](#)

Finite Type Invariants of w-Knotted Objects II: Tangles, Foams and the Kashiwara-Vergne Problem (joint with [Zsuzsanna Dancso](#), 57 pages, posted May 2014, updated October 2015, *Mathematische Annalen* **367** (2017) 1517-1586, partially replaces [WKO](#), [arXiv:1405.1955](#))

This is the second in a series of papers dedicated to studying w-knots, and more generally, w-



[WKO1.pdf](#)

Finite Type Invariants of w-Knotted Objects I: w-Knots and the Alexander Polynomial (joint with [Zsuzsanna Dancso](#), 52 pages, posted May 2014, updated April 2016, *Algebraic and Geometric Topology* **16-2** (2016) 1063-1133, partially replaces [WKO](#), [arXiv:1405.1956](#))

This is the first in a series of papers studying w-knots, and more generally, w-knotted objects (w-

$$\begin{aligned} w_{4/2,2}(\pi_{2n}(D)) &= 2R_n(D)e_2^n + \dots \\ &\updownarrow \\ JJ(h)(K) \cdot \bar{C}(h)(K) &= 1 \\ &\text{arXiv:1401.0754} \end{aligned}$$

Proof of a Conjecture of Kulakova et al. Related to the sl_2 Weight System (joint with Huan Vo, *European Journal of Combinatorics* **45** (2015) 65-70, [arXiv:1401.0754](#)).

In this article, we show that a conjecture raised in [KLMR] ([arXiv:1307.4933](#)), which regards the coefficient of the highest term when we evaluate the sl_2 weight system on the projection of a

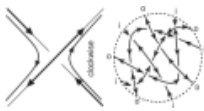


[paper's home](#)

Balloons and Hoops and their Universal Finite Type Invariant, BF Theory, and an Ultimate Alexander Invariant (56 pages, posted August 2013, updated November 2017, *Acta Mathematica Vietnamica* **40-2** (2015) 271-329, [arXiv:1308.1721](#))

Balloons are two-dimensional spheres. Hoops are one dimensional loops. Knotted Balloons and

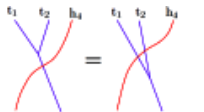
Khovanov Homology for Alternating Tangles (joint with [Hernando Burgos-Soto](#), *Journal of Knot Theory and its Ramifications* **23-2** (2014), 18 pages, posted May 2013, updated March 2014,



[arXiv:1305.1695](#)

[arXiv:1305.1695](#)).

We describe a "concentration on the diagonal" condition on the Khovanov complex of tangles,



[MetaMonoids.pdf](#)

Meta-Monoids, Meta-Bicrossed Products, and the Alexander Polynomial (joint with Sam Selmani, 15 pages, posted February 2013, updated February 2014, [Journal of Knot Theory and its Ramifications](#) **22-10** (2013), [arXiv:1302.5689](#)).

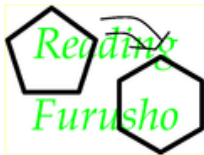
We introduce a new invariant of tangles along with an algebraic framework in which to understand



[paper's home](#)

Homomorphic Expansions for Knotted Trivalent Graphs (joint with [Zsuzsanna Dancso](#), 23 pages, posted March 2011, updated August 2012, [Journal of Knot Theory and Its Ramifications](#) **22-1** (2013), [arXiv:1103.1896](#)).

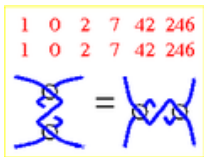
It had been known since old times that there exists a universal finite type invariant ("an expansion") Z^{old} for Knotted Trivalent Graphs (KTGs), and that it can be chosen to intertwine between some of



[arXiv:1010.0754](#)

Pentagon and Hexagon Equations Following Furusho (joint with [Zsuzsanna Dancso](#), 7 pages, posted October 2010, [Proceedings of the American Mathematical Society](#) **140-4** (2012) 1243-1250).

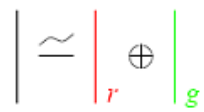
In [[arXiv:math/0702128](#)] H. Furusho proves the beautiful result that of the three defining equations for associators, the pentagon implies the two hexagons (see also [[Willwacher's arXiv:1009.1654](#)]).



[paper's home](#)

Some Dimensions of Spaces of Finite Type Invariants of Virtual Knots (joint with Iva Halacheva, Louis Leung, and Fionntan Roukema, 8 pages, posted September 2009, updated January 2011, [Experimental Mathematics](#) **20-3** (2011) 282-287, [arXiv:0909.5169](#)).

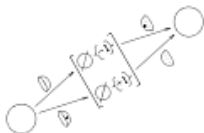
We compute many dimensions of spaces of finite type invariants of virtual knots (of several kinds) and the dimensions of the corresponding spaces of "weight systems", finding everything to be in



[paper's home](#)

The Karoubi Envelope and Lee's Degeneration of Khovanov Homology (joint with [Scott Morrison](#), 8 pages, posted June 2006, [Algebraic & Geometric Topology](#) **6** (2006) 1459-1469, [arXiv:math.GT/0606542](#)).

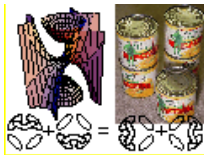
We give a simple proof of Lee's result from [arXiv:math.GT/0210213](#), that the dimension of the Lee variant of the Khovanov homology of an c -component link is 2^c , regardless of the number of



[paper's home](#)

Fast Khovanov Homology Computations (13 pages, posted June 2006, updated May 2007, [arXiv:math.GT/0606318](#), [Journal of Knot Theory and Its Ramifications](#), **16-3** (2007) 243-255).

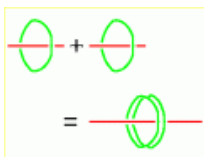
We introduce a *local* algorithm for Khovanov Homology computations - that is, we explain how it is possible to "cancel" terms in the Khovanov complex associated with a ("local") tangle, hence canceling the many associated "global" terms in one swoosh early on. This leads to a dramatic



[paper's home](#)

Khovanov's Homology for Tangles and Cobordisms (39 pages, posted October 2004, updated April 2006, [Geometry and Topology](#) **9** (2005) 1443-1499, [arXiv:math.GT/0410495](#)).

We give a fresh introduction to the Khovanov Homology theory for knots and links, with special emphasis on its extension to tangles, cobordisms and 2-knots. By staying within a world of topological pictures a little longer than in other articles on the subject, the required extension

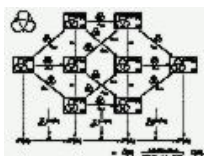


[paper's home](#)

Two Applications of Elementary Knot Theory to Lie Algebras and Vassiliev Invariants

(joint with [Thang T. Q. Lê](#) and Dylan P. Thurston, [Geometry and Topology](#) **7-1** (2003) 1-31, posted April 2002, [arXiv:math.QA/0204311](#)).

Using elementary equalities between various cables of the unknot and the Hopf link, we prove the Wheels and Wheeling conjectures of [[BGRT:WheelsWheeling](#)] and [[Deligne:Letter](#)], which give



[paper's home](#)

On Khovanov's Categorification of the Jones Polynomial (posted September 2001,

[Algebraic and Geometric Topology](#) **2-16** (2002) 337-370, [arXiv:math.QA/0201043](#), updated August 2004).

The working mathematician fears complicated words but loves pictures and diagrams. We thus give a no-fancy-anything picture-rich glimpse into Khovanov's novel construction of "the

Bracelets and the Goussarov Filtration of the Space of Knots (posted November 26, 2001, [Invariants of knots and 3-manifolds \(Kyoto 2001\)](#), [Topology and Geometry Monographs](#) **4**, 1-12, [arXiv:math.GT/0111267](#)).



[paper's home](#)

Following Goussarov's paper "Interdependent Modifications of Links and Invariants of Finite Degree" we describe an alternative finite type theory of knots. While (as shown by Goussarov) the



[paper's home](#)

A Rational Surgery Formula for the LMO Invariant (joint with [Ruth Lawrence](#), posted May 15, 2000, [Israel Journal of Mathematics](#) **140** (2004) 29-60, [arXiv:math.GT/0007045](#)). We write a formula for the LMO invariant of a rational homology sphere presented as a rational surgery on a link in S^3 . Our main tool is a careful use of the Århus integral and the (now proven) "Wheels" and "Wheeling" conjectures of B-N, Garoufalidis, Rozansky and Thurston. As steps, side



[paper's home](#)

Solving the Bible Code Puzzle (joint with [Brendan McKay](#), [Gil Kalai](#) and Maya Bar-Hillel; [Statistical Science](#) **14-2** (1999) 150-173)

A paper of Witztum, Rips and Rosenberg in the journal *Statistical Science* in 1994 made the extraordinary claim that the Hebrew text of the Book of Genesis encodes events which did not occur until millennia after the text was written. In reply, we argue that Witztum, Rips and

$$\begin{aligned} & (-1)^{|m|X|} \int^{(m)} G dX \\ & = (\det \Lambda)^m \int^{FG} G dX \end{aligned}$$

[AarhusIII.pdf](#)

The Århus integral of rational homology 3-spheres III: The Relation with the Le-Murakami-Ohtsuki Invariant (joint with [Stavros Garoufalidis](#), [Lev Rozansky](#) and Dylan P. Thurston, *Selecta Mathematica*, New Series **10** (2004) 305-324, [arXiv:math.QA/9808013](#)).

Continuing the work started in [Part I](#) and [Part II](#) of this series, we prove the relationship between the Århus integral and the invariant *LMO* defined by T.Q.T. Le, J. Murakami and T. Ohtsuki in q -

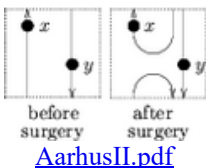


A story of
"Cooked Lists"
and
"Permutation Races."

[Chance.pdf](#)

The Torah Codes: Puzzle and Solution (joint with Maya Bar-Hillel and [Brendan McKay](#), [Chance](#) **11-2** (1998) 13-19)

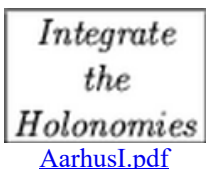
A plain-English account of some of our investigations into "Bible codes".



[AarhusII.pdf](#)

The Århus integral of rational homology 3-spheres II: Invariance and Universality (joint with [Stavros Garoufalidis](#), [Lev Rozansky](#) and Dylan P. Thurston, *Selecta Mathematica*, New Series **8** (2002) 341-371, [arXiv:math.QA/9801049](#)).

We continue the work started in [Part I](#), and prove the invariance and universality in the class of finite type invariants of the object defined and motivated there, namely the Århus integral of



Integrate
the
Holonomies

[AarhusI.pdf](#)

The Århus integral of rational homology 3-spheres I: A highly non trivial flat connection on S^3 (joint with [Stavros Garoufalidis](#), [Lev Rozansky](#) and Dylan P. Thurston, *Selecta Mathematica*, New Series **8** (2002) 315-339, [arXiv:q-alg/9706004](#)).

Path integrals don't really exist, but it is very useful to dream that they do and figure out the consequences. Apart from describing much of the physical world as we now know it, these dreams



[Wheels.pdf](#)

Wheels, Wheeling, and the Kontsevich Integral of the Unknot (joint with [Stavros Garoufalidis](#), [Lev Rozansky](#) and Dylan P. Thurston, posted March 1997, *Israel Journal of Mathematics* **119** (2000) 217-237, [arXiv:q-alg/9703025](#)).

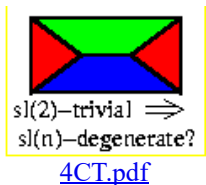
We conjecture an exact formula for the Kontsevich integral of the unknot, and also conjecture a formula (also conjectured independently by [Deligne](#)) for the relation between the two natural



[GT1.pdf](#)

On Associators and the Grothendieck-Teichmuller Group I (*Selecta Mathematica*, New Series **4** (1998) 183-212, June 1996, updated October 1998, [arXiv:q-alg/9606021](#)).

We present a formalism within which the relationship (discovered by Drinfel'd) between associators (for quasi-triangular quasi-Hopf algebras) and (a variant of) the Grothendieck-Teichmuller group becomes simple and natural, leading to a great simplification of Drinfel'd's original work. In

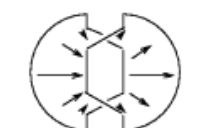


$sl(2)$ -trivial \Rightarrow
 $sl(n)$ -degenerate?

[4CT.pdf](#)

Lie Algebras and the Four Color Theorem (*Combinatorica* **17-1** (1997) 43-52, last updated October 1999, [arXiv:q-alg/9606016](#)).

Contains an appealing statement about Lie algebras that is equivalent to the Four Color Theorem. The notions appearing in the statement also appear in the theory of finite-type invariants of knots (Vassiliev invariants) and 3-manifolds.



An Elementary Proof That All Spanning Surfaces of a Link Are Tube-Equivalent (joint with [Jason Fulman](#) and [Louis H. Kauffman](#), June 1995, updated March 1998, *Journal of Knot Theory and its Ramifications* **7-7** (1998) 873-879).

The standard proof that the potential function provides a model for the Alexander-Conway

[tube.pdf](#)



[poly.pdf](#)

Polynomial Invariants are Polynomial (Mathematical Research Letters **2** (1995) 239-246, [arXiv:q-alg/9606025](#)).

Contains a proof that (as conjectured by Lin and Wang [[arXiv:dg-ga/9411015](#)] when a Vassiliev invariant of type m is evaluated on a knot projection having n crossings, the result is bounded by a constant times n^m . Thus the well known analogy between Vassiliev invariants and polynomials



[mmr.pdf](#)

On the Melvin-Morton-Rozansky Conjecture (joint with [Stavros Garoufalidis](#), July 1994, last updated January 1996, *Inventiones Mathematicae* **125** (1996) 103-133).

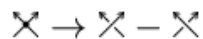
We prove a conjecture made by Melvin and Morton, saying that a certain specialization of the colored Jones polynomial is equal to the inverse of the Conway polynomial (in particular, the Conway polynomial is computable from the Jones polynomial and its cablings, something that was



[homotopy.pdf](#)

Vassiliev Homotopy String Link Invariants (February 1993, last updated January 1999, *Journal of Knot Theory and its Ramifications* **4-1** (1995) 13-32).

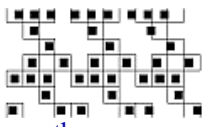
I show that the main conjectures of [On the Vassiliev Knot Invariants](#) become theorems when the attention is restricted to string links considered only up to *homotopy*. That is, the corresponding map into surfaces is injective (so all homotopy invariants come from surfaces), and Vassiliev homotopy



crossings
differences
are
derivatives
 $V^{(m)} \rightarrow V^{(m+1)}$
[paper's home](#)

On the Vassiliev Knot Invariants (August 1992, last updated January 2007, *Topology* **34** (1995) 423-472).

An introduction to Vassiliev invariants. Contains the definition, proofs that the various knot polynomials are Vassiliev invariants (appropriately parametrized and expanded), the basic constructions (of weight systems from Vassiliev invariants and from Lie algebras), a discussion of



[the paper](#)

Perceived Depth Images (appeared (in a shorter form) as *Random Dot Stereograms* in *The Mathematica Journal* **1-3** (1991) 69-75).

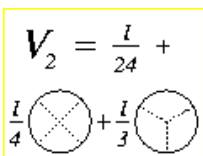
Describes a Mathematica package for creating perceived depth images - these things that look 3D when you look at them with your eyes crossed. For the mathematica package itself, click [here](#). For a primitive but working 9 line version of that package, click [here](#).



[NonCompact.pdf](#)

Perturbative Expansion of Chern-Simons Theory with Non-Compact Gauge Group (joint with [Edward Witten](#), *Communications in Mathematical Physics* **141** (1991) 423-440).

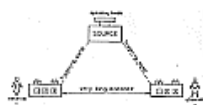
A discussion of the semi-classical approximation for Chern-Simons theory with a non-compact gauge group. After finding the correct gauge fixing, we discuss the somewhat non-standard eta invariant that enters the computation of the phase of the path integral, and a certain anomaly related to it.



[pcs.pdf](#)

Perturbative Chern-Simons Theory (43 pp, April 1990, last updated September 1995, *Journal of Knot Theory and its Ramifications* **4-4** (1995) 503-548).

Contains an introduction to perturbation theory in the context of Chern-Simons theory and knots, a discussion of the first order perturbation theory (linking, self-linking, and the torsion-related anomaly that forces the introduction of framings), a proof that the second order perturbation theory



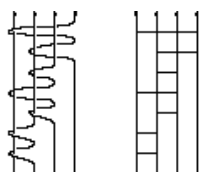
[the paper](#)

Two Examples in Non-Commutative Probability (*Foundations of Physics* **19** (1989) 97-104).

Mainly an exposition of the Bell inequality from the point of view of non-commutative (quantum) probability. Also contains a short discussion of the Heisenberg uncertainty principle from the same point of view.

**Refereed
conference
proceedings**

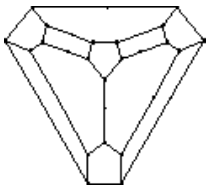
(active links and uncut abstracts at <http://drorbn.net/ARC18/PublicationList.html>)



[glN.pdf](#)

Vassiliev and Quantum Invariants of Braids (*Proceedings of Symposia in Applied Mathematics* **51** (1996) 129-144, Amer. Math. Soc., [arXiv:q-alg/9607001](#)).

Contains a proof of the fact that all Vassiliev invariants of braids come (in the natural sense) from $gl(N)$ and its representations, and thus, in the light the fact that Vassiliev invariants separate braids (see my paper [Vassiliev Homotopy String Link Invariants](#)), the $gl(N)$ invariants separate braids. A



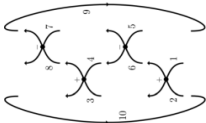
[nat.pdf](#)

Non-Associative Tangles (in *Geometric topology*, proceedings of the Georgia international topology conference, W. H. Kazez, ed., 139-183, Amer. Math. Soc. and International Press, Providence, 1997).

We give a *first* completely combinatorial construction of a universal Vassiliev invariant, along lines suggested by Drinfel'd's work on quasi-Hopf algebras (previous papers on the subject did not give a

Other publication output

(active links and uncut abstracts at <http://drorbn.net/ARC18/PublicationList.html>)



[arXiv:1708.04853](#)

A Polynomial Time Knot Polynomial (joint with [Roland van der Veen](#), 21 pages, posted August 2017).

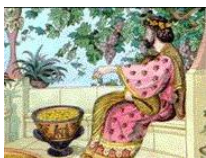
We present the strongest known knot invariant that can be computed effectively (in polynomial time).



[CDMReview.pdf](#)

Review of a Book by Chmutov, Duzhin, and Mostovoy (Bull. Amer. Math. Soc. **50** (2013) 685-690, posted February 2013).

Merely 30 years ago, if you had asked even the best informed mathematician about the relationship between knots and Lie algebras, she would have laughed, for there isn't and there can't be. Knots are



[paper's home](#)

On the Witztum-Rips-Rosenberg Sample of Nations (joint with [Brendan McKay](#) and Shlomo Sternberg, draft, April 1998; first edition: March 1998).

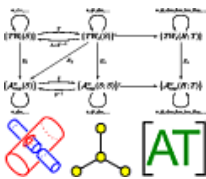
We study the Witztum-Rips-Rosenberg (WRR) sample of nations and find clear evidence that their results were obtained by selective data manipulation and are therefore invalid. Our tool is the study of variations - we vary the sample of nations in many ways, and find that the variations are almost



[paper's home](#)

Equidistant Letter Sequences in Tolstoy's "War and Peace" (joint with [Brendan McKay](#), draft, December 1997; first edition: September 1997).

In [WRR1], Witztum, Rips and Rosenberg found a surprising correlation between famous rabbis and their dates of birth and death, as they appear as equidistant letter sequences in the Book of Genesis. We make a smaller or equal number of mistakes, and find the same phenomenon in



[WKO4.pdf](#)

Finite Type Invariants of w-Knotted Objects IV: Some Computations (49 pages, posted November 2015, [arXiv:1511.05624](#))

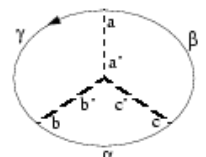
In the previous three papers in this series, [WKO1]-[WKO3], Z. Dancso and I studied a certain theory of "homomorphic expansions" of "w-knotted objects", a certain class of knotted objects in 4-

$$\begin{aligned} \mathcal{G}_4 \mathcal{E}_5 &= R_{811} \\ \mathcal{G}_5 \mathcal{E}_5 &= 2R_{311} \\ \mathcal{G}_6 \mathcal{E}_5 &= 3R_{311} + R_{2111} \\ \mathcal{G}_7 \mathcal{E}_5 &= 5R_{311} + R_{2111} \end{aligned}$$

[table.pdf](#)

Some Computations Related to Vassiliev Invariants (18 pp, last updated May 5, 1996, available online, not meant for publication).

Contains tables of dimensions of over 500 spaces of Chinese Characters, as well as some tables of dimensions of spaces of Vassiliev invariants of knots, braids, and string links. Also contains the decompositions into irreducibles of the representations of the symmetric groups naturally associated



[weights.pdf](#)

Weights of Feynman Diagrams and the Vassiliev Knot Invariants (22 pp, February 1991, last updated June 1995).

My first paper on Vassiliev invariants, the first place where the relation between Vassiliev invariants and Lie algebras was noticed, and the first place where it was shown that there are more than finitely many Vassiliev invariants (by showing that the coefficients of the Conway polynomial are