

Banana Seifert

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\People\\VanDerVeen"]
Once[<< KnotTheory`]
```

```
Out[*]=
```

C:\drorbn\AcademicPensieve\People\VanDerVeen

Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.

Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[*]:= SetAttributes[P, Orderless]
ProcessSigns[pd_] :=
  pd /. X[a_, b_, c_, d_] => If[PositiveQ[X[a, b, c, d]], Xp[a, b, c, d], Xm[a, b, c, d]]
SeifertCycles[pd_] := (Times @@ pd) /.
  {Xp[a_, b_, c_, d_] => P[a, b] P[c, d], Xm[a_, b_, c_, d_] => P[a, d] P[c, b]} //.
  {P[a___, b_] P[b_, c___] => P[a, b, c], x_^2 => x}
SeifertBycles[pd_] := (Times @@ pd) /.
  {Xp[a_, b_, c_, d_] => Q[a] Q[b] Q[d, c], Xm[a_, b_, c_, d_] => Q[a, d] Q[c] Q[b]} //.
  {Q[a___, b_] Q[b_, c___] => Q[a, b, c]}
CutList[pd_] := First /@ List @@ (SeifertCycles[pd])
IsIncoming_z_[XX_] :=
  If[(z == 1) || (Head[XX] == Xp && (z == 4)) || (Head[XX] == Xm && (z == 2)), True, False]
CutPD[pd_] := Module[{pd2 = pd, pos},
  Do[
    pos = Position[pd, c];
    If[IsIncoming_pos[[1, 2]] [pd[[pos[[1, 1]]]],
      Part[pd2, Sequence @@ pos[[1]]] = $[c];
      Part[pd2, Sequence @@ pos[[2]]] = $$[c],
      Part[pd2, Sequence @@ pos[[2]]] = $[c];
      Part[pd2, Sequence @@ pos[[1]]] = $$[c];

    , {c, CutList[pd]}];
  pd2]
SeifertBGraph[K_] :=
  Module[{pd2 = CutPD[ProcessSigns@PD[K]], Edges, Signs = <|>, XV, OV, V},
    XV =
      (List @@ pd2) /. {Xp[a_, b_, c_, d_] => {-a, b, -d}, Xm[a_, b_, c_, d_] => {-a, -b, c}};
    Edges = XV // Flatten;
    (List @@ pd2) /. {Xp[a_, b_, c_, d_] => (Signs = Append[Signs, {-a -> -1, b -> 1, -d -> -1}]},
      Xm[a_, b_, c_, d_] => (Signs = Append[Signs, {-a -> 1, c -> 1, -b -> -1}])};
    OV = If[Length[#] == 1, Switch[Head[#[[1]]], Integer, {-#[[1]], #[[1]]}, $, -#, $$, #],
      Join[If[IntegerQ[#[[1]]], {#[[1]], {}},
        -Most[#],
        If[IntegerQ[#[[1]], {-#[[1]], {}}, {}]]]
```

```

]

] & /@ (List@@ (SeifertBycles[pd2] /. x_<sup>2</sup> -> x) /. Q -> List);
{V = Join[OV, XV], Signs}
]

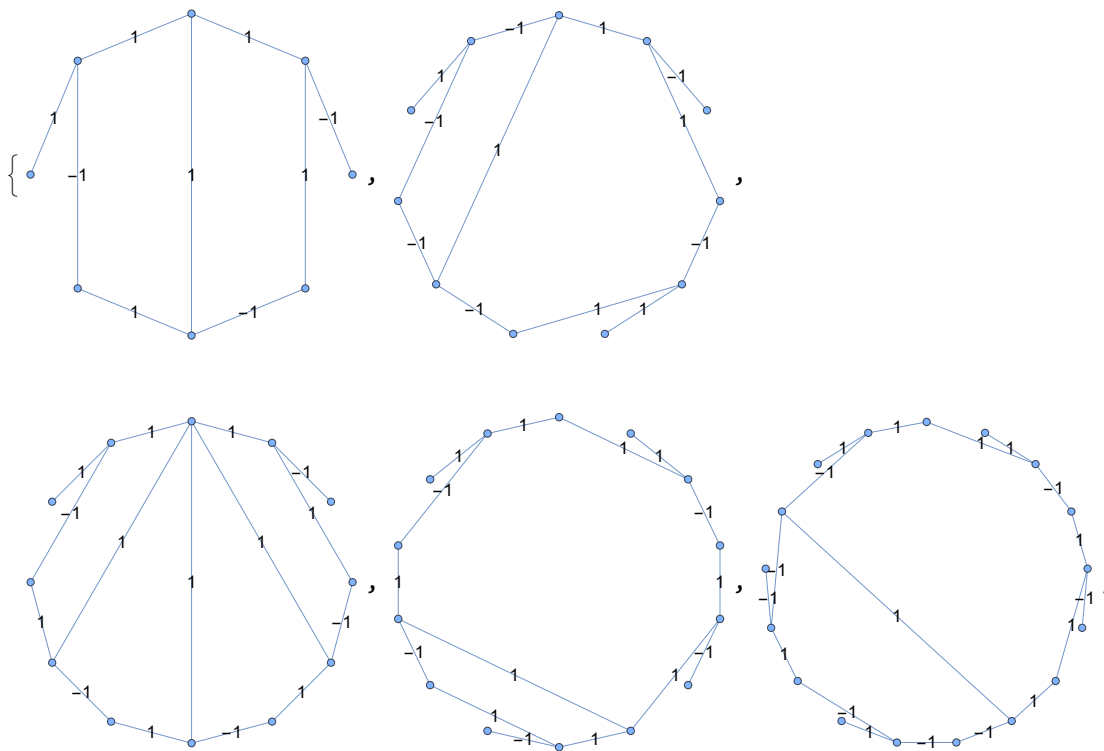
(*Warning! misrepresents the labels of multi-edges*)
DrawGraph[sb_] :=
  PlanarGraph[Table[Labeled[
    UndirectedEdge@@ (First /@ Position[sb[[1]], e]), sb[[2]][e]], {e, sb[[2]] // Keys}]]
]

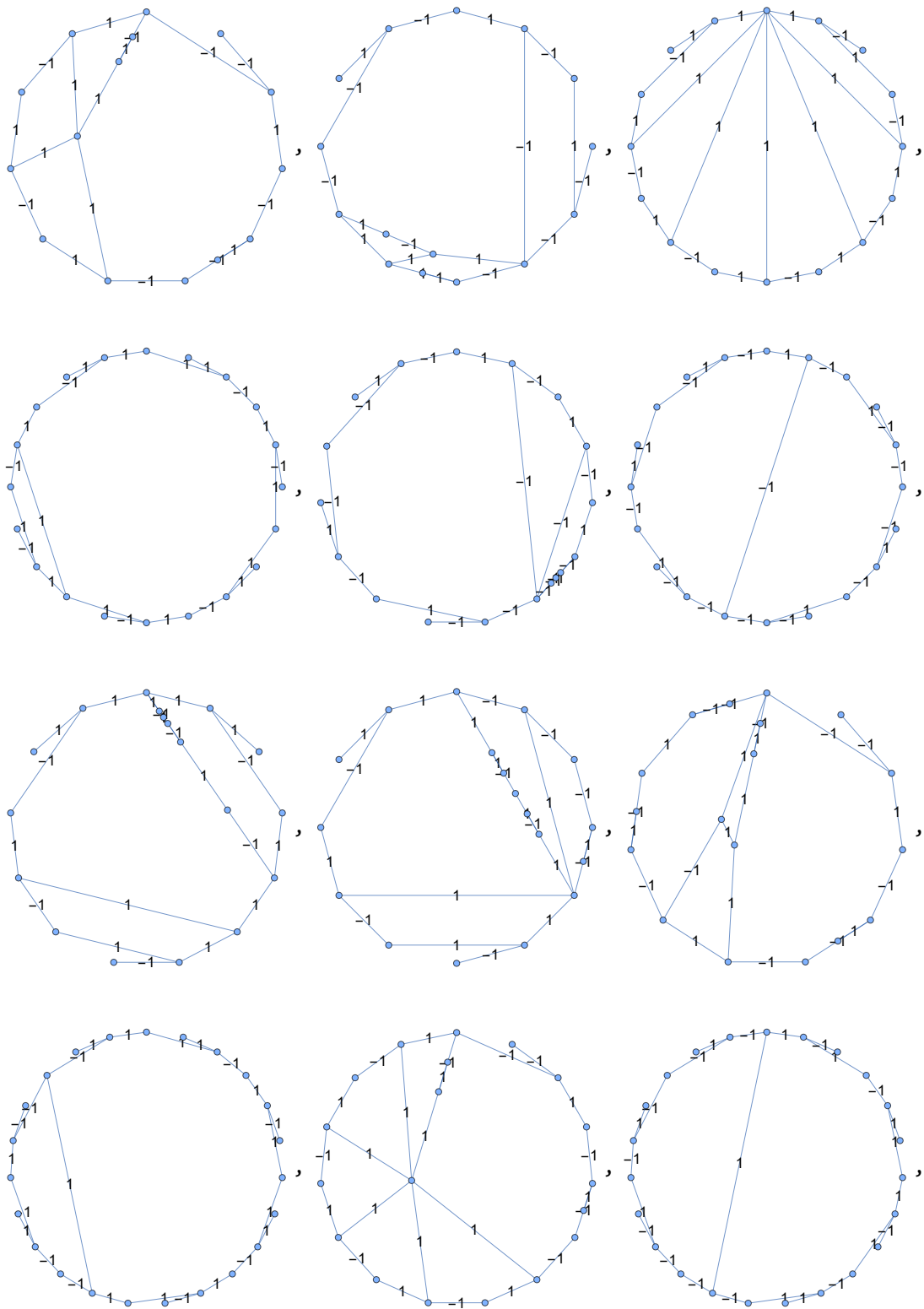
```

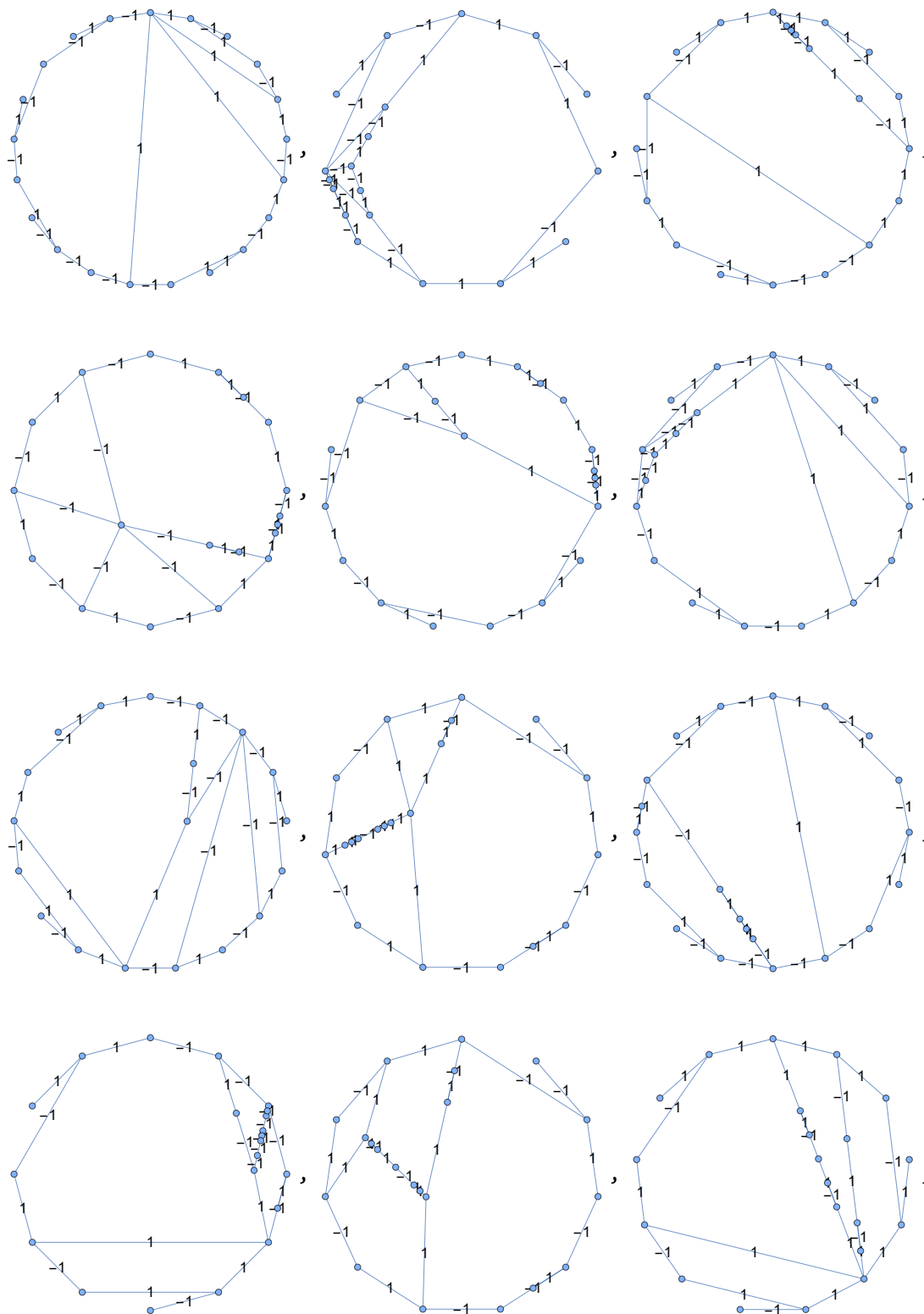
In[]:= DrawGraph[SeifertBGraph[#]] & /@ AllKnots[{3, 8}]

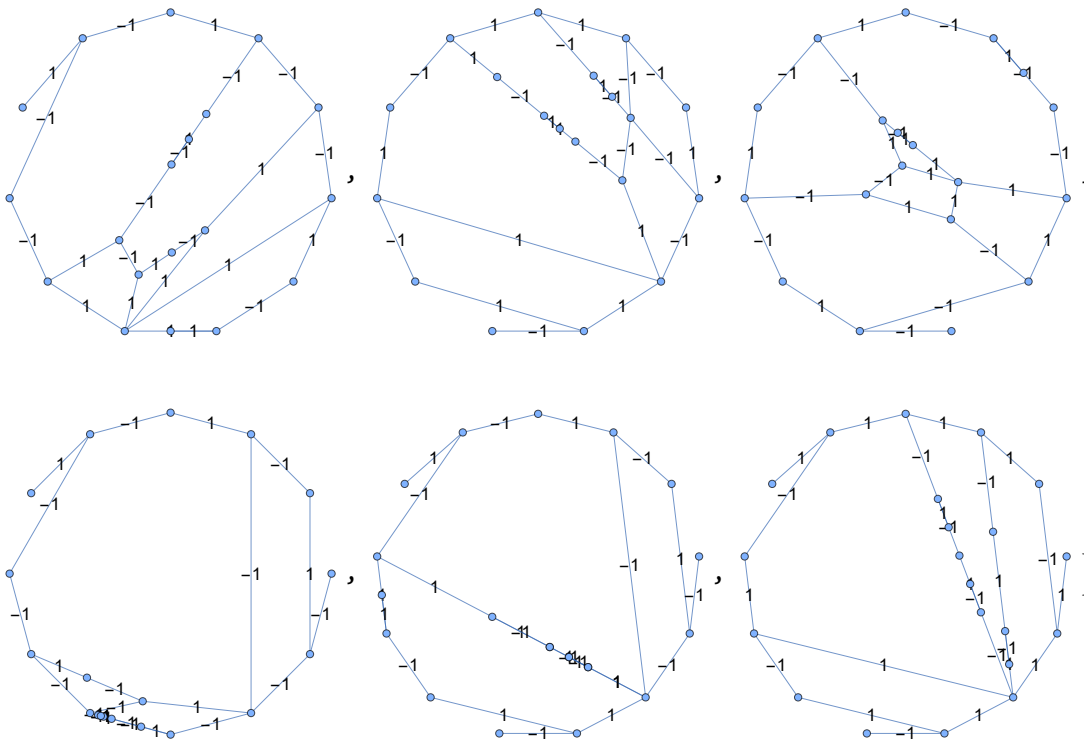
*** KnotTheory: Loading precomputed data in PD4Knots`.

Out[]:=









```

In[*]:= (*Get rid of leaves and degree 2 vertices*)
Prune[sb_] := Module[{sbnew = sb, Leaves = Flatten@Select[sb[[1]], Length[#] == 1 &]},
  Do[
    sbnew[[1]] = DeleteCases[sbnew[[1]], lv, {2}];
    sbnew[[2]] = Delete[sbnew[[2]], Key[lv]]
    , {lv, Leaves]];
  sbnew /. {{}} -> Nothing]]
Contract[sb_] :=
Module[{sbnew = sb, SmallV = First@Select[sb[[1]], Length[#] == 2 &], asso},
  asso = sbnew[[2]];
  asso [First@SmallV] = asso [First@SmallV] + asso [Last@SmallV];
  sbnew[[2]] = Delete[asso, Key[Last@SmallV]];

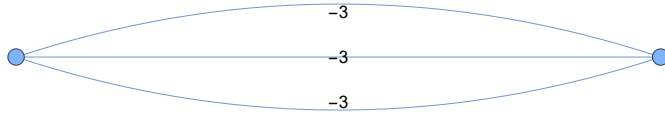
  sbnew[[1]] = DeleteCases[sbnew[[1]], SmallV];
  sbnew[[1]] = sbnew[[1]] /. {Last@SmallV -> First@SmallV};
  sbnew]

```

```
In[*]:= (*Not correct!*)
```

```
DrawGraph@{{ {6, -$[2], -4}, {- $[2], 6, -4}}, <|-4 → -3, 6 → 1, - $[2] → 1|>}
```

```
Out[*]=
```



```
In[*]:= (# → (Quiet@FixedPoint[Contract, Prune[SeifertBGraph[#]]]) & /@ AllKnots[{3, 7}])
```

```
Out[*]=
```

```
{Knot[3, 1] → {{ {- $[1], -5, -3}, {-3, -5, - $[1]}}, <|- $[1] → 1, -3 → 1, -5 → 1|>},
Knot[4, 1] → {{ {6, - $[2], -4}, {- $[2], 6, -4}}, <|-4 → -3, 6 → 1, - $[2] → 1|>},
Knot[5, 1] → {{ {- $[1], -7, -3, -9, -5}, {-3, -7, -4}, {-5, -10, - $[1]}, {-9, -4, -10}},
<|- $[1] → 1, -3 → 1, -5 → 1, -10 → 0, -7 → 1, -9 → 1, -4 → 0|>},
Knot[5, 2] → {{ {- $[3], -9, -7}, {- $[3], -7, -9}}, <|- $[3] → 1, -9 → 3, -7 → 1|>},
Knot[6, 1] → {{ {8, -9, - $[7]}, {- $[7], -9, 8}}, <|- $[7] → 3, 8 → 1, -9 → -3|>}, Knot[6, 2] →
{{ {8, -9, - $[1]}, {- $[1], -5, -11, -7}, {-5, -9, -6}, {-7, -12, 8}, {-11, -6, -12}},
<|- $[1] → 0, -5 → 1, -9 → -3, -7 → 1, 8 → 1, -12 → 0, -11 → 1, -6 → 0|>}, Knot[6, 3] →
{{ {7, -8, -3, -12}, {- $[6], -12, -10}, {-8, -4, -3}, {-10, -4, -11}, {- $[6], -11, 7}},
<|-4 → -2, -8 → -1, -3 → -1, -12 → 0, -10 → 1, - $[6] → 1, 7 → 1, -11 → 0|>},
Knot[7, 1] → {{ {- $[1], -9, -3, -11, -5, -13, -7}, {-3, -9, -4}, {-5, -12, -6},
{-7, -14, - $[1]}, {-11, -4, -12}, {-13, -6, -14}}, <|- $[1] → 1, -3 → 1,
-5 → 1, -12 → 0, -7 → 1, -14 → 0, -9 → 1, -11 → 1, -4 → 0, -13 → 1, -6 → 0|>},
Knot[7, 2] → {{ {- $[3], -11, -9}, {- $[3], -9, -11}}, <|- $[3] → 1, -11 → 5, -9 → 1|>},
Knot[7, 3] → {{ {- $[3], -11, -5, -14, -10}, {-10, -4, - $[3]}, {-12, -14, -5},
{-4, -12, -11}}, <|-10 → -1, - $[3] → -1, -14 → -3, -12 → 0, -5 → -1, -4 → 0, -11 → -1|>},
Knot[7, 4] → {{ {- $[5], -14, -10}, {-10, -14, - $[5]}}, <|- $[5] → -1, -14 → -3, -10 → -3|>},
Knot[7, 5] → {{ {- $[3], -11, -9}, {- $[1], -5, -13, -7}, {- $[3], -9, - $[1]}, {-7, -13, -8},
{-11, -8, -5}}, <|- $[1] → 1, - $[3] → 1, -5 → 1, -7 → 1, -13 → 1, -11 → 1, -8 → 0, -9 → 1|>},
Knot[7, 6] → {{ {- $[1], -11, -13}, {11, -11, -7, -3, -9}, {-3, -7, - $[1]}, {-13, 11, -9}},
<|- $[1] → 1, -3 → 1, -9 → -3, -13 → -1, 11 → 1, -11 → 2, -7 → 1|>}, Knot[7, 7] →
{{ {6, -13, -14}, {12, -13, -9, - $[1]}, {- $[1], -5, -11}, {-5, -9, 6}, {-11, -14, 12}},
<|- $[1] → 0, -5 → 1, 6 → 1, -9 → -3, -11 → 1, 12 → 1, -14 → -1, -13 → -2|>}}
```

Seifert surfaces for the Rolfsen table (warning twists on multi-edges are displayed incorrectly!)

```
In[*]:= (# → (DrawGraph@Quiet@FixedPoint[Contract, Prune[SeifertBGraph[#]]]) & /@
```

```
AllKnots[{3, 10}])
```

```
Out[*]=
```



