$$P = \epsilon(P_2 + P_4)$$

$$\begin{pmatrix} 1 & 1-t \\ 0 & t \end{pmatrix}$$

$$\downarrow$$

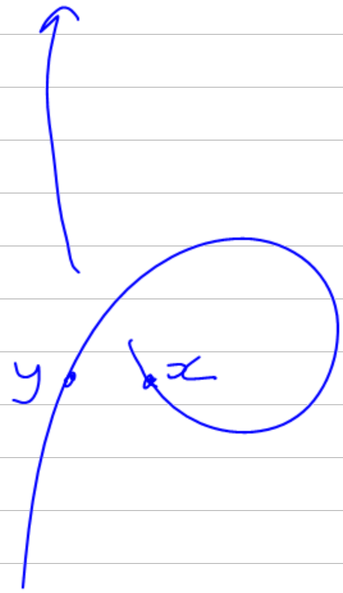
$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\sigma_{ij} \longmapsto \begin{pmatrix} 1 & & & & & 0 \\ & 1 & & & & \\ & & 1 & 1-t & & \\ 0 & & 0 & t & & \\ & & & & 1 & \\ 0 & & & & & 1 \end{pmatrix}$$

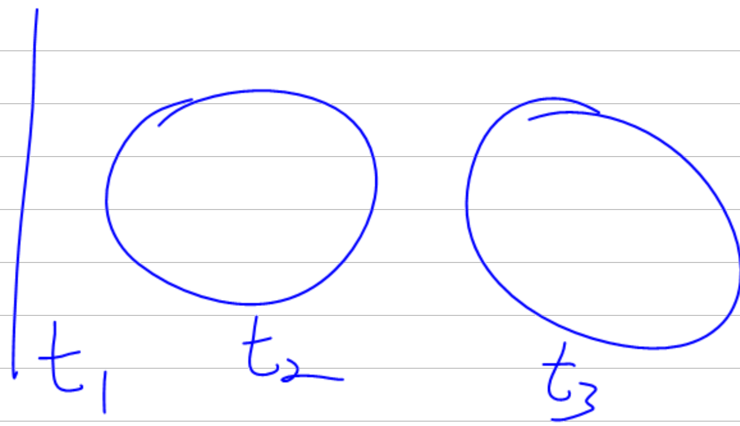
$$t_{ij} = \begin{pmatrix} 0 & & 0 & & \\ & 1 & & -1 & \\ & & 0 & & \\ & -1 & & 1 & \\ 0 & & & & 0 \end{pmatrix}$$

$$R = \mathbb{E}\left((e^{t+\epsilon} - 1)(P_1 - P_2) \supset G_2\right)$$

$$R \sim e^{yx+ba}$$

$e^W =$ 

$\longrightarrow \mathbb{E}\left(yx + ba + \underset{\text{stuff}}{a^b} + p\right)$



$$[x, ay] = xy + at$$
$$\qquad\qquad\qquad \| $$
$$\qquad\qquad\qquad [a, xy]$$

$$W = ta + yx \qquad [x, y] = t$$

$$a = \frac{W}{t} - \frac{y}{t}x \qquad [x, p] = 1$$

$$\underset{p}{y}$$

$$F(\epsilon) = \int_0^\infty \frac{e^{-x}}{1+\epsilon x}\,dx \sim \int_0^\infty dx \sum_{n=0}^\infty e^{-x}(-\epsilon x)^n$$

$\epsilon \geq 0$ $\qquad$ $|x| < \frac{1}{\epsilon}$

$$= \sum_{n=0}^\infty (-\epsilon)^n \int_0^\infty e^{-x} x^n \, dx$$

$$= \sum_{n=0}^\infty (-1)^n n! \, \epsilon^n \qquad \sum \frac{1}{n!}\epsilon^n$$

$\sin 100$

Aug 11, 2020

Q If P_1 is $2k+2$ docile, show that so is

$$\log(\varnothing(e^{P_1})) \in \mathcal{U}(-)[[\epsilon]]$$

If P_2 is $4k$-docile, so is

$$\log(\varnothing(P_2))$$

$$\begin{tikzcd}
\mathcal{S} \arrow[r, "?"] \arrow[d, "\varnothing"] & \mathcal{S} \arrow[d, "\varnothing"] \\
\mathcal{U} \arrow[r, "\log"] & \mathcal{U}
\end{tikzcd}$$

$$\mathcal{O}(e^{\mu xy}) = \sum \frac{\mu^{3} x^n y^n}{n!} \xrightarrow{\log} \log(\mu+1)\cdot xy$$

in U $\quad e^{\lambda xy} = \mathcal{O}\left(e^{(e^{\lambda}-1)xy}\right)$

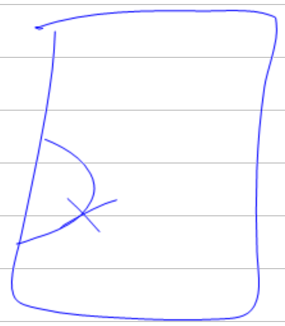
$\log_u \mathcal{O}\left(e_s^{\lambda Q}\right)$ is quadratic?

$$\partial_\lambda : \mathcal{O}(e^{\lambda Q})^{-1}\,\mathcal{O}(Q e^{\lambda Q})$$

$$e^{P_0 + tP_1 + t^2 P_2}$$

where $wt(P_k) \le 2k+2$ $\qquad P_{\text{linear}}$

at $k=0$ $\quad wt(P_0) \le 2$ $\qquad e(t)a$, $w+2$



EDDO

no $t^2 a$

no $\alpha\beta$

neither

Aug 17, 2020

$$Q \longmapsto Q'$$

$$\underset{\text{no}}{\underline{L}} + \underset{(n-k)k}{\underline{Q}} + \underline{P}$$

$$\underline{\underline{e^Q p}} \sim \underline{\underline{e^{Q+p}}}$$

$$\langle a_1 a_2 a_3 ,\ e^{q_{ij} x_i y_j} p(\bar{x}, \bar{y}) \rangle$$

n^2

$$\frac{\partial}{\partial \alpha} \rho(A) \qquad \deg \rho = n$$

$$\frac{\partial}{\partial \alpha}\left(\sum_{k=-n}^{n} \lambda_k e^{k\alpha} \right) = \sum_{k=-n}^{n} k\lambda_k e^{k\alpha}$$

$$\frac{d}{dx} f = \lim \frac{f(x+h) - f(x)}{h}$$

$$\frac{\varsigma}{\varsigma x} f = f(x+1) - f(x)$$

$y^n \to n y^{n-1}$

$$\operatorname{ad}_x f(y) = b f'(y)$$

$$\operatorname{ad}_x f(a) = \cancel{a} f(a+1) - f(a)$$

$$a^{(n)} = a(a-1)(a-2) \ldots (a-n+1)$$

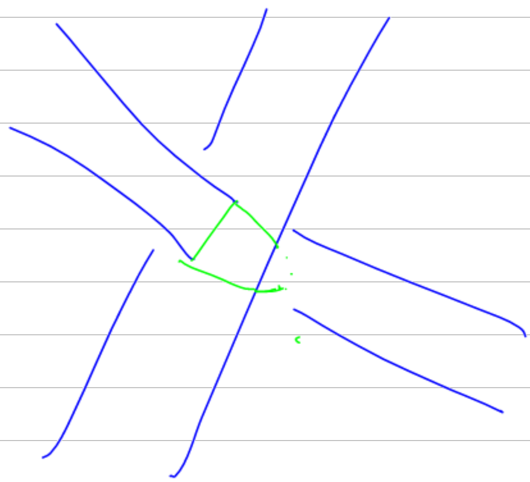
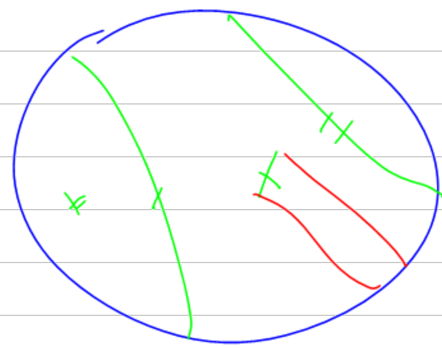
$$L : a^n \longmapsto a^{(n)}$$

$$g(L) = \sum \frac{1}{n!} \alpha^n a^{(n)}$$

$$= (1+\alpha)^{\wedge}$$

$f: \mathbb{R}^2 \to \mathbb{R}$ ☺

$Df = f \quad (f)' = (1 \quad 1)$

$$u = \mathrm{ABS}(u) \Rightarrow u \geq 0$$

$$\frac{1}{1-\alpha} = \frac{1}{-(\alpha + \frac{\alpha^2}{2} +}$$

$$= \frac{-1}{\alpha}\left(\frac{1}{1+\frac{\alpha}{2}}\right)$$

$$\langle \tfrac{1}{\alpha}, \alpha^n \rangle = \partial_\alpha^n \tfrac{1}{\alpha}\Big|_{\alpha=0}$$ Boom!

$$\left\langle \frac{1}{\alpha}, e^{\lambda a} \right\rangle = e^{\lambda \partial_\alpha} \frac{1}{\alpha} \Big|_{\alpha=0}$$

$$= \frac{1}{\alpha+\lambda}\Big|_{\alpha=0} = \frac{1}{\lambda} \underset{\lambda<0}{=} \int_0^\infty e^{\lambda a}\, da$$

$$\left\langle \frac{1}{\alpha}, f \right\rangle \rightsquigarrow \int_{-\infty}^{\infty} \frac{\hat{f}(\lambda)}{i\lambda}\, d\lambda$$

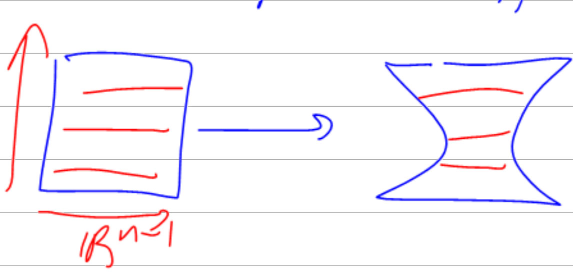
$$e^{Q_1} /\!/ e^{Q_2} = e^{Q} \qquad \frac{1}{1-FG} = \sum (FG)^n$$

$$e^{Q} \qquad \rho$$

---

cov if g is l.p.

$$g\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} \cancel{x} \\ z \\ x_n \end{pmatrix}$$

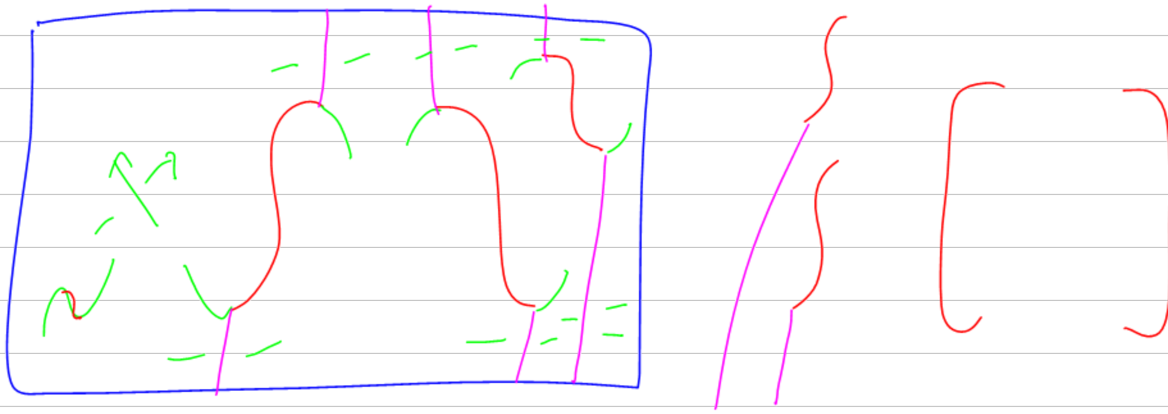


$$(x_1, \dots, x_n) \longmapsto (y_1, \dots, y_n)$$

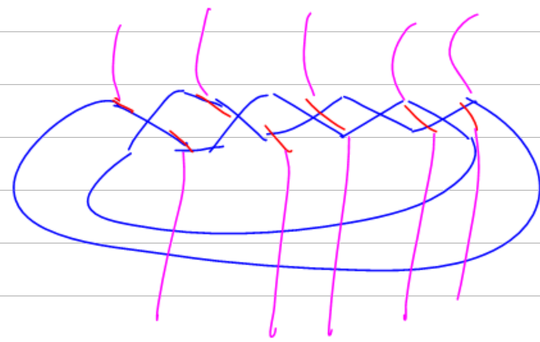
$$(x_1, \dots, x_{n-1}, y_k)$$

Knots $\xrightarrow{\text{Alex}}$ Braids

R-moves $\longleftarrow$ M-moves



$n \cdot \sqrt{n}$



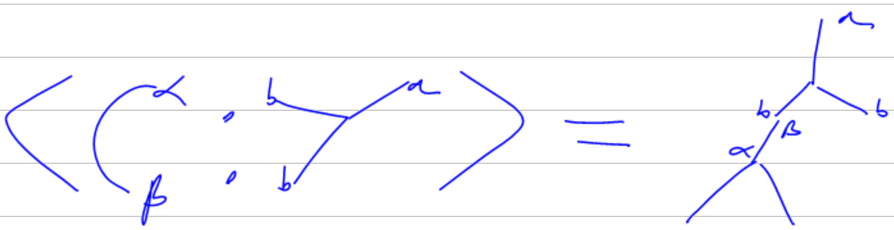
linear.

$$e^{L+\lambda_8} = F_L(\lambda_8) \qquad \text{to deg } 8$$

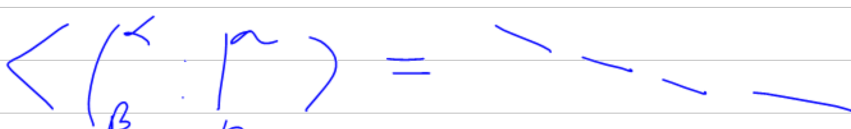
$x \quad \bar{x} \quad \tilde{x}$

$R \quad \bar{R} \quad \tilde{R}$

$$\partial_x \partial_y e^z = \partial_x\left((\partial_y z) e^z\right)$$



$$\begin{array}{c|cc} & \alpha & \beta \\ \hline \alpha & 0 & 1 \\ \beta & 1 & 0 \end{array}$$



$$(w, A) \longmapsto w \Lambda^*\left(\frac{A}{w}\right)$$

1. no dens.
2.

$$R \times M_{n\times n}(R) \xrightarrow[\text{linear}]{\text{non}} \mathrm{End}(\Lambda^*(R^n))$$

$\circlearrowleft m^{ij}_k$ $\circlearrowleft tr_i$ $\qquad$ $\circlearrowleft m^{ij}_k$ linear $\quad$ $\circlearrowright tr_i$

$$\mathbb{R}^n \longrightarrow S^*(\mathbb{R}^n)$$

$\circlearrowleft P$ $\qquad$ $\circlearrowleft L$

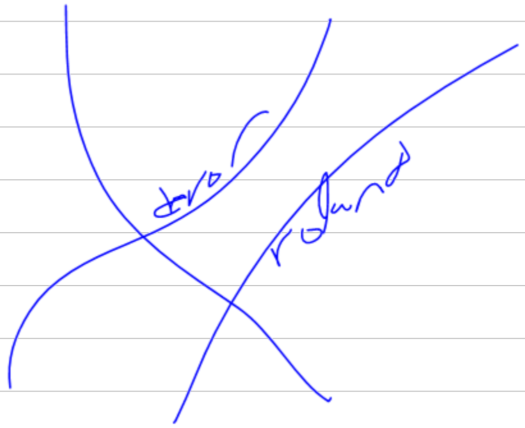
$$\mathrm{Hom}(SX, SY) \sim S(X^*, Y)$$

$$\mathrm{Hom}(\Lambda X, \Lambda Y) \sim \Lambda(X^*, Y)$$

$$R \times M_{s\times s}(R) \longrightarrow \mathrm{Hom}(\Lambda X, \Lambda X) = \Lambda(X^*, X)$$

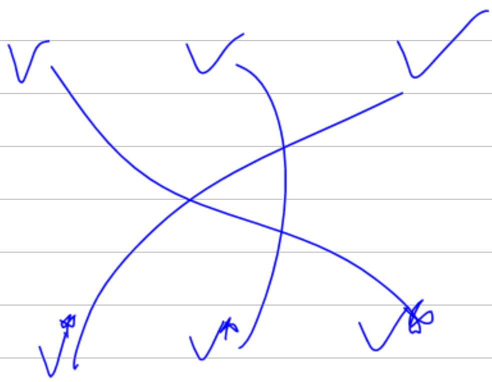
$\circlearrowleft m^{ij}_k$ $\qquad$ $\circlearrowleft$ zip w/ x^n

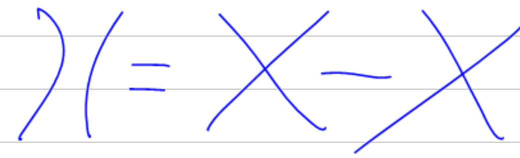
$$(w, A) \longmapsto w e^{(x^*)^T A x}$$

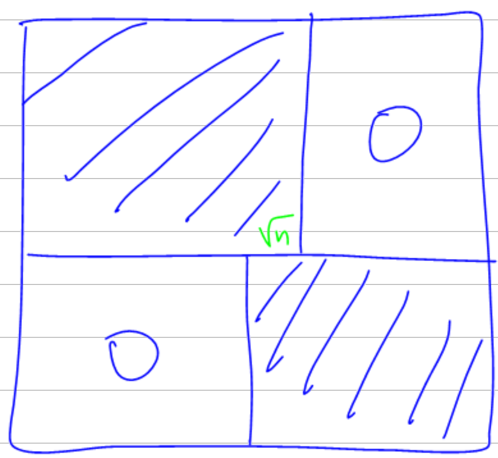


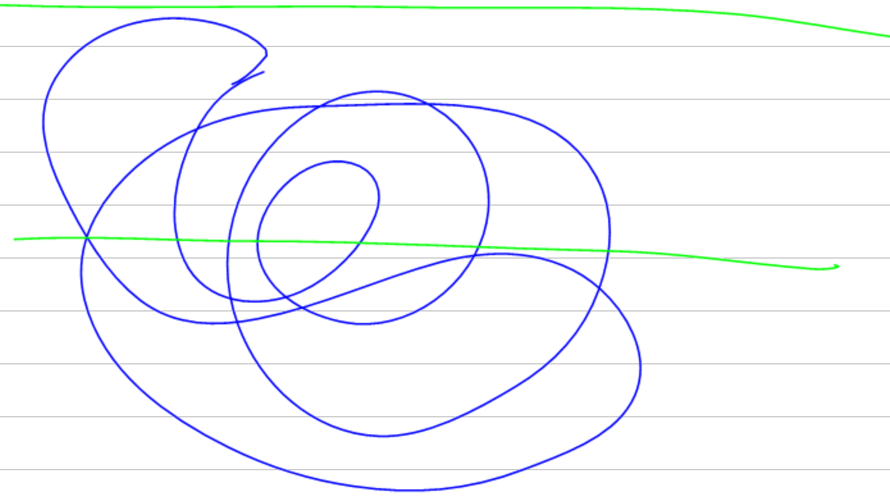
$$\begin{array}{c} A^{\otimes 3} \\ \downarrow \rho \\ V^{\otimes 3} \\ \otimes \\ (V^*)^{\otimes 3} \end{array}$$

$$A \to V^* \otimes V$$

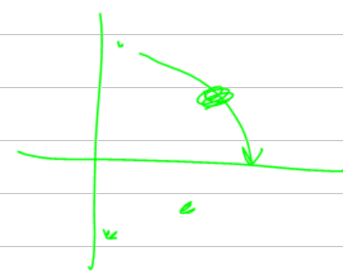




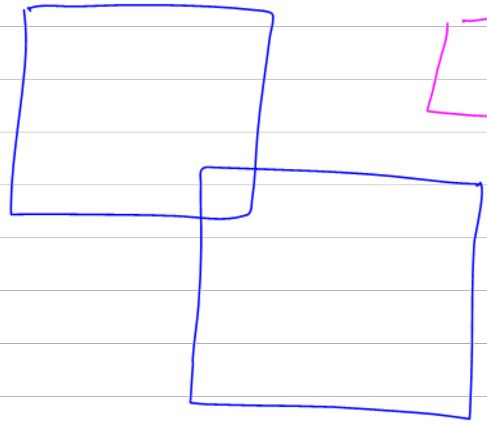


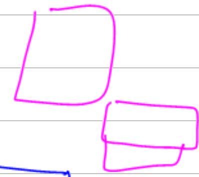


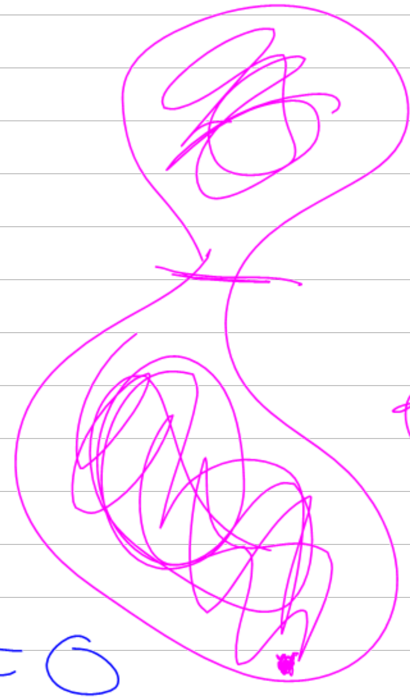
$$\frac{1}{(2\pi)^{n/2}} \int e^{-\frac{1}{2}\lambda_{ij} x^i x^j} = \frac{1}{\sqrt{\det \Lambda}}$$

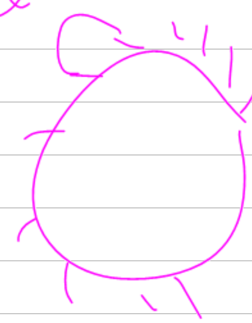


$$\frac{1}{\sqrt{2\pi}} \int e^{\frac{i\lambda}{2} x^2 - \epsilon x^2} = \frac{1}{\sqrt{i\lambda}} \to \frac{1}{\sqrt{\lambda}} e^{\mp i \frac{\pi}{4} \operatorname{sign} \lambda}$$









$[a, x] = x$

even odd

$\{x, x\} = 0$

$x^2 = 0$

$\mathbb{Q}[a] \otimes \langle 1, x \rangle$ $\quad \frac{e^s - 1}{s}$

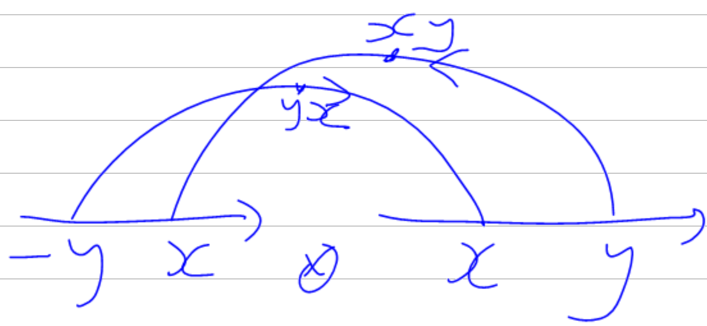
$y\ b\ a\ x$ $\quad \{y, x\} = b + c a$

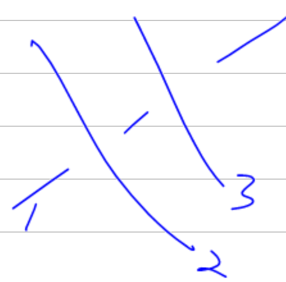
even

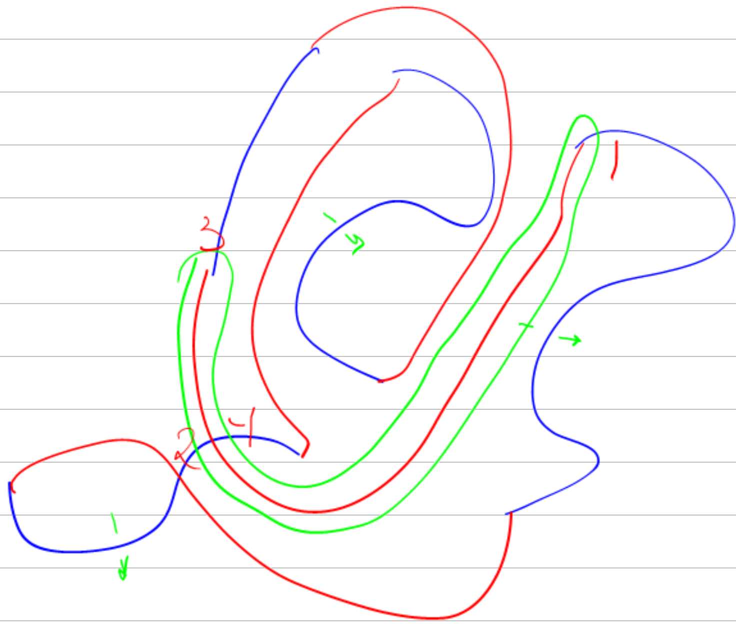
odd

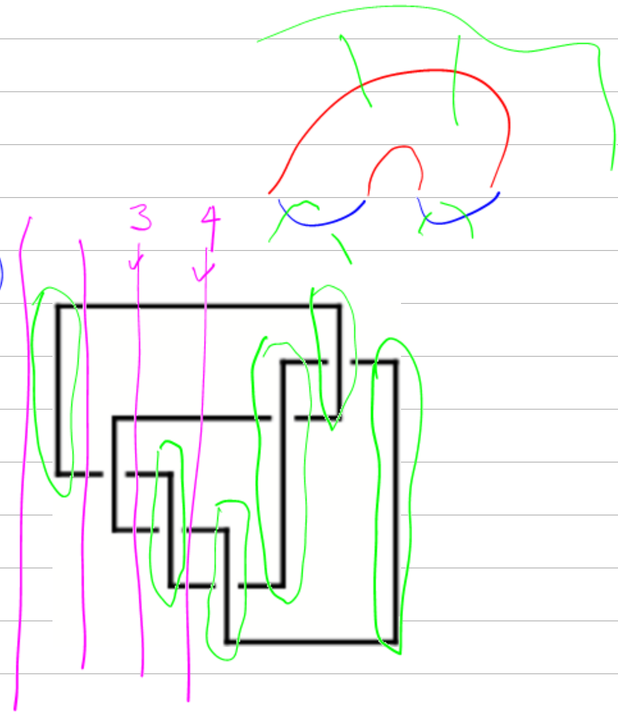
$r = ba + yx \quad R = e^r$

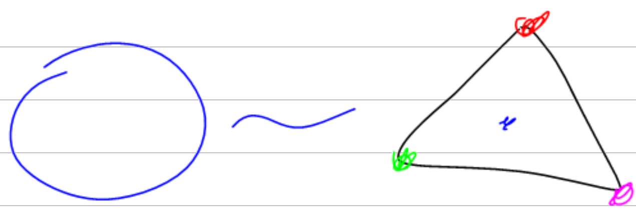
$U = \mathbb{Q}[a, b] \otimes \langle 1, x, y, yx \rangle$

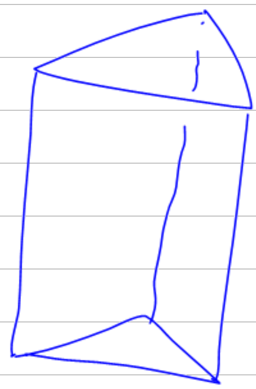


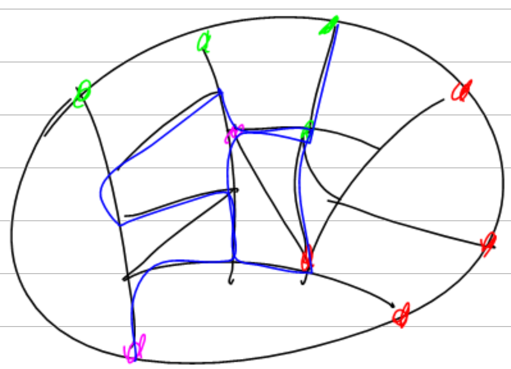




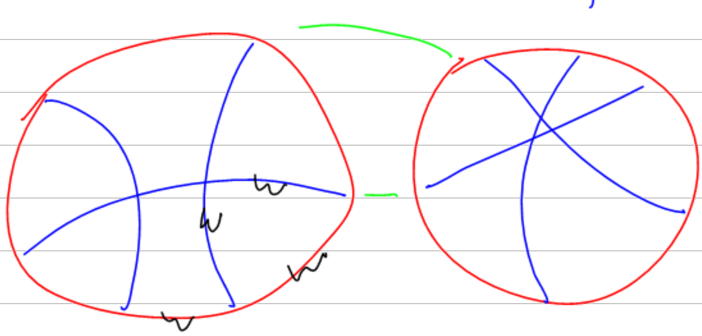


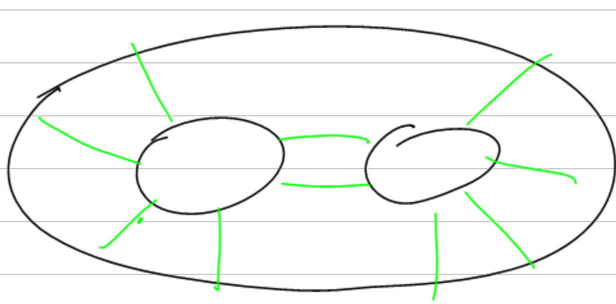






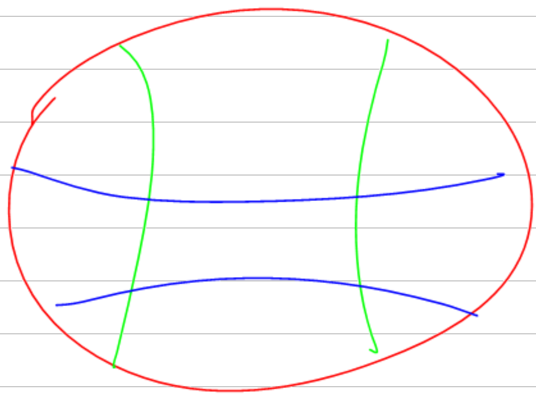
$$2 \perp 2 = 4$$





$A^{\otimes S}$

$A^{\otimes E} / A^{\otimes V}$



$R = C^{\sim 3}$

$$\mathbb{Q}[a] \xrightarrow[F]{e^{\alpha x}} \mathbb{Q}[x]$$

$$a^n \longrightarrow 0$$

$$F(a^n) = x \not\to 0$$

am $\qquad A = U\left(\begin{matrix} x, a \\ [a, x] = x \end{matrix}\right)$

$a \cdots x$

$$x a^n = \pm x + \dots$$

$$\mathrm{Hom}_c(\mathbb{Q}[[z_1]] \to \mathbb{Q}[[z_2]]) \sim$$

$$\mathbb{Q}[[z_2]][[\zeta_1]]$$

$$\mathrm{CHom}_{\mathbb{Q}[[\hbar]]}(\mathbb{Q}[z][[\hbar]] \circlearrowleft)$$

$$\simeq \mathbb{Q}[z][[\xi]][[\hbar]]$$

$am = \dots\dots e^{\alpha\xi}$

$\cancel{\mathbb{Q}[[a,x]]}$ $\quad e^{x} \quad e^{\hbar x}$

$\wr$

$$U(a,x/[a,x]=x)[[\hbar]]$$

$$[x,y]=b+\underline{\underline{\epsilon}}a \qquad e^{ab}$$

$$\mathbb{Q}[[\alpha,\xi]]$$

$$U_0 \subset U_1 \subset \dots \subset U_k \subset \dots$$

$$U_0^* \leftarrow U_1^* \leftarrow \leftarrow .$$

$$\begin{array}{ccc} \mathbb{Q}[x] & \xrightarrow{\mathbb{Q}[y][[\xi]]} & \mathbb{Q}[y] \\ \wr & & \wr \\ U & & U \end{array}$$

$$\mathbb{D} = QU = \mathcal{D}(\mathbb{A}, \mathbb{B})$$

$$dm^{ij}_k \quad d\Delta \quad ds$$

$$dm_\lambda = \Delta\Delta // P_\lambda P_\lambda // mm$$

$$\downarrow \qquad\qquad \downarrow$$

$$dm$$

$$g(\Delta\Delta // PP // mm) = g(\Delta\Delta) // g(PP) // g(mm)$$

in $\mathbb{Q}[z, \hbar^{\pm}][[\xi]]$

$$\alpha \quad b = \alpha^* \qquad B = e^{-\hbar}b$$

$$\mathbb{A} \otimes \mathbb{B} \longrightarrow \mathbb{Q}_\hbar$$

$$(a, b) = 1 \quad (x, y) = 1$$

$$\langle x^n, y^n \rangle = [n]!_\hbar$$

$$(y^{17}, \cdot)^{-1} (\hbar^{500} \ldots)$$

a^{18}, x^{18}

$$x^n \longrightarrow 0$$

$$|F(z)| \leq \underbrace{|F|}\,|z|$$

$$\begin{tikzcd} K \arrow[r, squiggly] \arrow[r, bend right, "z"'] & \tilde{A} \arrow[r, "g"] & U(g)[[h]] \arrow[loop below, "\cdot h"] \end{tikzcd}$$

$$\mathrm{Hom}_{Q[[h]]}\left(Q[z][[h]] \circlearrowright\right) \overset{?}{\longleftrightarrow} Q[z][[\zeta]][[h]]$$

$$\mathrm{Hom}_{\underbrace{Q[[h]]}_{R}}\left(Q[[h]][z] \circlearrowright\right) \longleftrightarrow Q[[h]][z][[\zeta]]$$

$$T(U) = R_{12}\, U_{34} \,//\, m_B^{13} m_A^{24} = I$$

$$T(U+\delta U) = T(U) + \underline{\underline{d}}T(\delta U) + o(\delta U)$$

$$\xrightarrow{a}\ \xrightarrow{b}\ \xrightarrow{c}$$

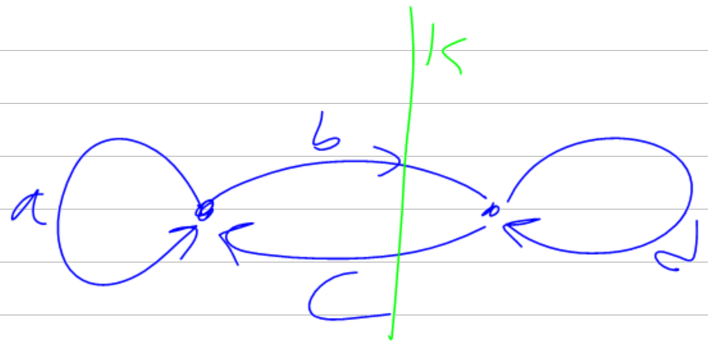
$ab \quad (ab)c \leftarrow$

$bc \quad a(bc) \leftarrow$

$$\underline{\mathrm{Hom}(V,W)} \cong \mathrm{Hom}(W^*,V^*)^*$$

$$(V^*\otimes W)[[h]]$$

$$S(p,q) \xrightarrow{\mathrm{Sym}} U([p,q]=1)$$



$$\rtimes \langle K\rangle / K^2=1$$

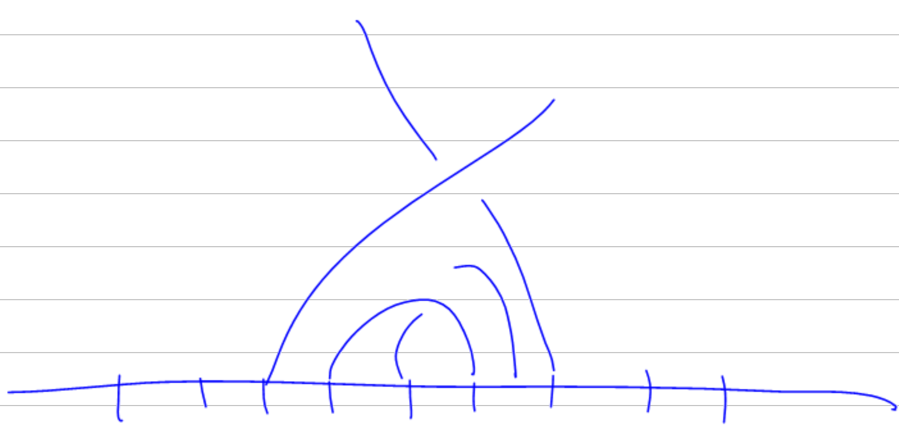
$GL(2)$ $\quad \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

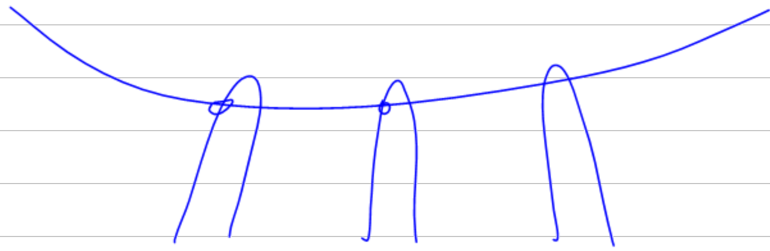
$Ka=aK \quad Kd=dK \quad Kb=-bK$

$\Delta a \neq a\otimes 1 + K\otimes a$

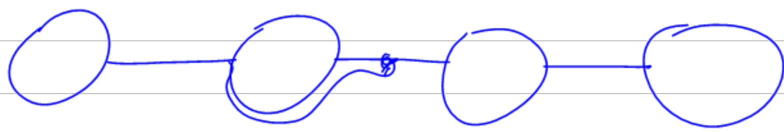
The Sweedler Algebra:

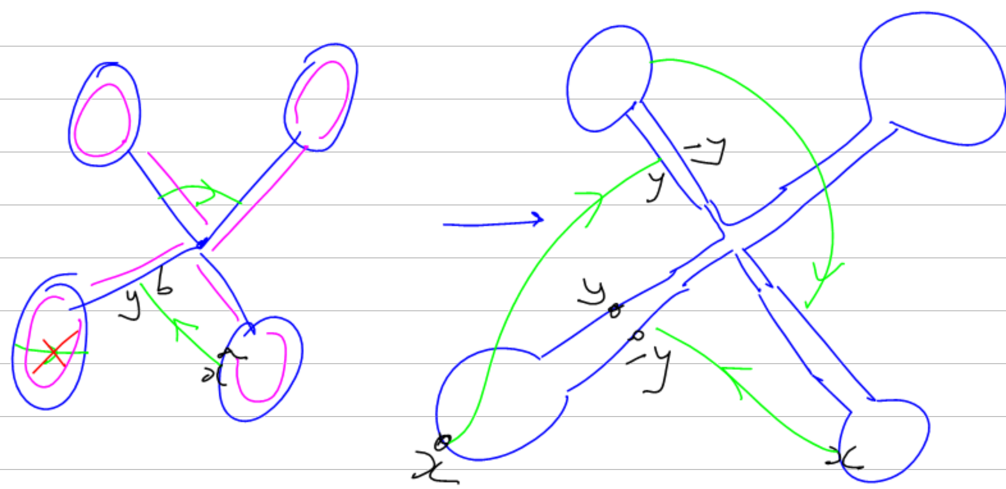
$s^2=1 \quad w^2=0 \quad sw=-ws$

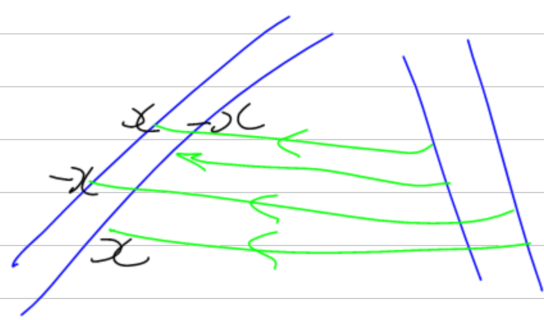




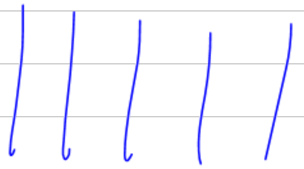
$$A_a(t) = f_1(t)\, f_2(t^{-1})$$

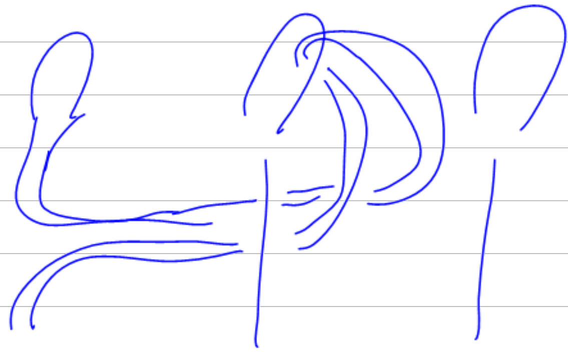






$$\begin{tikzcd}
 & A(\uparrow_{2n}) \arrow[dl, "K"'] \arrow[dr, "\tau"] & \\
A(\uparrow_n) & & A(\uparrow_n)
\end{tikzcd}$$





$$x \quad (= \emptyset(\ldots))$$

$$e(\lambda) = e_m^{\lambda x} = \emptyset(e^{f(\lambda)})$$

$$e(\lambda+\mu) = e(\lambda)e(\mu)$$

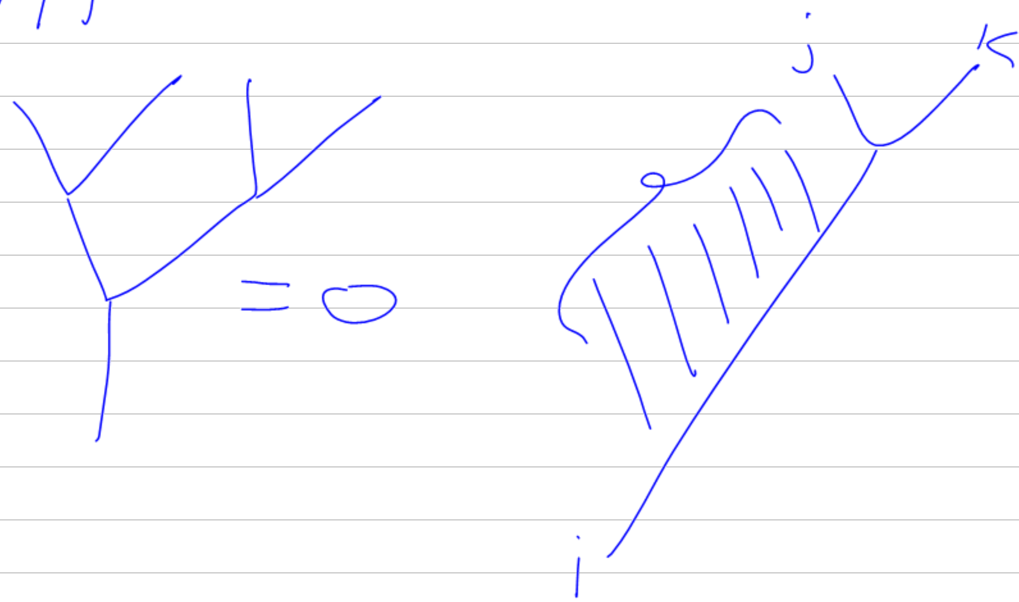
$$\emptyset(e^{f(\lambda+\mu)}) = \emptyset(e^{f(\lambda)})\emptyset(e^{f(\mu)})$$

$$= \emptyset(e^{h(\lambda,\mu)})$$

$$\partial_\mu f(\lambda+\mu)\big|_{\mu=0} = \partial_\mu h(\lambda,\mu)\big|_{\mu=0}$$

$$e^{\vdash\!\dashv} = e^{\vdash\!\rightarrow\!\dashv \, + \, \vdash\!\leftarrow\!\dashv}$$

KBH



$a_{i\_} := i^2$

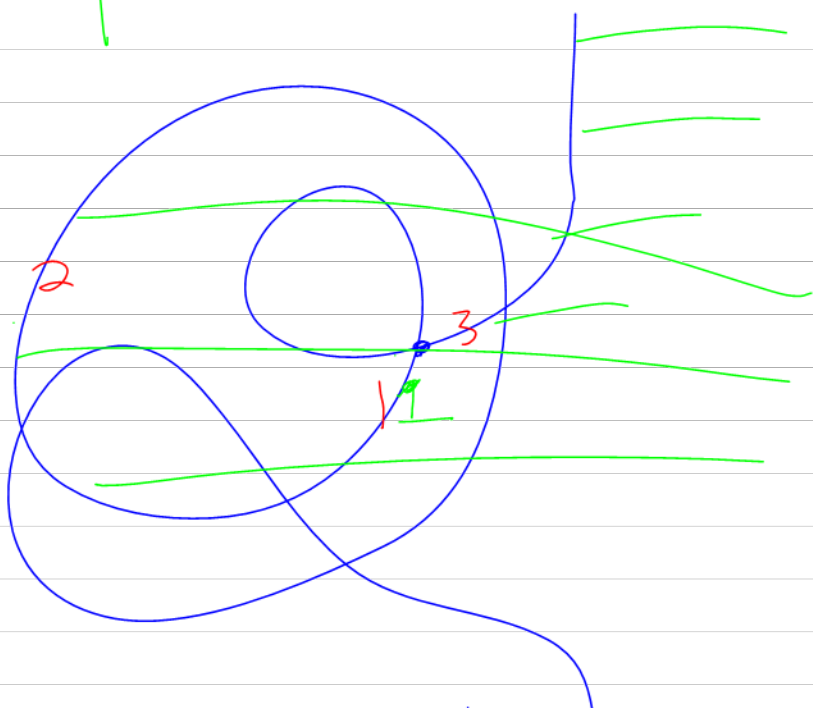
Subscript[a, i_] := i^2

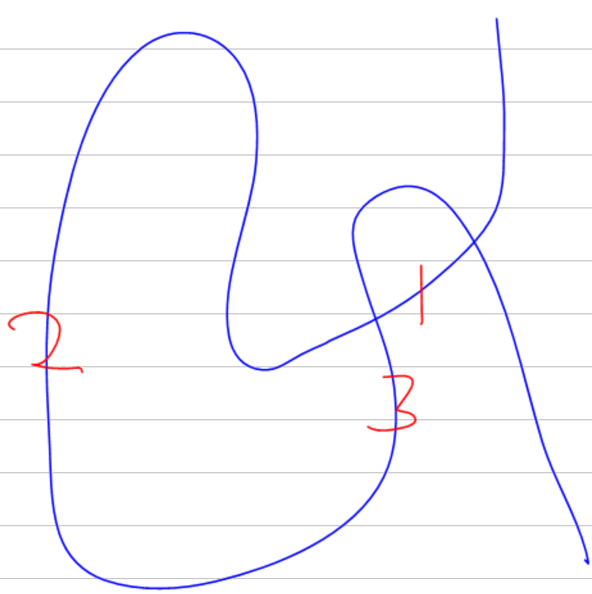
a /: $a_{i\_} := i^2$

S[k_] := S[n, k]

$S[n\_, k\_] := S(n,k) = \_\ \_\ \_\ \_$

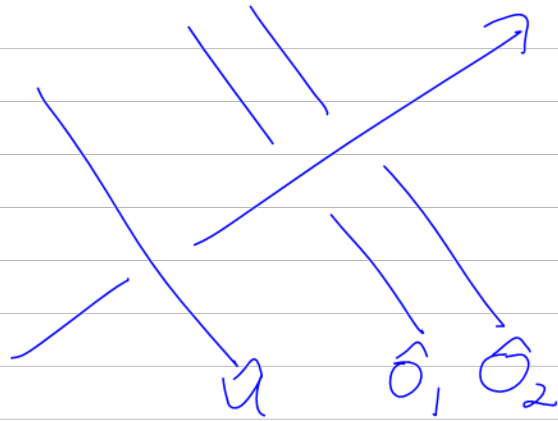
$\mathcal{C}\_ // bm\_ := \mathcal{C} // bm\_ = \ldots$



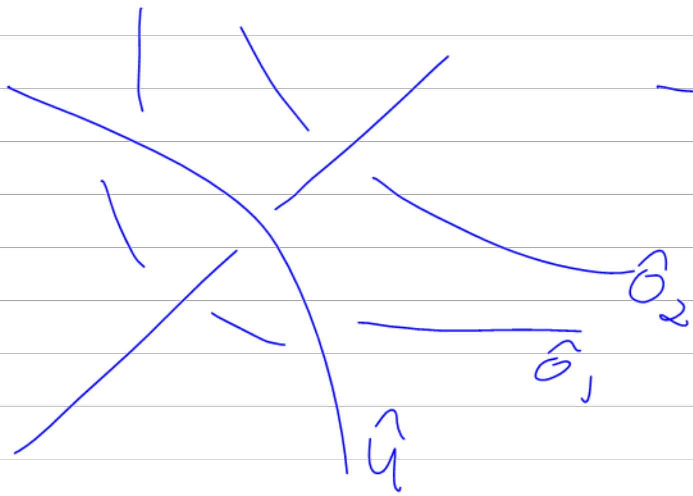


$\underline{W} e^{Q+r}$

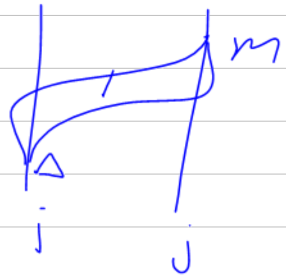
$\det Q \cdot (- - .$

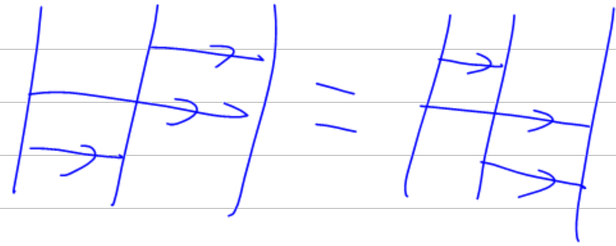


$u \; O_1 O_2$

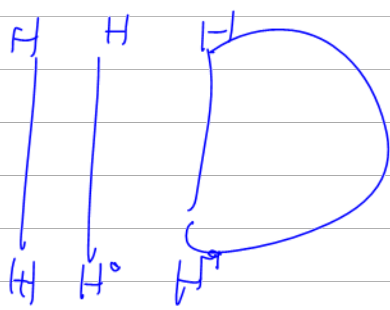


σ_{ij}

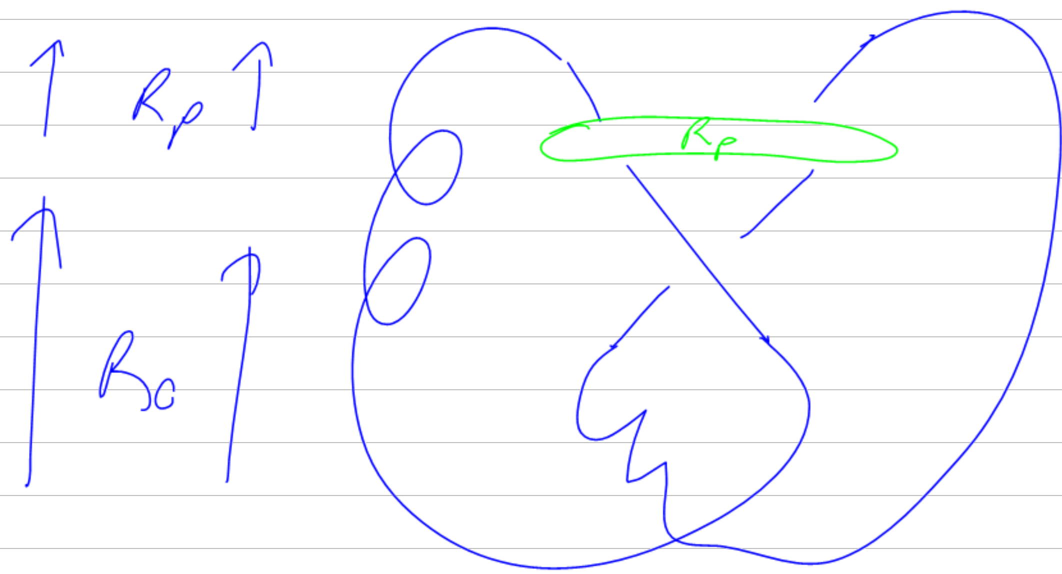




$$H^{*}\otimes\cdots\otimes H^{*}\otimes H\otimes\cdots\otimes H$$



$$\begin{pmatrix}\alpha&\beta&\gamma\\ \delta&\epsilon&\eta\\ \lambda&\mu&\nu\end{pmatrix}\longrightarrow \beta\delta-\gamma\mu+\nu$$



$$\left\langle \sum \pi_i p_i + \xi_i x_i \; ; \; mm \cdot \prod R \right\rangle$$

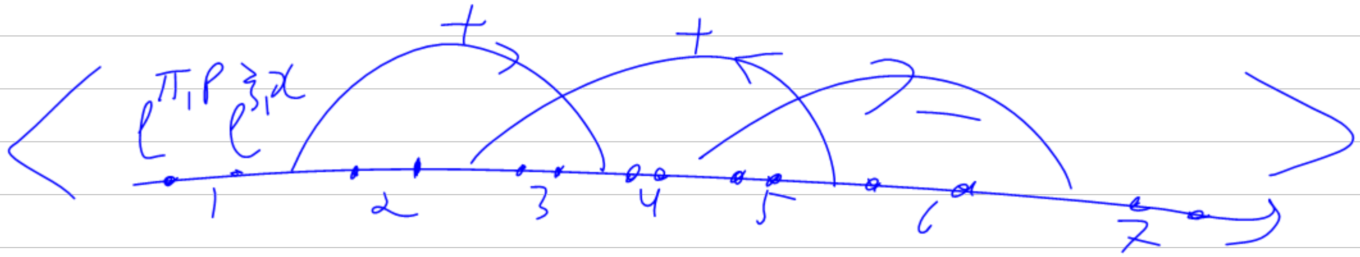
$$\langle F : G + p \rangle \qquad p_i p_j x_j^2$$

$$= \langle (I + FG)^{-1} : p \rangle$$

$$R_{ij} = R^{\circ}_{ij}\left(1 + p_i p_j x_j^2\right)$$

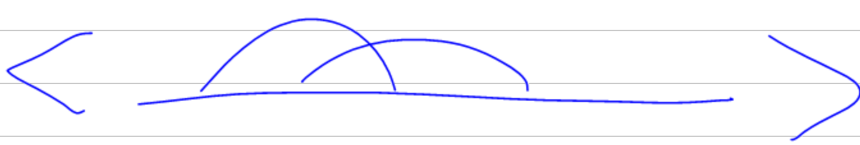
$$\Downarrow$$

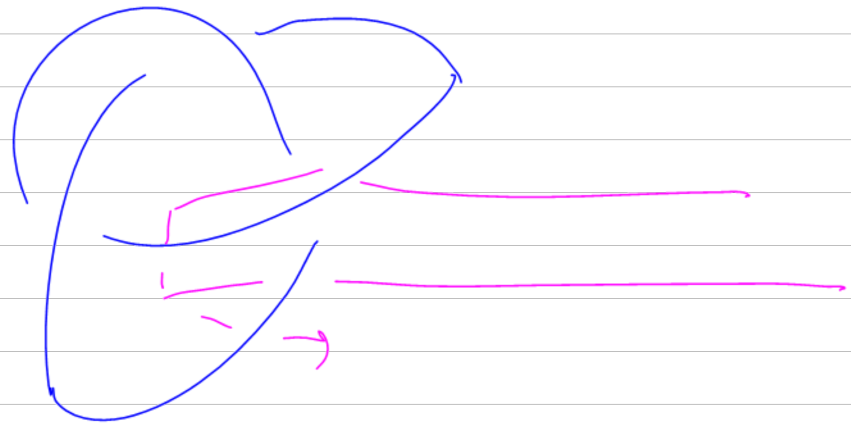
$$2\vartheta_{ij}\,\vartheta_{jj} \qquad p_i p_j x_j x_j$$

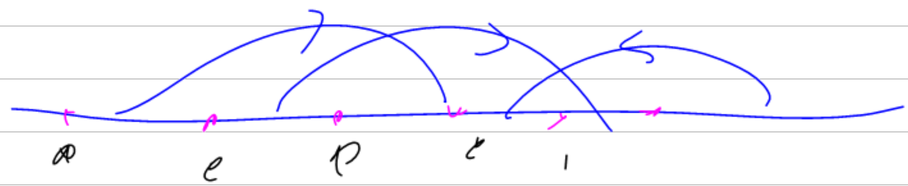


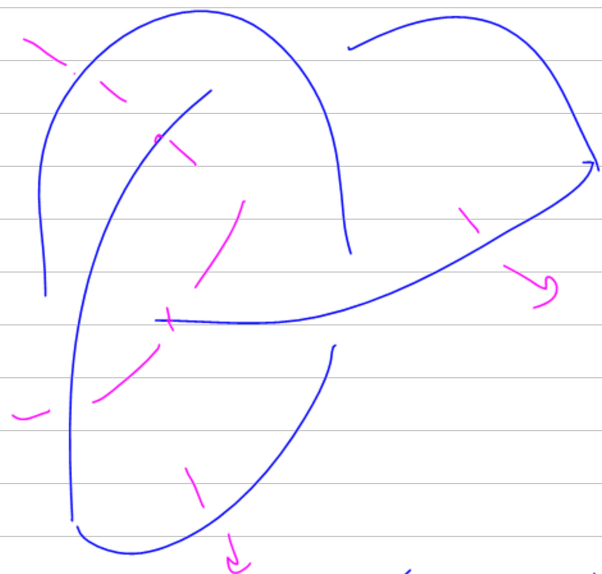
$$\|$$

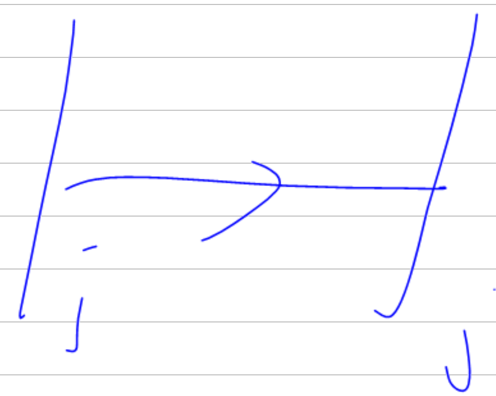
$$\exp\left(\sum \vartheta_{ij}\,\pi_i\,\xi_j\right)$$











$$R_{ij} = e^{(T-1)(P_i - P_j)x_j}$$

$$\frac{df}{dp} = [x, f(px)]$$

$$\left(\sum A^n\right)_{ij} = (1-A)^{-1}_{ij}$$

$\partial w = v$

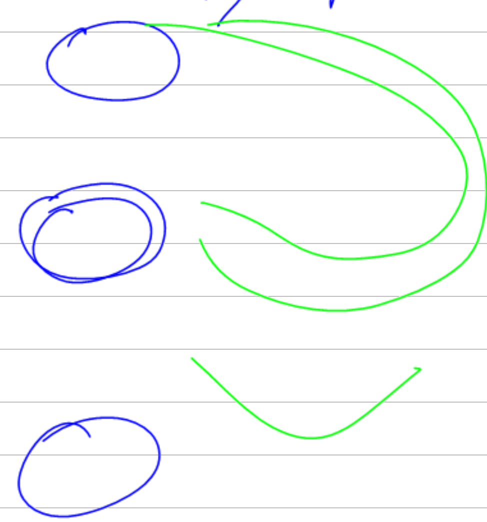
$x, y \in H_1(\tilde{C})$ $\qquad lk(u,v) = \mathrm{int}(u,w)$

torsion over $\mathbb{Z}[T^{\pm 1}]$

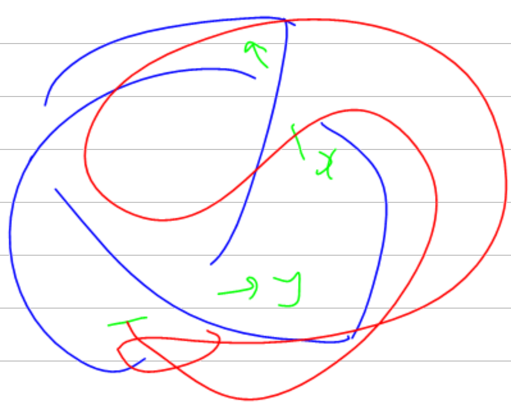
$\exists z$ s.t. $\partial z = p y \qquad p \in \mathbb{Z}[T^{\pm 1}]$

$$\langle x, y \rangle = \text{Equiv-int}(x, z)/p$$

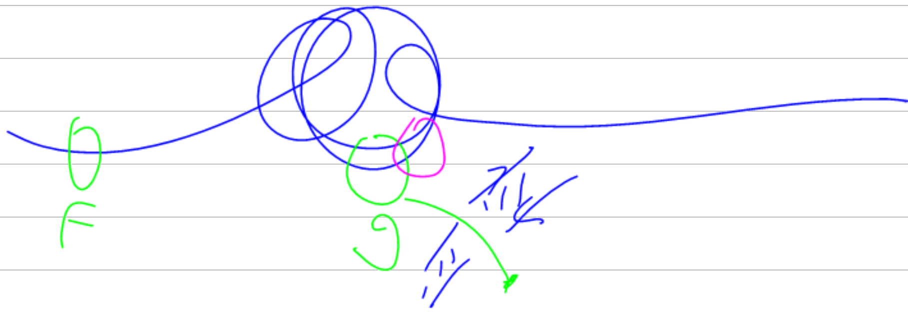
$$= \sum_k T^k \,\mathrm{int}(T^k x, z)/p$$



$(T + 2 + T^{-1}) \bigcirc_y$



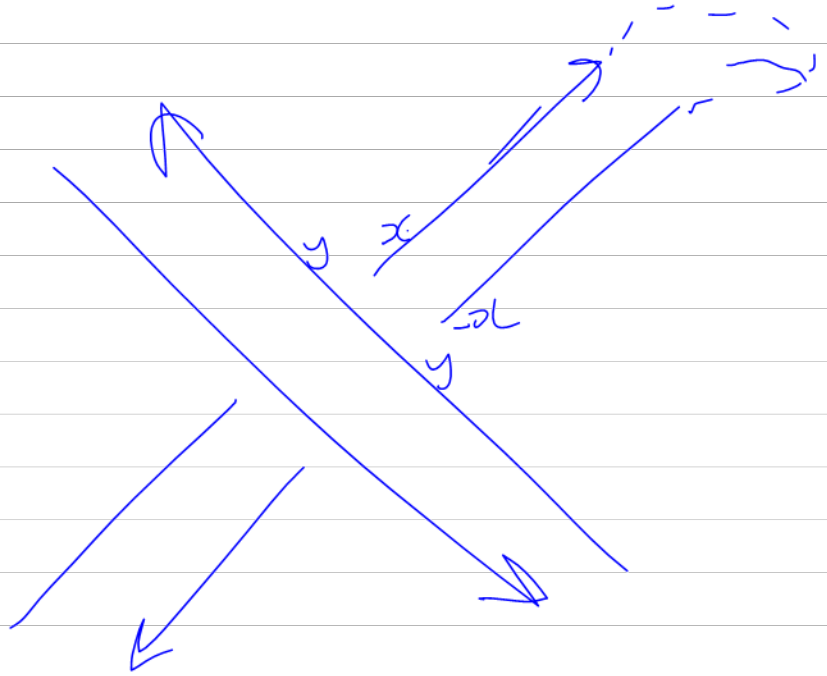
$$\boxed{G_{sl_2}} = \boxed{G}\,\boxed{G}\,\boxed{G}$$

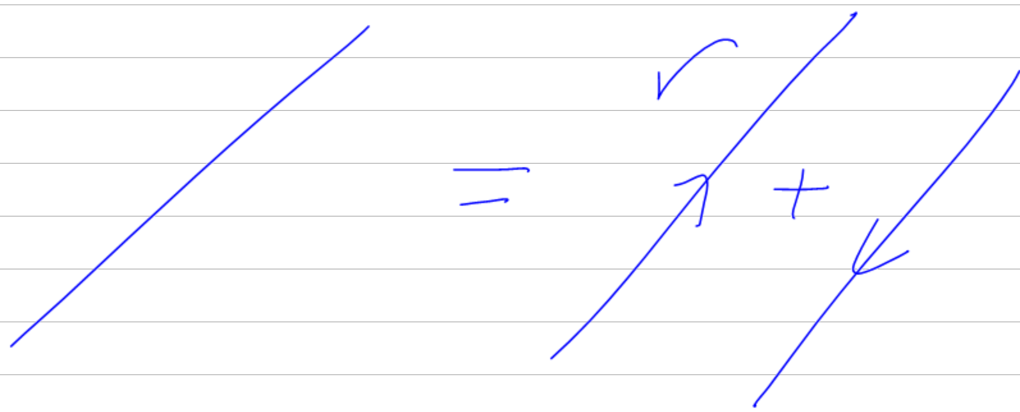


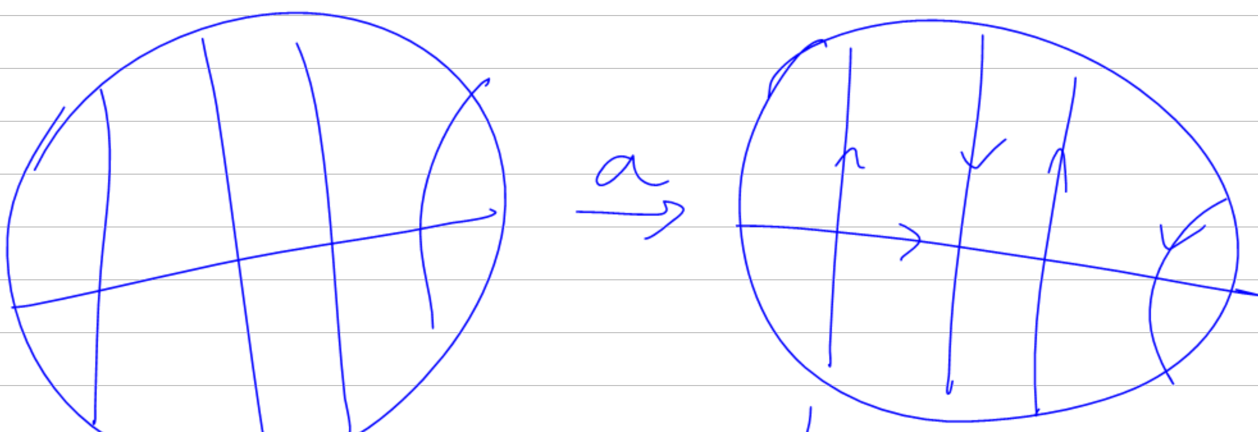
$$G^{ij}_{\alpha\beta}(- \quad - \quad)$$

$$[e_{ijk}, e_{in}] \in \langle e_{in} \rangle$$

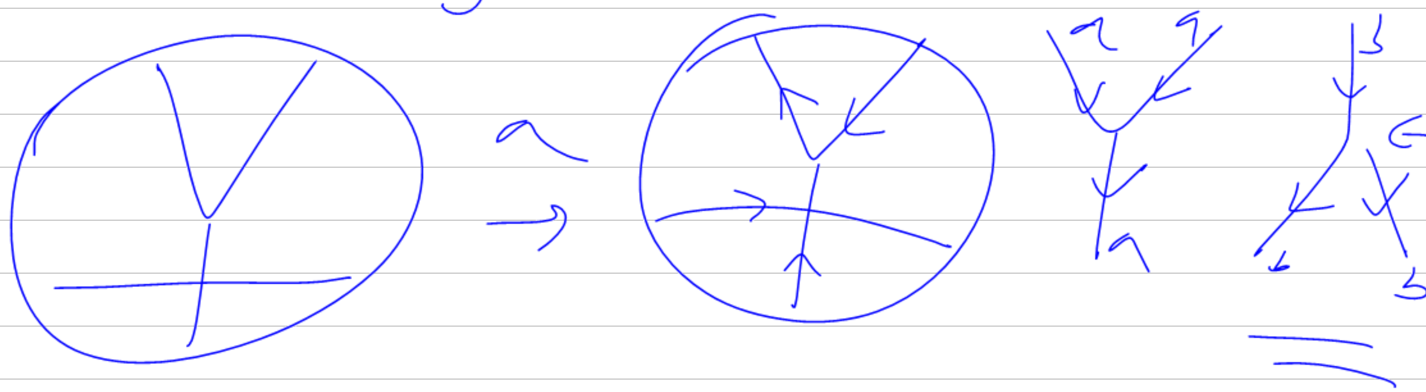
$\langle m^{ij}_k$ am aΔ
bm bΔ

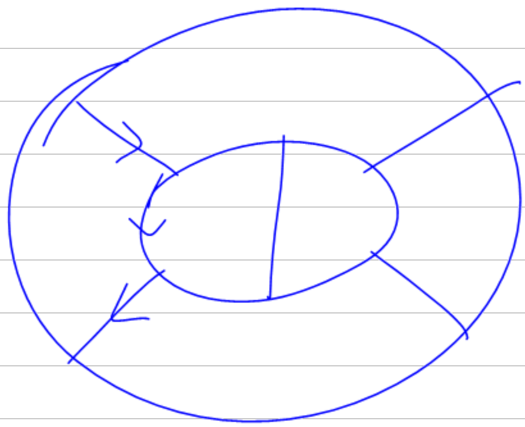






$$g = a \oplus b$$





$0 \qquad \partial_{x_1} \quad y\partial_y$

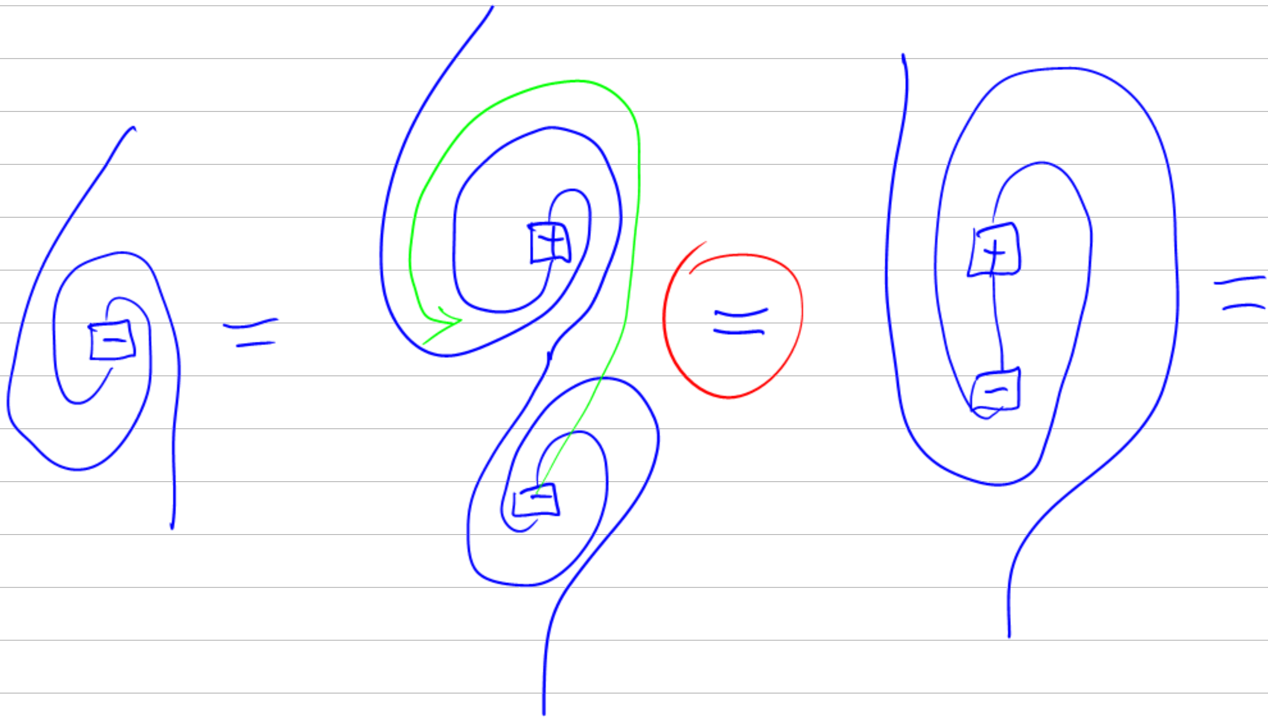
$\downarrow$

K_t

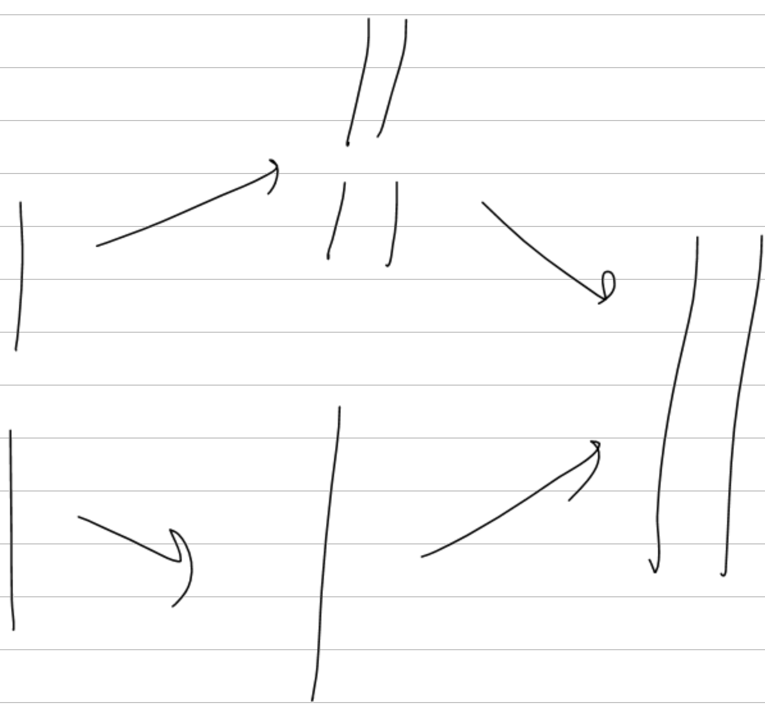
$\downarrow$

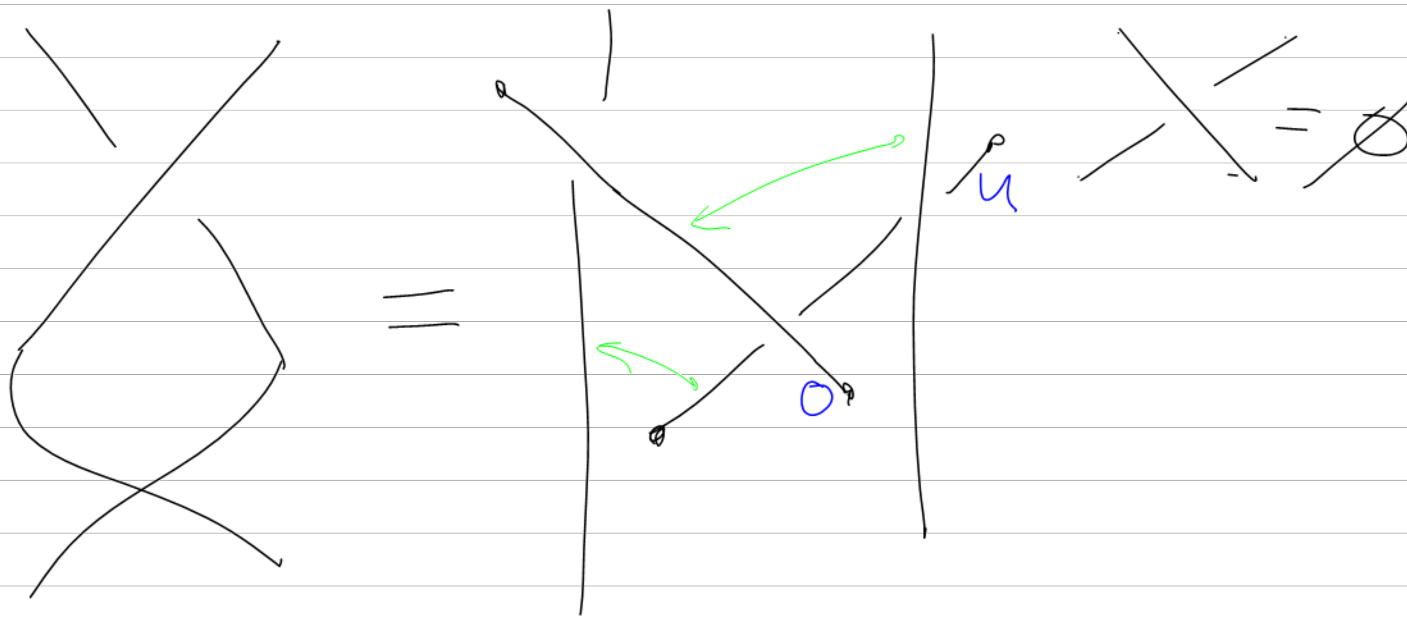
$Q[y]$

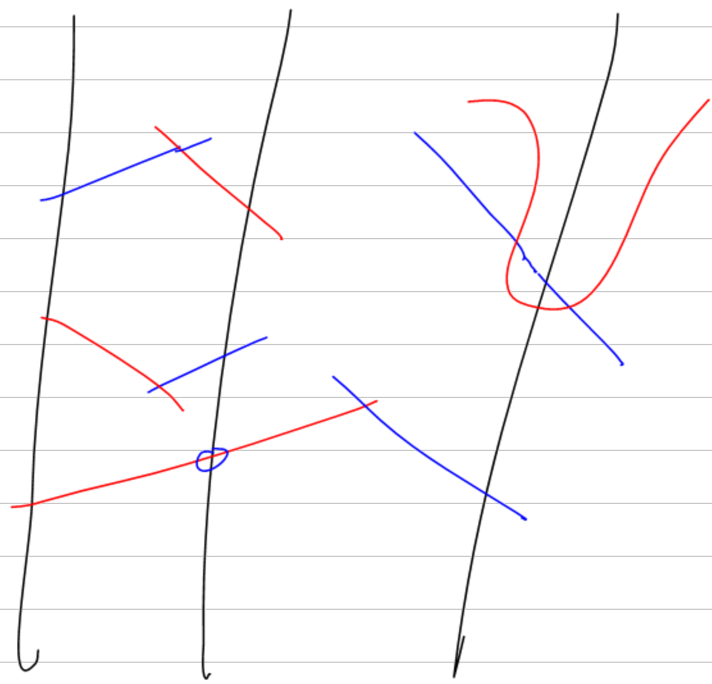
$\downarrow$



$$0 = \int \partial_x \left(e^{Q} \rho \right) = \int e^{Q} \underbrace{\left(\rho\, \partial_x Q + \partial_x \rho \right)}$$







$$e^{\sum_c R^s_{ij}}\Big|_{\substack{p_i \to \partial/\partial\pi_i \\ x_i \to -\ -}} e^{\vartheta_{ij}\xi_i\pi_j}\Big|_0$$

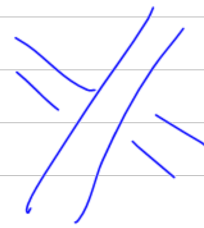
$$R^s_{ij} \in \mathbb{Q}[T^{\pm 1}, p_i p_j x_i x_j, \epsilon]$$

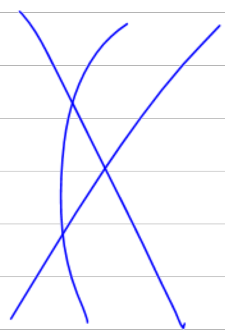
$$p_i p_j x_i x_j \longrightarrow \overset{7}{g_{ii}\, g_{jj}} + \overset{3}{g_{ij}\, g_{ij}}$$

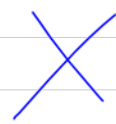
$$g_{ii} \downarrow x_i p_i \qquad g_{jj} \downarrow x_j p_j$$

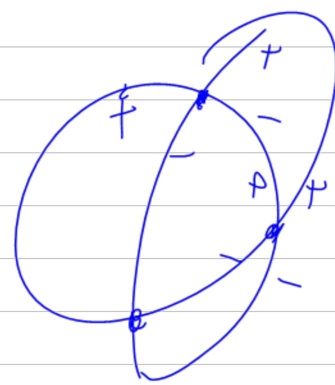
$$\left\langle \begin{matrix} g_{ik} g_{jl} \\ - g_{jk} g_{il} \end{matrix} \right\rangle \longrightarrow Q[g_{ij}] \longrightarrow Q[p_i, x_j]$$

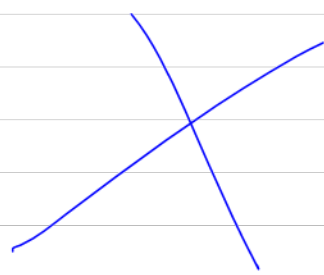
$$Q[g_{ij}] \downarrow R$$









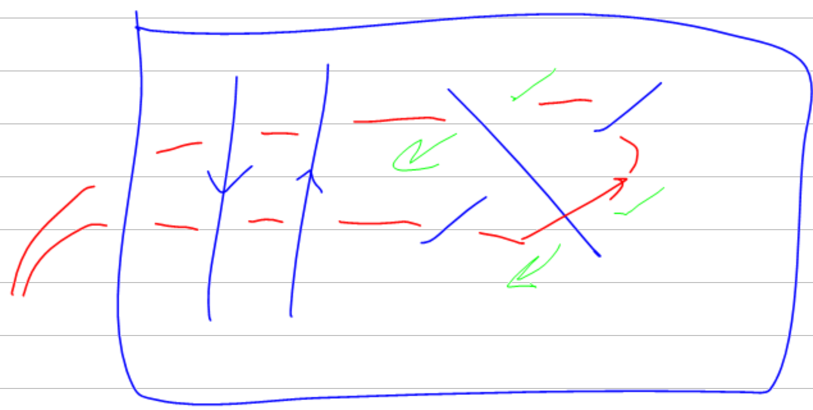


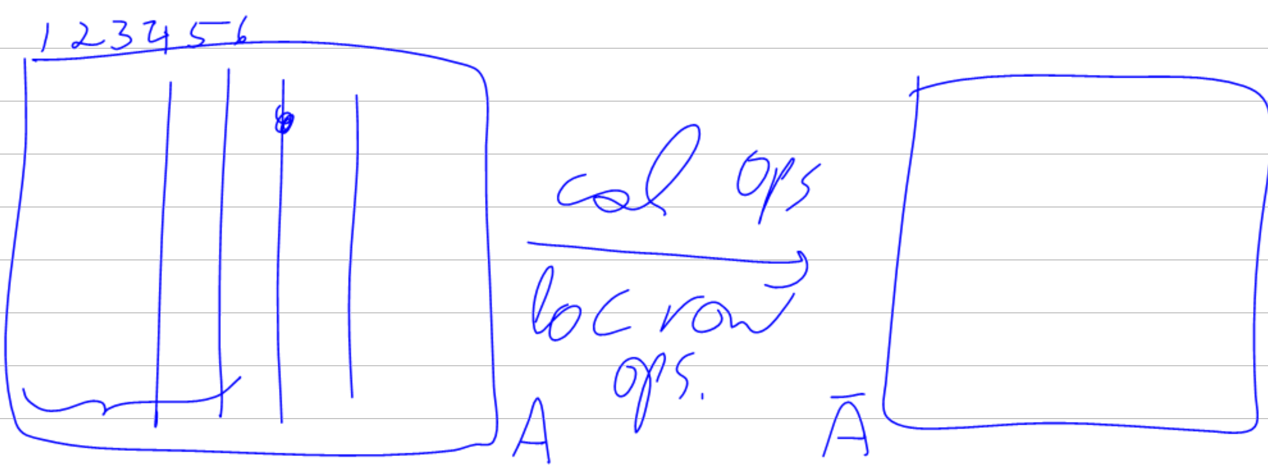
$R_1(g_{ij})$ $\qquad$ $R_1(g_{i'j'}) \longleftarrow$ Gram

$R_1(x_i p_j)$ $\qquad$ $R_1(x_{i'} p_{j'}) \longleftarrow$ Heis

$$G_{ij} = \begin{Bmatrix} g_{ii} & g_{ij} \\ g_{ji} & g_{jj} \end{Bmatrix}$$

$$\begin{tikzcd}
\mathbb{Q}[G_{ij}]^{\otimes 2} \arrow[r, "m"] \arrow[d, "\text{not } 1\text{-}1"] & \mathbb{Q}[G_{ijkl}] \\
\mathbb{Q}[p_{ij}, x_{ij}]^{\otimes 2} \arrow[r, "m"] & \mathbb{Q}[p_{ijkl}, x_{ijkl}] \arrow[u, "\text{Feynman}"']
\end{tikzcd}$$





$$\begin{pmatrix} \frac{1}{1-t} & & 0 \\ & \ddots & \\ & & \frac{1}{1-t} \end{pmatrix}$$

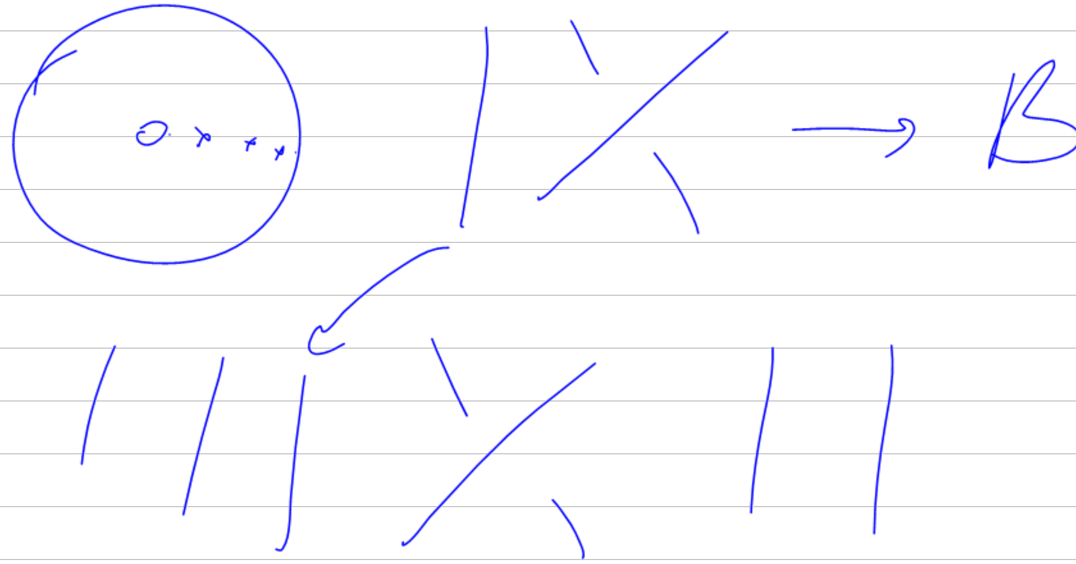
$$\langle a, b, c : a^b = c \rangle$$

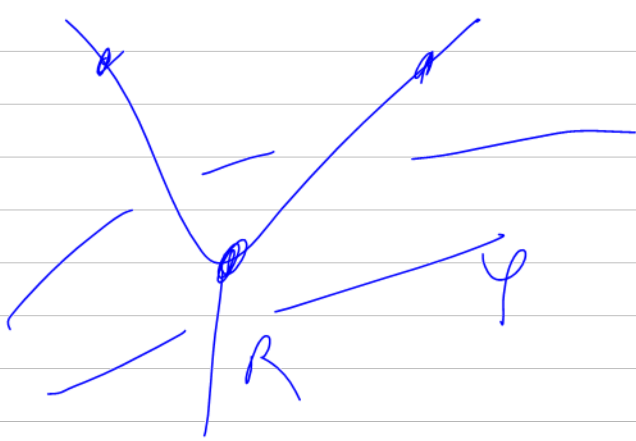
$$\downarrow$$

$$a^{(b^{-1})} = c$$

$$\begin{pmatrix} A \end{pmatrix} \longrightarrow \begin{pmatrix} \overline{A} \end{pmatrix}$$

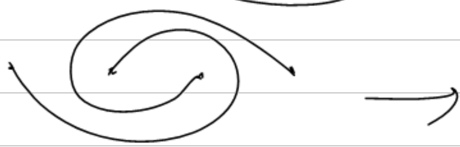
$$G \qquad\qquad\qquad G$$

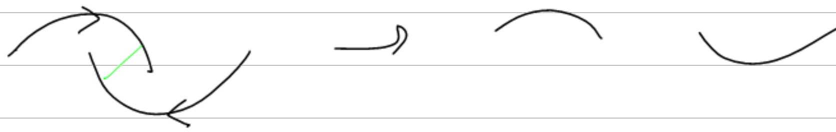


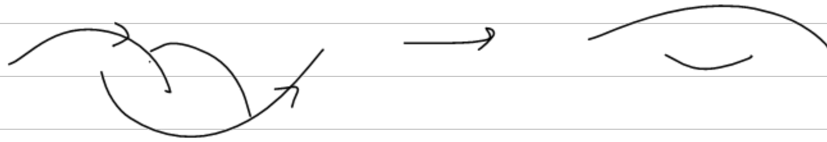


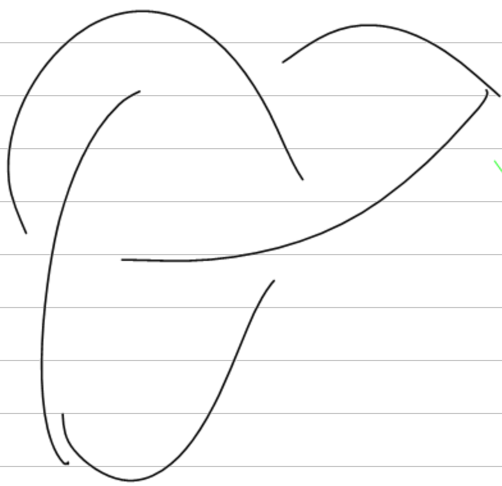
$$(\Delta\otimes 1)R = R^{13}R^{23}$$

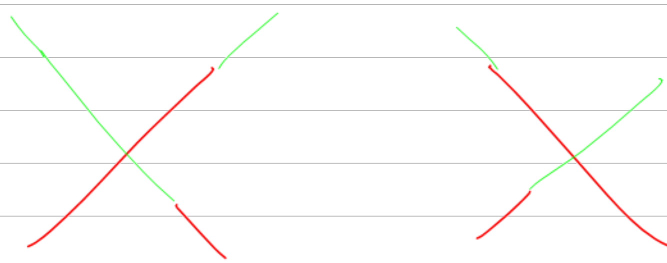
$$\frac{\partial r_i}{\partial g_j}$$









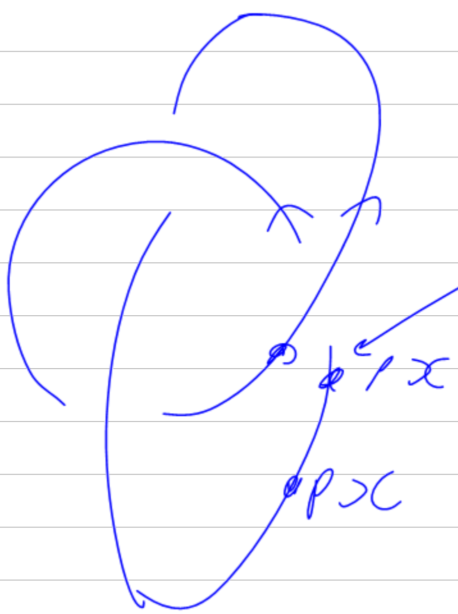


$B_n \circlearrowright V$ Burau.

$$\mathbb{Q}B_n \ltimes \overbrace{H(V \oplus V^*)}^{A}[[\epsilon]]_{doc} \underset{J}{\overset{\pi}{\rightleftarrows}} \mathbb{Q}B_n$$

$$J: B_n \longrightarrow H(p_i, x_i)$$

$$J(\beta_1 \cdot \beta_2) = \left(J(\beta_1) // \beta_2\right) \cdot J(\beta_2)$$

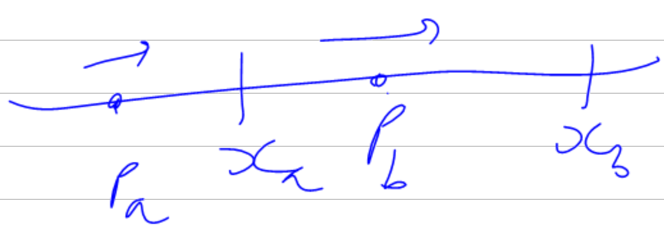


$e^{\pi p} e^{\xi x}$

$r_s(\varphi_i, \varphi_j, i, j)$

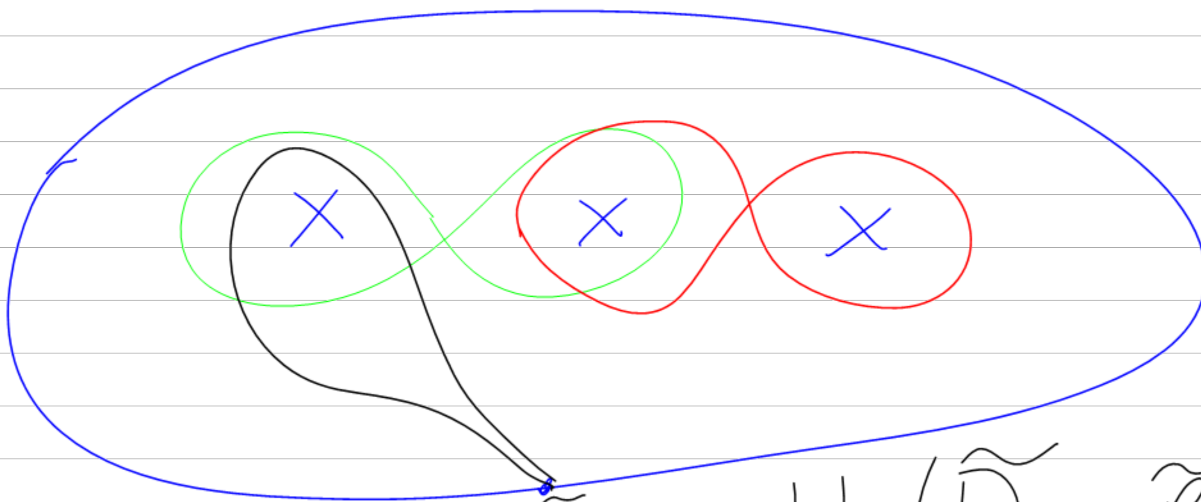
$r_s(0\;0\;i\;j)$

γ



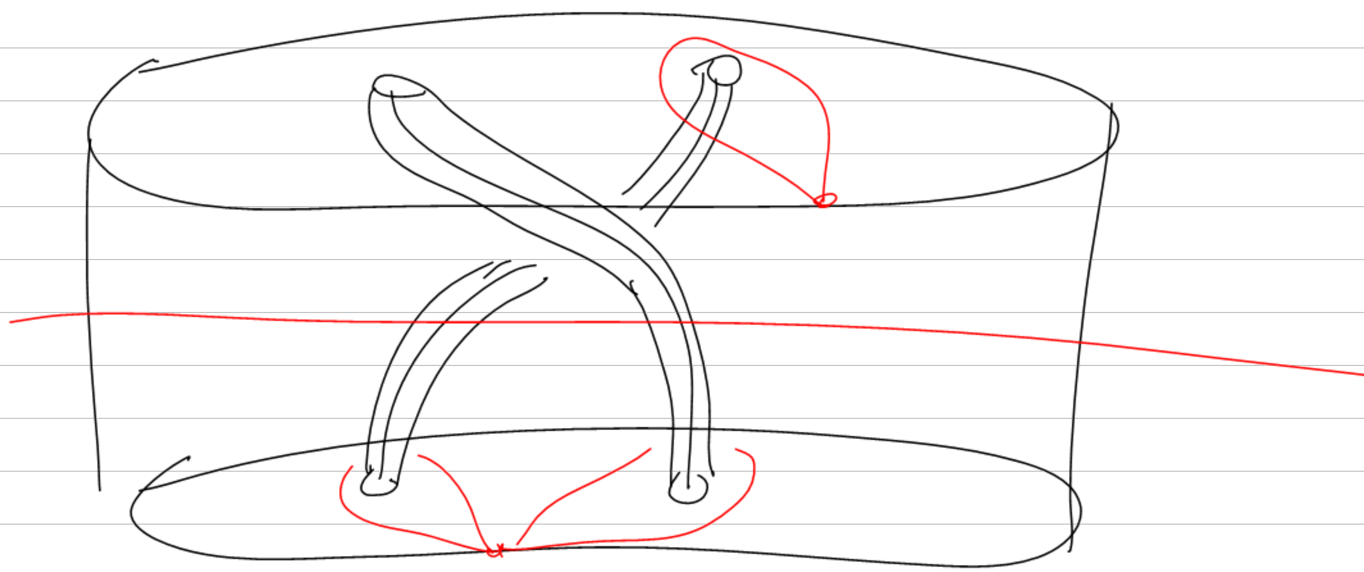
$\vartheta_{\alpha\beta} \qquad p_\alpha \rightsquigarrow p^a_\alpha \ p^b_\alpha$

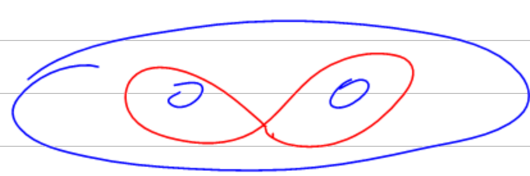
$$e^{\sum_{\alpha,\beta} \vartheta_{\alpha\beta}(p^a_\alpha + p^b_\alpha)\cdot(x^a_\beta + x^b_\beta) + -\sum_\alpha p^b_\alpha x^a_\alpha}$$

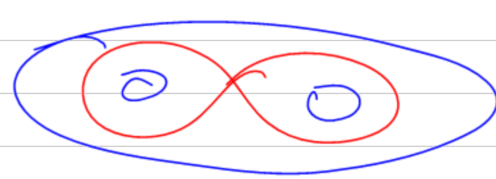


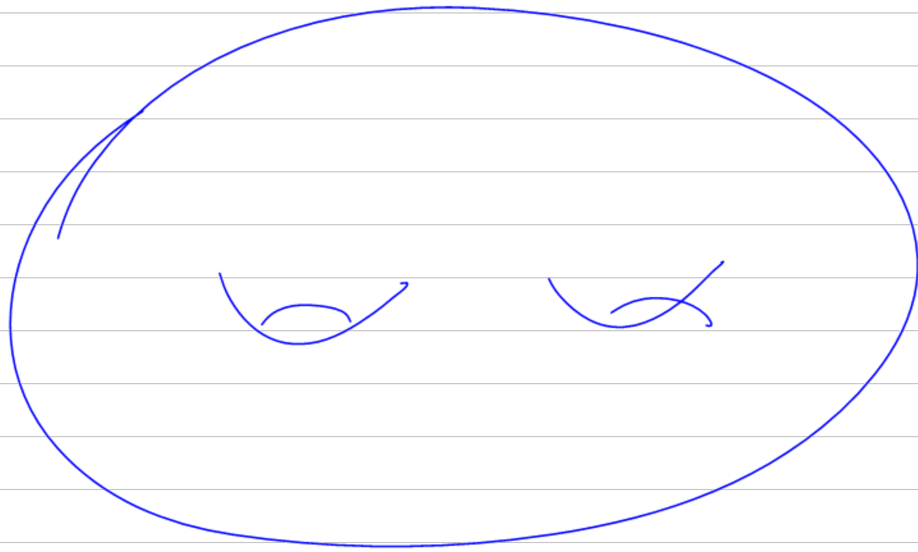
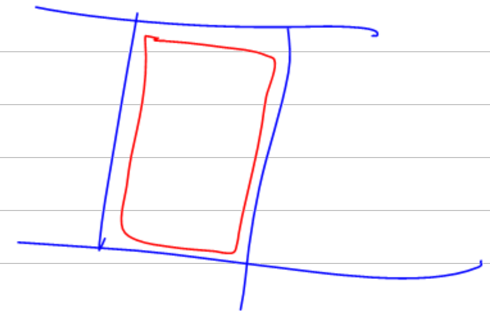
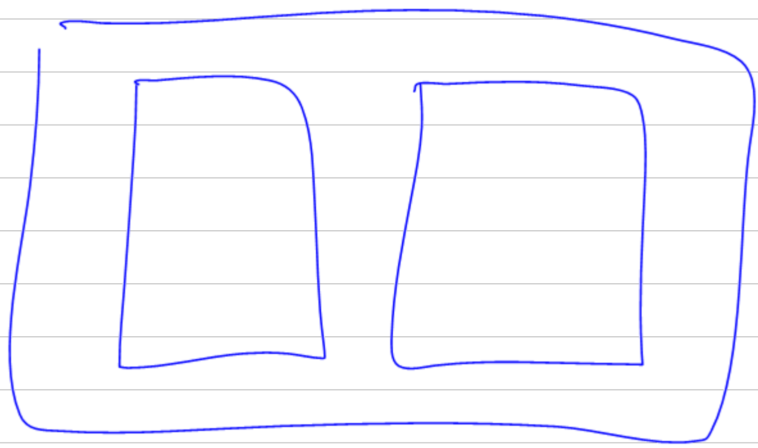
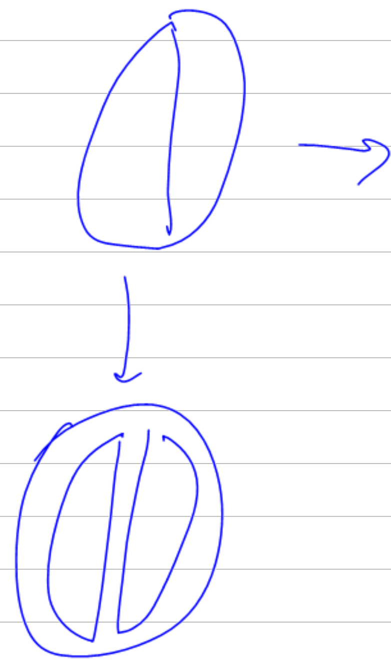
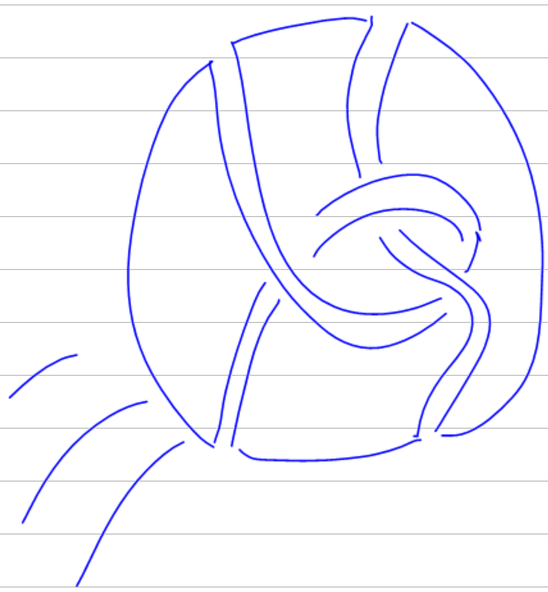
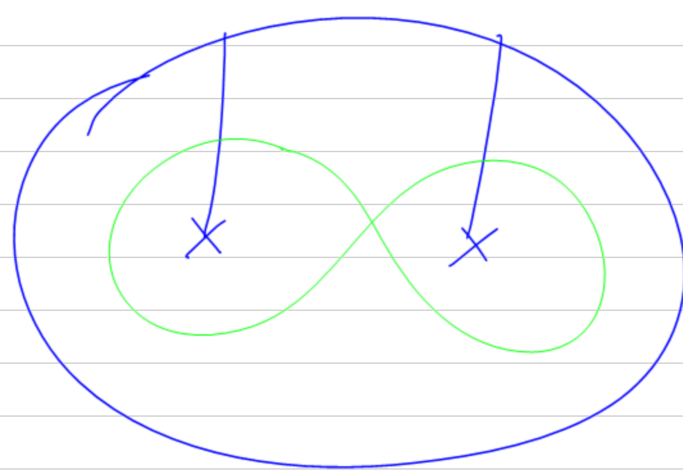
$\tilde{p}$

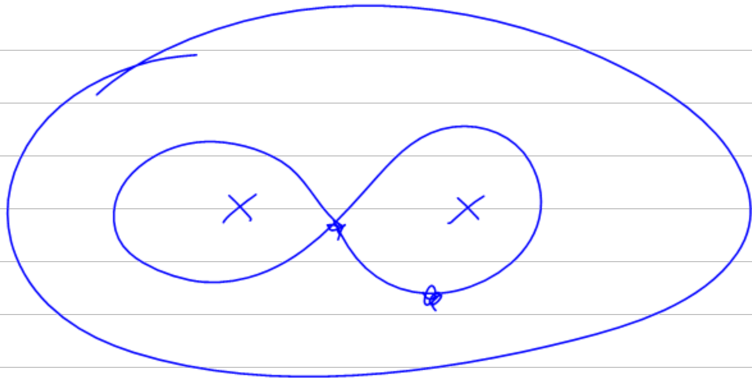
$H_1(\tilde{D}_n, \tilde{p})$

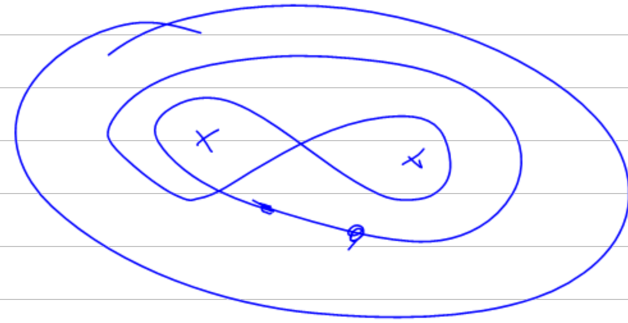


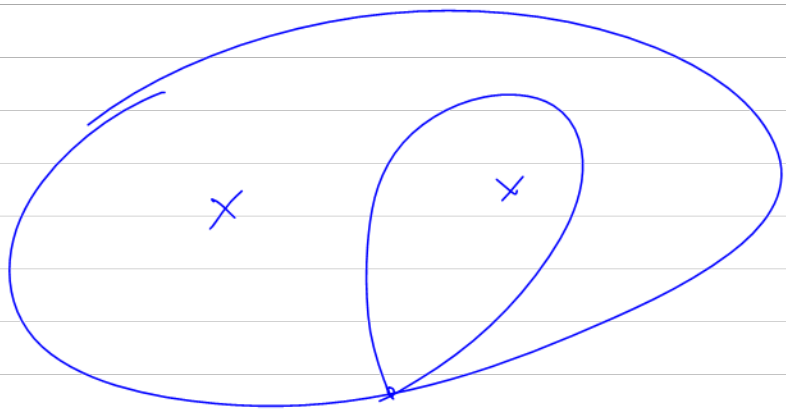


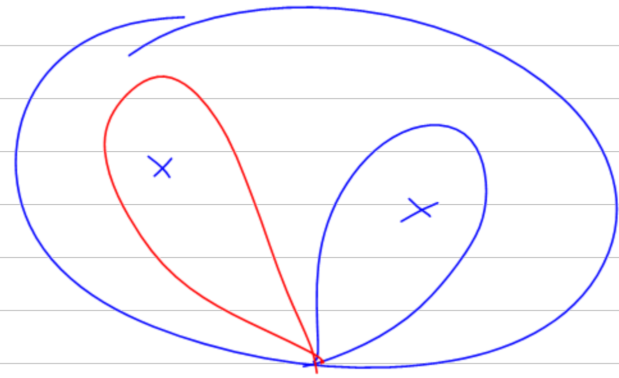


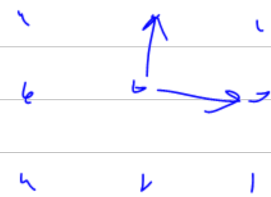











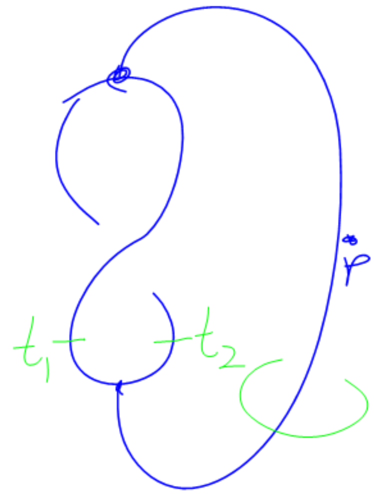






Let H be a genus g handlebody in $\mathbb{R}^3$, let $\Sigma = \partial H$, and let $p \in \Sigma$ be a basepoint (think "H is a tubular neighborhood of a pinched tangle T"). Let $C = (\operatorname{int} H)^c$ and let $\tau\colon \pi_1(C) \to H^1(H;\mathbb{Z}) \cong \mathbb{Z}^g$ be induced by Alexander (?) duality. Let $\tilde{\Sigma} \subset \tilde{C}$ be the τ-covers of $\Sigma \subset C$, respectively, with covering projections ϕ. Let $\tilde{p} = \phi^{-1}(p)$; it is a copy of $\mathbb{Z}^g$. Let $R \coloneqq \mathbb{Z}H^1(H;\mathbb{Z}) \cong \mathbb{Z}[T_i^{\pm 1}]_{i=1}^g$ be the group ring of $H^1(H;\mathbb{Z})$, the ring of Laurent polynomials in variables $T_1, \dots, T_g$, and note that $H_1(\tilde{\Sigma}, \tilde{p})$ and $H_1(\tilde{C}, \tilde{p})$ are R-modules.

Definition 1. $A(H) \coloneqq \ker i_*\colon \underbrace{H_1(\tilde{\Sigma}, \tilde{p})}_{2g} \to H_1(\tilde{C}, \tilde{p})$, *the Alexander module of H.*

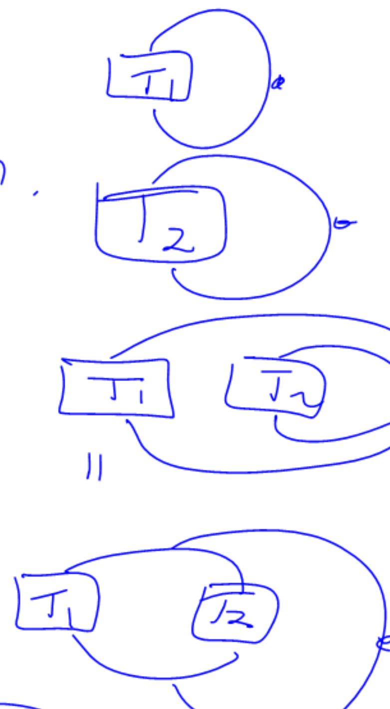


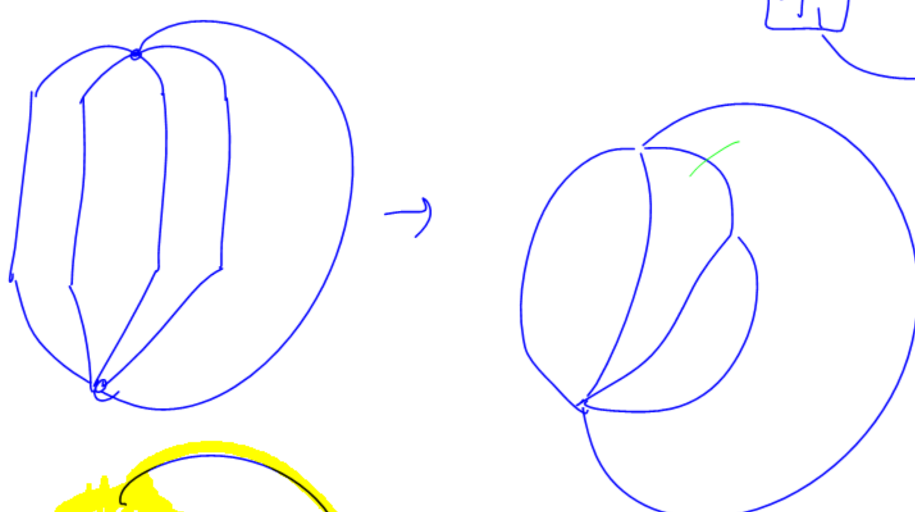
Def 2 < >

Thm 1 A(H) is Lagrangian.

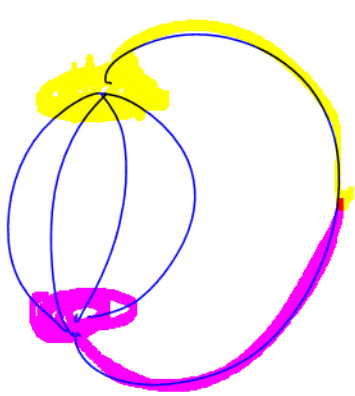
Thm2 behavior under ⊗

Thm3 Behavior under amputations.

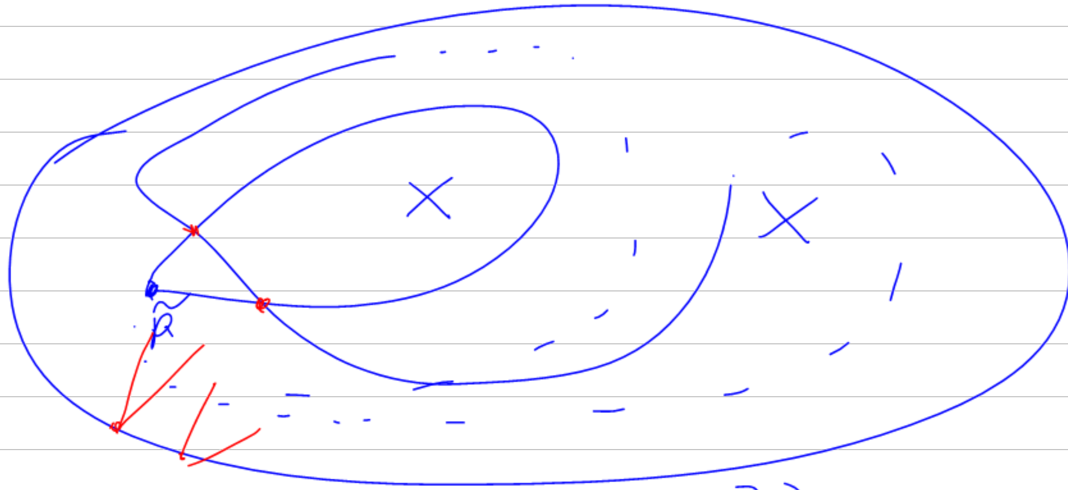




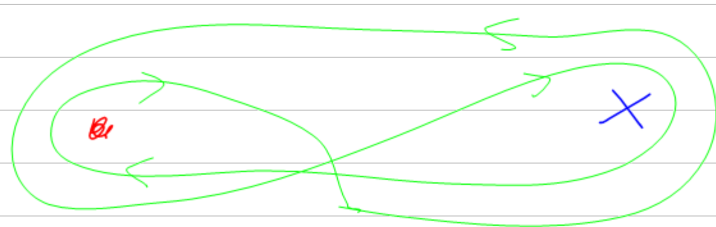
Def

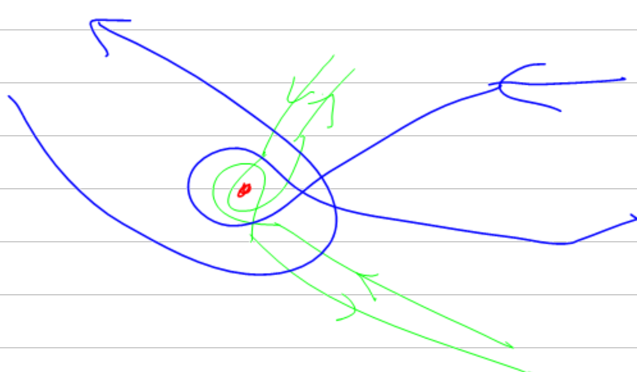


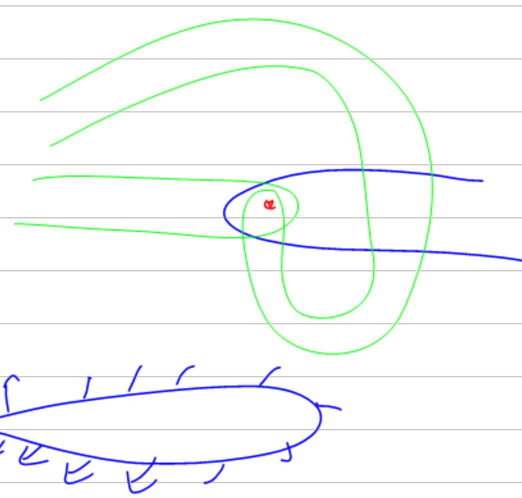
⟺ Gassner & unitarity

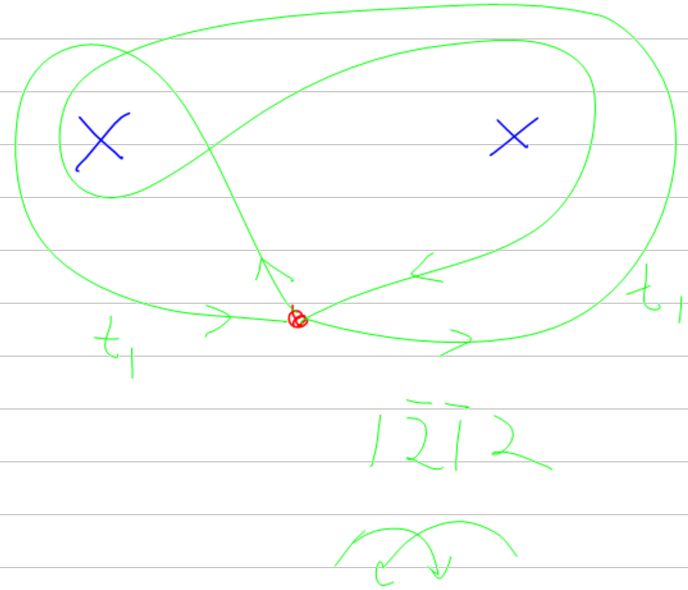


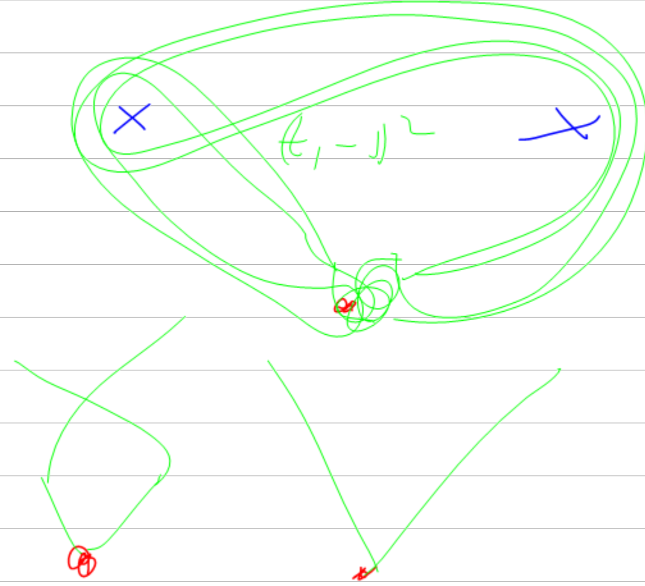
$$H_1(\tilde{D}_2, \tilde{p})^{\otimes 2} \longrightarrow R$$





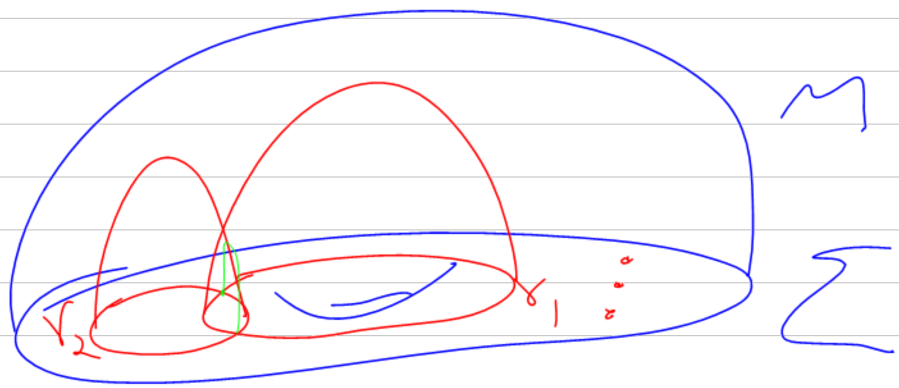


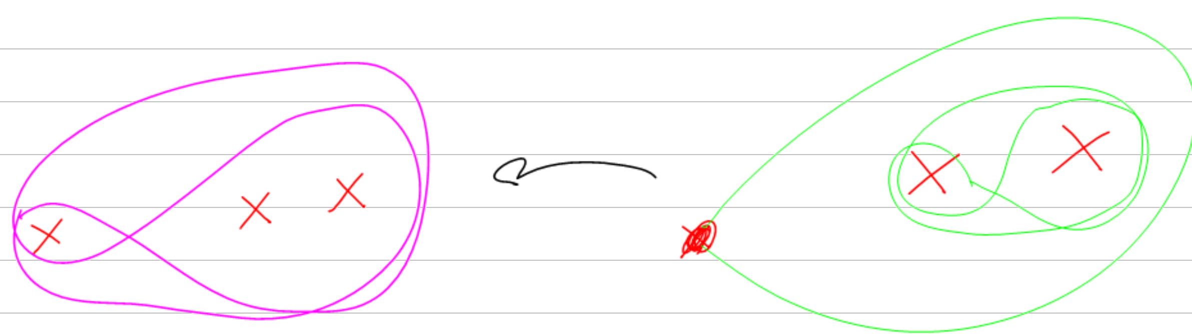


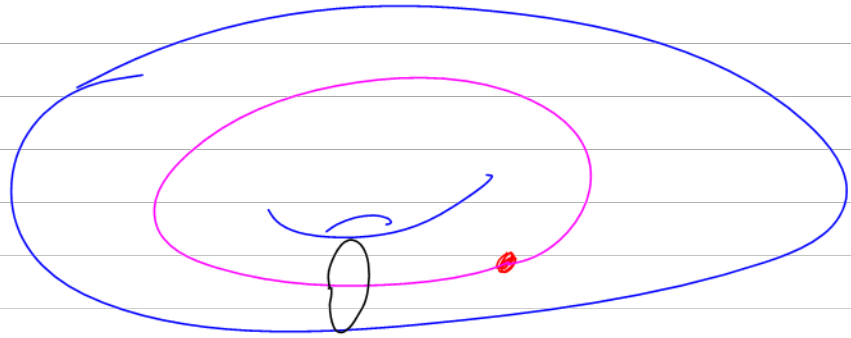


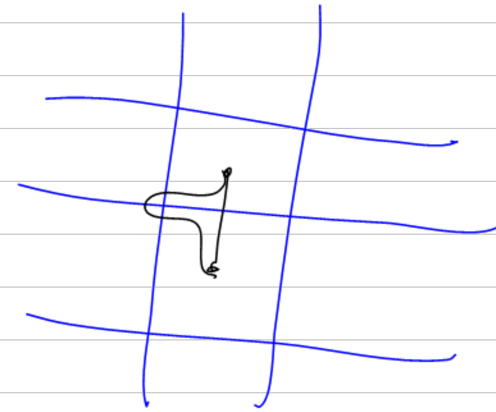
$$\Sigma = \partial M^3 \xrightarrow{i} M^3$$

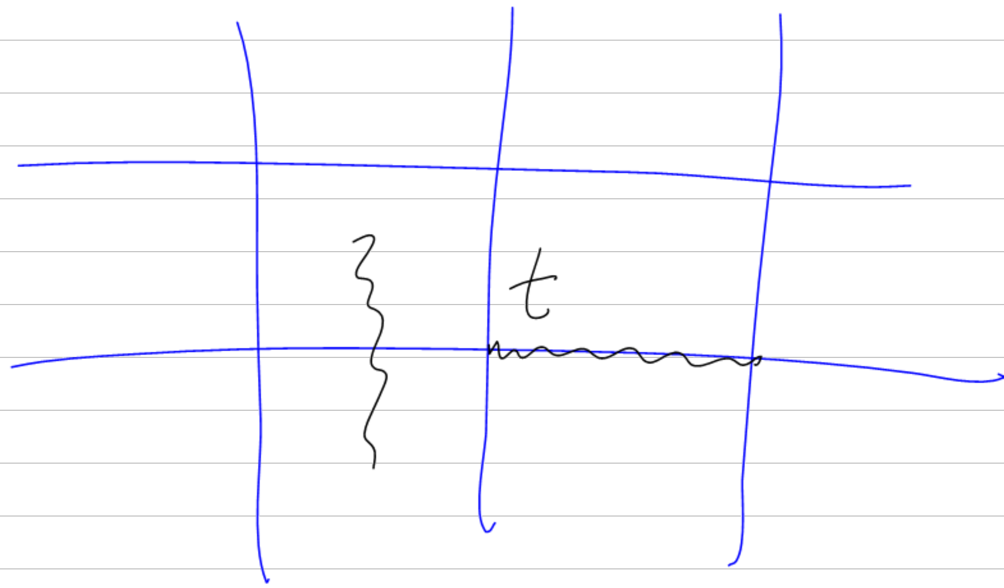
$\operatorname{Ker} i_* \subset H_1(\partial M)$ is isotropic
rel. intersection of $\partial M^3 = \Sigma$

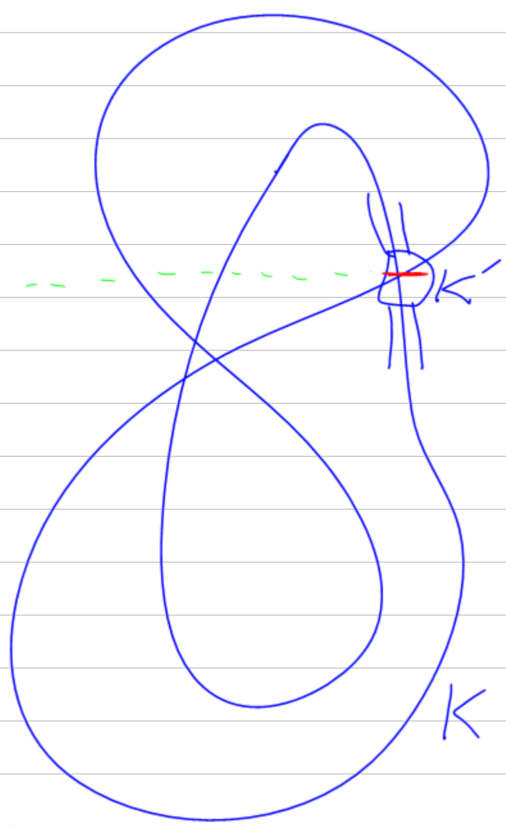


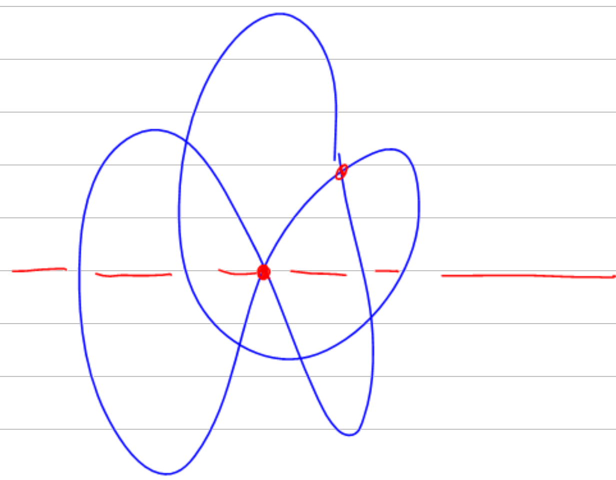




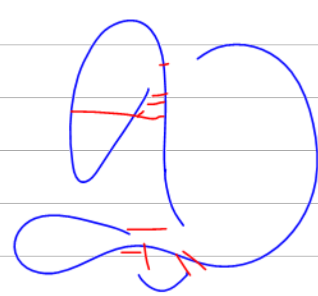








$$\varphi(K) = \sum_{\substack{\text{smoothings}\\ \text{of } K}}$$

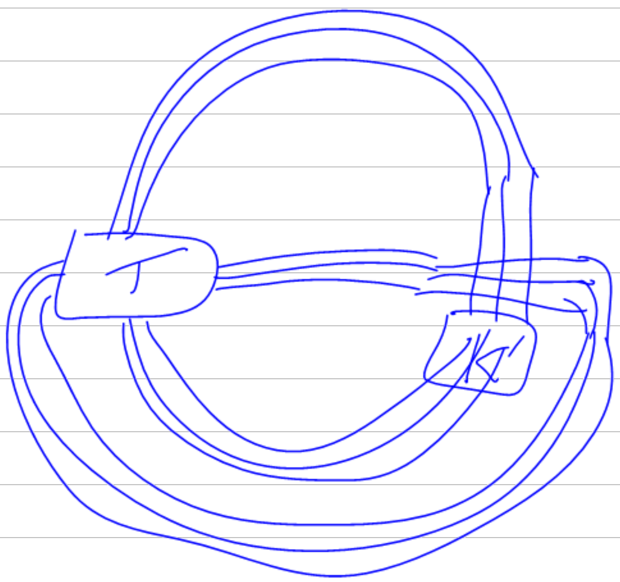


$\mathbb{Z}V \xrightarrow{[\;]} V \qquad \mathbb{Z}\mathbb{Z}S \neq \mathbb{Z}S$

$D \in GD \qquad V = \mathbb{Z}(GD)$

$$Fi(D) = \sum_{a \in D} \sum_{D' \subset D \setminus a} D' \in \mathbb{Z}\mathbb{Z}(GD)$$

$$\sum_\alpha a_\alpha x^{W(\sum_\beta b_{\alpha\beta} D_{\alpha\beta})} \longleftarrow\!\shortmid \sum_\alpha a_\alpha \Big(\sum_\beta b_{\alpha\beta} D_{\alpha\beta}\Big)$$



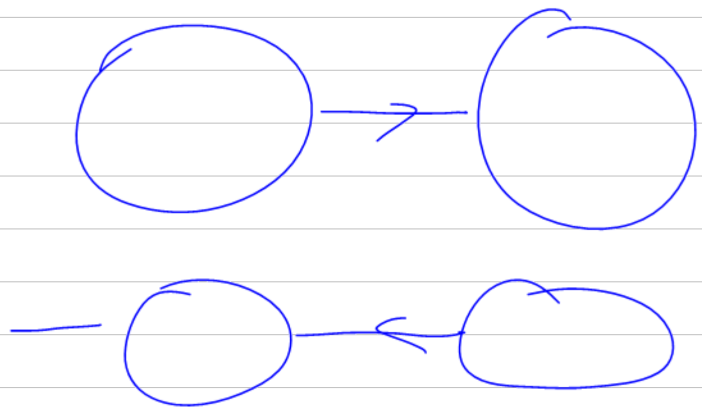
$$Fi_v(K) = \sum_{c \in K} (-1)^c x^{V(K \setminus c)}$$

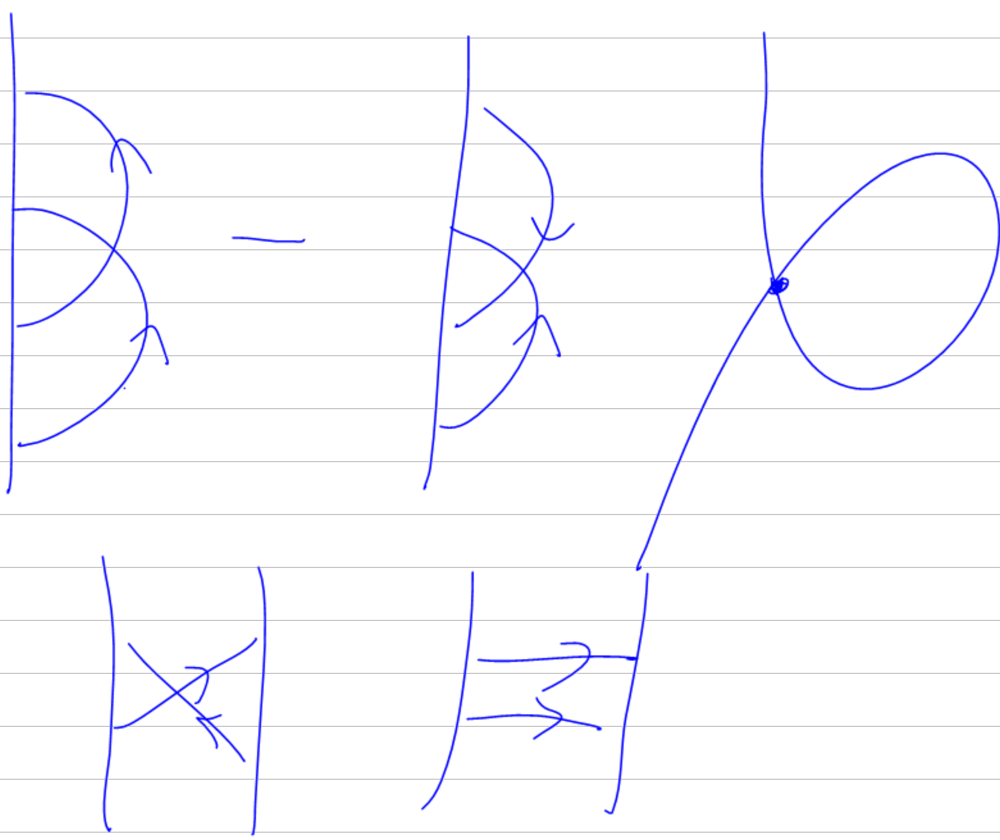
$$R = \mathbb{Z}[t^{\pm 1}]$$

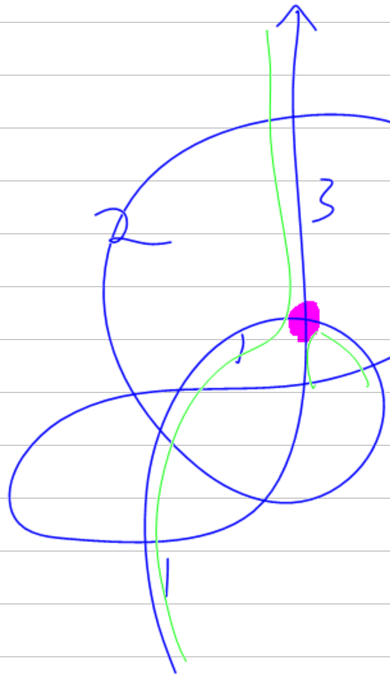
$$\begin{array}{c} H_1(\tilde{A}) \oplus H_1(\tilde{B}) \\ 0 \end{array} \longrightarrow H_1(\widetilde{A \cup B}) \longrightarrow H_0(\widetilde{A \cap B}) \longrightarrow H_0(\tilde{A}) \oplus H_0(\tilde{B}) \longrightarrow H_0(\widetilde{A \cup B})$$

$$\longrightarrow R/t^{pq}-1 \xrightarrow{(1,1)} R/t^{p}-1 \oplus R/t^{q}-1 \xrightarrow{(1,-1)} \underline{\underline{R/t-1}}$$

$$\Delta = \frac{(t^{pq}-1)(t-1)}{(t^{p}-1)(t^{q}-1)} = \frac{1+t^{q}+t^{2q}+\dots+t^{(p-1)q}}{1+t+\dots+t^{p-1}}$$





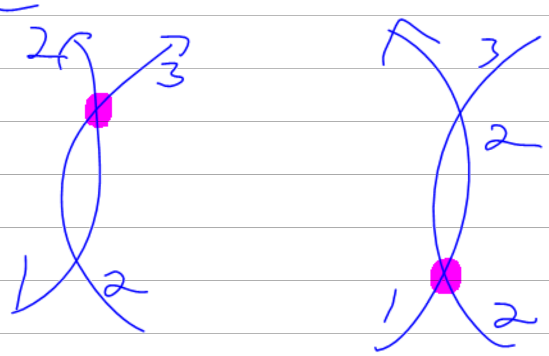


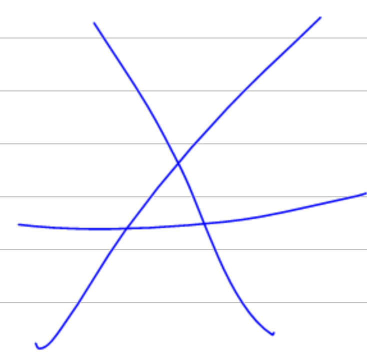
$l_{12}+l_{32}=$

$l_{21}+l_{23}$

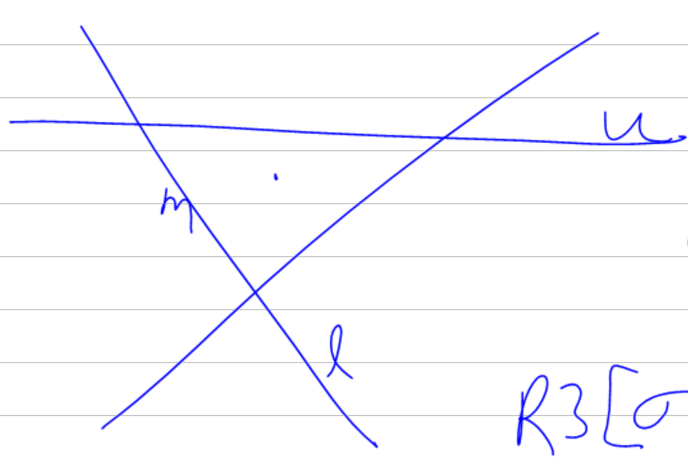
l_{ij} # of times i goes over j

$$\sum_c (-1)^c x^{\sum a^t_{ij} l_{ij}}$$





$a_{12} = a_{23}$



$\overbrace{\text{u : 2 \quad +}}^{6 \quad 8}$

R3: m : 1 −

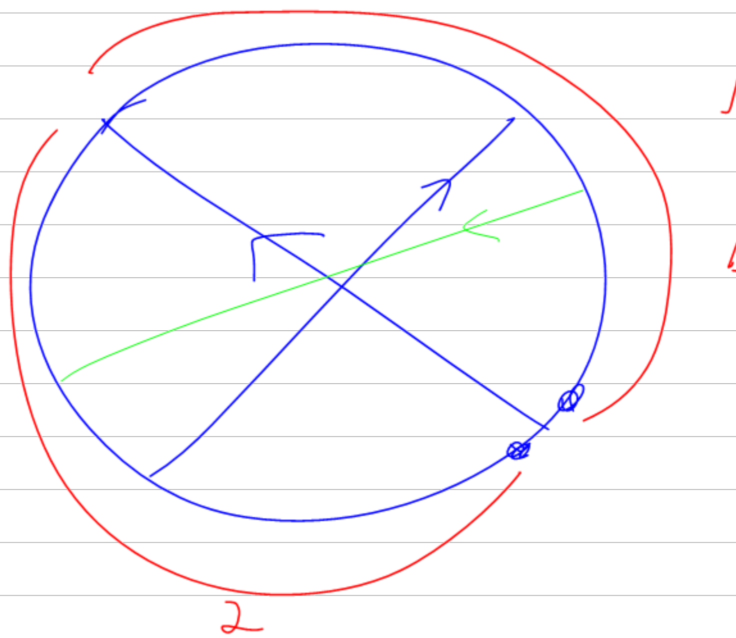
l : 3 +

$R3[\sigma, S_{1\ldots3}]=$

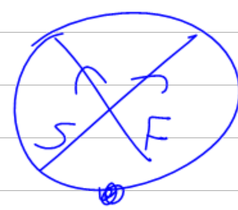
$$A \otimes A \xrightarrow{m} A$$

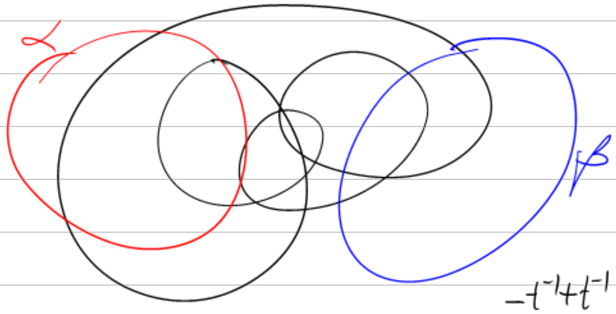
$$A^{\otimes 3} \longrightarrow A^{\otimes 3}$$

$$\begin{tikzcd}
A \otimes A \otimes A \arrow[rr, "\tilde{\sigma}"] \arrow[d, "m \otimes 1"'] & & A \otimes A \otimes A \arrow[d, "1 \otimes m"] \\
A \otimes A \arrow[r, "m"] & A & A \otimes A \arrow[l, "m"']
\end{tikzcd}$$

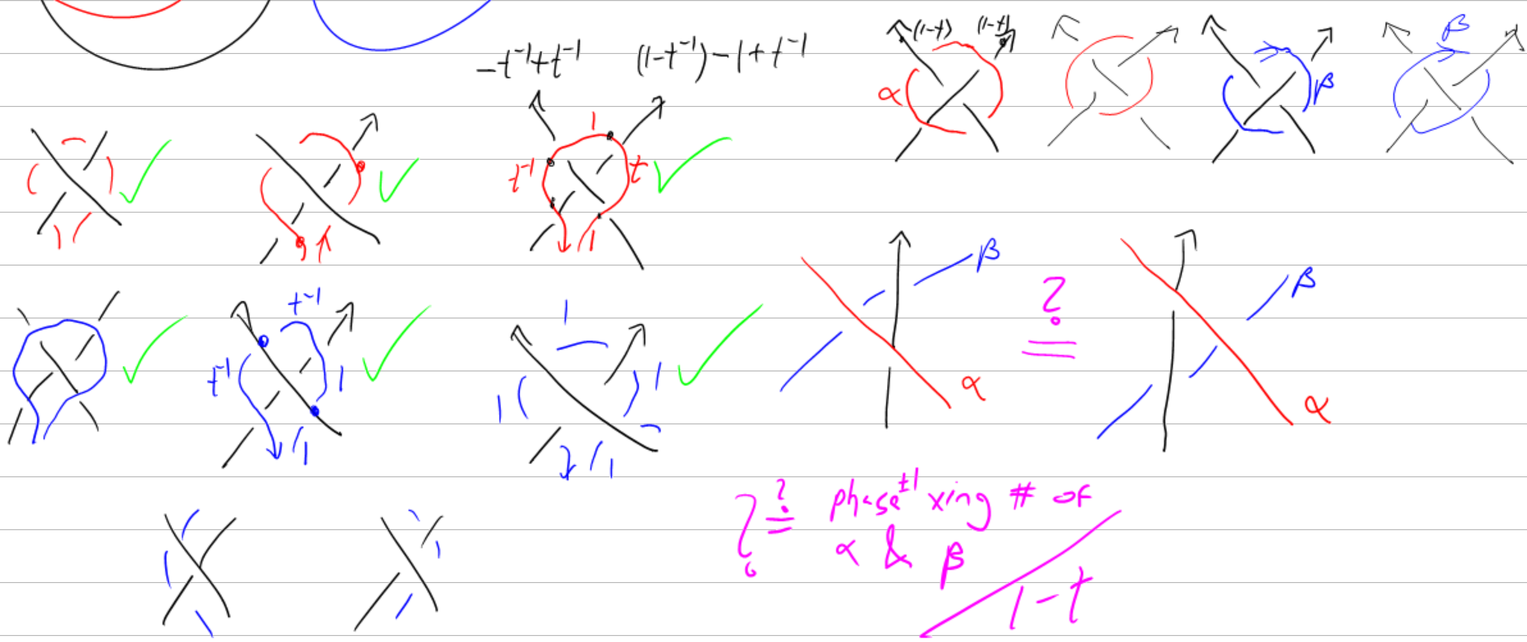


$$\Delta = -\ell_{21} + \ell_{12}$$

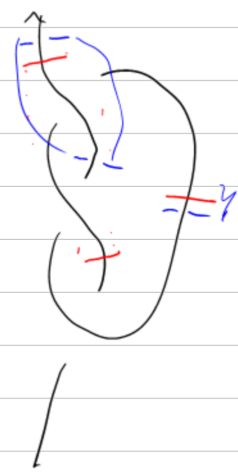




230228b Given a diagram D for a long K, the phase ϕ along a curve $\gamma \subset D^c$ multiplies by T^s whenever γ passes over D with sign s. **Conj.** $\mathrm{lk}_K(\alpha,\beta) = \langle\text{flow: generated by } \alpha, \text{ measured by } \beta\rangle + ?$. α generates $\pm\phi$ flow when it runs over D. β $\pm\phi^{-1}$-measures flow when it runs under D.

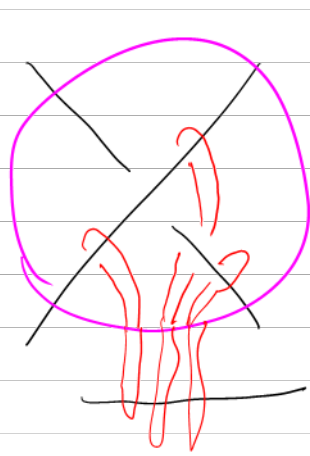


$? \overset{?}{=} \dfrac{\text{phase}^{\pm 1} \text{ xing \# of } \alpha \ \& \ \beta}{1-t}$



α_i: lower clasper

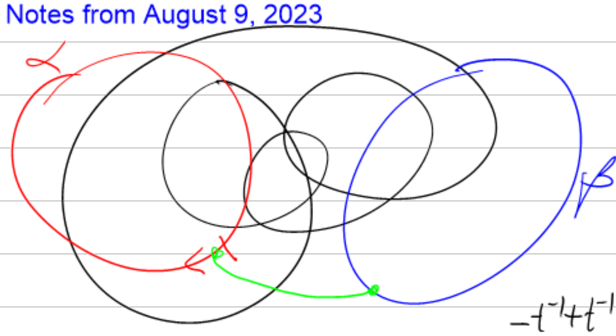
β_j: upper clasper



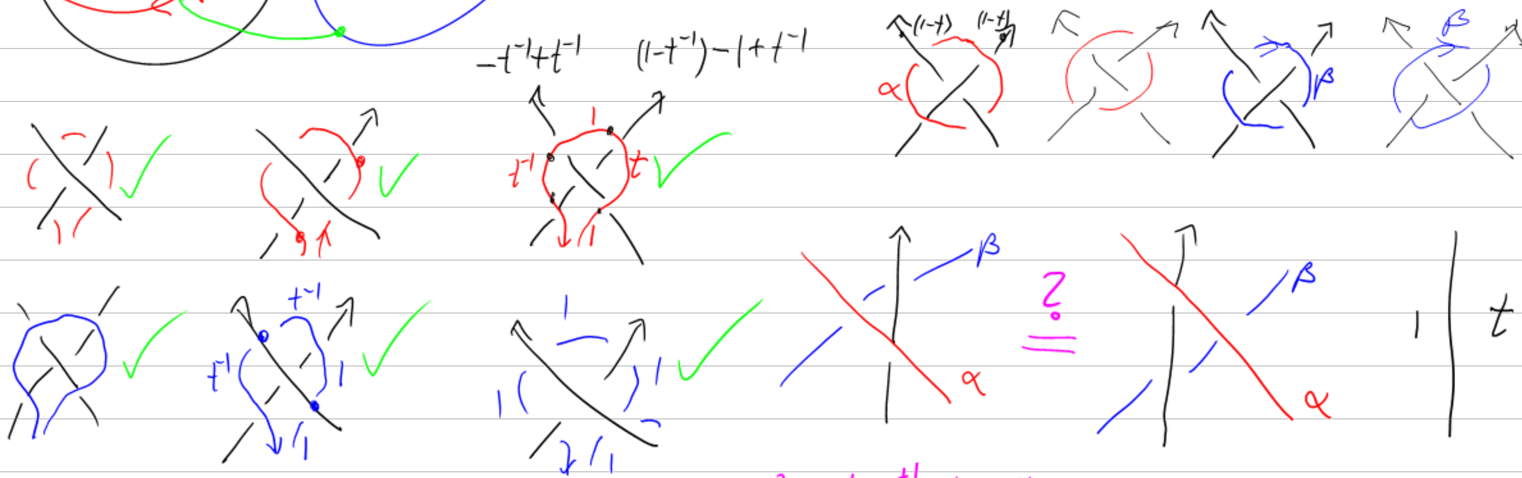
$$\exists \text{ map } \left\{ \text{3-comp classical} \right\} \longrightarrow \left\{ \begin{matrix} \text{3-components} \\ \text{virtuals} \end{matrix} \right\} \Big/ \begin{matrix} \text{OC on red} \\ \text{UC on blue} \\ \text{homotopy on red \& blue} \end{matrix}$$

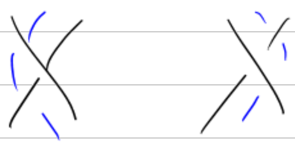
$$(1-T) = (1-T) \qquad (1-T^{-1}) = \underset{\uparrow}{-}\,\underset{\phi}{T^{-1}}(1-T)$$

Notes from August 9, 2023

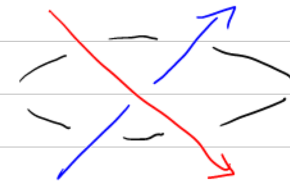


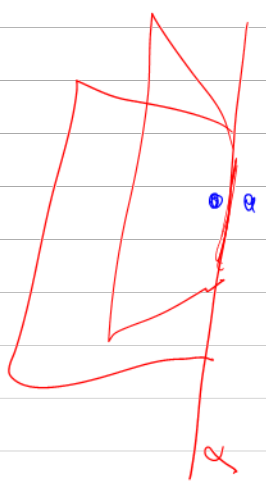
230228b In VanDerVeen_Journal: Given a diagram D for a long K, the phase ϕ along a curve $\gamma \subset D^c$ multiplies by T^s whenever γ passes over D with sign s. **Conj.** $\mathrm{lk}_K(\alpha,\beta) = \langle$flow: generated by α, measured by $\beta\rangle$ + $\langle$phased α over β count$\rangle$/(1−t). α generates $\pm\phi$-flow when it runs over D. β measures $\pm\phi^{-1}$-flow when it runs under D.

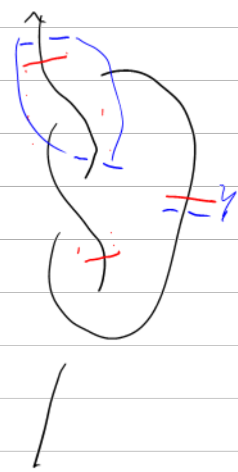




$? \overset{?}{=} \dfrac{\text{phase}^{\pm 1} \text{ xing \# of } \alpha \ \& \ \beta}{1-t}$

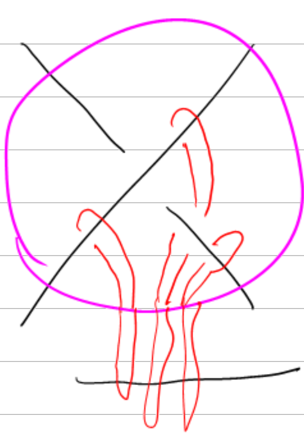


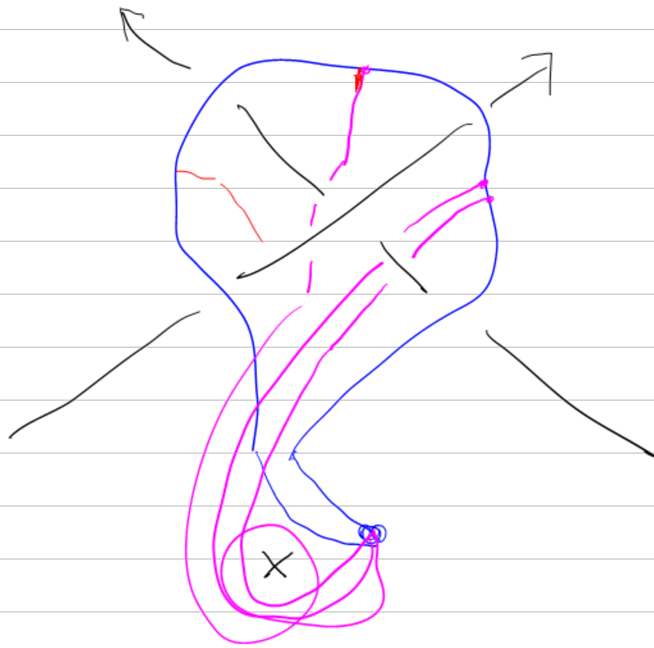


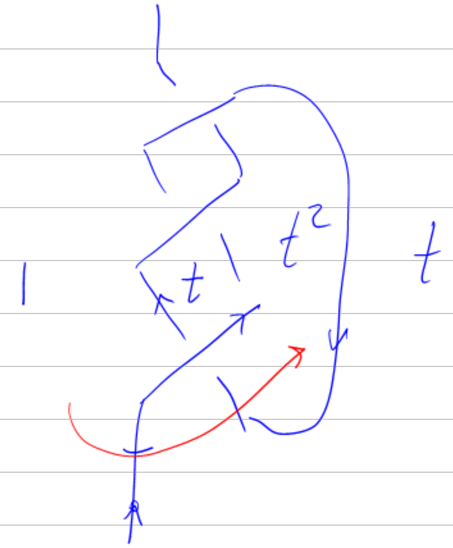


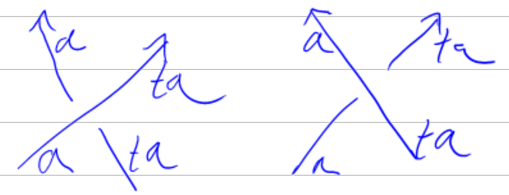
α_i: lower clasper

β_j: upper clasper

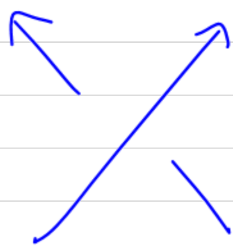


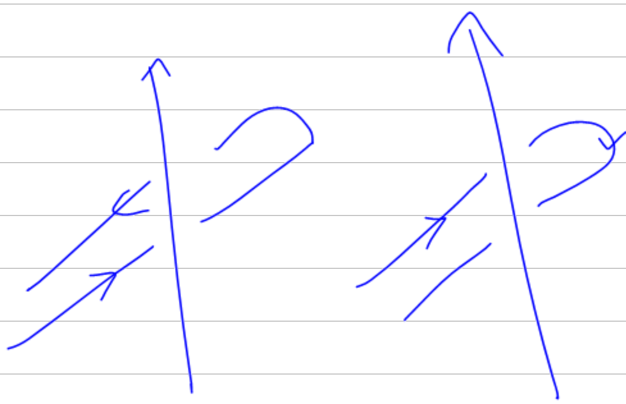






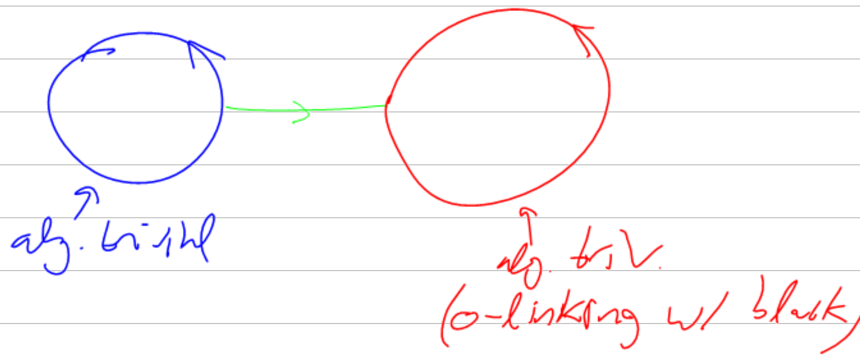
$$R = a \otimes b \qquad R^{-1} = a \otimes S(b)$$



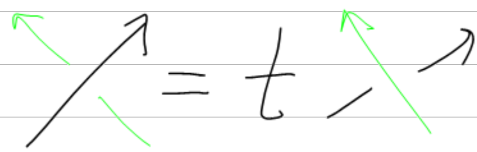


Expansion for virts/braid like $\overset{?}{\Longrightarrow}$ Expansion for rotational virtuals.

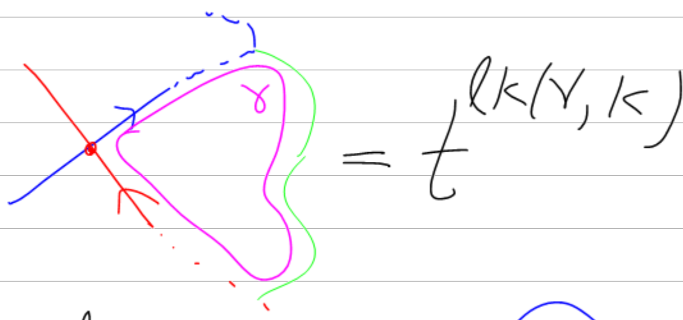
Properties of equivariant linking numbers in the background of a black knot:

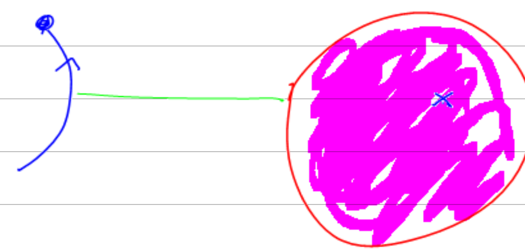
1. An invariant of 

2. green is blind to red, blue & itself

3. 

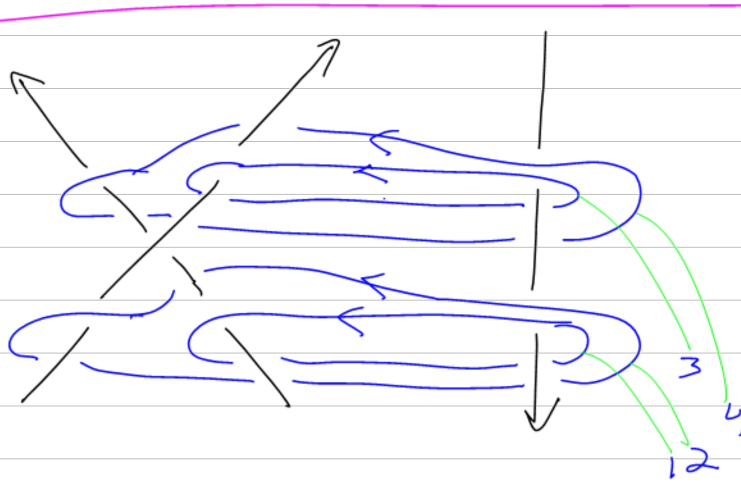
4. blue and red are blind to themselves.

5. 



6. "Bilinear". 

7. Conjugation of blue/red multiplies by a power of t.



$$b_1 = b_4$$

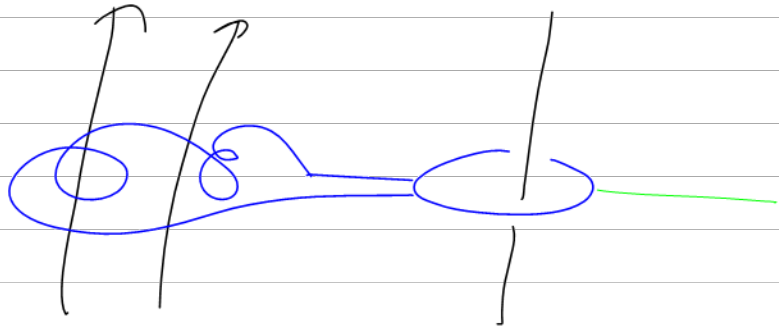
$$b_2 \stackrel{?}{=} t b_3 + (1-t) b_4$$

$$\Updownarrow$$

$$b_2 - t b_3 = b_1 - t b_4$$

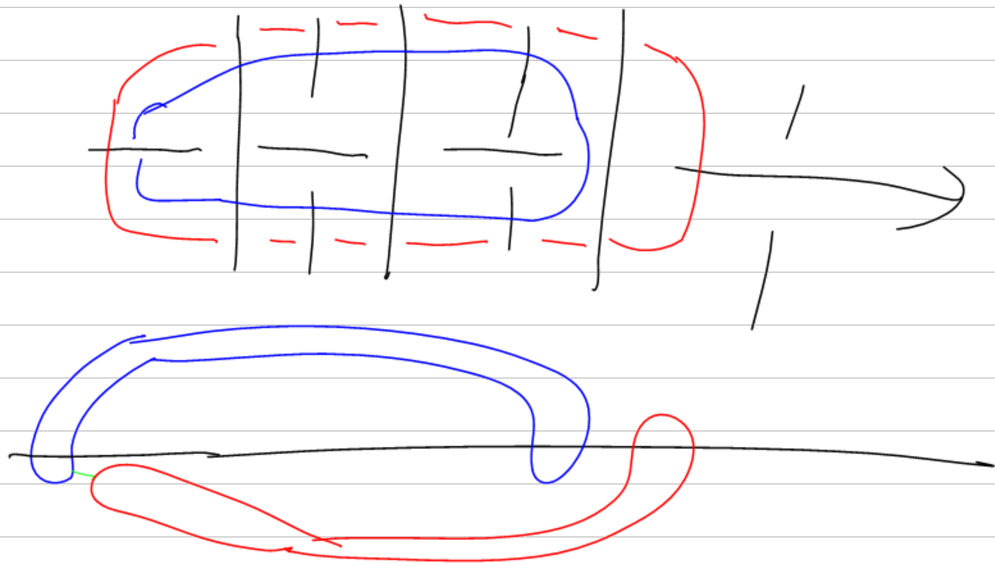
$$\Updownarrow$$

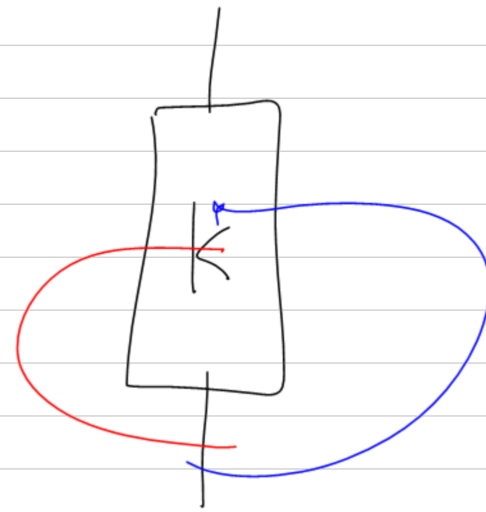
$$b_2 + t b_4 = b_1 + t b_3$$



$\gamma^a \alpha \qquad \gamma^b \beta$

$$\gamma^{a+b} \alpha\beta = \gamma^b \gamma^a \alpha \gamma^{-b} \gamma^b \beta$$
$$= (\gamma^a \alpha)^{\gamma^{-b}} \gamma^b \beta$$

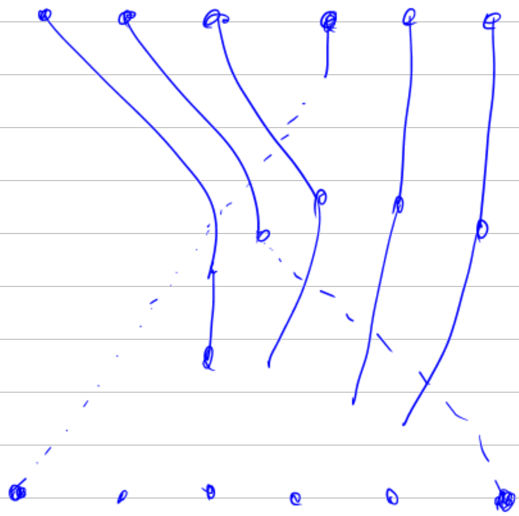


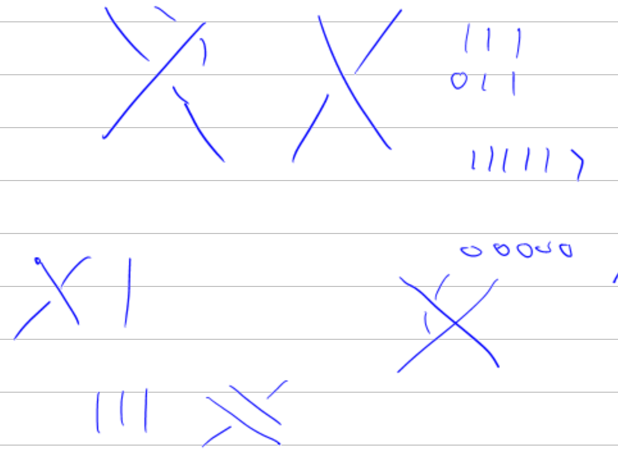


$GA = I \qquad LA = D$

$L = DG$

<u>conj</u> $A \otimes \bar{A} \cong \Lambda \otimes_{\Lambda_0} B$ where B is a Λ_0-module.



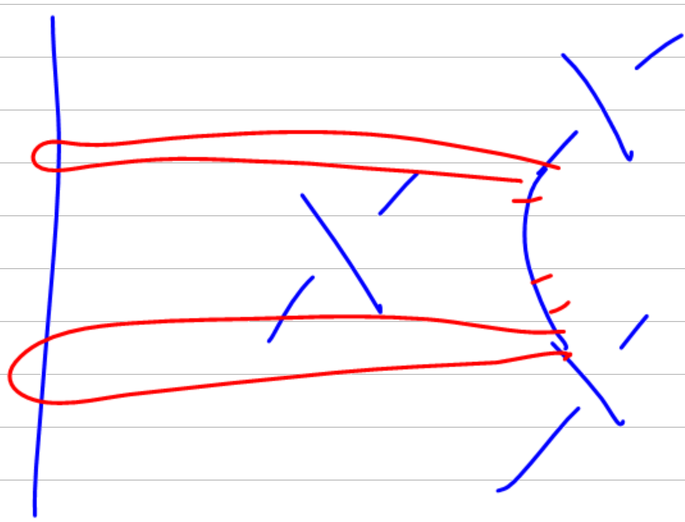


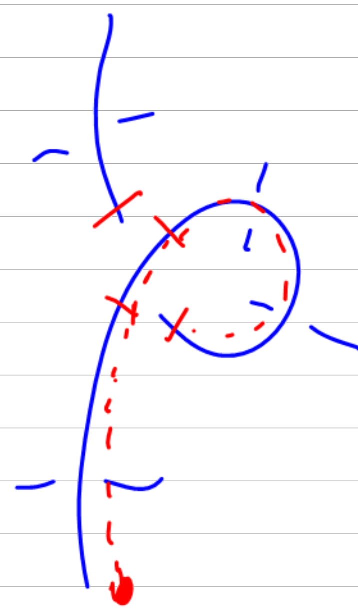
$1+3+3+2+2+2=13$

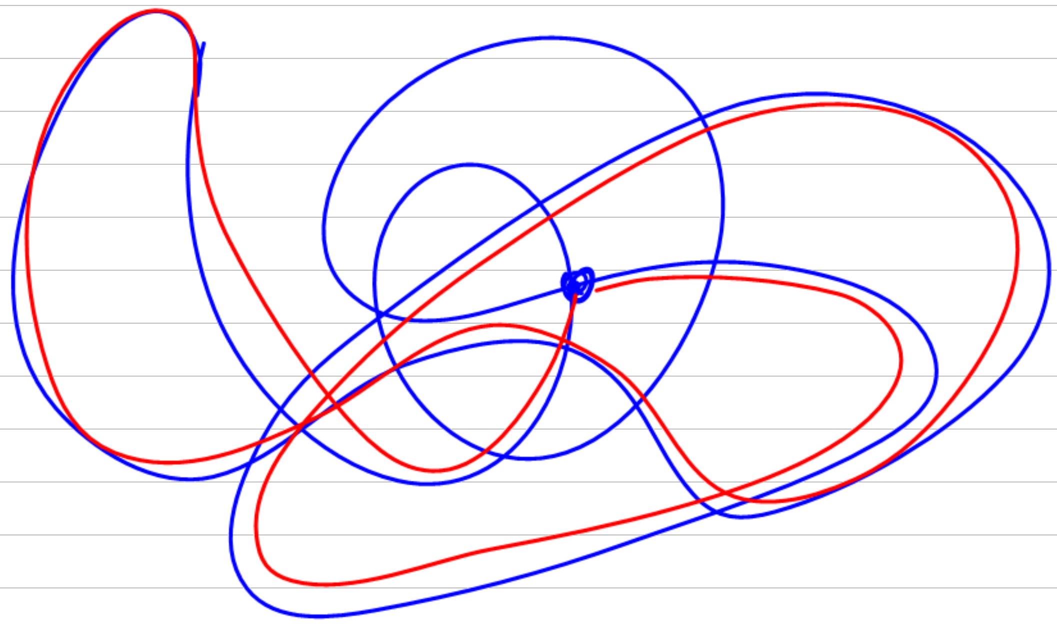
$$\begin{tikzcd}
R_1 \arrow[r, "A_1"] & G_1 \arrow[dr] & \\
 & & A \\
R_2 \arrow[uu, "Q"] \arrow[r, "A_2"] & G_2 \arrow[uu, "P"] \arrow[ur] &
\end{tikzcd}$$

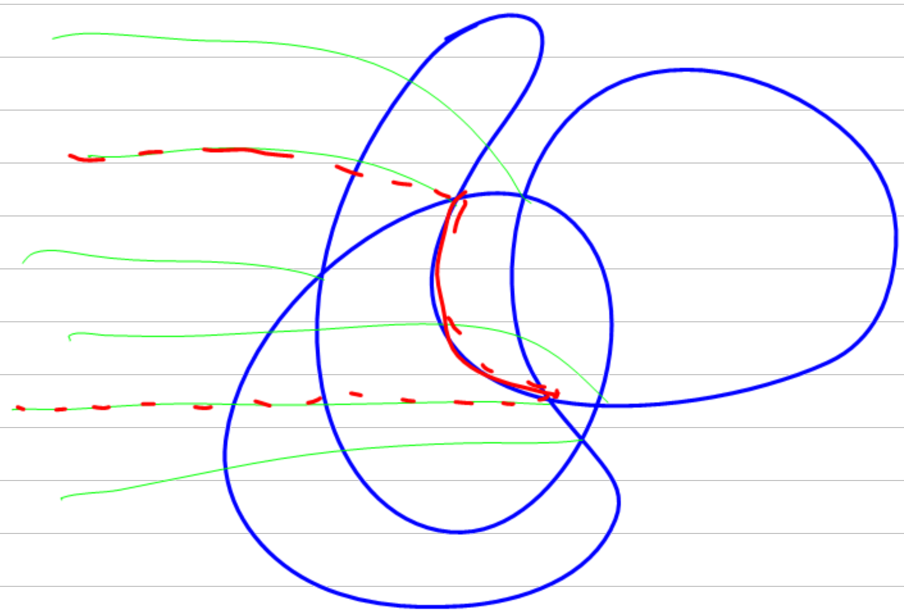
upper

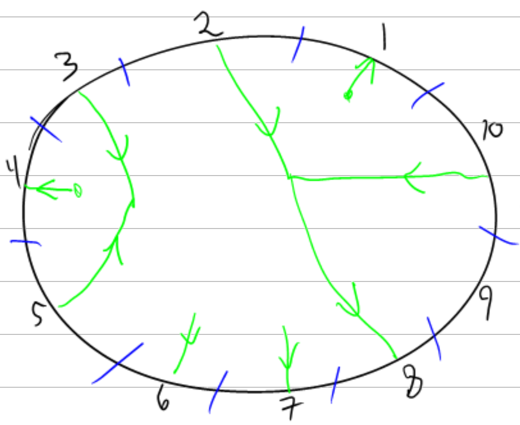
lower



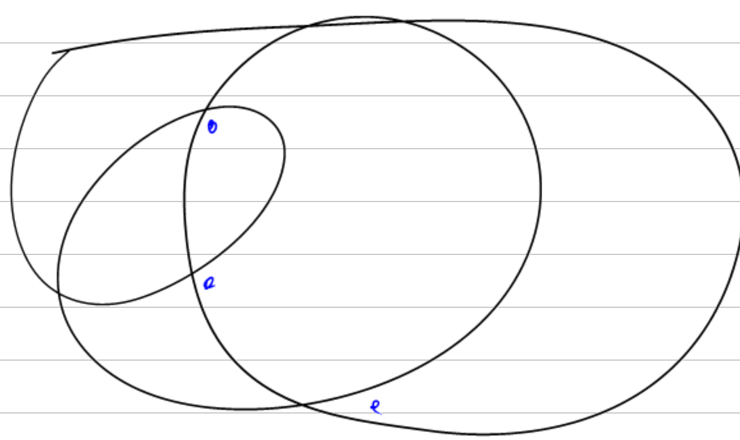


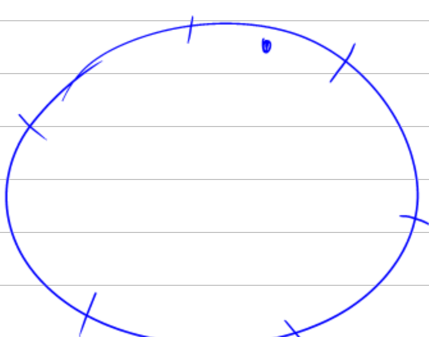


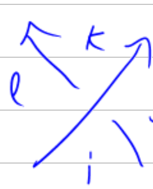




$\mathbb{Z}[T^{\pm 1}, m]$. $\longleftrightarrow$ $Y_8[2,10]\,Y[3,5]$.
$Y_1[\ ]\,Y_4[\ ]\,Y_6[\ ]\,Y_9[\ ]$

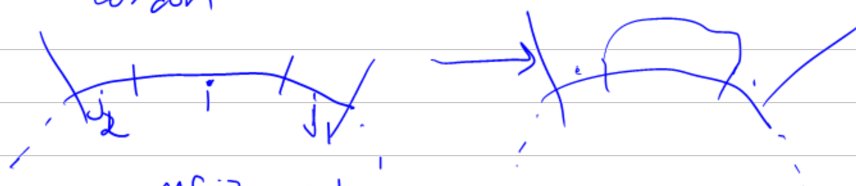




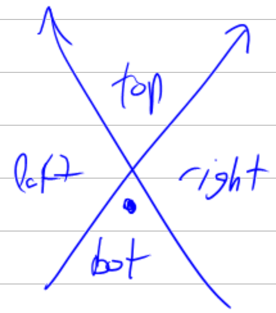


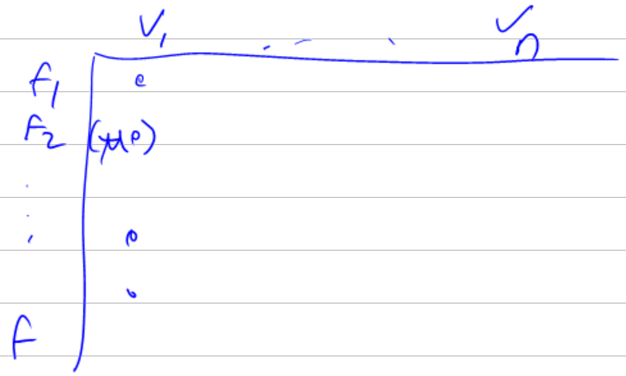
; $M^T M[i] + M[j] + M[l] + M[k] \cdot T$

cordon



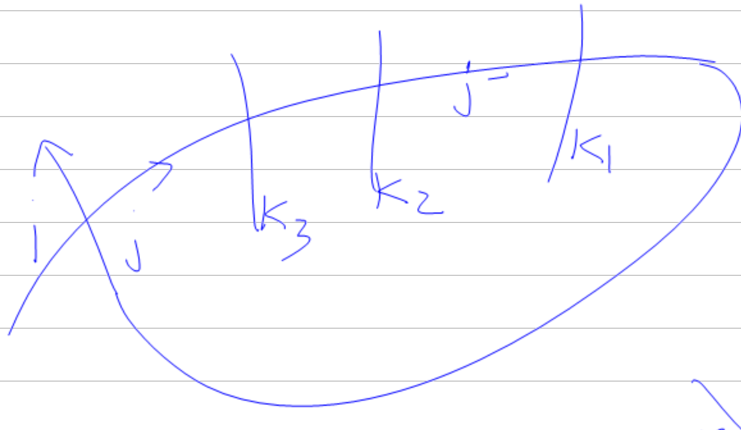
$M[i] \longrightarrow 1$


top
left
right
bot


v_1 ... v_n
f_1
f_2
f

1/8				
~~1/4~~ 0	~~1/8~~ 0	0		
~~1/2~~ 0	~~1/4~~ 0	0	0	
1	~~1/2~~ 0	1/4	0	

$$\sum_{n=2}^{\infty} \frac{n+1}{2^n} < 2$$



$$g_{j\beta} - g_{i\beta} = \underbrace{(g_{j\beta} - g_{j^-\beta})}_{=(J-1)\,g_{k\beta}} + \cdots$$