

$$I^2 = \langle (g_i - 1)(g_j - 1) \rangle$$

$$F - 1 = \prod_{\alpha=1}^K g_{i_\alpha} - 1 = \sum_{\beta=1}^K \prod_{\alpha=1}^{\beta-1} g_{i_\alpha} (g_{i_\beta} - 1)$$

$$(F - 1)(F' - 1) = \sum c_\alpha (g_{i_\alpha} - 1) d_\alpha (g_{j_\alpha} - 1)$$

$$(g - 1) d = \sum e_\alpha (g_{i_\alpha} - 1)$$

$$(g d - 1) - (d - 1)$$

$$\sigma((F - 1)(F' - 1)) = (\sigma F - 1)(\sigma F' - 1)$$

$$\sigma g_{ij} = (\sigma g_i - 1)(\sigma g_j - 1)$$

Aug 10, 2020.

$$g_{kj} g_i = (g_k - 1)(g_j - 1) g_i = (g_k - 1)(g_j g_i - g_i)$$

$$= (g_k - 1)(g_j g_i - 1 - (g_i - 1))$$

$$= (g_k - 1)(g_j(g_i - 1) + (g_j - 1) - (g_i - 1))$$

$$= (g_k - 1)g_j(g_i - 1) + g_{kj} - g_{ki}$$

$$= (g_k g_j - 1 - (g_j - 1))(g_i - 1) + g_{kj} - g_{ki}$$

$$= \underline{g_k} g_j g_i + g_{ki} - g_j 1 + g_{kj} - g_{ki}$$

Q. How does σ_{ij} act on $(\text{Aug } F_n) \otimes_{F_n} V$

Ans. $\sigma_{ij}(\tau \otimes v) = \underset{\text{wada}}{\sigma_{ij}(\tau)} \otimes \underset{\substack{B/G \\ t(t_i)}}{\sigma_{ij}(v)}$

August 13, 2020:

Bellingeri-Soulie, contains stuff about Wada: <https://arxiv.org/pdf/2001.04272.pdf>

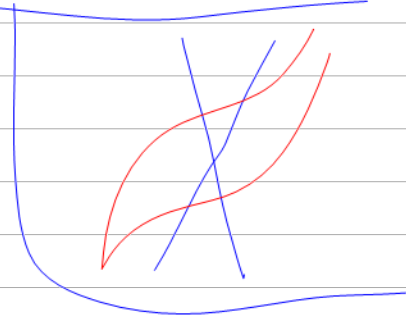
$$G = N \rtimes H$$

<http://www.math.toronto.edu/~drorbn/Talks/Georgetown-1503/>

Aug 17, 2020

$$\mathbb{Q}G/I^{n+1} \quad I^n/I^{n+1}$$

$$I^n ?$$



$$I = \langle g-1 \rangle \quad \bar{g} = g-1 \quad g = \bar{g}+1$$

$$\mathbb{Q}G = \langle 1, \bar{g} : g \neq 1 \rangle = \left\{ \begin{matrix} 1, \bar{g}_1, \bar{g}_2 \\ \bar{g}_1, \bar{g}_2, \bar{g}_3 \end{matrix} \right\}$$

$$I = \langle \bar{g} \rangle = \text{words of length at least 1 in } \langle \bar{g} \rangle$$

$$I^n = \text{words of length at least } n \text{ in } \langle \bar{g} \rangle$$

$$\underbrace{\quad}_n \mid \underbrace{\quad}_{\text{rest}}$$

$$I^n = \langle \text{words of length } n \rangle \cdot \mathbb{Q}F_n$$

$$= \mathbb{Q}F_n \langle \text{words of length } n \rangle$$

$$n=5$$

$$a \cdot nancy = anancy$$

Aug 31, 2020

$$\begin{array}{c} \text{FA} \langle a_1, \dots, a_n \rangle \\ \parallel \\ A \end{array} \xrightarrow[a_i \mapsto 1]{\epsilon} \mathbb{Q}$$

$$I = \ker(\epsilon)$$

$$\bar{a}_i = a_i - 1$$

$$A = \text{FA} \langle \bar{a}_i \rangle$$

$$I = \langle \text{words of length} > 0 \rangle$$

$$\mathbb{Q}FG \langle g_1, \dots, g_n \rangle = \text{FA} \langle g_i^{\pm 1} \rangle / g_i g_i^{-1} = 1$$

$$= g_i^{-1} g_i$$

$$= \text{FA} \langle \bar{g}_i^{\pm 1} \rangle / (\bar{g}_i + 1)(\bar{g}_i^{-1} + 1) = 1$$

$$= \dots$$

$$\bar{g}_i \bar{g}_i^{-1} + \bar{g}_i + \bar{g}_i^{-1} + 1 = 1$$

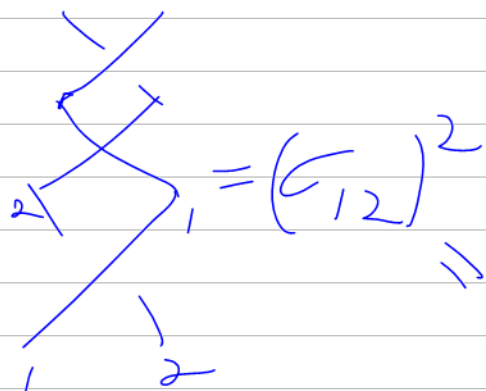
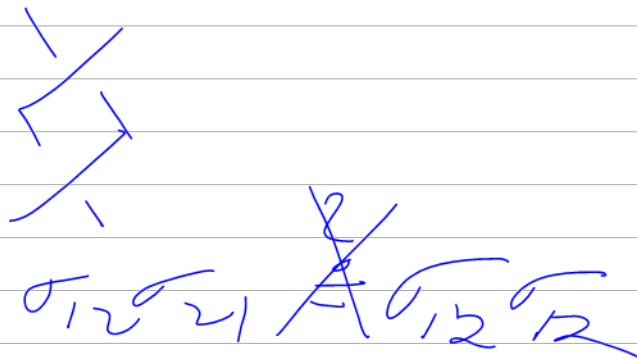
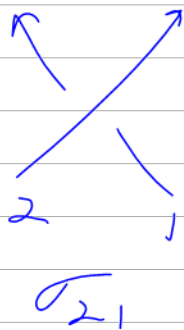
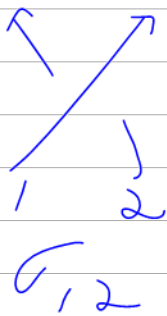
$$\bar{g}_i + \bar{g}_i^{-1} = -\bar{g}_i \bar{g}_i^{-1} \in I^2$$

Magnus Expansion: $\mathbb{Z}: \mathbb{Q}FG_n \rightarrow \prod_{i=1}^n \frac{\mathbb{Z}}{I^{i+1}}$

$$\frac{\bar{g}_i^{-1}}{I} = -\frac{\bar{g}_i}{I} - \frac{\bar{g}_i \bar{g}_i^{-1}}{I^2}$$

$$\hat{\text{FA}} \langle \bar{g}_i \rangle$$

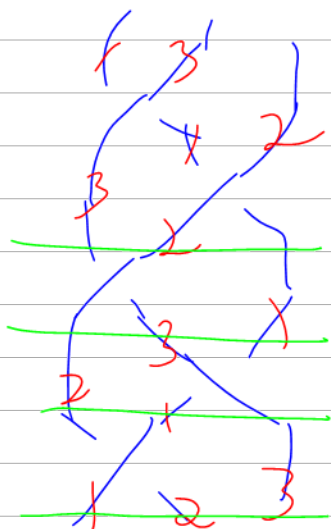
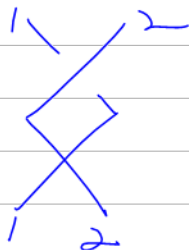
$$\bar{g}_i = t_i \quad \text{"FA} \langle t_i \rangle$$



$$\sigma_{12} = \sigma_1 \tau_1$$

$$\sigma_1 \tau_1 \sigma_1 \tau_1$$

$$\sigma_{21} = \tau_1 \sigma_1$$



$$\sigma_{31}$$

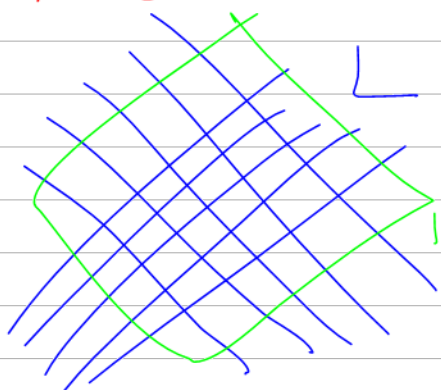
$$\sigma_{21}$$

$$\sigma_{23}$$

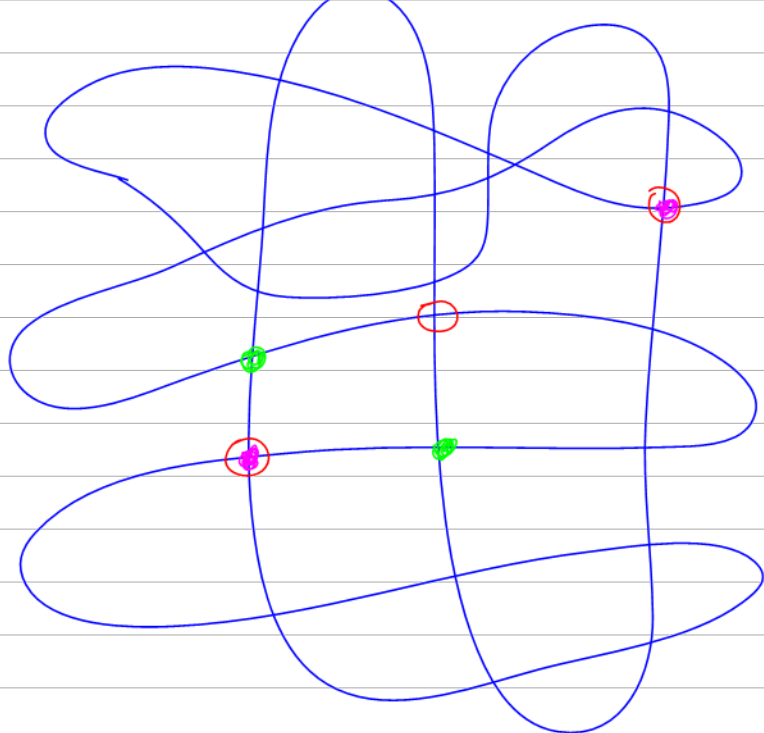
$$\sigma_{31}^{-1}$$

$$\sigma_{12}$$

$$\rho_1 = A^{-1} \rho_2 A$$



$$j=2$$



$$W(\beta^k) = kW(\beta)$$

$$\beta^k = \gamma^{-1} \beta^k \gamma$$

$$kW(\beta) = W(\beta^k) = W(\gamma^{-1} \beta^k \gamma)$$

$$= \pm 2 + W(\beta^k) = \pm 2 + kW(\beta)$$

$$\beta \in B_n$$

$$W(\beta) = \sum_{\emptyset \neq A \subseteq \underline{n}} (-1)^{|A|+n} W(\beta_A) / |A|$$

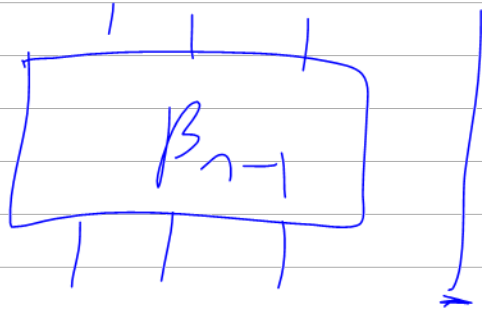
$$W(\gamma) = \frac{1}{2} - 0 - 0 = \frac{1}{2} \quad W(\Delta_n) = \binom{n}{2}$$

$$w(\Delta_n) = \sum_{k=1}^n (-1)^{k+n} \binom{n}{k} \binom{k}{2} \frac{1}{k}$$

$$= \frac{(-1)^n}{2} \sum_{k=1}^n (-1)^k (k-1) \binom{n}{k} =$$

$$= \frac{(-1)^n}{2} \left[\frac{d}{dx} (1-x)^n - (1-x)^n \right]_{x=1} - (-1)$$

$\sum_{k=0}^n (-1)^k \binom{n}{k} x^k$

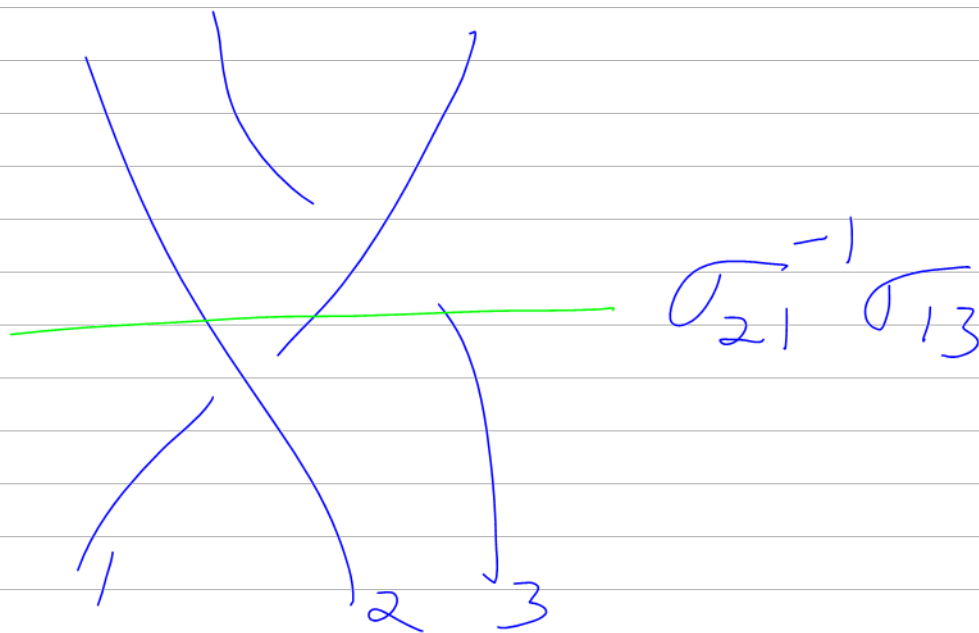


$$w(\beta) = \frac{(-1)^n}{n} \sum_{\emptyset \neq A \subset \underline{n}} (-1)^{|A|} w(\beta_A)$$

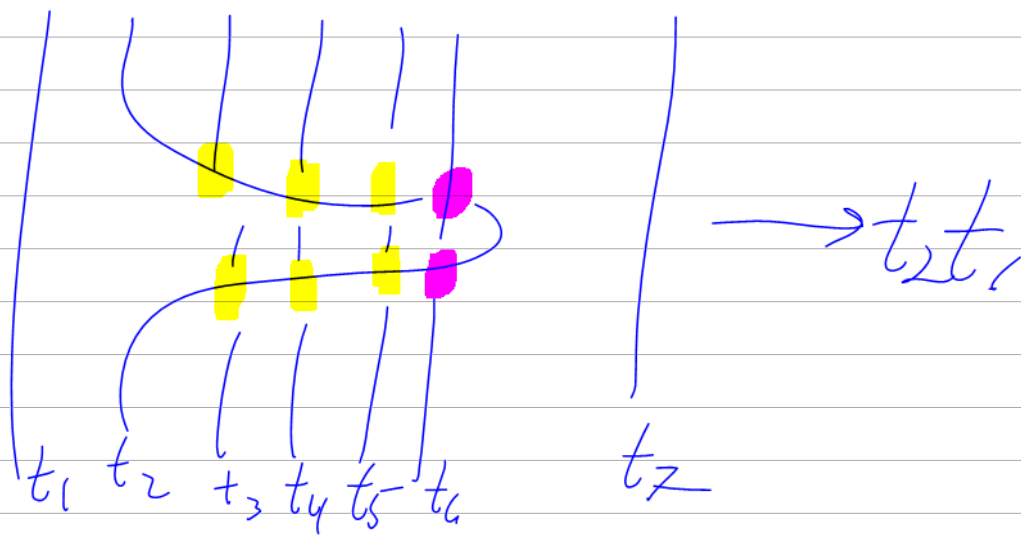
$$w(\Delta) = \frac{(-1)^n}{n} \sum_{k \geq 1} \binom{n}{k} (-1)^k \binom{k}{2}$$

$$= \frac{(-1)^n}{2n} \frac{d^2}{dx^2} (1-x)^n \Big|_{x=1}$$

$$\text{Exp}((t-1)t - (t^{-1}-1)t^2(t-1))$$



bas / $t_i \xrightarrow{\text{green}} \mathbb{Z}$



$$\begin{aligned} \mathbb{Z} : g_i &\longmapsto 1 + \bar{g}_i = 1 + t_i \\ g_i^{-1} &\longmapsto \frac{1}{1+t_i} = 1 - t_i + t_i^2 - t_i^3 + \dots \end{aligned}$$

$$\mathbb{Z} : FG_n \xrightarrow{1-1} \widehat{FA}_n$$

$$\widehat{FG}_n = \varprojlim_{p \rightarrow \infty} QFG_n / I^p$$

$$\mathbb{Z} : \widehat{FG}_n \xrightarrow{\sim} \widehat{FA}_n$$

$$\begin{aligned} \times \quad x^\infty &= x \cdot x \cdot x \cdot \dots \\ \checkmark \quad e^x &= \sum \frac{x^n}{n!} \end{aligned}$$

Words in t_i of length $\geq p$

$$\varprojlim_{q \geq p} I^p / I^q = \widehat{I}_G^p \xrightarrow{\sim} \widehat{I}_A^p = \prod_{q \geq p} \underbrace{I^q / I^{q+1}}_{\text{words of length } q}$$

$$\{(w_p, w_{p+1}, w_{p+2}, \dots)\}$$

magnus karrass solitar

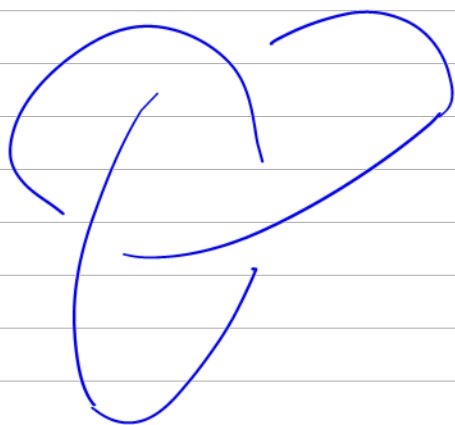
Injectivity of $\mathbb{Z}$: If $w = g_{i_1}^{k_1} \dots g_{i_n}^{k_n}$, $n \geq 1$, $\forall \alpha \ i_\alpha \neq i_{\alpha+1}$ ~~$k_\alpha \neq 0$~~ $\neg \tau s$, $\mathbb{Z}(w) \neq 1$.

PF Consider the coeff of $t_{i_1} \dots t_{i_n}$ in $\mathbb{Z}(w)$. It is $k_1 \cdot k_2 \cdot \dots \cdot k_n \neq 0$ $\square$

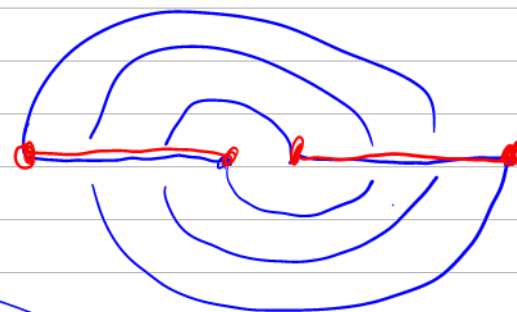
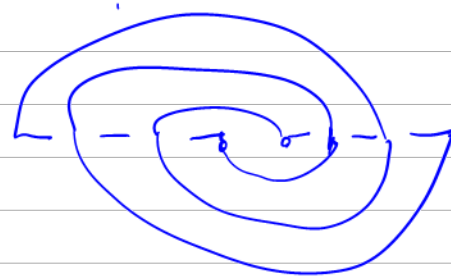
$$\begin{pmatrix} t_i & t_i(1-t_j) \\ 0 & t_i^2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & (1-t_j) \\ 0 & t_i \end{pmatrix}$$

$$\begin{pmatrix} 1-t_i & \\ & 1-t_j \end{pmatrix}$$



bridge # = 3



$$\rho: \text{PwB}_n \rightarrow \text{End}(V) ; k \quad F_n \triangleleft \text{PwB}_n$$

$$\rho^{+k}: \text{PwB}_n \rightarrow \text{End}(I^k \otimes_{F_n} V)$$

$$g_j = \sigma_{j,0}$$

$$\rho^{+k}(\beta)(x \otimes v) = x^\beta \otimes \rho(\beta)v$$

$$0 \rightarrow A \rightarrow A \oplus B \rightarrow B \rightarrow 0$$

$$\downarrow 0 \quad \downarrow \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \downarrow 0$$

$$0 \rightarrow C \rightarrow C \oplus D \rightarrow D \rightarrow 0$$

Transgression map

