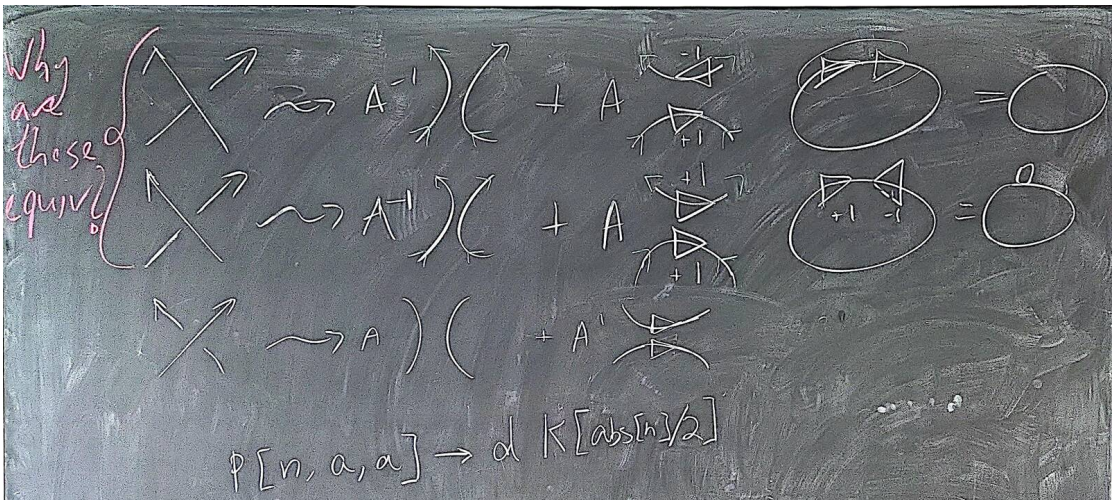
Pensieve Header: A very short implementation of the Arrow Polynomial.

https://drorbn.net/bbs/show?shot=SantosK-250717-132117.jpg :



In[•]:=
```
<< KnotTheory`
```

```
Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at http://katlas.org/wiki/KnotTheory.
```

In[•]:=
```
pd = PD[Knot[3, 1]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[•]=
```
PD[X[1, 4, 2, 5], X[3, 6, 4, 1], X[5, 2, 6, 3]]
```

In[•]:=
```
pd /. x : X[i_, j_, k_, l_] :> If[PositiveQ[x],
   A P0[i, j] P0[l, k] + A^-1 P1[l, i] P-1[j, k],
   A^-1 P0[i, l] P0[j, k] + A P1[i, j] P-1[k, l]
  ]
```

Out[•]=
$$\text{PD}\left[\frac{P_0[1,5]\,P_0[4,2]}{A}+A\,P_{-1}[2,5]\,P_1[1,4],\ \frac{P_0[3,1]\,P_0[6,4]}{A}+A\,P_{-1}[4,1]\,P_1[3,6],\ \frac{P_0[2,6]\,P_0[5,3]}{A}+A\,P_{-1}[6,3]\,P_1[5,2]\right]$$

```
In[•]:= Expand[Times @@ pd /. x : X[i_, j_, k_, l_] :> If[PositiveQ[x],
          A P0[i, j] P0[l, k] + A^-1 P1[l, i] P-1[j, k],
          A^-1 P0[i, l] P0[j, k] + A P1[i, j] P-1[k, l]
         ]]
```

Out[•]=

$$\frac{P_0[1,5]\,P_0[2,6]\,P_0[3,1]\,P_0[4,2]\,P_0[5,3]\,P_0[6,4]}{A^3} +$$
$$\frac{P_{-1}[2,5]\,P_0[2,6]\,P_0[3,1]\,P_0[5,3]\,P_0[6,4]\,P_1[1,4]}{A} +$$
$$\frac{P_{-1}[4,1]\,P_0[1,5]\,P_0[2,6]\,P_0[4,2]\,P_0[5,3]\,P_1[3,6]}{A} +$$
$$A\,P_{-1}[2,5]\,P_{-1}[4,1]\,P_0[2,6]\,P_0[5,3]\,P_1[1,4]\,P_1[3,6] +$$
$$\frac{P_{-1}[6,3]\,P_0[1,5]\,P_0[3,1]\,P_0[4,2]\,P_0[6,4]\,P_1[5,2]}{A} +$$
$$A\,P_{-1}[2,5]\,P_{-1}[6,3]\,P_0[3,1]\,P_0[6,4]\,P_1[1,4]\,P_1[5,2] +$$
$$A\,P_{-1}[4,1]\,P_{-1}[6,3]\,P_0[1,5]\,P_0[4,2]\,P_1[3,6]\,P_1[5,2] +$$
$$A^3\,P_{-1}[2,5]\,P_{-1}[4,1]\,P_{-1}[6,3]\,P_1[1,4]\,P_1[3,6]\,P_1[5,2]$$

```
In[•]:= Expand[Times @@ pd /. x : X[i_, j_, k_, l_] :> If[PositiveQ[x],
          A P0[i, j] P0[l, k] + A^-1 P1[l, i] P-1[j, k],
          A^-1 P0[i, l] P0[j, k] + A P1[i, j] P-1[k, l]
         ]] //. {
  Pα_[i_, j_] Pβ_[j_, k_] :> Pα+β[i, k],
  Pα_[i_, j_] Pβ_[k_, j_] :> Pα-β[i, k],
  Pα_[j_, i_] Pβ_[j_, k_] :> P-α+β[i, k]
 }
```

Out[•]=

$$\frac{P_0[2,4]^2}{A} + A\,P_0[2,6]^2\,P_0[4,4] + \frac{P_0[3,3]\,P_0[4,4]}{A^3} + \frac{P_0[4,6]^2}{A} + \frac{P_0[6,2]^2}{A} +$$
$$A\,P_0[2,2]\,P_0[6,4]^2 + A\,P_0[4,2]^2\,P_0[6,6] + A^3\,P_0[2,2]\,P_0[4,4]\,P_0[6,6]$$

```
AP[K_] := Module[{t},
  t = Expand[Times @@ PD[K] /. x : X[i_, j_, k_, l_] :> If[PositiveQ[x],
          A P0[i, j] P0[l, k] + A^-1 P1[l, i] P-1[j, k], A^-1 P0[i, l] P0[j, k] + A P1[i, j] P-1[k, l]
         ]] //. {
       Pα_[i_, j_] Pβ_[j_, k_] :> Pα+β[i, k],
       Pα_[i_, j_] Pβ_[k_, j_] :> Pα-β[i, k], Pα_[j_, i_] Pβ_[j_, k_] :> P-α+β[i, k]
      } /. {
      Pα_[k_, k_] :> (-A^2 - A^-2) VAbs@α/2, P_[_, _]^2 → (-A^2 - A^-2) V0
     } /. {V0 → 1, Pα_[i_, j_] /; i > j :> P-α[j, i]};
  Factor[t / (-A^2 - A^-2)]
 ]
```

```
In[•]:= AP[Knot[3, 1]]
```

Out[•]=

$$\frac{-1 - A^8 + A^{12}}{A^5}$$

```
In[•]:= AP[PD[X[2, 4, 3, 1], X[3, 1, 4, 2]]]
```

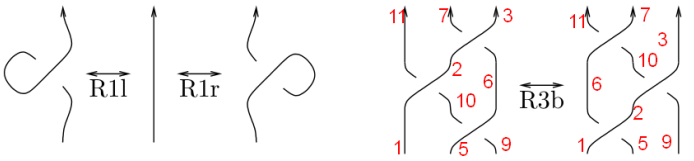
Out[•]=

$$-\frac{-1 - A^2\, V_1 + A^6\, V_1}{A^2}$$

```
In[•]:= AP@PD[X[2, 6, 3, 1], X[4, 1, 5, 2], X[5, 4, 6, 3]]
```

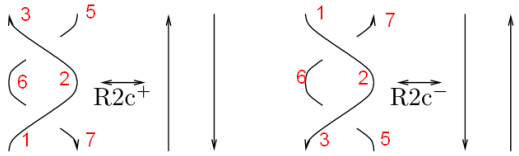
Out[•]=

$$-\frac{V_1^2 + A^4\, V_1^2 - A^4\, V_2}{A^3}$$



R1-left and R1-right

braid-like R3



counterclockwise and clockwise cyclic R2

```
In[•]:= AP@PD[X[5, 3, 6, 2], X[6, 1, 7, 2]]
```

Out[•]=

$$-\frac{A^2\, P_0[1, 3]\, P_0[5, 7]}{1 + A^4}$$

```
In[•]:= AP@PD[X[5, 2, 6, 3], X[6, 2, 7, 1]]
```

Out[•]=

$$-\frac{A^2\, P_0[1, 3]\, P_0[5, 7]}{1 + A^4}$$

```
In[•]:= lhs = AP@PD[X[9, 6, 10, 5], X[10, 2, 11, 1], X[6, 3, 7, 2]]
```

Out[•]=

$$-\frac{1}{1 + A^4}\, A \Big(A^4\, P_0[1, 11]\, P_0[3, 9]\, P_0[5, 7] + P_{-2}[9, 11]\, P_{-1}[3, 7]\, P_1[1, 5] + A^2\, P_{-1}[7, 11]\, P_0[3, 9]\, P_1[1, 5] + A^2\, P_{-1}[3, 7]\, P_0[1, 11]\, P_1[5, 9] + P_{-1}[7, 11]\, P_1[5, 9]\, P_2[1, 3]\Big)$$

In[•]:=
```
rhs = AP@PD[X[5, 2, 6, 1], X[9, 3, 10, 2], X[10, 7, 11, 6]]
```

Out[•]=

$$-\frac{1}{1+A^4}\,A\left(A^4\,P_0[1,11]\,P_0[3,9]\,P_0[5,7]+P_{-2}[9,11]\,P_{-1}[3,7]\,P_1[1,5]+A^2\,P_{-1}[7,11]\,P_0[3,9]\,P_1[1,5]+A^2\,P_{-1}[3,7]\,P_0[1,11]\,P_1[5,9]+P_{-1}[7,11]\,P_1[5,9]\,P_2[1,3]\right)$$

In[•]:=
```
lhs == rhs
```

Out[•]=
```
True
```