$L: V \to W$ $\quad L \in V^* \otimes W$ $\quad V^* = \langle \varphi_i \rangle$ $\quad V = \langle v^j \rangle$ $\quad \varphi_i(v^j) = \delta_i^j$

$W = \langle w^i \rangle$

$$L = \sum_{ij} A^i_j \varphi_i \cdot w^j$$

$$L(v^{i'}) = \sum_{ij} A^i_j \varphi_i(v^{i'}) \cdot w^j = \sum_j A^{i'}_j w^j$$

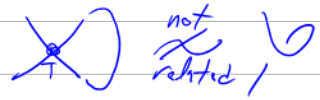
Tensor: "A matrix w/ any number of subscripts or superscripts"

$$T_{ijkl} \rightsquigarrow \sum_{ij,kl} T_{ijkl} w^i \otimes w^j \otimes w^k \otimes w^l \in W^{\otimes 4}$$

$$T^{ij}_{kl} \longrightarrow C^i_k = \sum_j T^{ij}_{kj} \quad \text{"Tensor contractions"}$$

Example $\sum_i A^i_i = tr(A)$

T^{ij}_{kl}

$C^i_k = \sum_j T^{ij}_{kj}$ 

$T^{ij}_{kl} \cdot A^m_n$

$\times \frac{217}{67}$

---

xings#1 $\quad$ xings#n

$a_1 a_2 a_3 \ldots a_n = \Delta$

$(a_1 + \epsilon b_1)(a_2 + \epsilon b_2 + \ldots) \ldots (a_n + \epsilon b_n) = \Delta + \epsilon \Delta_1 + \epsilon^2 (\Delta_2)$

$\sum_{\text{xings}}$ all is a_i's w/ one b_i

$\sum_{\text{xings} \times \text{xings}}$ all a_i's w/ two b_i's

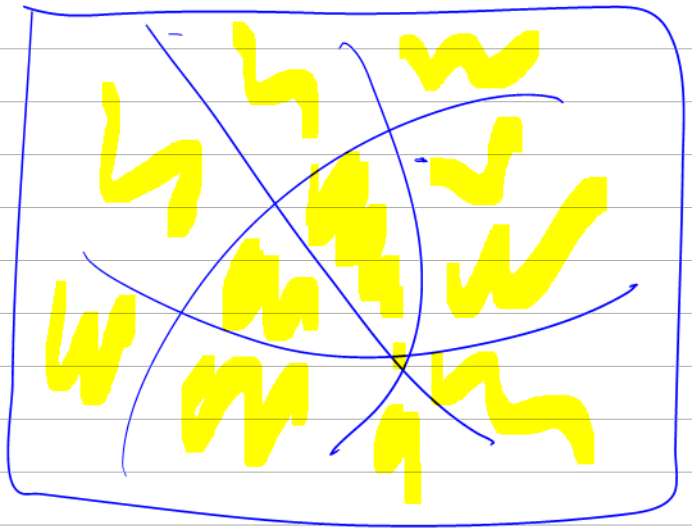
http://drorbn.net/oa22

$n \gg 1$ $\quad e^{\sqrt{n}} > n^{20}$ Theory

$e^{\sqrt{n}} < n^{20}$ practice

$\sqrt{n} \doteq 20 \log n \quad \frac{\sqrt{n}}{\log n} = 20$

$n \sim 46{,}000$



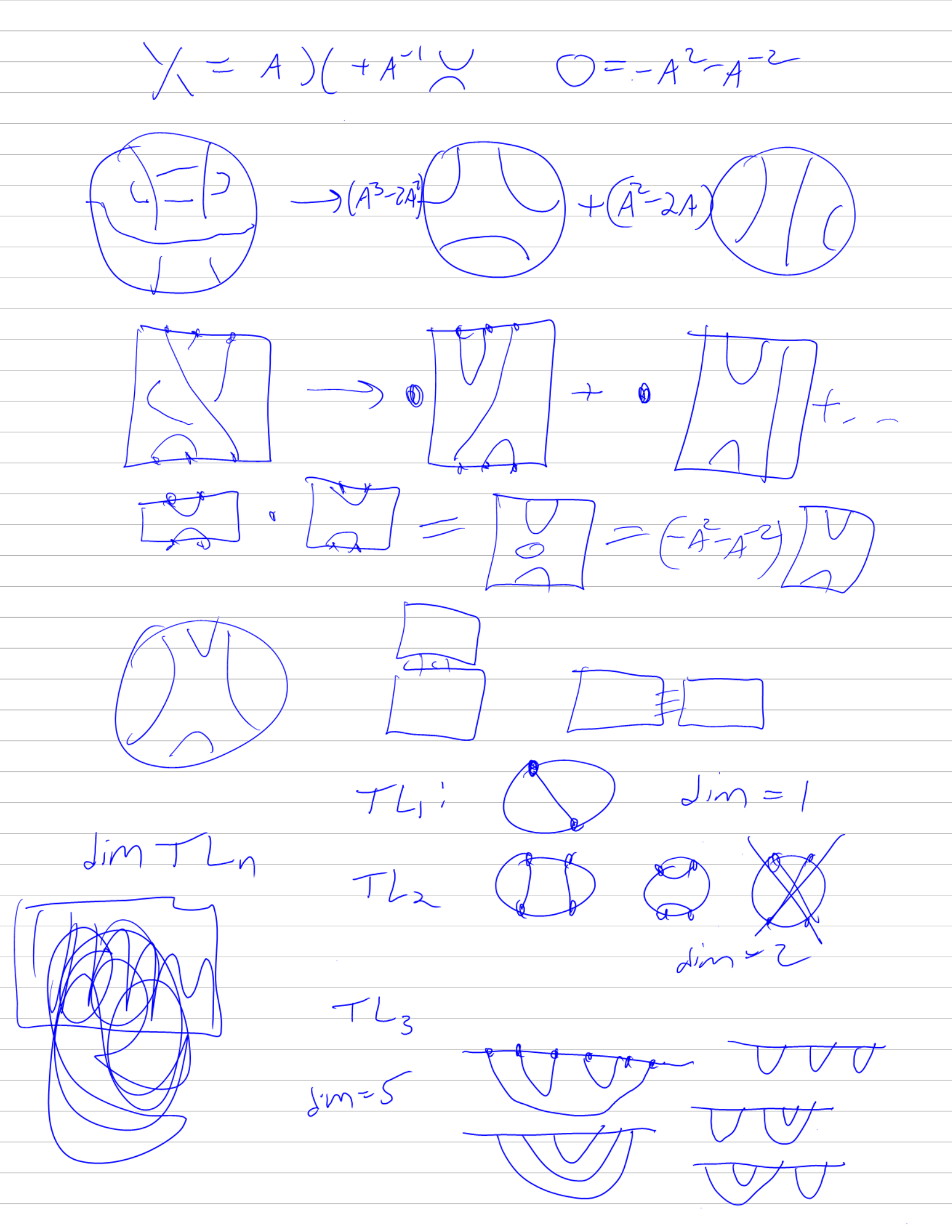
3 vectors in $\mathbb{R}^3$ are lin dep generically.

Cayley-Hamilton Thm: $A \in M_{n\times n}(\mathbb{C})$

$\chi(\lambda) = \det(\lambda I - A)$

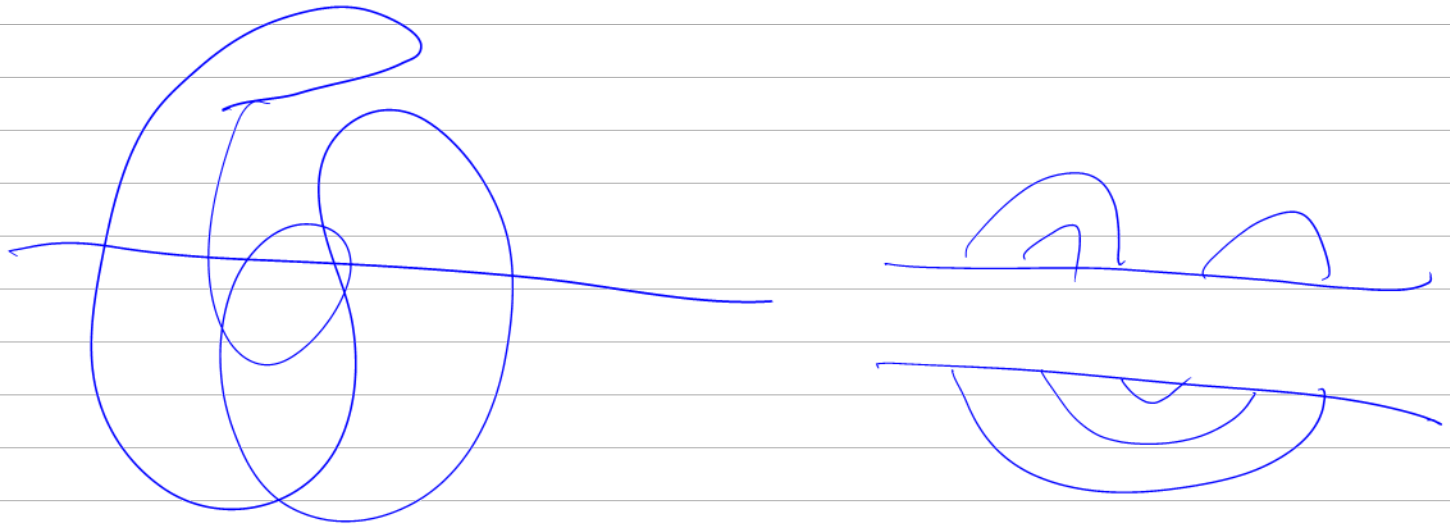
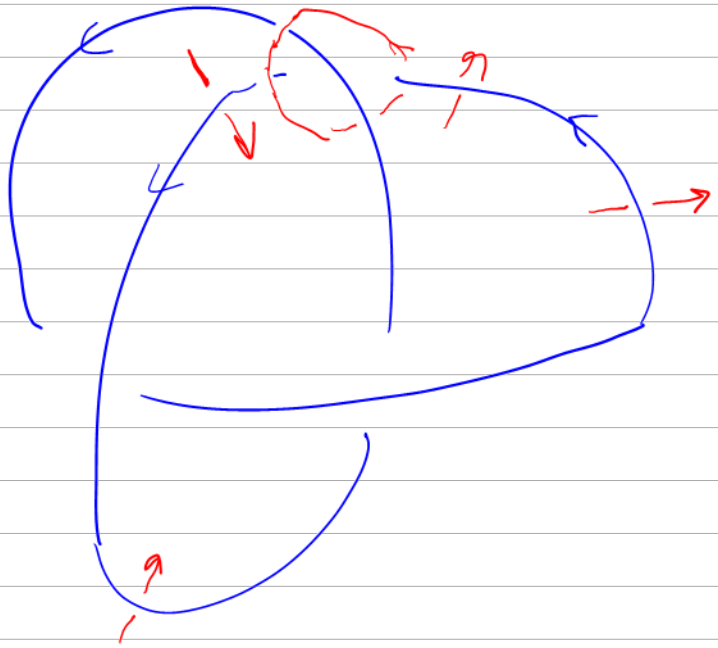
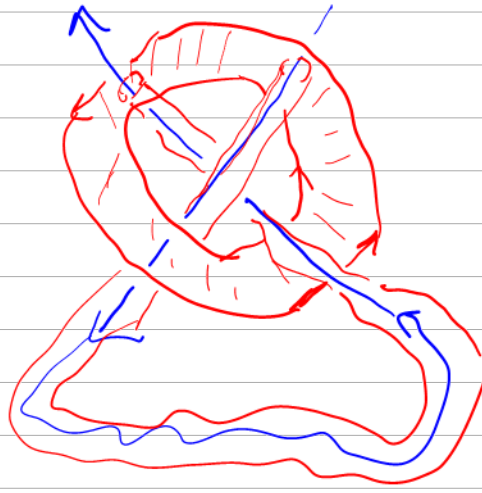
Then $\chi(A) = 0$.

generically $A = P \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix} P^{-1}$


$\times = A)(+ A^{-1} \smile$
$\bigcirc = -A^2 - A^{-2}$
$\to (A^3 - 2A)$
$+ (A^2 - 2A)$
$+ \cdots$
$= (-A^2 - A^{-2})$
TL_1 ; $\dim = 1$
$\dim TL_n$
TL_2
$\dim = 2$
TL_3
$\dim = 5$

$\dim TL_n = C_n$ catalan$(n) \sim 4^n$

$\dim TL_4 = 14$ $\dim TL_5 = 42$



$C_0 =$ $\dim C_0 = 1$