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HomflyptCoefficient[K_PD, n_] :=
Module[{m, L, OK, UnderCrossingQ, P, Linking},
m = 2 Crossings[K];
L = NormalizePD[K];
OK = OrderKnot[L];
UnderCrossingQ[i_] := (First @ OK[i] == i);
P[k_, i_, j_] := P[k, i, j] = Expand @ Module[{t},
Piecewise[{
{If[k == 0, 1, 0], j - i ≤ 4},
{P[k, i + 1, j], t = FirstEdge @@ OK[i][[{2, 4}]]};
Not @ UnderCrossingQ[i] || Not[i < t ≤ j]}},
If[PositiveQ[OK[i]],
-a-2 P[k, i + 1, j] + a-1 P[k, i, j, t],
-a2 P[k, i + 1, j] + a P[k, i, j, t]
]];
Linking[i_, j_, t_] :=
Sum[If[UnderCrossingQ[1] &&
(i + 1 ≤ FirstEdge @@ OK[1][[{2, 4}]] ≤ t - 1),
If[PositiveQ[OK[1]], 1, -1], 0], {1, t + 1, j - 1}];
P[0, i_, j_, t_] := (-a-2)Linking[i,j,t] (a + a-1)
P[0, i + 1, t] P[0, t + 1, j];
P[k_, i_, j_, t_] :=
Module[{FlipCheck, e, Smooth, D, T, D0},
FlipCheck[X[a_, b_, c_, d_]] := (e = FirstEdge[b, d];
(a < i && Not[a < e < i]) || (a ≥ j && a > e > i));
Smooth[D_, s_] := Module[{r, d1, d2, d3, A},
r = FirstEdge @@ OK[s][[{2, 4}]]];
d1 = i + s - (t + 1); d2 = d1 + t - (r + 1);
d3 = d2 + r - (i + 1);
A = AssociationThread[
Range[0, i] ~Join~ Range[t + 1, s] ~Join~
Range[r + 1, t] ~Join~ Range[i + 1, r] ~Join~
Range[s + 1, m - 1] →
Range[0, i] ~Join~ Range[i, d1] ~Join~
Range[d1, d2] ~Join~ Range[d2, d3] ~Join~
Range[d3, m - 5]];
D /. X[a_, b_, c_, d_] :=
If[MatchQ[X[a, b, c, d], OK[s]], ## &[],
X[A[a], A[b], A[c], A[d]]];
T[j] := (a + a-1) Sum[P[2 r, i + 1, t] P[k - 2 r, t + 1, j],
{r, 0,  $\frac{k}{2}$ }]];
T[s_] :=
If[UnderCrossingQ[s] &&
i + 1 ≤ FirstEdge @@ OK[s][[{2, 4}]] ≤ t - 1,
D0 = Smooth[D, s];
D = D /. OK[s] := FlipCrossing[OK[s]];
If[PositiveQ[OK[s]],
a-1 HomflyptCoefficient[D0, k - 2] - a-2 T[s + 1],
a HomflyptCoefficient[D0, k - 2] - a2 T[s + 1]],
T[s + 1]];
D =
L /. x_X := If[MatchQ[x, OK[i]], ## &[],
If[FlipCheck[x], FlipCrossing[x], x]];
T[t + 1]
];
P[n, 0, m]
];

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