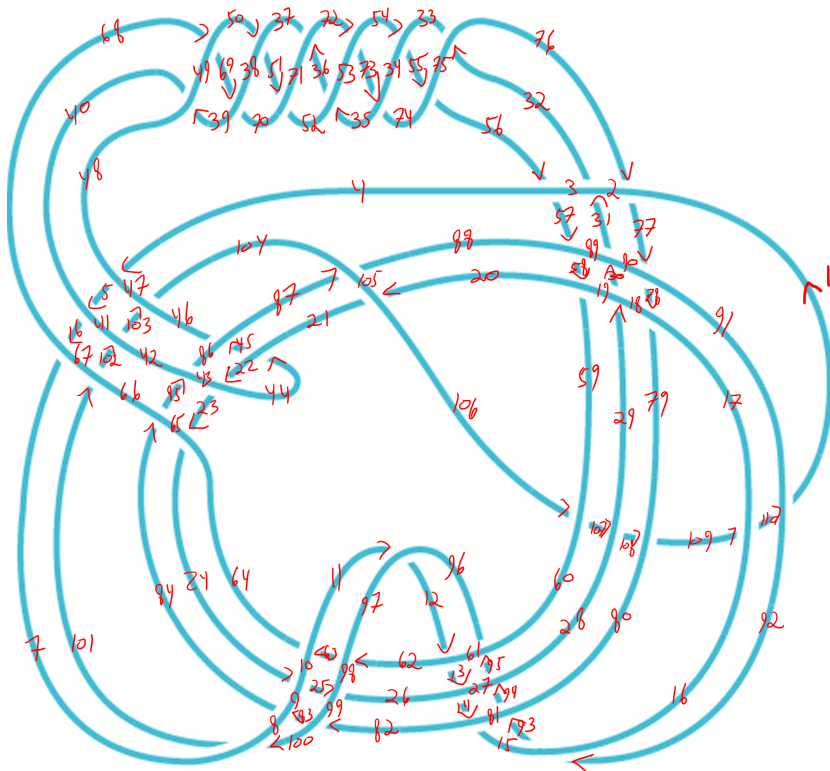Pensieve header: A fast Jones program for Stavros. Based on pensieve://Classes/23-FastComputations/.

In[ ]:= **<< KnotTheory`**

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

In[ ]:= **PK = PD[**
    X[4, 48, 5, 47], X[5, 40, 6, 41], X[6, 68, 7, 67], X[11, 97, 12, 96],
    X[12, 62, 13, 61], X[13, 26, 14, 27], X[14, 82, 15, 81], X[20, 105, 21, 106],
    X[22, 43, 23, 44], X[23, 65, 24, 64], X[24, 9, 25, 10], X[25, 99, 26, 98],
    X[29, 18, 30, 19], X[30, 90, 31, 89], X[31, 2, 32, 3], X[32, 76, 33, 75],
    X[35, 53, 36, 52], X[36, 72, 37, 71], X[39, 49, 40, 48], X[44, 21, 45, 22],
    X[45, 87, 46, 86], X[50, 38, 51, 37], X[51, 70, 52, 71], X[54, 34, 55, 33],
    X[55, 74, 56, 75], X[56, 4, 57, 3], X[57, 88, 58, 89], X[58, 20, 59, 19],
    X[62, 97, 63, 98], X[63, 11, 64, 10], X[68, 49, 69, 50], X[69, 39, 70, 38],
    X[72, 53, 73, 54], X[73, 35, 74, 34], X[76, 2, 77, 1], X[77, 90, 78, 91],
    X[78, 18, 79, 17], X[82, 99, 83, 100], X[83, 9, 84, 8], X[84, 65, 85, 66],
    X[85, 43, 86, 42], X[87, 105, 88, 104], X[92, 16, 93, 15], X[93, 80, 94, 81],
    X[94, 28, 95, 27], X[95, 60, 96, 61], X[100, 8, 101, 7], X[101, 66, 102, 67],
    X[102, 42, 103, 41], X[103, 46, 104, 47], X[106, 60, 107, 59], X[107, 28, 108, 29],
    X[108, 80, 109, 79], X[109, 16, 110, 17], X[110, 92, 1, 91]
    ];

```mathematica
In[∘]:= J1[pd_] := Module[{w, p, t1, t2, t3, t4, t5},
    w = Plus @@ (pd /. {
        X[i_, j_, k_, l_] /; j - l == 1 ⋁ l - j > 1 :> 1,
        X[i_, j_, k_, l_] /; l - j == 1 ⋁ j - l > 1 :> -1
      });
    SetAttributes[p, Orderless];
    t1 = pd /. X[i_, j_, k_, l_] :> A p[i, j] p[k, l] + B p[i, l] p[j, k];
    t2 = Expand[t1 /. PD → Times];
    t3 = t2 //. p[i_, j_] p[j_, k_] :> p[i, k];
    t4 = t3 /. {p[i_, i_] :> d, p[i_, j_]^2 :> d};
    t5 = Expand[t4 /. {B → 1 / A, d → -A^2 - A^-2}];
    Simplify[(-A^3)^-w  t5/(-A^2 - A^-2)  /. A → q^(-1/4)]
  ]
```

```mathematica
In[∘]:= pd = PD@Knot[3, 1]
```

⟨⋯⟩ KnotTheory: Loading precomputed data in PD4Knots`.

```mathematica
Out[∘]=
    PD[X[1, 4, 2, 5], X[3, 6, 4, 1], X[5, 2, 6, 3]]
```

```mathematica
In[∘]:= pd /. {
    X[i_, j_, k_, l_] /; j - l == 1 ⋁ l - j > 1 :> 1,
    X[i_, j_, k_, l_] /; l - j == 1 ⋁ j - l > 1 :> -1
   }
```

```mathematica
Out[∘]=
    PD[-1, -1, -1]
```

```mathematica
In[∘]:= w = Plus @@ (pd /. {
      X[i_, j_, k_, l_] /; j - l == 1 ⋁ l - j > 1 :> 1,
      X[i_, j_, k_, l_] /; l - j == 1 ⋁ j - l > 1 :> -1
     })
```

```mathematica
Out[∘]=
    -3
```

```mathematica
In[∘]:= SetAttributes[p, Orderless];
```

```mathematica
In[∘]:= t1 = pd /. X[i_, j_, k_, l_] :> A p[i, j] p[k, l] + B p[i, l] p[j, k]
```

```mathematica
Out[∘]=
    PD[B p[1, 5] p[2, 4] + A p[1, 4] p[2, 5],
     A p[1, 4] p[3, 6] + B p[1, 3] p[4, 6], B p[2, 6] p[3, 5] + A p[2, 5] p[3, 6]]
```

In[ ]:= **t2 = Expand[t1 /. PD → Times]**

Out[ ]=

$A B^2 p[1, 4] p[1, 5] p[2, 4] p[2, 6] p[3, 5] p[3, 6] +$
$A^2 B p[1, 4]^2 p[2, 5] p[2, 6] p[3, 5] p[3, 6] + A^2 B p[1, 4] p[1, 5] p[2, 4] p[2, 5] p[3, 6]^2 +$
$A^3 p[1, 4]^2 p[2, 5]^2 p[3, 6]^2 + B^3 p[1, 3] p[1, 5] p[2, 4] p[2, 6] p[3, 5] p[4, 6] +$
$A B^2 p[1, 3] p[1, 4] p[2, 5] p[2, 6] p[3, 5] p[4, 6] +$
$A B^2 p[1, 3] p[1, 5] p[2, 4] p[2, 5] p[3, 6] p[4, 6] +$
$A^2 B p[1, 3] p[1, 4] p[2, 5]^2 p[3, 6] p[4, 6]$

In[ ]:= **t3 = t2 //. p[i_, j_] p[j_, k_] ⧴ p[i, k]**

Out[ ]=

$A^3 p[1, 4]^2 p[2, 5]^2 p[3, 6]^2 + A^2 B p[3, 6]^2 p[4, 5]^2 +$
$A^2 B p[2, 5]^2 p[4, 6]^2 + B^3 p[3, 5]^2 p[4, 6]^2 + 3 A B^2 p[5, 6]^2 + A^2 B p[1, 4]^2 p[5, 6]^2$

In[ ]:= **t4 = t3 /. {p[i_, i_] ⧴ d, p[i_, j_]^2 ⧴ d}**

Out[ ]=

$3 A B^2 d + 3 A^2 B d^2 + B^3 d^2 + A^3 d^3$

In[ ]:= **t5 = Expand[t4 /. {B → 1 / A, d → -A^2 - A^{-2}}];**
Simplify[(-A^3)^{-w} $\dfrac{t5}{-A^2 - A^{-2}}$ /. A → q^{-1/4}]

Out[ ]=

$\dfrac{-1 + q + q^3}{q^4}$

```mathematica
In[*]:= J2[pd_] := Module[{w, front, TL, todo, v, x, t1, t2, t3, t4, B, d},
          w = Plus @@ (pd /. {
                X[i_, j_, k_, l_] /; j - l == 1 ∨ l - j > 1 :> 1,
                X[i_, j_, k_, l_] /; l - j == 1 ∨ j - l > 1 :> -1
              });
          SetAttributes[p, Orderless];
          front = {};
          TL = 1;
          todo = List @@ pd;
          v[x_] := Length[front ⋂ List @@ x];
          While[Length[todo] > 0,
            Echo[v /@ todo];
            x = Echo@RandomChoice[MaximalBy[todo, v]];
            t1 = TL (x /. X[i_, j_, k_, l_] :> A p[i, j] p[k, l] + B p[i, l] p[j, k]);
            t2 = Expand[t1];
            t3 = t2 //. p[i_, j_] p[j_, k_] :> p[i, k];
            t4 = t3 /. {p[i_, i_] :> d, p[i_, j_]^2 :> d};
            TL = Expand[t4 /. {B → 1 / A, d → -A^2 - A^-2}];
            todo = Complement[todo, {x}];
            front = Complement[front ⋃ List @@ x, front ⋂ List @@ x]
          ];
          Simplify[(-A^3)^-w  TL/(-A^2 - A^-2)  /. A → q^(-1/4)]
        ]
```

```mathematica
In[*]:= pd = PD@Knot[4, 1]
```

```mathematica
Out[*]=
        PD[X[4, 2, 5, 1], X[8, 6, 1, 5], X[6, 3, 7, 4], X[2, 7, 3, 8]]
```

```mathematica
In[*]:= w = w = Plus @@ (pd /. {
                X[i_, j_, k_, l_] /; j - l == 1 ∨ l - j > 1 :> 1,
                X[i_, j_, k_, l_] /; l - j == 1 ∨ j - l > 1 :> -1
              })
```

```mathematica
Out[*]=
        0
```

```
In[◦]:= SetAttributes[p, Orderless];
        front = {}
        TL = 1
        todo = List @@ pd
```

```
Out[◦]=
        {}
```

```
Out[◦]=
        1
```

```
Out[◦]=
        {X[4, 2, 5, 1], X[8, 6, 1, 5], X[6, 3, 7, 4], X[2, 7, 3, 8]}
```

```
In[◦]:= v[x_] := Length[front ⋂ List @@ x]; (* v for "value" *)
```

```
In[◦]:= v /@ todo
```

```
Out[◦]=
        {0, 0, 0, 0}
```

```
In[◦]:= MaximalBy[todo, v]
```

```
Out[◦]=
        {X[4, 2, 5, 1], X[8, 6, 1, 5], X[6, 3, 7, 4], X[2, 7, 3, 8]}
```

```
In[◦]:= x = RandomChoice[MaximalBy[todo, v]]
```

```
Out[◦]=
        X[8, 6, 1, 5]
```

```
In[◦]:= t1 = TL (x /. X[i_, j_, k_, l_] :> A p[i, j] p[k, l] + B p[i, l] p[j, k])
```

```
Out[◦]=
        B p[1, 6] p[5, 8] + A p[1, 5] p[6, 8]
```

```
In[◦]:= t2 = Expand[t1];
        t3 = t2 //. p[i_, j_] p[j_, k_] :> p[i, k];
        t4 = t3 /. {p[i_, i_] :> d, p[i_, j_]^2 :> d};
        TL = Expand[t4 /. {B → 1 / A, d → -A^2 - A^-2}]
```

```
Out[◦]=
        p[1, 6] p[5, 8]
        ─────────────── + A p[1, 5] p[6, 8]
               A
```

```
In[◦]:= todo = Complement[todo, {x}]
```

```
Out[◦]=
        {X[2, 7, 3, 8], X[4, 2, 5, 1], X[6, 3, 7, 4]}
```

```
In[◦]:= front = Complement[front ⋃ List @@ x, front ⋂ List @@ x]
```

```
Out[◦]=
        {1, 5, 6, 8}
```

```
In[◦]:= v /@ todo
```

```
Out[◦]=
        {1, 2, 1}
```

In[◦]:= `MaximalBy[todo, v]`

Out[◦]=

$\{X[4, 2, 5, 1]\}$

In[◦]:= `x = RandomChoice[MaximalBy[todo, v]]`

Out[◦]=

$X[4, 2, 5, 1]$

In[◦]:= `t1 = TL (x /. X[i_, j_, k_, l_] :> A p[i, j] p[k, l] + B p[i, l] p[j, k])`

Out[◦]=

$(A\,p[1, 5]\,p[2, 4] + B\,p[1, 4]\,p[2, 5])\left(\dfrac{p[1, 6]\,p[5, 8]}{A} + A\,p[1, 5]\,p[6, 8]\right)$

In[◦]:= `t2 = Expand[t1];`
`t3 = t2 //. p[i_, j_] p[j_, k_] :> p[i, k];`
`t4 = t3 /. {p[i_, i_] :> d, p[i_, j_]^2 :> d};`
`TL = Expand[t4 /. {B -> 1 / A, d -> -A^2 - A^{-2}}]`

Out[◦]=

$\dfrac{p[2, 8]\,p[4, 6]}{A^2} + p[2, 4]\,p[6, 8] - A^4\,p[2, 4]\,p[6, 8]$

In[◦]:= `todo = Complement[todo, {x}]`

Out[◦]=

$\{X[2, 7, 3, 8], X[6, 3, 7, 4]\}$

In[◦]:= `front = Complement[front ⋃ List @@ x, front ⋂ List @@ x]`

Out[◦]=

$\{2, 4, 6, 8\}$

In[◦]:= `v /@ todo`

Out[◦]=

$\{2, 2\}$

In[◦]:= `MaximalBy[todo, v]`

Out[◦]=

$\{X[2, 7, 3, 8], X[6, 3, 7, 4]\}$

In[◦]:= `x = RandomChoice[MaximalBy[todo, v]]`

Out[◦]=

$X[6, 3, 7, 4]$

In[◦]:= `t1 = TL (x /. X[i_, j_, k_, l_] :> A p[i, j] p[k, l] + B p[i, l] p[j, k])`

Out[◦]=

$(B\,p[3, 7]\,p[4, 6] + A\,p[3, 6]\,p[4, 7])\left(\dfrac{p[2, 8]\,p[4, 6]}{A^2} + p[2, 4]\,p[6, 8] - A^4\,p[2, 4]\,p[6, 8]\right)$

```
In[ ]:= t2 = Expand[t1];
        t3 = t2 //. p[i_, j_] p[j_, k_] :> p[i, k];
        t4 = t3 /. {p[i_, i_] :> d, p[i_, j_]^2 :> d};
        TL = Expand[t4 /. {B -> 1 / A, d -> -A^2 - A^-2}]
```

$$Out[ ]= \quad -\frac{p[2, 8]\, p[3, 7]}{A^5} + \frac{p[2, 8]\, p[3, 7]}{A} - A^3\, p[2, 8]\, p[3, 7] + A\, p[2, 7]\, p[3, 8] - A^5\, p[2, 7]\, p[3, 8]$$

```
In[ ]:= todo = Complement[todo, {x}]
```

$$Out[ ]= \quad \{X[2, 7, 3, 8]\}$$

```
In[ ]:= front = Complement[front ⋃ List @@ x, front ⋂ List @@ x]
```

$$Out[ ]= \quad \{2, 3, 7, 8\}$$

```
In[ ]:= v /@ todo
```

$$Out[ ]= \quad \{4\}$$

```
In[ ]:= MaximalBy[todo, v]
```

$$Out[ ]= \quad \{X[2, 7, 3, 8]\}$$

```
In[ ]:= x = RandomChoice[MaximalBy[todo, v]]
```

$$Out[ ]= \quad X[2, 7, 3, 8]$$

```
In[ ]:= t1 = TL (x /. X[i_, j_, k_, l_] :> A p[i, j] p[k, l] + B p[i, l] p[j, k])
```

$$Out[ ]= \quad (B\, p[2, 8]\, p[3, 7] + A\, p[2, 7]\, p[3, 8])$$
$$\left(-\frac{p[2, 8]\, p[3, 7]}{A^5} + \frac{p[2, 8]\, p[3, 7]}{A} - A^3\, p[2, 8]\, p[3, 7] + A\, p[2, 7]\, p[3, 8] - A^5\, p[2, 7]\, p[3, 8]\right)$$

```
In[ ]:= t2 = Expand[t1];
        t3 = t2 //. p[i_, j_] p[j_, k_] :> p[i, k];
        t4 = t3 /. {p[i_, i_] :> d, p[i_, j_]^2 :> d};
        TL = Expand[t4 /. {B -> 1 / A, d -> -A^2 - A^-2}]
```

$$Out[ ]= \quad -\frac{1}{A^{10}} - A^{10}$$

```
In[ ]:= todo = Complement[todo, {x}]
```

$$Out[ ]= \quad \{\}$$

```
In[ ]:= front = Complement[front ⋃ List @@ x, front ⋂ List @@ x]
```

$$Out[ ]= \quad \{\}$$

$In[•]:=$ **Simplify$\left[\left(-A^3 \right)^{-w} \dfrac{TL}{-A^2 - A^{-2}} \ /. \ A \to q^{-1/4} \right]$**

$Out[•]=$

$$1 + \frac{1}{q^2} - \frac{1}{q} - q + q^2$$

$In[•]:=$ **Timing@J2[PK]**

» {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

» X[103, 46, 104, 47]

» {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}

» X[4, 48, 5, 47]

» {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}

» X[39, 49, 40, 48]

» {2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1,
    0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}

» X[5, 40, 6, 41]

» {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0,
    0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0}

» X[102, 42, 103, 41]

» {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1,
    0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}

» X[87, 105, 88, 104]

» {1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0,
    1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}

» X[45, 87, 46, 86]

» {1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1,
    1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}

» X[85, 43, 86, 42]

» {1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0,
    1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}

» X[44, 21, 45, 22]

» {1, 0, 0, 0, 0, 2, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,
    1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}

» X[22, 43, 23, 44]

» {1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,
    1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}

» X[20, 105, 21, 106]

» {1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1,
   1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0}

» X[69, 39, 70, 38]

» {1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1,
   1, 1, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0}

» X[68, 49, 69, 50]

» {2, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0,
   1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0}

» X[50, 38, 51, 37]

» {2, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 1,
   1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0}

» X[6, 68, 7, 67]

» {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 1,
   1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 2, 1, 0, 0, 0, 0}

» X[101, 66, 102, 67]

» {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 1,
   1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0}

» X[100, 8, 101, 7]

» {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0,
   1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0}

» X[51, 70, 52, 71]

» {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 1,
   1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0}

» X[36, 72, 37, 71]

» {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 2, 0, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0}

» X[84, 65, 85, 66]

» {0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 2, 0, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0}

» X[35, 53, 36, 52]

» {0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 2, 1, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0}

» X[23, 65, 24, 64]

» {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 2, 1, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0}

» X[72, 53, 73, 54]

» {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 0, 1, 2, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0}

» X[73, 35, 74, 34]

» {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 1, 1, 1, 1, 0, 1, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0}

» X[54, 34, 55, 33]

» {0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 2, 1, 1, 1, 0, 1, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0}

» X[55, 74, 56, 75]

» {0, 0, 0, 0, 1, 0, 0, 0, 0, 2, 2, 1, 1, 0, 1, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0}

» X[56, 4, 57, 3]

» {0, 0, 0, 0, 1, 0, 0, 0, 1, 2, 2, 1, 0, 1, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0}

» X[83, 9, 84, 8]

» {0, 0, 0, 0, 2, 0, 0, 0, 1, 2, 2, 1, 0, 1, 0, 0, 0, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0}

» X[24, 9, 25, 10]

» {0, 0, 0, 0, 1, 0, 0, 1, 2, 2, 1, 0, 2, 0, 0, 0, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0}

» X[32, 76, 33, 75]

» {0, 0, 0, 0, 1, 0, 0, 2, 2, 1, 0, 2, 1, 0, 0, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0}

» X[31, 2, 32, 3]

» {0, 0, 0, 0, 1, 0, 1, 2, 1, 0, 2, 2, 0, 0, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0}

» X[63, 11, 64, 10]

» {1, 0, 0, 0, 1, 0, 1, 2, 1, 1, 2, 0, 0, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0}

» X[76, 2, 77, 1]

» {1, 0, 0, 0, 1, 0, 1, 2, 1, 1, 1, 0, 2, 0, 0, 0, 0, 1, 0, 0, 0, 1}

» X[82, 99, 83, 100]

» {1, 0, 0, 1, 2, 0, 1, 2, 1, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1}

» X[57, 88, 58, 89]

» {1, 0, 0, 1, 2, 0, 2, 2, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1}

» X[25, 99, 26, 98]

» {1, 0, 1, 1, 0, 2, 2, 2, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1}

» X[58, 20, 59, 19]

» {1, 0, 1, 1, 1, 2, 2, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 1}

» X[62, 97, 63, 98]

» {2, 1, 1, 1, 1, 2, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 1}

» X[30, 90, 31, 89]

» {2, 1, 1, 1, 2, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 1}

» X[29, 18, 30, 19]

» {2, 1, 1, 1, 2, 1, 0, 0, 0, 0, 2, 1, 0, 0, 1}

» X[11, 97, 12, 96]

» {2, 1, 1, 2, 1, 0, 0, 0, 1, 2, 1, 0, 0, 1}

» X[106, 60, 107, 59]

» {2, 1, 1, 2, 1, 0, 0, 0, 2, 2, 0, 0, 1}

» X[107, 28, 108, 29]

» {2, 1, 1, 2, 1, 0, 0, 1, 2, 1, 0, 1}

» X[77, 90, 78, 91]

» {2, 1, 1, 2, 0, 0, 1, 2, 1, 0, 2}

» X[78, 18, 79, 17]

» {2, 1, 1, 0, 0, 1, 2, 2, 1, 2}

» X[108, 80, 109, 79]

» {2, 1, 1, 0, 1, 1, 2, 2, 2}

» X[12, 62, 13, 61]

» {2, 1, 0, 1, 1, 3, 2, 2}

» X[95, 60, 96, 61]

» {2, 1, 0, 1, 2, 2, 2}

» X[109, 16, 110, 17]

» {2, 1, 1, 1, 2, 3}

» X[110, 92, 1, 91]

» {2, 1, 2, 1, 2}

» X[13, 26, 14, 27]

» {2, 2, 1, 3}

» X[94, 28, 95, 27]

» {2, 2, 2}

» X[14, 82, 15, 81]

» {3, 3}

» X[93, 80, 94, 81]

» {4}

» X[92, 16, 93, 15]

Out[ ]=

$$\left\{3.35938, \; -\frac{1 - 3\,q + q^2 + q^3 - 2\,q^4 + q^5 + q^7 - q^8 + q^{10} - q^{11} + q^{13} - q^{14} - q^{15} + q^{16} - q^{18} + q^{20} + q^{25} - q^{26}}{q^2}\right\}$$